Impact of Assimilating Radar Data With 3D Thermodynamic Fields in an Ensemble Kalman Filter: Proof of Concept and Feasibility

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ABSTRACT

This study examined the impact of assimilating 3D temperature and water-vapor information in addition to radar observations in a multiscale weather system. A frontal system with extremely heavy rainfall over northern Taiwan was selected. Using the WRF–LETKF Radar Assimilation System, we performed three sets of observing system simulation experiments to assimilate radar observations with or without thermodynamic variables obtained using different methods. First, assimilating the radar data for 2 h showed better structure and short-term forecast than 1 h. Second, we assimilated radar data and thermodynamic variables from a perfect model simulation. The results of the analysis revealed that when a precipitation position error occurred in the background field, assimilating thermodynamic information with the radar data could correct the dynamic structure and shorten the spin-up assimilation period, resulting in substantial improvements to the quantitative precipitation forecast. Third, we applied a thermodynamics retrieval algorithm for a feasibility study. With a warm and wet bias of the retrieved fields, assimilating the temperature data had significant impact on the mid-level of stratiform areas and the forecast of the heavy rainfall was consequently improved. Assimilating the water vapor information helped reconstruct the range and intensity of the cold pool, but the improvement of rainfall forecast was limited. The optimal results of analysis and short-term forecast were achieved when both retrieved temperature and water vapor fields were assimilated. In conclusion, assimilating thermodynamic variables in the precipitation system is feasible for shortening the spin-up period of data assimilation and improving the analysis and short-term forecast.

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1. Introduction

Severe precipitation represents a critical threat to life and property worldwide. An effective tool for monitoring and forecasting severe precipitation is a weather radar, which provides high temporal and spatial sampling. Over the last two decades, data assimilation (DA) methods have been applied to objectively combine radar data and numerical weather predictions to obtain the optimal analysis fields and improve the quantitative precipitation forecast (QPF) in high-impact weather systems. With three-dimensional variational (3DVar) data assimilation algorithms, many studies have verified that the assimilation of radar data can improve short-term forecasts (Xiao et al. 2005; Xiao and Sun 2007; Sugimoto et al. 2009; Chung et al. 2009). The 3DVar, based on climatological background error covariances, has long been used for assimilating radar data in operational institutes (Lindskog et al. 2004; Montmerle and Faccani 2008; Xiao et al. 2008). Because flow-dependent error structures change rapidly in space and time at the convective scale (Brousseau et al. 2011; Chung et al. 2013; Jacques et al. 2017), the disadvantage of 3DVar is that a static background error structure is assumed and applied through the cycling process.

To account for the evolution of the system in time and space, four-dimensional variational (4DVar) data assimilation methods have been developed and applied at the convective scale. For instance, the Variational Doppler Radar Analysis System (VDRAS) assimilates radar data at the convective scale and has been applied to different real cases (Sun and Crook 1997, 2001; Sun and Zhang 2008; Sun et al. 2010; Tai et al. 2011; Chang et al. 2014; Chang et al. 2016). The weather and research forecast (WRF) model includes the option to assimilate 4D radar data (Sun and Wang 2014). The Japan Meteorological Agency has also developed a 4DVar system to assimilate radar data (Koizume et al. 2005; Kawabata et al. 2011). Although the 4DVar algorithm implicitly considers the time evolution of the state in space, it is usually computation and labor intensive to respectively run and develop a required complex adjoint model. Ensemble-based DA systems have also been widely used for radar DA (Snyder and Zhang 2003; Tong and Xue 2005; Yussouf et al. 2013; Chang et al. 2014; Tsai et al. 2014; Wheatley et al. 2015; Jones et al. 2016; Miyoshi et al. 2016; You et al. 2020; Wu et al. 2020; Tsai and Chung 2020; Do et al. 2022). The ensemble scheme uses multiple forecasts and their statistics to capture the complex
uncertainty of the atmosphere. The method does not require an adjoint model, and the assimilation process can maintain flow-dependent error structures (Chung et al. 2013; Ménétrier et al. 2015; Chen et al. 2021). Studies using various types of DA systems, such as variational, ensemble, and hybrid systems (Wang et al. 2007; Gao and Stensrud 2014; Li et al. 2012), have overwhelmingly demonstrated that assimilating radar data improves the performance of short-term forecasts at the convective scale.

In recent years, much research has been conducted on assimilating additional high-resolution information with radar data to improve storm-scale numerical weather predictions (NWPs). Using a 3DVar system with observing system simulation experiments (OSSEs), Ge et al. (2013) investigated the impact of assimilating different state variables at the convective scale; their results demonstrated that dynamic variables such as horizontal wind play a major role in analyzing storm structure. Moreover, they revealed that thermodynamic variables such as temperature and humidity are more effective than hydrometeor variables for reconstructing severe storms because hydrometeors are primarily indicators of thermodynamical processes; without supporting wind, temperature, and humidity information, hydrometeors rapidly evaporate or precipitate. Many studies have attempted to include thermodynamic information into radar data. For example, Wattrelot et al. (2014) assimilated retrieved humidity profiles and radar data by using a 1D + 3DVar assimilation method. Their results revealed the positive impact of analysis and short-term forecasts. Kerr et al. (2015) evaluated the effect of assimilating both cloud-top temperature from satellite data and radar observations, whereas Caumont et al. (2016) assimilated retrieved temperature and humidity profiles from ground-based microwave radiometers. Other strategies, such as using vertical integrated liquid water content or differential reflectivity columns to modify humidity or temperature or both, have recently been tested (Carlin et al. 2017; Lai et al. 2018), demonstrating substantial utility in terms of high-impact weather events.

The potential impact of assimilating both radar data and retrieved thermodynamic information remains unclear, and most studies have focused on areas of strong convection. Themens and Fabry (2014) described the potential benefit of providing temperature and humidity information over 3D domains for mesoscale forecasting, and high-density thermodynamic information has been successfully retrieved from radar data (Liou et al.
2019; Feng and Fabry 2016; Ellis and Vivekanandan 2010) despite some concerns related to accuracy and bias.

Since the impact of assimilating 3D thermodynamic variables inside severe weather systems had not been completely investigated, we conducted a series of OSSEs to explore this issue both for analysis and short-term forecast. First, high-density temperature and/or humidity fields were assimilated along with radar data simulated from a truth run. This was to evaluate and illustrate the added value of assimilating thermodynamic information to the ensemble-based DA system without the uncertainty due to thermodynamic retrieval algorithms. Second, a terrain-permitting thermodynamic retrieval scheme (TPTRS; see Liou et al. 2019) was applied to retrieve temperature and water vapor assimilated to the DA system. Considering the uncertainty of the retrieval scheme, it is able to understand the feasibility of assimilating the thermodynamic information. A case study of a frontal system was selected to examine and demonstrate the impact of assimilating these observations in different areas of the precipitating weather system.

