NOTES AND CORRESPONDENCE

Vertical Structure of Time-Dependent Flow for Viscosity that Depends on Both Depth and Time

JAMES R. BAKER

Department of Physics and Astronomy, University of Wyoming, Laramie 82071

THOMAS F. JORDAN

Department of Physics, University of Minnesota, Duluth 55812

9 February 1981 and 13 September 1981

ABSTRACT

A previously developed eigenfunction expansion, that describes horizontal current as a function of depth and time, is extended to include any eddy viscosity given as a product of a function of depth and a function of time.

Recently, we developed an eigenfunction expansion that gives the solution of linear hydrodynamic equations for horizontal current as a function of depth and time when the eddy viscosity is given as a function of depth (Jordan and Baker, 1980a; Baker and Jordan, 1980). Here we extend that to include any viscosity given as a product of a function of depth and a function of time.

This will allow a better approximation of the actual situation where the viscosity depends on the flow and therefore changes in time (Thomas, 1975). Models of storm surges (Jelesnianski, 1970; Forristall, 1974) might be improved simply by giving the viscosity a time dependence corresponding to that of the wind force. A periodic time dependence could be used in models of seiches or tides. Csanady and Shaw (1980) found and analyzed the solution for drift current caused by a suddenly applied constant wind stress, for viscosity that depends on time but not depth, in the limit of infinitely deep water; they use it to describe a turbulent Ekman layer. Madsen (1977) had found a solution describing a turbulent boundary layer near the surface by using viscosity that increases linearly with depth but does not depend on time, also for infinitely deep water. Our formulas give solutions for viscosity that depend on both depth and time, in water of finite depth.

The inspiration for this work came (after publication of our papers) from learning of the work of Nihoul (1977) who found solutions with time-dependent viscosity using eigenfunctions the same as we used for zero bottom friction.

Let the eddy viscosity be $\eta(t)\nu(z)$ with ν a positive function of depth and η a positive function of time. The differential equation for the current [replacing Eq. (1) of Jordan and Baker, 1980a] is

$$\frac{\partial w}{\partial t} = -ifw + \eta \frac{\partial}{\partial z} \nu \frac{\partial w}{\partial z} - q.$$

The boundary condition at the surface [replacing Eq. (2) of Jordan and Baker, 1980a] is

$$\eta\left(\nu \frac{\partial w}{\partial z}\right)(z = 0) = F.$$

The boundary condition for linear bottom friction [replacing Eq. (3) of Jordan and Baker, 1980a] is

$$\eta[\nu(\partial w/\partial z)](z = -H) = \eta b w(z = -H).$$

For $b = 0$ this is the condition for zero bottom friction [that replaces Eq. (5) of Jordan and Baker, 1980a]. Otherwise the problem is the same as considered by Jordan and Baker (1980a). The other bottom boundary conditions that may be used are $w(z = -H) = 0$ for zero bottom current or $w(z = -H + \epsilon H) = 0$ for a turbulent boundary layer at the bottom. It is assumed, as before, also that $w(t = 0) = 0$.

A solution of this problem is unique. That can be shown as before (Jordan and Baker, 1980b). Including the time-dependent viscosity requires only straightforward modifications in the proof.

0022-3670/81/121673-02$04.50

In terms of the eigenfunction expansion, the solution is

$$w(z, t) = \sum_{n=0}^{\infty} B_n f_n(z) \int_0^t q(t - \tau) \times \exp - (if\tau) \exp - \left[\lambda_n \int_{t-\tau}^{t} \eta(\theta) d\theta\right] d\tau + \frac{F(t)}{\eta(t)} w_s(z) - \frac{F(t)}{\eta(t)} \sum_{n=0}^{\infty} D_n f_n(z) + \sum_{n=0}^{\infty} (if + \lambda_n) D_n f_n(z) \int_0^t F(t - \tau) \times \exp - (if\tau) \exp - \left[\lambda_n \int_{t-\tau}^{t} \eta(\theta) d\theta\right] d\tau$$

[which replaces Eq. (7) of Jordan and Baker (1980a)] where $w_s(z)$, $f_n(z)$, λ_n, B_n and D_n are the same as before.

Acknowledgment. Part of this work was supported by the U.S. Environmental Protection Agency, Grant R805667-01-0.

REFERENCES

Baker, J. R., and T. F. Jordan, 1980: Vertical-structure functions for time-dependent flow in a well-mixed fluid with turbulent boundary layers at the bottom and top. *J. Phys. Oceanogr.*, **10**, 1691–1694.

Csanady, G. T., and P. T. Shaw, 1980: The evolution of a turbulent Ekman layer. *J. Geophys. Res.*, **85**, 1537–1547.

Forristall, G. Z., 1974: Three-dimensional structure of storm-generated currents. *J. Geophys. Res.*, **79**, 2721–2729.

Jelesnianski, C. P., 1970: Bottom stress time history in linearized equations of motion for storm surges. *Mon. Wea. Rev.*, **98**, 462–478.

Jordan, T. F., and J. R. Baker, 1980a: Vertical structure of time-dependent flow dominated by friction in a well-mixed fluid. *J. Phys. Oceanogr.*, **10**, 1091–1103.

——, and ——, 1980b: Current as a function of depth in the water: Solution of a linear model. U.S. Environ. Protection Agency Tech. Rep.

Madsen, O. S., 1977: A realistic model of the wind-induced Ekman boundary layer. *J. Phys. Oceanogr.*, **7**, 248–255.

Nihoul, J. C. J., 1977: Three-dimensional model of tides and storm surges in a shallow well-mixed continental sea. *Dyn. Atmos. Oceans*, **2**, 29–47.

Thomas, J. H., 1975: A theory of steady wind-driven currents in shallow water with variable eddy viscosity. *J. Phys. Oceanogr.*, **5**, 136–142.

A Deep, Thick, Isopycnal Layer within an Anticyclonic Eddy

ZACHARIAH R. HALLOCK,[1] WILLIAM J. TEAGUE AND ROBERT D. BROOME

U.S. Naval Oceanographic Office, NSTL Station, MS 39522

27 April 1981 and 8 September 1981

ABSTRACT

An anticyclonic mesoscale eddy, encountered during a hydrographic survey in the Sargasso Sea in September 1979, is described. Embedded in the eddy at a depth of 800 m was an isopycnal lens of anomalous water which extended 150 m vertically and ~40 km horizontally. The source of the lens is not known but may be related to frontal processes in the area.

1. Introduction

In August, 1979, a hydrographic survey was conducted in the Sargasso Sea near 31°N, 72°30′W by the U.S. Naval Oceanographic Office aboard USNS *Kane*. The survey consisted of a grid of CTD (conductivity–temperature–depth) stations and XBT (expendable bathythermograph) drops over an area of ~100 n mi square. A total of 31 CTD stations were occupied and 150 XBT drops were made.

Analysis of data revealed an anticyclonic eddy embedded in a larger anticyclonic field. Also present was a surface front just west of the eddy and oriented roughly north–south. Most notable was a peculiar lens of homogeneous water in the main thermocline near the center of the eddy.

[1] Present affiliation: Naval Ocean Research and Development Activity, NSTL Station, MS 39529.

2. Observations

Data were acquired with a Neil Brown Instrument Systems Mark III CTD and with Sippican T-7 XBT probes. CTD data were recorded directly on computer-compatible magnetic tape and XBT data on digital cassettes.

Separation of CTD stations was 20 n mi on the first, third and fifth east–west transects (see Fig. 1) and 40 n mi on the remainder. XBT drops were made every 10 n mi. Most CTD casts were to a depth of 2000 m; several were to the bottom, ~5000 m.