

ON THE RELATIONSHIP BETWEEN GEOSTROPHIC AND SURFACE WIND AT SEA

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ABSTRACT

The ratio between observed surface and geostrophic wind speed has been investigated from observations at the German Bight, taking geostrophic wind and the air-sea temperature difference as parameters. The ratio decreases with increasing geostrophic wind and increasing stability. While stability is an important parameter for light to moderate winds, variation of the ratio with geostrophic wind speed cannot be neglected, taking the full range of geostrophic wind speeds into consideration. From the Navier-Stokes equations, such a variation is to be expected. For light winds, the (local) surface wind may exceed the (mesoscale) geostrophic wind. Both effects together can be described approximately by a linear relation between the surface wind and geostrophic wind, with a slope of 0.56 and a constant term $b > 0$ varying with stability. The residual error was 2 m/s. Variation with latitude is inferred from the Navier-Stokes equations.

1. INTRODUCTION

The relationship between the surface wind speed and the geostrophic wind speed as a function of simple parameters is of great interest for various reasons. Momentum exchange between the air and the water generates and maintains surface waves and drives ocean currents. Nearly all our knowledge about the interactions between air and sea is related to the surface wind speed. But usually, the geostrophic wind is better known than the actual wind. With the sparse data from the oceans, the pressure field can be interpolated with more confidence than the wind field due to the greater variability of the latter. Furthermore, numerical weather forecasting provides us with information of the pressure field over the ocean, which then could be converted into the field of surface wind. Vice versa, for achieving reliable numerical predictions for a longer period, there is need to express the air-sea interactions in terms of the geostrophic rather than the surface wind.

The relation between surface and geostrophic wind speed has been investigated for a long time, and there are a number of famous names associated with such studies. Yet, no generally accepted relationships have evolved (Roll 1965). One reason for this is that, at sea, sufficiently

numerous and accurate-enough pressure measurements are lacking to determine the geostrophic wind reliably. Therefore, some investigations relied on observations at coastal stations though, due to the considerable change of the surface roughness, these cannot be considered as representative of the wind field at sea. Another reason is that, even for steady unaccelerated flow, the ratio of surface wind speed to geostrophic wind speed depends at least on the geostrophic wind speed, the stability of the density stratification, and the Coriolis parameter. Other variables may be of importance (e.g., the height of low-level temperature inversions). What really renders such investigations difficult is that these different variables are correlated and that their correlations change with location and season.

The purpose of this paper is to investigate how the surface wind at sea may be derived from the surface-pressure field as a function of other simple parameters (e.g., the air-sea temperature difference). Recently, Findlater et al. (1966) made a much more extensive study with a similar aim. They replaced the geostrophic wind by the observed 900-mb wind and took the mean lapse rate (surface to 900 mb) as a measure of stability. Their paper includes a detailed discussion as to what extent this choice of variables may influence their results. Additionally, it may be noted that the 900-mb wind is a local variable,

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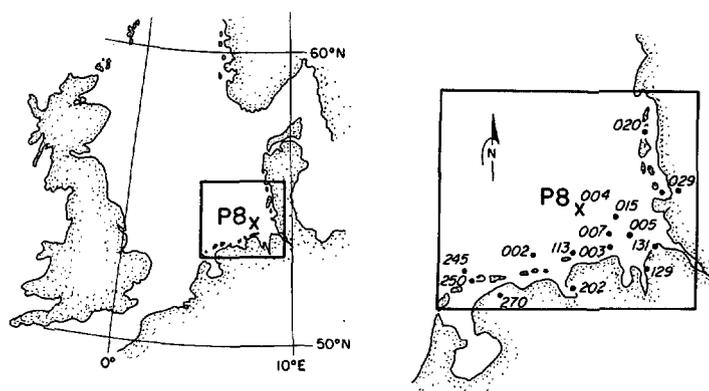


FIGURE 1.—Location of the lightship *P8* and surface-pressure stations in the German Bight.

while the geostrophic wind is derived from the pressure field in an area of order $200 \text{ km} \times 200 \text{ km}$, and may be considered as a mesoscale or synoptic variable. Their material, therefore, excludes one aspect of the surface and geostrophic wind relationship. It is interesting to note that the derived standard deviation of the surface to 900-mb wind ratio at ocean weather ships *I* and *J* is 24 and 29 percent, respectively, which is higher than that obtained with our choice of variables.

