is seen that all entries under A, B and C are of order 10^{-3} dyn cm^{-3} or less. Incidentally, in the cross-stream direction the forces are typically 10^{-2} dyn cm^{-3}, so that the forces involved in the downstream direction are at least an order of magnitude smaller than those in the cross-stream direction.

The role played by the vertical advection term is to be noted.

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On the Importance of the Significant Slope in Empirical Wind-Wave Studies

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ABSTRACT

The significant slope of a random wave field is found to be an important parameter in empirical wind-wave studies. This significant slope S_s is defined as $S_s = (\overline{\zeta^2})^{1/2}/\lambda_0$, with $\overline{\zeta^2}$ as the mean-square surface elevation and λ_0 as the wavelength corresponding to the waves at the peak of the spectrum. With this parameter, the relationship between $\tilde{E}$ and $\tilde{n}$ is reduced to an identity expressing a pure geometric measure of the sea state, because $\tilde{E}\tilde{n}^4 = (2\pi S_s)^2$. By applying the significant slope as a parameter explicitly, we proposed that the traditional empirical formulas relating the nondimensional energy $\tilde{E}$, fetch $\tilde{x}$ and frequency $\tilde{n}$ be combined into a single unified relationship as $\tilde{E}\tilde{n}/\tilde{x} = (9/40)S_s^{9/4}$. This unified empirical formula governs the wind-wave data equally as well in the field as in the laboratory.

1. Introduction

The shape of the wave is a very important parameter in the studies of wave dynamics. For a single train of sinusoidal waves, the shape is characterized by the slope of the wave ϵ, i.e., $\epsilon = ak$, with a as the amplitude and k as the wavenumber. Although, by definition, ϵ is only a geometrical similarity parameter, it measures the degree of nonlinearity of the waves that governs the wave propagation and wave-wave interactions. Since the dynamics of the waves in a random wave field are governed by the same set of equations, a similar kind of geometrical similarity parameter also should play an important role. By analogy, we designate this geometrical similarity parameter as the significant slope S_s which is defined as

$$S_s = (\overline{\zeta^2})^{1/2}/\lambda_0, \quad (1)$$

with $\overline{\zeta^2}$ as the mean square surface elevation and λ_0, the wavelength corresponding to the waves at the peak of the spectrum.

The importance of this geometric parameter in wind-wave studies has been demonstrated by Huang and Long (1980) and Huang *et al.* (1981), who showed that S_s was the controlling parameter in determining the statistical properties and the spectral function of the wind-wave field. These results are in agreement with the theoretical results of Longuet-Higgins (1963) and Hasselmann *et al.* (1976).

As can be seen easily, the definition of the significant slope depends only on the wave characterics. Therefore S_s is an internal parameter, as contrasted with the external ones, which are defined by wind speed, fetch, etc. Even though the wind-wave properties can be successfully characterized by the internal parameter which measures the integrated effects of wind, fetch, duration, etc., it still would be necessary to establish rules governing the variation of this internal parameter as a function of the environmental variables. This last step is the core of the wind-wave generation problem. A quantitative solution, however, has yet to be achieved. For lack

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of such a solution, the relationship between the internal and external variables would have to be empirical. In this short note we will review some of the past results, and then propose a unified empirical formula that includes the significant slope of the wave field as an explicit parameter. Since the significant slope is a measure of the nonlinearity of the wave field, the inclusion of S_s enables the formula to be valid over a much wider range of wave conditions. Comparison with both laboratory and field data shows that the unified formula applies equally well to both conditions.

2. A unified empirical formula

In empirical studies of wind-wave problems, the nondimensionalized fetch $\tilde{x}$, frequency $\tilde{n}$ and energy $\tilde{E}$ are used routinely. The definitions of these parameters are

$$\left.\begin{aligned} \tilde{x} &= gx/u_*^2 \\ \tilde{n} &= n_0 u_*/g \\ \tilde{E} &= \overline{\zeta^2} g^2/u_*^4 \end{aligned}\right\}, \qquad (2)$$

with x as the fetch, u_* the friction velocity, n_0 the frequency at the spectral peak, and g the gravitational acceleration. All these quantities are defined by external variables. However, since S_s is similar to ϵ, which is an indicator for nonlinearity, and which controls wave-wave interactions or the energy transfer among different components in a spectrum, wave breaking and wind-wave interaction (see, e.g., Phillips, 1977), it is only natural that this geometrical similarity parameter should be included explicitly in the formulation of the empirical relationships. Unfortunately, it has been largely neglected.