The remainder of the manuscript is organized as follows: in Section 2, the ensemble DA system and related observation operators are introduced; in Section 3, an overview of the rainfall event is provided; in Section 4, the design of the OSSEs is described; in Section 5, the results of the analysis and short-term forecast are illustrated; in Section 6, the results of assimilating thermodynamic variables from a retrieval algorithm and the performance of the short-term forecast are presented. Finally, the summary and conclusion are provided in Section 7.

2. Methodology

a. WRF local ensemble transform Kalman filter radar assimilation system

The local ensemble transform Kalman filter (LETKF) algorithm was used to conduct a series of OSSEs. Similar to other ensemble Kalman filter (EnKF) methods such as the Ensemble Square Root Filter (EnSRF, Whitaker and Hamill 2002), the Ensemble Adjustment Kalman Filter (EAKF, Anderson 2001), and the Ensemble transform Kalman filter (ETKF, Bishop et al. 2001), LETKF belongs to the family of deterministic EnKFs. LETKF independently updates the analysis at each model grid point according to its nearby background information (a previous short-range forecast ensemble) and observations. One
main advantage of LETKF is that it can be easily parallelized. Brief descriptions of LETKF’s features are provided here, and the details of the algorithm are available in the studies of Hunt et al. (2007) and Yang et al. (2009).

The final analysis can be obtained as follows:

(1) Estimate the state variables and their uncertainty by updating the ensemble mean and perturbations as

\[
\begin{align*}
\bar{x}_a &= \bar{x}_b + X_b \bar{w} \\
X_a &= X_b W
\end{align*}
\]

where \(\bar{x}\) is a column vector storing the ensemble mean of the model variables, \(X\) is a matrix in which the \(k\)th column stores the perturbation from the \(k\)th ensemble member, and the subscript \(b\) or \(a\) refers to the background or analysis, respectively. The analysis mean increment and perturbations are derived from the linear combination of background perturbations.

(2) The analysis mean weighted vector \(\bar{w}\) and the analysis perturbation weighted matrix \(W\) are defined as

\[
\begin{align*}
\bar{w} &= \bar{P}_a Y_b^T R^{-1} (y_o - \bar{y}_b) \\
W &= [(K - 1) \bar{P}_a]^1/2
\end{align*}
\]

where \(y_o\) is the observation vector and \(\bar{y}_b\) is the background ensemble mean in the observation space.

(3) The analysis error covariance matrix \(\bar{P}_a\) is defined and calculated in the ensemble space as

\[
\bar{P}_a = [(K - 1) I/\rho + Y_b^T R^{-1} Y_b]^{-1}
\]

where matrix \(Y_b\) represents the perturbations in the observation space, \(R\) is the observation error covariance matrix, \(I\) is an identity matrix, \(K\) is the ensemble size, and \(\rho\) is a multiplicative covariance inflation factor (Anderson 2001).

(4) The R-localization scheme (Hunt et al. 2007) models the observation error with a Gaussian function to propagate the information between the observation and analysis grid point. Figure 1 presents a schematic of a horizontal and vertical R-localization.

The LETKF system is coupled with the WRF model in the present study. The system was originally developed by Yang et al. (2009) for assimilating conventional observations, and Tsai et al. (2014) modified this system for radar DA at the convective scale, renamed

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it the WRF–LETKF radar assimilation system (WLRAS). The features and setup of this system are described as follows:

1. A mixed localization strategy (Tsai et al. 2014) was used to avoid unrealistic correlations and assign different error covariance localization radii to different model variables. In the present study, three horizontal radii of 36 (horizontal wind, U and V), 24 (temperature T, water vapor mixing ratio Qv, and cloud mixing ratio Qc), and 12 km (vertical velocity W, rain mixing ratio Qr, snow mixing ratio Qs, and graupel mixing ratio Qg) were used to update the model variables. The vertical localization radius was 4 km for all the variables.

2. A data quality control procedure was used to reject a particular observation if the innovation (the difference between the observation and background state) is three times larger than the observation error.

3. To address the underdispersive problem, a prior inflation factor $\rho$ was applied to every assimilation cycle, and an empirical value of 1.08 was used throughout the assimilation experiments.

4. In contrast to the findings of Tsai et al. (2014), all the observations assimilated in the WLRAS system were used to update all the model variables without switching off any error correlation.
b. Observation operator

Radial wind and radar reflectivity were assimilated in the study. Through a geometric relation in space, the radial wind is related to the three-dimensional wind. The forward model for assimilating radial wind is defined as

$$V_r = [ux + vy + (w - V_t)z](x^2 + y^2 + z^2)^{-\frac{1}{2}}$$

(6)

where \(x\), \(y\), and \(z\) are the Cartesian coordinates with respect to the radar site. \(V_t\) is calculated by assuming a Marshall–Palmer drop size distribution (Marshall and Palmer 1948):

$$V_t = 5.40\left(\frac{p_0}{\bar{p}}\right)^{0.4}(\rho_a q_r)^{0.125}$$

(7)

where \(q_r\) is the rainwater mixing ratio (a model variable), \(p_0\) is the surface pressure, \(\bar{p}\) is the base-state pressure, and \(\rho_a\) is the air density.

Radar reflectivity observations are linked to the hydrometeor variables in the numerical model. Instead of using the Lin scheme (Lin et al. 1983), as in the study of Tsai et al. (2014), the three-ice scheme of the Goddard cumulus ensemble (GCE, Tao et al. 2003)
was applied as the microphysical scheme in the study. The prognostic hydrometeor mixing ratio variables are rain \((q_r)\), cloud \((q_c)\), ice \((q_i)\), snow \((q_s)\), and graupel \((q_g)\). In accordance with Dowell et al. (2011), the forward model for synthetic reflectivity \((Z)\), which uses rainwater \((Z_r)\), snow \((Z_s)\), and graupel \((Z_g)\), is formulated as

\[
Z = Z_r + Z_s + Z_g .
\]  

(8)

When the GCE scheme is used, the relationships between the hydrometeor variables and reflectivity factor on the right-hand side of Eq. (8) are

\[
Z_r = 3.63 \times 10^9 (\rho_a q_r)^{1.75}
\]  

(9)

\[
Z_{s-wet} = 1.21 \times 10^{11} (\rho_a q_s)^{1.75}
\]  

(10)

\[
Z_{s-dry} = 2.79 \times 10^8 (\rho_a q_s)^{1.75}
\]  

(11)

\[
Z_g = 1.12 \times 10^9 (\rho_a q_g)^{1.75}
\]  

(12)

3. Description of the case study

A mesoscale convective system event accompanied by a frontal system on 11 June 2012 was selected for this study. The weather system resulted in total accumulated rainfall of more than 400 mm within 10 h, leading to flooding in northern Taiwan.