The following nomenclature is introduced to avoid misunderstanding. "Surface wind" U is the actual wind speed measured near the surface at the conventional anemometer level which may be taken as the 10-m height. "Geostrophic wind" U_g is the geostrophic wind speed at the surface, calculated from the pressure observations. This is a fictitious quantity (which cannot be observed). It should not be confused with the geostrophic wind at the top of the planetary boundary layer (which may be observed), since they are identical only if there is no thermal wind. However, this cannot be assumed a priori. The ratio of surface wind speed to geostrophic wind speed is sometimes called the "ratio." The speeds (i.e., the absolute values of the vectors) only are used in this paper because the angle between the actual surface wind and the geostrophic wind vector at sea is usually small and the coded directions do not permit a meaningful investigation of these small values.

Neiburger et al. (1948) suggest that, over land, the geostrophic wind is more suitable for the determination of the actual wind than the gradient wind. No attempt has been made to verify this for conditions over the sea or to investigate effects of fronts or nonstationarity.

2. THE OBSERVATIONS AND ANALYSIS

For obtaining reliable data, shipboard observations of the surface wind have to be used from an area where it is possible to calculate the geostrophic wind with some accuracy from pressure data. The lightship *P8* was selected for this investigation (fig. 1). *P8* is the outermost light vessel in the German Bight, at 54.3°N , 7.2°E ; the closest distance to any land is 65 km. For calculating the geostrophic wind, the recorded pressure data (Deutscher

Wetterdienst 1966a–1968, 1966b–1968; KNMI 1966–1968) from 15 lightships and coastal and island stations in the area between List (Sylt), Bremerhaven, and Terschelling were used. Only data in winter (1200 GMT) were used in this study because of the density of observations. After applying the method of least squares, a second-order surface was fitted to the pressure observations; and the derivative at the position of *P8* was used to calculate the geostrophic wind. This was compared with the local surface wind at *P8* (reported 10-min average), using air-sea temperature difference from the ships weather log as stability parameter.

We selected 438 cases between 1966 and 1968 in which the isobars showed little or no curvature in the entire area. Therefore, cases with fronts or noticeable curvature of isobars were deliberately excluded. Inclusion of these cases would be meaningful only if sound results are obtained from the simpler cases. The effects of fronts would have to be investigated by other methods. The main motivation for the restriction was to have as reliable pressure gradients as possible at position *P8* located near the long side of the triangular shaped area outlined by pressure stations. It should be noted that, in this investigation, the pressure field is a variable with observational errors, while, in application of the results, the pressure field may be a given variable (e.g., computed with a sufficient degree of accuracy and a grid width which should permit interpolation). Additionally, all cases have been disregarded where the geostrophic wind was below 5 m/s as, in these cases, the pressure gradient is too small to be determined reliably from these kinds of data (Wagner 1969).

The observations were grouped into three classes of about equal size according to the air-sea temperature difference to represent unstable, near-neutral, and stable stratification. The corresponding differences (air temperature minus sea temperature) are -2.7° , -0.2° , and $+1.7^\circ\text{C}$, respectively. Within the classes, group averages and standard deviations were determined for each 10 observations, ordered according to the geostrophic or the surface wind speed.

In figure 2, the ratio U/U_g is plotted versus the geostrophic wind speed for the three stability classes: unstable (circles), near-neutral (crosses), and stable (dots). For comparison, the ratio one would obtain for the same air-sea temperature difference, using values by Pierson et al. (1955) and the graph by Johnson (1955), are given by dotted lines and dashed curves, respectively. It can be seen from figure 2 that the influence of stability is described fairly well by Johnson and by Pierson et al. However, the dependence of the ratio on the geostrophic wind shows considerable discrepancy with our data, especially at low U_g . The solid curves are the relationships (1) as explained below.

Plotting the ratio U/U_g as a function of geostrophic wind speed and air-sea temperature difference is the conventional method of viewing the surface and geostrophic wind relationship. It gives the (nonlinear) regression of the ratio on U_g and implicitly of U on U_g . This presentation is not very advantageous, as abscissa and ordinate depend