Past examples of empirical relationships are

$$\tilde{n} = C_1 \tilde{x}^{-1/3}, \qquad (3)$$

$$\tilde{E} = C_2 \tilde{x}, \qquad (4)$$

$$\tilde{E} = C_3 \tilde{n}^{-3}, \qquad (5)$$

by Toba (1973), and

$$\tilde{n} = D_1 \tilde{x}^{-1/4}, \qquad (6)$$

$$\tilde{E} = D_2 \tilde{x}, \qquad (7)$$

$$\tilde{E} = D_3 \tilde{n}^{-4}, \qquad (8)$$

by Phillips (1977) with C_i and D_i (i = 1, 2, 3) in (3)–(8) as absolute constants. Not all the equations in the sets (3)–(5) and (6)–(8) are independent. For example, it can be shown easily that

$$\left.\begin{aligned} C_1^3 C_2 &= C_3 \\ D_1^4 D_2 &= D_3 \end{aligned}\right\}. \qquad (9)$$

Since Eqs. (4) and (7) are identical, the choice in the past has been between the −3rd or −4th power dependence of $\tilde{E}$ on $\tilde{n}$.

With the introduction of S_s as an explicit parameter in wind-wave studies, it becomes obvious that Eq. (8) cannot be true. This can be easily demonstrated by using the definitions of the parameters as in Eq. (2) and the dispersion relationship; then Eq. (8) becomes

$$\tilde{E}\tilde{n}^4 = (2\pi S_s)^2 = D_3. \qquad (10)$$

Eq. (10) requires that all the waves, under different wind and fetch conditions, to have identical geometric shape in the mean, a condition certainly unrealizable. Thus the invalidity of Eq. (8) is established.

Next, we examine Eq. (7). Since Eq. (4) is identical to (7), this examination would be a critical test to both sets of past empirical formulas. Published laboratory data by Hidy and Plate (1966), Mitsuyasu (1968), and Toba (1973), together with the field data from JONSWAP sub-group A as given by Müller (1976), are shown in Fig. 1 in comparison with our laboratory results discussed in detail by Huang and Long (1980) and Huang *et al.* (1981). The bulk of the data fits the straight line very well. But scattering is obvious, especially for the laboratory data. Judging the critical role (4) and (7) play in both sets, we decided to test the value of the constant in the equation in detail by rewriting the equation as

$$\tilde{E}/\tilde{x} = C. \qquad (11)$$

The constancy of C can be tested by plotting $\tilde{E}/\tilde{x}$ as a function of S_s. The results using the same data set are shown in Fig. 2.

Two features are evident in the figure. The first is a strong dependence of C on S_s. This is especially clear for the laboratory data covering a range of two decades of S_s values. For the field data the trend is also unmistakable, but because of the limited S_s range, the variation is not so clear as it is for the laboratory data. The data show that C may vary by as much as three to four decades for the laboratory values. Even for the field data, a decade of scattering can be detected. The second feature is the separation of field and laboratory data sets. On close examination of the data, we found that the difference in $\tilde{E}/\tilde{x}$ value between the laboratory- and field-data clusters was approximately one decade, which was precisely the same order as the differences in $\tilde{n}$. Guided by this observation, we propose the following formula

$$\tilde{E}\tilde{n}/\tilde{x} = (9/40) S_s^{9/4}, \qquad (12)$$

as an alternative to those of Toba and Phillips. Fig. (3) shows this relationship as a solid line drawn against field and laboratory data. The field and laboratory data were brought together. This shows that the field and laboratory results can be represented in a unified way by using the proper parameters. The key in this new formula is the introduction