According to an analysis of the surface weather map (see Ke et al. 2019, Fig. 2a), the Mei-Yu front remained north of Taiwan at 1200 UTC on June 11 (2000 LST), and the weather map at 850 hPa revealed a short-wave trough and abundance of water vapor over the ocean west of Taiwan (Ke et al. 2019, Fig. 2b). During this time, a strong and deep southwesterly jet was flowing from the surface to the mid-levels in the southern region of the frontal system. The environmental conditions ahead of the frontal system were highly amenable to the development of severe weather.

Figure 2 illustrates the evolution of the composite reflectivity observed by the RCWF radar at 1-h intervals from 1200 to 1600 UTC. At 1200 UTC, a line convection system with strong reflectivity (>40 dBZ) approached northern Taiwan. After the system landed over Taiwan, a convective cell with a line shape was triggered along and parallel to the northwest coast and then merged with the main precipitation system. This feature was repeated twice between 1400 and 1600 UTC while the main precipitating system remained quasi-stationary.

This event was poorly predicted by the Central Weather Bureau of Taiwan because a 5-km resolution numerical weather prediction model failed to accurately capture the precipitation system that had become stationary over northern Taiwan. Wang et al. (2016) studied this event with a higher resolution (1.5-km) numerical simulation to analyze its dynamics and thermodynamics. Despite their strategy of nesting and downscaling, they realized that improving the initial conditions for a very short-term QPF at the cloud-resolving scale was crucial. By using radiosondes, surface observations, and reanalysis data, Chen et al. (2018) demonstrated that high moisture content and the existence of a barrier jet accompanying a frontal system caused long-lived convection cells over Taiwan’s northwestern coast. Using a retrieval algorithm to obtain wind, pressure, and temperature data in three dimensions, Ke et al. (2019) examined the evolution of severe weather phenomena by using multiple Doppler radar observations. These studies highlighted the role of multiscale features (such as the location of the frontal system, the strength of the barrier jet, and the cold pool) and orographic interactions in the formation of the front over northern Taiwan.
Fig. 2. Composite radar observation reflectivity (shaded) in northern Taiwan from 1200 UTC to 1600 UTC.
4. Experimental design

a. OSSE experimental design with WRF

The WRF model V3.2.1 (Powers et al. 2017; Skamarock et al. 2008) was used to conduct a series of OSSEs. Two-way nesting was configured using a three-layer nested domain. The horizontal resolution was 27 (D01), 9 (D02), and 3 km (D03) with 251 × 261, 337 × 271, and 223 × 232 grid points, respectively (Fig. 3). The vertical model contained 51 levels, with the level top at 10 hPa. The parameterization schemes of the model were as follows: the rapid radiative transfer model for the physical parameterization of long-wave radiation (Mlawer et al. 1997); the Dudhia method for short-wave radiation (Dudhia 1989); and the Yonsei University scheme for boundary layer parameterization (Hong et al. 2006). In D01 and D02, the Grell–Dévényi ensemble scheme (Grell and Dévényi 2002) was used for cumulus parameterization; the microphysical GCE scheme was applied in all domains. The 40 members of the ensemble were generated from the WRF–3DVar random perturbations (Barker et al. 2004) in D01 and nested down to 9 km and 3 km resolutions. In this study, all the data were assimilated in the innermost domain (D03), and the changes were propagated through upscaling.

To mimic the actual uncertainty inherent in this event, two sets of initial conditions were used to differentiate the experiments from the truth run (Truth, Exp. 1, Table 1) and the control run (NoDA, Exp. 2). The 1° × 1° final operational global analysis (FNL) from the National Centers for Environmental Prediction (NCEP) for 0000 UTC on 11 June 2012 was applied to all the experimental tests. ERA-interim reanalysis data of a higher resolution (0.75° × 0.75°) were used simultaneously to launch the model as the truth run. Figure 4 presents the two initial conditions for the truth run (truth, Figs. 4a, c) and control run (NoDA, Figs. 4b, d). The synoptic features (Figs. 4a, b) of the southwestern wind, potential height, and potential temperature near Taiwan were similar in the two reanalysis fields at 850 hPa. However, the water vapor mixing ratio of the ERA-interim reanalysis (Fig. 4c) was slightly larger than that of the NCEP FNL (Fig. 4d) in the Taiwan area. According to the observations at 1400 UTC (Fig. 2c), the rainband of the truth run (Fig. 5a) had already landed in northern Taiwan, whereas the rainband of the control run initialized by NCEP (Fig. 5b) was still over the ocean. Furthermore, in terms of the QPF performance (not shown), the forecast launched by NCEP did not predict heavy rainfall over northern Taiwan,
whereas the simulation initialized by the ERA-interim reanalysis was able to closely reproduce the location and total accumulated rainfall of the surface observations in northern Taiwan.

Fig. 3. Domain nesting of the WRF model with horizontal grid spacing of 27 (251 × 261 points), 9 (337 × 271 points), and 3 (223 × 232 points) km, respectively.
Table 1. Summary of OSSE Experiments.

<table>
<thead>
<tr>
<th>No.</th>
<th>Experiment</th>
<th>Initial condition</th>
<th>Assimilation period</th>
<th>Assimilation Radar data</th>
<th>Assimilation thermodynamic information</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Truth</td>
<td>ERA-interim</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>NoDA</td>
<td>NCEP-FNL</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>ZVr</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>ZVr2h</td>
<td>NCEP-FNL</td>
<td>2 h</td>
<td>Z, Vr</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>ZVrT</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>T</td>
</tr>
<tr>
<td>6</td>
<td>ZVrQv</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>Qv</td>
</tr>
<tr>
<td>7</td>
<td>ZVrTQv</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>T, Qv</td>
</tr>
<tr>
<td>8</td>
<td>ZVrTQv2h</td>
<td>NCEP-FNL</td>
<td>2 h</td>
<td>Z, Vr</td>
<td>T, Qv</td>
</tr>
<tr>
<td>9</td>
<td>ZVrTR</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>Retrieved T</td>
</tr>
<tr>
<td>10</td>
<td>ZVrQvR</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>Retrieved Qv</td>
</tr>
<tr>
<td>11</td>
<td>ZVrTQvR</td>
<td>NCEP-FNL</td>
<td>1 h</td>
<td>Z, Vr</td>
<td>Retrieved T, Qv</td>
</tr>
</tbody>
</table>

Table 1. Summary of OSSE Experiments: Exp.1 (Truth) and Exp.2 (NoDA) are simulated from ERA-interim and NCEP-FNL reanalysis, respectively. There are three sets of data assimilation in the study: 1) Exps. 3 and 4 that only assimilate radar data (Z and Vr). 2) Exps. 5–8 that assimilate radar data with additional thermodynamic data (T and/or Qv) generated from Exp.1 (truth). 3) Exps. 9–11 that assimilate radar data with thermodynamic data retrieved via TPTRS.
Fig. 4. Reanalysis fields of (a), (b) potential height (blue line), potential temperature (red line), and wind vector (gray vector) at 850-hPa; (c), (d) water vapor mixing ratio at 925-hPa at 0000 UTC 11 June 2012. (a), (c) ERA-interim for the Truth run; (b), (d) NCEP for the NoDA and OSSE experiments.
Fig. 5. Reflectivity at 2.5-km height (shaded) in d03 shows the rainband (black-dashed line) located in northern Taiwan at 1400 UTC in (a) “Truth” simulated from the initial condition ERA-interim 0.75° × 0.75° and (b) NoDA simulated from the initial condition NCEP-FNL 1° × 1°. The black solid line in 5a is a vertical cross-section portion between (120.75°E, 25.93°N) and (121.18°E, 24.64°N). The dotted square shows the focused area in the study.