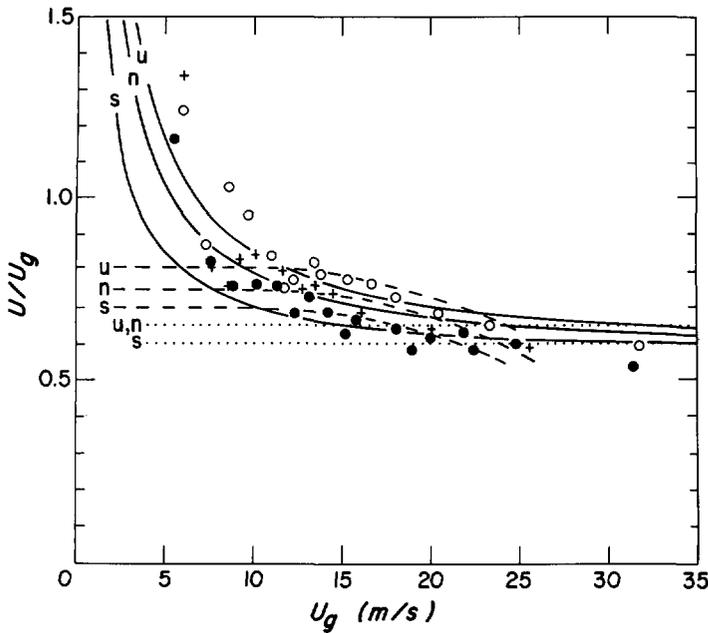


FIGURE 2.—Variation of the surface and geostrophic wind speed ratio versus geostrophic wind speed as a function of stability: unstable (u, circles), near-neutral (n, crosses), and stable (s, dots). The symbols denote means of 10 observations each from lightship P8. The full lines are from eq (1); the dashed lines, from Johnson (1955); and the dotted lines, from Pierson et al. (1955). The full curves may be thought of as representing the observations with systematic errors removed.

on the same variable, namely, U_g . Since the geostrophic wind is obtained from observations, it is subject to errors resulting in a systematic effect: if U_g is too small, U/U_g will be too large, and vice versa. On the average, U/U_g therefore will be too large for low geostrophic winds and too small for high geostrophic winds.

One method to assess this is to calculate the regressions both of U on U_g and of U_g on U . This is given in figure 3 for the three stability classes. This figure shows that the regressions are fairly linear. Therefore, only linear regressions have been computed, which are given in table 1 (where the regression of U_g on U has been inverted to allow easier comparison). Sverdrup (1916, see also Scripps Institution of Oceanography 1945) already used the two linear regressions to evaluate the relation between U and U_g , where both are subject to observational errors. Without further information, there is no way to know what part of the scatter is due to errors of U and what part to errors of U_g . Therefore, Sverdrup assumed that the slope a of the "most probable" linear relation $U = aU_g + b$ is equal to the ratio of the standard deviations of U and U_g . The constant term is given by $b = \bar{U} - a\bar{U}_g$, so that the line goes through (\bar{U}, \bar{U}_g) , where \bar{U} and \bar{U}_g are the respective means. This most probable relation is also given in table 1. These most probable slopes are not significantly different for the three classes and obviously do not vary systematically with stability. Therefore, for all practical purposes, it will be sufficient to use a mean value