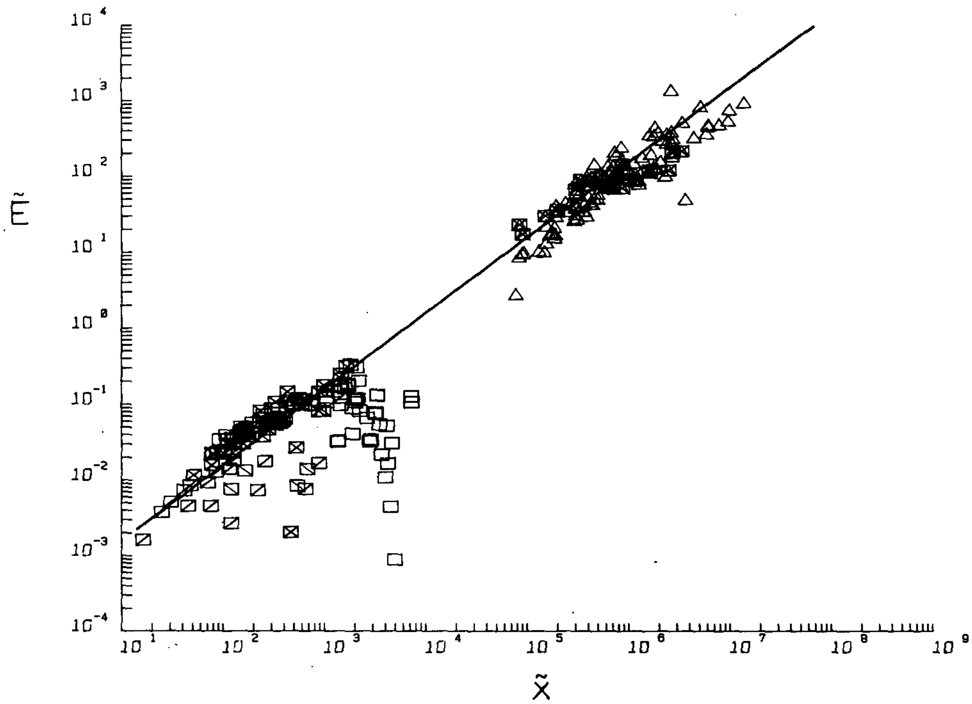

FIG. 1. Comparison of existing empirical relationship for fetch-limited wave field with data: $\tilde{E}$ vs $\tilde{x}$; solid line according to Eqs. (4) and (7) with constant value at 1.5×10^{-4}. Symbols for data for all figures except otherwise noted: □ Huang and Long (1980) and Huang *et al.* (1981); ⧄ Mitsuyasu (1968); ⧅ Hidy and Plate (1966); ⊠ Toba (1972); △ JONSWAP (1976).

of S_s. In principle, a unified formula also makes good sense because, in a wind-wave field, the wave energy, frequency, wind speed and fetch are all intertwined.

Eq. (12) forms the foundation governing the sea state under all conditions. By using some simple algebraic steps and the definition of S_s, one can easily write an alternative form of (12) as