**b. Synthetic radar data and thermodynamic variables**

To clearly examine the benefit and added value of providing 3D thermodynamic information at the convective scale, we assume that the temperature and humidity observations are obtained from the area of precipitation in the experiments. Therefore, the observation operator in these experiments was the mapping of the truth of temperature and water vapor mixing ratio on the model grid to the observation spaces. The simulated observations were generated from the truth run. In addition, considering the uncertainty of the observations, the truth was perturbed randomly according to the prescribed observation errors: 5 dBZ for reflectivity (Z) and 3 m s\(^{-1}\) for radial wind (Vr) as in Tsai et al. (2014), and 0.8 K for temperature and 0.8 g kg\(^{-1}\) for the water vapor mixing ratio. Radar observation data (reflectivity and radial velocity) of two radars in northern Taiwan were obtained up to a range of 230 km (180 km) from the RCWF (NCU-CPOL) site every 15 minutes on nine plan position indicator (PPI) elevations between 0.5° and 19.5°. The superobbing method (Alpert and Kumar 2007; Lindskog et al. 2004) was applied to reduce
the simulated observation count in this study. The data points were collected every 4 km in the radial direction and every 4° in the azimuthal direction in the PPI (Fig. 6) and then averaged through inverse distance weighting.

c. Thermodynamic variables retrieved via TPTRS

By taking 3D wind fields from two consecutive (15-min interval) of the truth, the 3D thermodynamic variables (T and Qv) inside the precipitation system were obtained using TPTRS (see appendix). The vertical structure of the retrieved temperature data (Fig. 7a) indicated that the mesoscale warm sector in the convection area and low-level cold pool area was well demonstrated, similar to that in the truth (not shown). The horizontal structure of the retrieved water vapor data at a height of 1.5 km (Fig. 7b) indicated that the wet area in front of the rainband and dry area behind the frontal system were well illustrated. The results of the temperature revealed that the root mean square error (RMSE) in the temperature was less than 0.5 K, and the bias was positive, meaning that the retrieved temperature was slightly warmer than that in the truth (Fig. 7c). The retrieved water vapor results indicated that the overall performance was slightly wetter than the truth model, with a correlation of 0.99 and an error of 0.56 g kg$^{-1}$ (Fig. 7d). This set of experiments assumed that the basic state (i.e. background fields) of the temperature and water vapor was error-free and the source of the error was from the perturbations of thermodynamic variables. The observation errors of the retrieved temperature and water vapor were set at 0.8 K and 0.8 g kg$^{-1}$, respectively (slightly larger than the root mean square error of retrieved T and Qv). This setup accorded with other studies of thermodynamic retrieval (Foerster and Bell 2017, Liou et al 2003, Liou et al. 2019, Feng et al. 2019). In general, the retrievals obtained a good quality of temperature and humidity fields similar as the unbiased thermodynamic field but with slight bias.

d. Experiments setup

The assimilation experiments in this study were conducted in three sets (Table 1). In the first set, both reflectivity and radial velocity were assimilated for 1 h (ZVR) and 2 h (ZVR2h). In the second set, the thermodynamic fields of temperature and water vapor generated from the truth run were additionally assimilated for 1 h (ZVRT adding temperature; ZVRQ adding humidity field; ZVRTQ adding both temperature and
humidity) and 2 h (ZVrTQv2h). Without any bias, the first and second sets of experiments can investigate the added value of assimilating 3D thermodynamic fields in the severe weather system. In the final set, to mimic bias that could be existed in observations retrieved from remote sensing, a retrieved 3D temperature and water vapor information with warm and wet bias from section 4c were assimilated (ZVrTR, ZVrQvR, and ZVrTQvR).

The flowcharts for all the experiments are presented in Fig. 8. The initial conditions derived from the ERA-interim and NCEP analyses were launched at 0000 UTC on 11 June 2012 and then integrated. The cycled DA started at 1300 (1200) UTC, assimilating observations every 15 min for a 1-h (2-h) assimilation period. Once the final analysis was obtained at 1400 UTC, a 3-h deterministic forecast was initialized using the ensemble mean to examine the QPF performance.

Fig. 6. The super observation points (gray) of simulated observations averaging 4-km in the radial direction and 4° in the azimuthal direction on every sweep from RCWF (121.77ºE, 25.07ºN) and NCU-CPOL (121.18ºE, 24.97ºN). The asterisks (*) are the radar locations of RCWF (white) and NCU-CPOL (black).
Fig. 7. (a) Vertical cross-section of retrieved temperature perturbation along dashed line in (b) at 1400 UTC. (b) Retrieved water vapor at 1.5-km height at 1400 UTC. (c) Vertical profile of retrieval temperature RMSE and BIAS. (d) Scatter plot of water vapor between truth model and retrieved by TPTRS.
5. Results of the OSSEs and assimilating retrieved thermodynamic variables

In this section, the results of Exps. 3-8 are presented using the 3-km resolution domain (D03). In addition, D03 is horizontally magnified to highlight northern Taiwan (see black dotted line in Fig. 5). First, the analysis at 1400 UTC after the cycling procedure is examined. Second, the performance of the QPF launched by the ensemble mean analysis is investigated.

a. Performance of the cycling process

The RMSE of reflectivity, radial velocity, temperature, and water vapor for Exps. 3-8 were presented in Fig. 9 during the assimilation period. The reflectivity error was similar for all the experiments (Fig. 9a). The performance of ZVrT, ZVrQv, ZVrTQv, and ZVrTQv2h (dashed lines in Fig. 9a) which additionally assimilated thermodynamic variables were better than ZVr. ZVr2h which had a longer assimilation period achieved a similar error at the final analysis. The best result is ZVrTQv2h which assimilated more thermodynamic variables and radar data than the other experiments. On the other hand, the
RMSEs of radial velocity (Fig. 9b) are similar for all the experiments when radial wind information is assimilated.