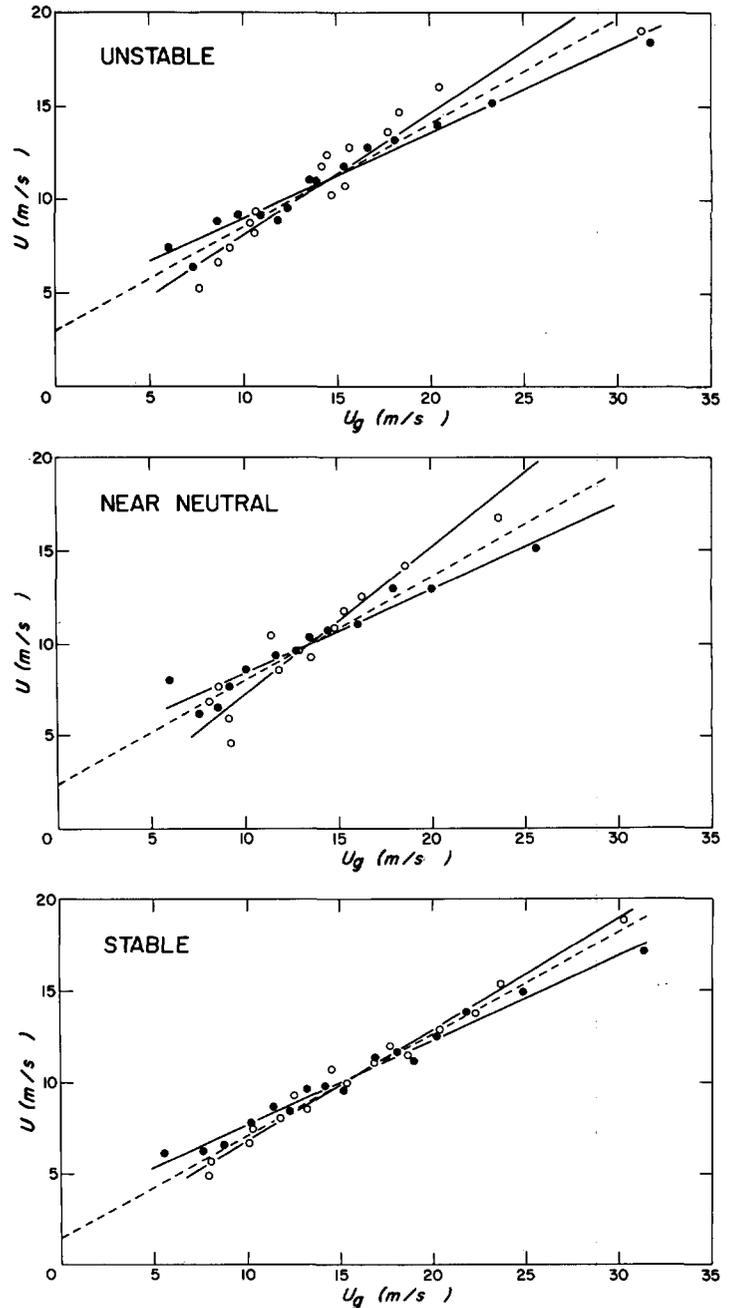


FIGURE 3.—Regression between surface and geostrophic wind speed for three stability classes. The full lines give linear regression on U_g and U ; the dashed lines represent eq (1). Averages of 10 observations each are indicated by dots and circles, grouped according to geostrophic and surface wind, respectively.

TABLE 1.—Relation between surface wind speed U and geostrophic wind speed U_g for three classes of stability from observations of light vessel P8 in the German Bight

Condition	Density stratification		
	Unstable	Near-neutral	Stable
Regression of U on U_g	(m/s) $U = 0.46 U_g + 4.4$	$0.45 U_g + 3.9$	$0.46 U_g + 3.1$
Regression of U_g on U	(m/s) $.66 U + 1.5$	$.80 U - 0.7$	$.60 U + 0.8$
Most probable straight line	(m/s) $.55 U + 3.1$	$.60 U + 1.9$	$.53 U + 2.0$
Air-sea temperature difference ($^{\circ}\text{C}$)	-2.7	-0.2	+1.7
Mean surface wind (m/s)	11.5	9.93	10.35
Mean geostrophic wind (m/s)	14.65	13.36	15.77

of the slope for all stabilities. This gives

$$\begin{aligned} \text{unstable} \quad U &= 0.56 U_g + 3.0 \quad (\text{m/s}), \\ \text{near-neutral} \quad U &= 0.56 U_g + 2.4, \\ \text{and} \\ \text{stable} \quad U &= 0.56 U_g + 1.5. \end{aligned} \quad (1)$$

When using both regressions, the systematic influences of random observational errors of both variables have been minimized. The fact that the straight lines intersect the U axis at U significantly (at the 0.5% level) greater than zero suggests that the surface wind exceeds the geostrophic wind at low wind speeds and that the ratio U/U_g is not constant.

3. DISCUSSION

Can the observations be explained or supported by other information or simple theoretical considerations? Take the Navier-Stokes equations for a steady, horizontally homogeneous flow (f indicating Coriolis parameter; ρ , density; τ_{xz} and τ_{yz} , stress components; u , v , and u_g, v_g , components of surface and geostrophic wind, respectively):

$$f\rho(v-v_g) = -\frac{\partial\tau_{xz}}{\partial z} \quad (2)$$

and

$$f\rho(u-u_g) = \frac{\partial\tau_{yz}}{\partial z}.$$

As the derivatives of the stress are small, it would be convenient if they could be determined from gross features. Replace the unknown derivatives by the difference quotient of the appropriate component of the total stress at the surface and a virtual height H :

$$\frac{\partial\tau_{xz}}{\partial z} = \frac{\tau_{xz}}{H} \quad \text{and} \quad \frac{\partial\tau_{yz}}{\partial z} = \frac{\tau_{yz}}{H}.$$