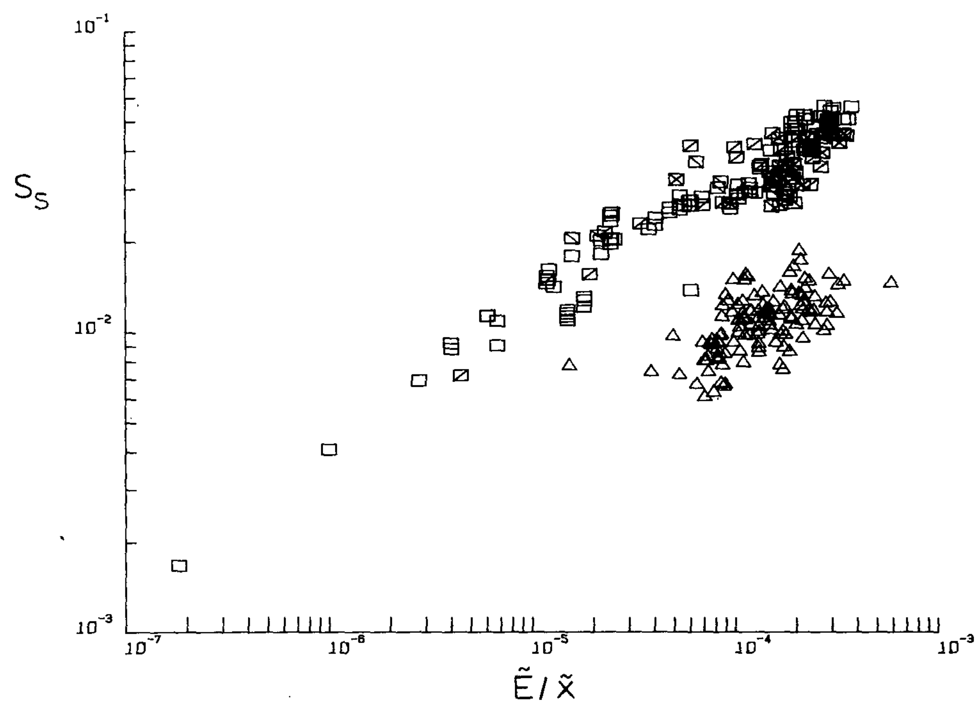

FIG. 2. Variation of the empirical constant in existing relationships as a function of the significant slope.

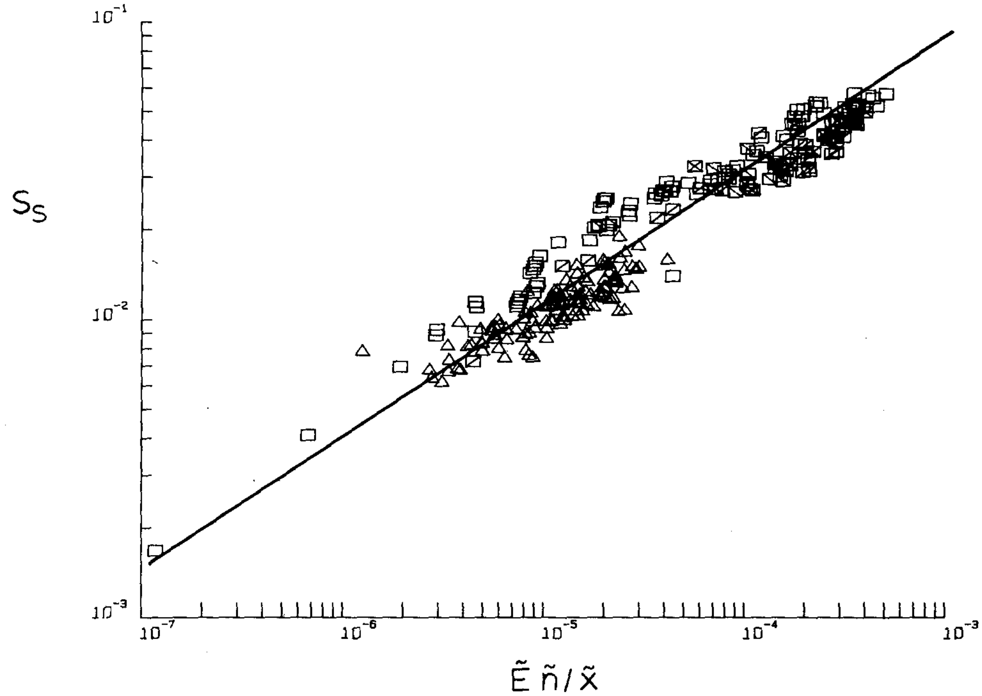

FIG. 3. Comparison of the new unified empirical relationship for a fetch-limited wave field with data. Solid line according to Eq. (12).

$$\tilde{n}^3\tilde{x} = 160\pi^2/9S_s^{1/4}. \quad (13)$$

This expression is similar to (3) except that $\tilde{n}^3\tilde{x}$ is not a constant anymore. Neither (12) nor (13) can be derived from simple algebraic manipulations from the existing formulas. However, judging from the forms of Eqs. (3) and (13), any improvement over (3) will be marginal because even for a 10^2 range of change in S_s, the change in C_1 is only $\pm 16\%$. Here Toba's formula seems to work better than Phillips's, but the unified formula works the best. From JONSWAP data, the value of S_s is always between 0.005 and 0.02. Then according to (13), the value of $\tilde{n}^3\tilde{x}$ will be between 660 and 466. The corresponding C_1 value in (3) would be between 8.71 and 7.76. Although a direct comparison here would be meaningless because the coefficient suggested is not an absolute constant as required in (3), nevertheless, the close agreement of the value to that from past data is reassuring.