The RMSE of temperature and water vapor were presented in Fig. 9c and Fig. 9d, respectively. When assimilating 3D temperature in ZVrT and ZVrTQv (red and green dashed lines in Fig. 9c), the error of temperature decreased rapidly in the early assimilating period compared to other experiments. In addition, ZVrQv gradually improved the error of temperature during the cycling process and had a similar error at the final analysis to ZVrT, ZVrTQv, and ZVrTQv2h. This indicated that via background error covariance in the analysis steps, assimilating water vapor information could improve temperature field. On the other hand, experiments of assimilating radial wind and/or reflectivity (solid lines in Fig. 9c) had larger error in the final analysis. Result of ZVr2h (black dashed line) showed that the error of temperature could not further improve by assimilating radial wind and reflectivity in a longer assimilation period. When examining the RMSE of water vapor, experiments of assimilating radial wind and/or reflectivity (solid lines in Fig. 9d) had similar result. Smaller error of water vapor was illustrated when water vapor information was assimilated (green and blue dashed lines in Fig. 9d). Assimilating 3D temperature (ZVrT, red dashed line) could not have similar error of water vapor compared to ZVrQv, ZVrTQv, and ZVrTQv2h. Results of Fig 9c (blue dashed line) and 9d (red dashed line) indicated that assimilating 3D water vapor information was more crucial and effective than assimilating temperature field to obtain the optimal analysis.
Fig. 9. Root mean square error in assimilation period (a) Z (unit: dBZ); (b) Vr (unit: m s\(^{-1}\)); (c) T (unit: K); (d) Qv (unit: g kg\(^{-1}\)).

**b. Performance of the final analysis in OSSEs**

Figure 10 depicts the convergence field at a height of 1-km (1400 UTC). A strong convergence area was located northwest of Taiwan in the truth (Fig. 10a). However, the convergence field was unclear in NoDA (Fig. 10b), indicating a forecast position error. When radar data (radial wind and reflectivity) were assimilated in ZVr and ZVr2h, the convergence field demonstrated that the position error had been largely corrected, triggering convection in the appropriate place (Figs. 10c, d). The strength of the convergence field in ZVr2h (Fig. 10d) was the most intense, similar to that in the truth (Fig. 10a). Thus, to obtain the optimal analysis in a multiscale weather system such as a frontal system, increasing the assimilation period is beneficial. However, when assimilating additional information, such as temperature and/or water vapor, for the 1-h assimilation in
ZVrT, ZVrQv, and ZVrTQv (Figs. 10e, f, and g), the results revealed that a comparable intensity of the convergence was achieved near the northwest coast (truth, Fig. 10a). For 2-h assimilation, ZVrTQv2h (Fig. 12h) shows the convergence range is more similar to the truth (Fig. 12a) than ZVr2h (Fig. 12d) by additionally assimilating thermodynamic variables information.

The reflectivity at a height of 2.5 km in the final analysis (1400 UTC) for true run and the experiments is illustrated in Fig. 11. Compared to the truth (Fig. 11a), assimilating both reflectivity and radial wind (Fig. 11c) provided a superior rainband structure. When Z and Vr were both assimilated for 2 h, as indicated in Fig. 11d, the rainband structure was even more similar to that of the truth run (Fig. 11a). Thus, longer assimilation lengths for Z and Vr further improved the analysis of the precipitation system. On the other hand, when additional information related to the thermodynamics field was assimilated, the rainband position error and precipitation intensity error were minimized (Figs. 11e, f, and g), demonstrating a similar performance to that in ZVr2h (Fig. 11d). Notably, the result of ZVrT (Fig. 11e) captured the strong convective rainband because of the assimilation of radial wind and reflectivity in 1 h with the addition of temperature. The horizontal reflectivity fields with assimilating additional water vapor showed the strong convective line-shaped rainband similar to the result of ZVrT (Fig. 11e). The overestimated reflectivity was also reduced near the north-west area (ZVrQv, Fig. 11f and ZVrTQv, Fig. 11g). The results indicated that providing thermodynamic information shortened the cycling process and assisted in improving the analysis of the strong convective rainband (Figs. 11e, f, and g). Moreover, the rainband structure of ZVrTQv2h (Fig. 11h) is closer to the truth (Fig. 11a) than the other experiments.

Figure 12 provides a cross section of reflectivity at 1400 UTC in the vertical structure (15 km × 333 km, see the solid line in Fig. 5a) across the rainband between 120.75°E–25.93°N and 121.50°E–23.00°N. In the truth, the reflectivity in Fig. 12a clearly demonstrates the structure of the rainband, revealing both a strong convective (Z > 40dBZ at X = 110–150 km) and stratiform (X = 0–110 km) region. Figure 12b depicts the spurious convection (X = 0–10 km) caused by the initial position error from the NoDA. When Z and Vr were both assimilated, the rainband shifted to the correct position (ZVr, Fig. 12c, and ZVr2h, Fig. 12d), and in particular, the area of strong convection was accurately captured. However, because of the initial position error in the background state, relatively strong
precipitation (Z = 25–35 dBZ at X = 0–10 km) behind the main precipitation system could not be removed properly, even for the longer assimilation window in ZVr2h (Fig. 12d). The additional temperature and/or humidity information (Figs. 12e, f, and g) was able to reduce these incorrect areas of precipitation. In the strong convective area, reflectivity values were above 45 dBZ and close to the truth when additional thermodynamic variables were assimilated (ZVrT, ZVrQV, and ZVrTQv, Figs. 12e, f, and g). ZVrTQv2h (Fig. 12h) shows the strong reflectivity values at the convective area are more similar to the truth than ZVr2h (Fig. 12d).

We further investigated the cross sections of the vertical velocity and temperature fields, revealing the impact of assimilating thermodynamic variables. Figure 13a presents the results of the truth run as a reference. The maximum updraft of the vertical velocity (shaded) was observed near a height of 6 km in the convective area; relatively weak vertical velocity was noted in the stratiform area. The temperature perturbation (contours) indicated a warm core (solid lines) at the upper layer in the convection area and a cold pool (dashed lines) at a low level of 3 km in the stratiform area. In NoDA, no updraft or cold pool signature was observed (Fig. 13b). According to the stable and stratified features (warm above and cold below, refer to Fig. 13a) in the stratiform region, assimilating both reflectivity and radial velocity for 1 h (ZVr, Fig. 13c) and 2 h (ZVr2h, Fig. 13d) did not suitably modify the vertical temperature structure. Assimilating the thermodynamic information (ZVrT, ZVrQv, ZVrTQv, and ZVrTQv2h, Figs. 13e, f, g, and h), by contrast, strengthened the intensity of the vertical velocity (with an upward motion of up to 9 m s\(^{-1}\)) and temperature (with positive perturbations of up to 3 K) more effectively than extending the assimilation period (ZVr2h, Fig. 13d). When temperature and/or humidity information (Figs. 13e, f, and g) was assimilated, the stratiform region could be entirely reconstructed. To explain this result, single observation tests of temperature and horizontal wind are applied in the stratiform area (not shown). The results illustrated that assimilating the temperature field alone propagated the information and adjusted the stratiform area more effectively than solely assimilating the horizontal winds.