As the surface stress varies approximately as the square of the mean wind speed, one may write $\tau_{xz} = \kappa\rho uU$ and $\tau_{yz} = \kappa\rho vU$. Equation (2) then becomes

$$v-v_g = -\kappa uU/fH \quad (3)$$

and

$$u-u_g = \kappa vU/fH.$$

It then follows that

$$U/U_g = (1 + \kappa^2 U^2 / f^2 H^2)^{-1/2}. \quad (4)$$

The drag coefficient κ and the virtual height H vary with wind speed and stability as the state of turbulence changes. The observed variation of the U/U_g ratio cannot be explained in terms of the variation of the drag coefficient alone. The dependence of the latter on stability (Hase

1968) tends to increase the ratio for stable stratification and decrease it for the unstable. A constant or slightly increasing drag coefficient would give too steep a decrease of the ratio with wind speed if the virtual height would not increase with wind speed. The variation of the ratio can be explained by variation of the virtual height that, on the average, increases slightly (less than linear) with wind and is smaller under stable than under unstable conditions. The ratio would be constant only if the virtual height would increase linearly with wind speed. Though due to lack of suitable observations, variation of H cannot as yet be investigated; eq (4) may be used to infer the variation of the ratio U/U_g with latitude.

Still, this approach does not explain the ratios of U/U_g greater than one. The observed ratios at low wind speeds therefore appear to be systematically too high. There are a number of possibilities to explain why the observed wind at low wind speeds may exceed the geostrophic wind speed: unsteady flow, wind driven from below by waves, increase of geostrophic wind speed with height and mixing, or erroneous recording of actual wind speed due to ship motions. The following, however, seems to be the most likely explanation. The geostrophic wind as used here is an areal average, while the observed surface wind is a local variable. Especially under conditions of weak pressure gradients, wind induced by circulations of smaller scale (e.g., convection) and even turbulence may be more noticeable. These local surface winds do not average out if the speed only is averaged and not the vector wind. It is, therefore, to be expected that, under conditions of weak pressure gradients, the surface wind appears to be increased compared to the geostrophic wind due to local variability. The Navier-Stokes equations cannot predict this if used with variables not compatible in scale. The increase, as a measure of steadiness of the wind below the grid-size scale, will depend on a variety of factors such as grid size, stability, or climatic regime. It may not be easily parameterized, but it is hoped that it will not be dynamically very important as it is noticeable only with light winds. For practical purposes, eq (1) may be used as a convenient tool.

It should be understood that the linear relation is used only as an approximation. As has been shown, eq (4) alone does not yield the functional form of the surface and geostrophic wind relationship. With a subject like ours, where the observational scatter is large, a presentation that yields an approximately linear dependence is convenient as it allows easy application of the method of least squares. The relations (1) approximate the surface and geostrophic wind relationship in two respects. They take care of the underestimation of the local wind from the mesoscale pressure field due to local variability, and they approximate the more complicated behavior to be expected (e.g., from eq 4). The constant term in eq (1), for this reason, may be said to be only a formal parameter. Even so, eq (1) will give a better description of the surface and geostrophic wind relationship than a constant ratio

TABLE 2.—Variation of a linear surface and geostrophic wind relation $U = a U_g + b$ with Coriolis parameter, calculated with the aid of eq (4) from eq (1)

Latitude (deg.)	20	30	40	50	60	70	80/90	Class
Slope a	0.27	0.38	0.47	0.54	0.59	0.62	0.64	
Constant term b (m/s)	4.8	4.0	3.5	3.1	2.8	2.6	2.5	Unstable
	3.6	3.2	2.8	2.5	2.3	2.1	2.0	Near-neutral
	2.2	1.9	1.7	1.6	1.4	1.3	1.3	Stable

U/U_g . As a formal parameter, the constant term will not have direct physical significance but will depend on the state of turbulence.

It is desirable to explain why the results derived here differ from earlier results. Most determinations of the geostrophic wind at sea used the analyzed synoptic weather charts. Due to lack of pressure observations, drawing of isobars was usually aided by the observations of surface wind speed and direction. This will bias the derived geostrophic wind somewhat, depending on the method of analysis. At any rate, deriving geostrophic wind from drawn isobars will yield more uncertainties than objective numerical calculation.