Another alternative way to write Eq. (12) is

$$S_s = \left(\frac{160\pi^2}{9\tilde{n}^3\tilde{x}}\right)^4. \quad (14)$$

This formula would enable us to compute the significant slope of a wave field as a function of $\tilde{x}$ and $\tilde{n}$. This coupled with the empirical relationship relating n_0 to wind speed U as

$$n_0 = g/U, \quad (15)$$

will make spectral calculation according to internal variables possible as proposed by Huang *et al.* (1981).

3. Conclusion

The steepness of the wave always plays a critical role in wave dynamics studies. This was clearly realized in all the theoretical studies (see, for example, Phillips, 1977). But experimental studies have been slow to use this important steepness parameter. Recently, experimental data were analyzed specifically to test the importance of the geometrical similarity parameter. The results indicated that the steepness of the waves are an important parameter for determining the statistical properties of the random surface (Huang and Long, 1980) and the energy spectral level (Huang *et al.*, 1981). For a random wave field the measure of this geometric similarity is the significant slope. Using this parameter explicitly, we reduced the existing empirical formulas relating $\tilde{E}$, $\tilde{n}$ and $\tilde{x}$ into a single unified formula. Significantly, with S_s the past discrepancies between the field and laboratory data were reconciled successfully. The empirical formula also enables one to relate the internal parameter to the external ones. Thus using the spectral model depending on an internal parameter for wave prediction becomes a real possibility.

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Practical Conversion of Pressure to Depth

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ABSTRACT

A conversion formula between pressure and depth is obtained employing the recently adopted equation of state for seawater (Millero *et al.*, 1980). Assuming the ocean of uniform salinity 35 NSU and temperature 0°C the following equation is proposed, namely, $z = (1 - c_1)p - c_2p^2$. If p is in decibars and z in meters $c_1 = (5.92 + 5.25\sin^2\phi) \times 10^{-3}$, where ϕ is latitude and $c_2 = 2.21 \times 10^{-6}$. To take account of the physical conditions in the water column a dynamic height correction is to be added but for many purposes this may be ignored.

This note updates the conclusions reported by Saunders and Fofonoff (1976) and results from the adoption of a more accurate description of the equation of state of seawater (Miller *et al.*, 1980) than was available previously.

Integration of the hydrostatic equation yields a conversion from depth z to pressure p of the form

$$\int_0^z g dz = \int_0^p \alpha dp$$

where α, the specific volume, is the reciprocal of density and p is reckoned zero at the surface. We allow for an increase of gravitational acceleration g with depth, viz $g = g_s + \gamma z$, where g_s, the surface value, is a function of latitude ϕ and is given by

$$g_s = 9.780318(1 + 5.3024 \times 10^{-3}\sin^2\phi - 5.9 \times 10^{-6}\sin^2 2\phi)\ [\mathrm{m\ s^{-2}}]$$

and γ has a value 2.226×10^{-6} s^{-2}. Hence

$$(g_s + \tfrac{1}{2}\gamma z)z = \int_0^p \alpha dp.$$

If z is replaced by p in the correction term ($\gamma z \approx \gamma' p$) with $\gamma' = 2.226 \times 10^{-6}$ db^{-1} m s^{-2}, then

$$z = \int_0^p \alpha dp/(g_s + \tfrac{1}{2}\gamma' p).$$

Given a set of observations of temperature and salinity versus pressure in the water column, the above equation may be evaluated numerically. Near the surface, data should be no more than 50 db apart and deeper the interval should be no more than 200 db. In the absence of such data (or to check such a calculation) the following procedure is recommended. The equation $\alpha = \alpha(35, 0, p) + \delta$ defines δ, the specific-volume anomaly, where $\alpha(35, 0, p)$ is the specific volume of the standard ocean (taken by convention to have salinity 35 and temperature 0°C). Consequently,

$$\int_0^p \alpha dp = \int_0^p \alpha(35, 0, p)dp + \int_0^p \delta dp.$$

We now consider the contribution from the first integral for which the new equation of state is

$$\alpha(35, 0, p) = \alpha(35, 0, 0)\left[1 - \frac{p}{(K + Ap + Bp^2)}\right],$$

where $\alpha(35, 0, 0)$ is 0.972662×10^{-3} m^3 kg^{-1}, K the secant bulk modulus is 21582.27 bar, and A and B,

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