Since temperature structure in the precipitation system was improved when assimilating thermodynamic variables, the performance of hydrometeor variables was further examined. The spatial correlation coefficient (SCC) is defined by the following equation:
\[ SCC = \frac{\sum_{i=1}^{N}(x_{\text{exp}} - \bar{x}_{\text{exp}})(x_{\text{truth}} - \bar{x}_{\text{truth}})}{\sqrt{\sum_{i=1}^{N}(x_{\text{exp}} - \bar{x}_{\text{exp}})^2 \cdot \sum_{i=1}^{N}(x_{\text{truth}} - \bar{x}_{\text{truth}})^2}} \]  

(13)

where \( x \) refers to the hydrometeor variables (Qr, Qs, or Qg) and \( \text{exp} \) is an experiment. We define relative SCC (RSCC) by the following equation:

\[ RSCC = \frac{SCC_{\text{exp}} - SCC_{ZVr}}{SCC_{ZVr}} \times 100\%. \]  

(14)

If RSCC is positive, the experiment has positive improvement compared with ZVr. Figure 14 presents improvement of each experiment (ZVr2h, ZVrT, ZVrQv, ZVrTQv, and ZVrTQv2h) for the reflectivity-related hydrometeor variables (mixing ratio of rain, graupel, and snow) compared with ZVr in which Vr and Z were assimilated. The results revealed that all the hydrometeor variables were improved by approximately 10% when the assimilation period was extended. Assimilating temperature information led to additional improvements above freezing for the mixing ratios of graupel and snow. However, when humidity was assimilated with radial wind and reflectivity, the mixing ratios of rain, graupel, and snow were improved by more than 20%. In particular, the mixing ratio of rain exhibited a more than 40% improvement when assimilating water vapor (ZVrQv and ZVrTQv). The best experiment is ZVrTQv2h, in which the RSCC scores are all higher than 0.3. Thus, radar DA with additional thermodynamic variables also improved the distribution of hydrometeor variables when warm core and cold pool were better reproduced in the final analysis.
Fig. 10. The low-level convergence field (shaded, units: $10^{-4}$ s$^{-1}$) and wind vectors are shown at a 1-km height of 1400 UTC and focus on a small area in northern Taiwan from d03: (a) Truth; (b) NoDA; (c) ZVr; (d) ZVr2h; (e) ZVrT; (f) ZVrQv; (g) ZVrTQv; (h) ZVrTQv2h.

Fig. 11. Reflectivity field on 2.5-km height at 1400 UTC. (a) Truth; (b) NoDA; (c) ZVr; (d) ZVr2h; (e) ZVrT; (f) ZVrQv; (g) ZVrTQv; (h) ZVrTQv2h.
Fig. 12. Vertical cross-section of reflectivity (shaded colors) at 1400 UTC as the black solid line in Fig. 5a: (a) Truth; (b) NoDA; (c) ZVr; (d) ZVr2h; (e) ZVrT; (f) ZVrQv; (g) ZVrTQv; (h) ZVrTQv2h.
Fig. 13. Vertical velocity (shaded) and Potential temperature perturbation (contours, solid lines are positive values while dashed line are negative values with contours of -1.5, -1.2, -0.5, 1.0, 3.0, 5.0 K) at 1400 UTC shown on vertical cross-sections as in Fig. 5a: (a) Truth; (b) NoDA; (c) ZVr; (d) ZVr2h; (e) ZVrT; (f) ZVrQv; (g) ZVrTQv; (h) ZVrTQv2h.
Fig. 14. Improvement of the final analysis at 1400 UTC in spatial correlation coefficient of hydrometeor variables compared with ZVr. Qg, Qs, and Qr refer to the mixing ratios of graupel, snow, and rain.

c. Performance of short-term forecast in OSSEs

The short-term forecast was launched by the ensemble mean analysis at 1400 UTC, and the results of 3-h accumulated rainfall are illustrated in Fig. 15. The truth in Fig. 15a reveals extreme heavy rainfall along the coast of northern Taiwan. The rainfall accumulation in NoDA (Fig. 15b) was low, and the area of precipitation was over the ocean because this experiment employed the mean forecast from the random perturbations without DA. The results of ZVr (Fig. 15c) and ZVr2h (Fig. 15d) displayed the correct position of the total accumulated rainfall. ZVr2h demonstrated a higher level of rainfall accumulation in northern Taiwan because of the use of a longer assimilation period than in ZVr. More observation information for model assimilation had a positive impact on the QPF, but the intensity of the precipitation was weaker than that in the truth (Fig. 15a). On the other hand, the QPF was improved when the thermodynamic variables were assimilated (Figs. 15e, f, and g). Figure 15g (ZVrT) depicts the rainfall accumulation area, revealing improved values compared with ZVr2h (Fig. 15d). Thus, the additional temperature information shortened the spin-up assimilation period. In ZVrQv in which humidity information was assimilated (Fig. 15f), the location and intensity of the rainfall were very close to the truth and the overestimation of rainfall in the northwest was mitigated. For
ZVrQv2h (Fig. 15h), the coverage and intensity of the rainfall were the best performance presented in rainfall over 90 mm among all the experiments.

The Fractions Skill Score (FSS, Roberts and Lean 2008) of the short-term forecast rainfall accumulation for 1 and 3 h through the OSSEs is illustrated in Fig. 16. FSS is a spatial neighborhood technique, defined by the following equation:

\[
FSS = 1 - \frac{\frac{1}{N} \left( P_{fcs} - P_{obs} \right)^2}{\frac{1}{N} \left( \sum P_{fcs} \right)^2 + \frac{1}{N} \left( \sum P_{obs} \right)^2}
\]  

(14)

where \( N \) is the number of total observation points and \( P \) is the probability of achieving a value over a threshold around a grid point. In this study, rainfall accumulation to be verified was 0.01–70 mm, and the range distance was 24 km. If FSS is 1, the forecast is accurate. An FSS of zero indicates no skill for the rainfall accumulation forecast. In the OSSE skill scores, only the 1-h assimilation of radar data failed to achieve a high score (scores for ZVr were less than 0.5). ZVr2h had a higher forecast score (over 0.5) than ZVr because more radar data were assimilated. In the OSSE with temperature (ZVrT), the forecast skill score was similar to that for the experiment with 2 h of assimilated radar data (ZVr2h). This result revealed that adding 3D temperature assimilation improved the performance of radar DA for short-term rainfall forecasting and shortened the spin-up assimilation period from 2 to 1 h. The best result is ZVrTQv2h that shows the skill score higher than 0.3 till forecast 3-h by assimilating more thermodynamic variables. The results of the OSSEs in which water vapor was added revealed that a wetter water vapor field could greatly improve the rainfall forecast score in the 1–3 h QPF. This also highlights that water vapor was key to prolonging the rainfall forecast results in the frontal event in this study.
Fig. 15. Rainfall accumulation in northern Taiwan in d03 from 1400 UTC to 1700 UTC. (a) Truth; (b) NoDA; (c) ZVr; (d) ZVr2h; (e) ZVrT; (f) ZVrQv; (g) ZVrTQv; (h) ZVrTQv2h.
d. Analysis performance of assimilating thermodynamic variables retrieved via TPTRS