4. VARIATION WITH LATITUDE

It is important to realize that, even for given geostrophic wind speed and air-sea temperature difference, the surface and geostrophic wind relationship depends on latitude. The often cited sources (Johnson 1955 and Pierson et al. 1955) do not mention this. Findlater et al. (1966) found that the ratios of the surface to 900-mb wind at ocean weather ships I and J "bear nearly the same ratio to each other as do the sines of the latitudes of the two positions." Gordon (1952) inferred the dependence of U/U_g on latitude from a climatological study. The observed variation was ascribed by Lettau (1959) to latitudinal variations of stability. In Lettau's treatment, dependence of the ratio on latitude enters only indirectly through the empirical dependence of the geostrophic drag coefficient on the surface Rossby number $R_o = U_g/z_0f$. As z_0 (or rather $\ln z_0$) at sea is about as unknown a quantity as the U/U_g ratio, it seems more reliable to revert to the Navier-Stokes equations.

Assuming that the frictional term is independent of the variation of the Coriolis parameter f , since turbulence is a small-scale process, from eq (4) one should expect the ratio to vary proportionally to $(1 + \text{factor}/f)^{-1/2}$. The "factor" then could be determined as a function of wind speed and stability from the observations, perhaps as given by eq (1). Without further information, one cannot infer to what extent the constant term in eq (1) is representing local variability of wind or is a formal parameter. Consequently, it is open to question whether or not this constant term should be included while calculating the

variation of the ratio with latitude from eq (4). Two cases of extreme possibilities are (1) the constant term is independent of latitude or (2) it varies with latitude, and U/U_g is calculated from eq (1), including the constant. The parameters of the resulting relation of the form $U = aU_g + b$ are given in table 2. The slope as listed in table 2 is calculated for case (1), but the slope can also be used for case (2), with only slight deviations for low wind speeds, disregard of which results in errors of U smaller than 1 m/s. The constant term is given for case (2); the italicized values would hold for all latitudes in case (1). This treatment does not include variations of local variability with different climatic regimes. It would be interesting to test the validity of the proposed reduction to other latitudes with other material (e.g., by Findlater et al. 1966 and Aagaard 1969); but due to the different choice of parameters and their different way of presentation, this is not possible (they did not include regression of U_g on U , which seems to be essential to eliminate systematic errors).

It can be deduced from table 2 that the variation of the ratio U/U_g with latitude ψ is somewhat weaker than with sine ψ , but is considerably stronger than through the dependence on the surface Rossby number after Lettau. The variation of the ratio U/U_g with latitude is about as marked as its dependence on stability and wind speed and therefore should not be neglected in practice.

5. RELIABILITY

An estimate of the accuracy of the determination of the surface wind from the geostrophic wind can be obtained from the set of observations mentioned in section 2. The standard deviations of the ratio U/U_g (determined for groups of 10 observations each) amounted to 20 percent of the ratio in the mean over all wind speeds. There was a systematic variation with wind speed, the percentage error was slightly higher (25%) at low wind speeds ($U_g < 15$ m/s) and decreased to about 15 percent at higher geostrophic winds. Using eq (1), the root-mean-square (rms) deviation from the straight lines was 2.1 m/s, only slightly higher than the rms deviation from the best fitting straight lines, which was 2.0 m/s. These rms deviations are approximately independent of wind speed, with the possible exception for higher wind speeds ($U_g \geq 25$ m/s). This information, however, "paints a picture" of the likely error, which may be too pessimistic. In both cases, the scatter has been attributed solely to errors of the surface wind. Yet, there is considerable error in the determination of the geostrophic wind from the pressure field. A rms error of 0.2 to 0.3 mb of the pressure observations could explain the observed scatter. Neither an error of 2 m/s for the surface wind nor an error 0.2 or 0.3 mb for the pressure observation is far from what one has to expect from routine observations and therefore cannot be ruled out. Nevertheless, it may be presumed that, given the pressure field

without errors, it will be possible to determine the surface wind with an accuracy better than 2 m/s. It should be noted that "surface wind" here is the usual 10-min average synoptic observation at one point. If one deals with averages for areas of, say, 200 km \times 200 km, the scatter may be considerably less. This argument does not include variations of the "constants" in the surface and geostrophic wind relationship, say, with season or latitude.

6. CONCLUSIONS

It has been shown that, over sea, the surface to geostrophic wind relationship—for a given stability—is not constant but depends on the geostrophic wind speed and latitude. As a practical tool, a linear U, U_g relation can be used, the coefficients of which vary with stability and latitude. Part of the variation of the ratio with wind speed could be explained by the fact that, in common usage, the surface wind is a local scale variable but the geostrophic wind is a mesoscale variable.

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