Temperature and water vapor information are retrieved from model truth (Exp.1) by the retrieval algorithm TPTRS (Section 4c). The three experiments called ZVrTR, ZVrQvR, and ZVrTQvR are listed in Table 1. At the final analysis time, the low level convergence fields at 1-km height and the reflectivity at 2.5-km are shown in Fig. 17. Figures 17a–c present the low-level convergence fields in the final analysis (1400 UTC). The convergence of ZVrTR (Fig. 17a) revealed comparable intensity to ZVrT (Fig. 10e). The reflectivity field in Fig. 17d shows that the intensity of the strong convection was similar to ZVrT (Fig. 11e), but the line-shaped convective rainband was not broadening enough. In the strong convection in northern Taiwan, the cross section of vertical velocity (Fig. 17g) reveals the upward motion was stronger (up to 8 m s$^{-1}$) than ZVrT (Fig. 13e). The overestimate of the upward motion in ZVrTR can be attributed to the warm bias. In ZVrQvR, the convergence (Fig. 17b) inland was weaker than ZVrQv (Fig. 10f). The reflectivity (Fig. 17c) shows similar intensity and pattern to ZVrQv at convective areas in northern Taiwan (Fig. 11f). The cold pool area (Fig. 17h) is wider than ZVrQv, which indicates that assimilating water vapor has more impact on low-level cooling than assimilating temperature. When the experiment assimilated both temperature and water vapor from the retrieval algorithm, the convergence (Fig. 17c) of ZVrTQvR was weaker than ZVrTQv (Fig. 10g). The reflectivity in Fig. 17f shows that the analysis could better represent the line-shaped area than ZVrTR (Fig. 17d) and ZVrQvR (Fig. 17e). At the final analysis time, the results revealed the cold pool at the low level (Fig. 17f) and the upward motion is lower than 5 m s$^{-1}$. Among the
three experiments, assimilating retrieved temperature enhanced the warm core structure at the mid-level and strengthens the upward motion in strong convection areas. In contrast, assimilating retrieved water vapor improved the intensity of cold pools near the surface. Overall, the results demonstrated that assimilated retrieved thermodynamic fields had a positive impact on the final analysis of the severe weather system.

![Graphs showing ZVrTR, ZVrQvR, and ZVrTQvR](image)

Fig. 17. Final analysis fields at 1400 UTC. ZVrTR (a, d, g), ZVrQvR (b, e, h), and ZVrTQvR (c, f, i). (a), (b), (c) same as Fig.10. (d), (e), (f) same as Fig.11. (g), (h), (i) same as Fig.13.

e. Short-term forecast performance of assimilating thermodynamic variables retrieved via TPTRS

By launching the model from the ensemble mean analysis in Section 5d, the benefit of assimilating retrieved thermodynamic information for the QPF was examined. For the

3-h total accumulated rainfall, ZVrTR (Fig. 18a) could produce the local maximum rainfall over 50 mm. Even though the retrieved temperature field had a warm bias compared with the truth, ZVrTR could better predict the local maximum of the heavy rainfall with shorter DA period (1-h) compared with ZVr2h (Fig. 15d). This was due to the warm core at mid-level and more intense upward motion in the final analysis. In ZVrQvR, more precipitation occurred in the first hour (not shown), but the amount of precipitation (Fig. 18b) was less than the truth and ZVrQv (Fig. 15f) for 3-h QPF. The optimal result came from ZVrTQvR (Fig. 18c), whose accumulated rainfall revealed the most realistic pattern and coverage over northern Taiwan. In addition, the amount of the precipitation was up to 70 mm though it was underestimated compared to the truth (Fig. 15a).

The FSSs of the short-term forecast for 0-1 and 0-3 h accumulated rainfall were displayed in Fig. 19. Among three experiments, assimilating additional retrieved temperature with radar data (ZVrTR) showed much more improvements of the short-term forecast in the first hour of the QPF (Fig. 19a) because of the strong upward motion effect by the retrieved temperature with warm bias. For 3-h forecast, the FSS showed the benefit of assimilating both retrieval temperature and water vapor (ZVrTQvR), especially in the heavy rainfall condition (threshold over 30 mm).

The feasibility study shows that when assimilating the retrieved thermodynamic fields with the warm bias, providing temperature information has more QPF improvement than assimilating retrieved water vapor. This is because more intense upward motion and warm core are reproduced in the analysis when assimilating retrieved temperature. Overall, when both retrieved temperature and water vapor are assimilated, the result shows the best impact to improve the QPF after 3 h.
Fig. 18. Same as Fig. 15, but for (a)ZVrTR; (b) ZVrQvR; (c) ZVrTQvR.

Fig. 19. FSSs of (a) 1-h and (b) 3-h rainfall accumulation from 1400 UTC by deviation distance of 24-km.

6. Summary and conclusion

In this study, we examined the impacts of assimilating 3D thermodynamic fields with radar data (radial wind and reflectivity). A series of OSSEs were designed, and three sets of DA experiments were conducted using the WLRAS ensemble DA system. Different types of thermodynamic variables were obtained from the truth or retrieved from a retrieval algorithm. A frontal system that brought extremely heavy rainfall to northern Taiwan on 11 June 2012 was investigated. The results of both analysis and very short-term forecast were examined through the three sets of experiments, resulting in the following conclusions:

(1) In this particular case, which clearly demonstrated a precipitation position error in the background, assimilating radar observations of radial wind and reflectivity promptly corrected or reduced the position error of the convergence zone and rain bands. In addition, when dealing with a multiscale weather system, such as the frontal system in this study, increasing the assimilation period helped improve the analysis and short-term forecast. However, improvements in the analysis of the stratiform area were limited and the 3-h QPF was underestimated.

(2) In the OSSEs, providing additional information regarding the unbiased temperature and/or humidity fields in the multiscale precipitation system shortened the spin-up assimilation period as expected. In addition, the upward motion and warm core in
the mid-level and the cold pool in the stratiform areas were more improved through additional thermodynamic variables than through additional cycles of radial wind and reflectivity assimilation. The improvement in the stratiform region further benefited the hydrometeor variables, which were strongly associated with the vertical structure of temperature. The results demonstrated that assimilating thermodynamic variables achieved greater improvements (0–3 h) in extremely heavy rainfall prediction than when only assimilating radar data. In addition, assimilating humidity information alone yielded a stronger accumulated rainfall performance than assimilating the 3D temperature field alone.

(3) In the study, we discuss the feasibility of assimilating retrieved 3D thermodynamic fields with radar data. When the biased thermodynamic fields were obtained by a retrieval algorithm, assimilating temperature information reproduced a stronger upward motion and warm core at the convective area of the final analysis, and improved the very short-term QPF at the first hour. Assimilating humidity field helped to reconstruct the coverage and intensity of the cold pool near the surface. Assimilating both temperature and water vapor could optimally shorten a spin-up assimilation period to improve the QPF up to 3 h.

Overall, this study demonstrated that assimilating 3D thermodynamic fields with radial wind and reflectivity was crucial to improve the analysis and QPF of a convective scale weather system. In addition, it was possible to assimilate 3D thermodynamic fields from weather radars. One should notice that the experiments of assimilating retrieved thermodynamics fields were nearly perfect scenarios since the basic state of thermodynamics was error-free. Further tests to alleviate this assumption are necessary. As a next step, we are preparing to assimilate the following information in the WLRAS: 1) low-level information from surface observations and 2) humidity information from radar refractivity and/or dual-wavelength retrievals. The evaluation and verification of QPFs will be examined with real cases of severe weather systems such as typhoon and thunderstorm.

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**Data Availability Statement**

The WRF output and code of assimilation used in this study are not currently available in a publicly accessible repository. The reanalysis data used in this study were obtained from ERA-interim (https://www.ecmwf.int/en/forecasts/datasets/reanalysis-datasets/era-interim) and NCEP-FNL (DOI: 10.5065/D6M043C6)
APPENDIX

In this appendix, the methods for 3D temperature and water vapor data retrieval are introduced.

1. Terrain-Permitting Thermodynamic Retrieval Scheme (TPTRS)

The terrain-permitting thermodynamic retrieval scheme (TPTRS, Liou et al., 2019) is an algorithm for immediately retrieving the 3D pressure and temperature fields for complex terrain using wind information. In the algorithm, three basic equations of motion employ momentum equations to obtain thermodynamic fields (Liou et al. 2003), including moisture-related contributions:

\[
\frac{1}{\theta_v \theta_0} \left[ \frac{\partial u}{\partial t} + V \cdot \nabla u - f v + turb(u) \right] = - \frac{\partial \pi'}{\partial x} \equiv -F \tag{1}
\]

\[
\frac{1}{\theta_v \theta_0} \left[ \frac{\partial v}{\partial t} + V \cdot \nabla v + f u + turb(v) \right] = - \frac{\partial \pi'}{\partial y} \equiv -G \tag{2}
\]

\[
\frac{1}{\theta_v \theta_0} \left[ \frac{\partial w}{\partial t} + V \cdot \nabla w + turb(w) + g(\text{Ref.}) \right] = - \frac{\partial \pi'}{\partial z} + g \frac{\theta'_c}{\theta_v \theta_0} \equiv -H \tag{3}
\]

\[
u \frac{\partial \theta'_v}{\partial x} + v \frac{\partial \theta'_v}{\partial y} + w \frac{\partial \theta'_v}{\partial z} + w \frac{\partial \theta_0}{\partial z} + S = 0 \tag{4}
\]

where \( S \) is the total effect of the temporal variation, diffusion, and source/sink achieved through microphysical processes (Liou, 2001). The \( S \) term is treated as a retrievable parameter in the study, and no additional parameterizations were applied.

Furthermore, the contributions of vapor, cloud, and rainwater are included to estimate the buoyancy force. In the retrieval scheme, a normalized pressure (\( \pi \)) is obtained (Exner function), and it is defined as

\[
\pi = C_p \left( \frac{P}{P_0} \right)^{R \theta_v} \tag{5}
\]

where \( P \) is the pressure, \( P_0 = 100 \text{ kPa} \), \( R \) (unit: J kg\(^{-1}\) K\(^{-1}\)) is the gas constant, and \( C_p \) (unit: J kg\(^{-1}\) K\(^{-1}\)) is the specific heat capacity at a constant pressure.

The virtual potential temperature (\( \theta_{vir} \)) and virtual cloud potential temperature perturbation (\( \theta'_c \), see Roux 1985) are defined by

\[
\theta_{vir} = \theta (1 + 0.61 q_{wv}) \tag{6}
\]

\[
\theta'_c = \theta' + (0.61 q'_{wv} - q_c) \theta_0 \tag{7}
\]

where \( \theta_0 \) is potential temperature, \( q'_{wv} \) (unit: g kg\(^{-1}\)) is the perturbation of the water vapor mixing ratio based on its basic statistics, and \( q_c \) (unit: g kg\(^{-1}\)) is the cloud water mixing
ratio. In (3), \( q_r \) refers to the rainwater mixing ratio, which can be estimated using the relationship with radar reflectivity (\( \eta \) in dBZ, Sun and Crook 1997):

\[
\eta = 43.1 + 17.5 \log(\rho q_r) \tag{8}
\]

where \( \rho \) is the air density.

2. Moisture and temperature adjustment scheme

The moisture adjustment method was based on that of Liou et al. (2014), who proposed a simple and effective approach for adjusting temperature and moisture fields through iterative methods. In the process, the water vapor mixing ratio perturbation is initially set to 0, and then the temperature is converted to temperature \( T \) through equation (7), where the virtual cloud potential temperature is the thermodynamic field derived from the temperature perturbation field and the cloud-water mixing ratio is derived from the model. The model is treated as DA and uses the inverted surface temperature and pressure to calculate the dew-point temperature:

\[
T_d = \frac{B}{\ln \ln (A \varepsilon / q_{wv}^P)} \tag{9}
\]

\[
A = 2.533 \times 10^8 \text{ kPa} \tag{10}
\]

\[
B = 5.417 \times 10^3 \text{ K} \tag{11}
\]

\[
\varepsilon = 0.622 \tag{12}
\]

\[
q_{wv} = q_{wv}' + q_{v0}. \tag{13}
\]

When the radar reflectivity exceeds 10 dBZ, this area is regarded as saturated. The value of the saturated water vapor mixing ratio on these saturated grid points is calculated, the water vapor mixing ratio of the basic state is subtracted to obtain a new \( q_{wv}' \), and the result is substituted into formula (7) to obtain a new potential temperature perturbation field \( \theta' \). Subsequently, the difference between the newly obtained water vapor perturbation and potential temperature perturbation and the original water vapor perturbation field and potential temperature perturbation field is calculated. If the difference between the two is less than the threshold, the calculation is stopped and the water vapor adjustment iterative process is complete, resulting in a new \( q_{wv}' \) and \( \theta' \). The threshold values are \( 5 \times 10^{-5} \) for the temperature field and \( 5 \times 10^{-2} \) for the water vapor mixing ratio.

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