Numerical Simulations of Hurricane-Generated Currents

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ABSTRACT

The work described here involves the application of a three-dimensional numerical circulation model to the hindcasting of currents generated during two storms—Tropical Storm Delia and Hurricane Anita. Reasonably high-quality current and other data were collected during these two storms and are reported in the literature. The hindcasted results of the two storms are used to assess the accuracy of the model and to explore the relative importance of the forcing mechanisms which affect water circulation during a tropical storm event. The observed currents from both data sets indicate that the bottom shear stress is much larger than the surface shear stress during the storms, implying that a forcing mechanism other than the local wind is important in driving flow during the storm. Two possible mechanisms have been suggested in the literature—flow driven by pressure gradients and unsteady flow effects created by wind waves. The circulation model is used to study the first mechanism and the results indicate that the pressure gradient is not the primary reason for the large bottom shear stress for either storm. The absence of other explanations would strongly suggest that waves are the dominant reason for the large bottom shear stress displayed in the data. Further evidence supporting this hypothesis is found by successfully using a bottom-layer wave/current model by Grant and Madsen (1980) to predict the maximum bottom shear stress that occurred during both storms. The boundary-layer model is also used to explain the difference between the bottom-friction coefficients deemed most appropriate for the two hindcasts. This latter finding suggests that including wave effects in the bottom-boundary-layer dynamics can improve current forecasts and hindcasts with the circulation model by a factor of more than 2 in shallower waters. The findings are also relevant to the majority of finite-depth circulation models.

1. Introduction

Reasonably high-quality data sets taken during Tropical Storm Delia and Hurricane Anita are available in the literature and offer an opportunity 1) to verify a recently developed 3-D numerical model and 2) to explore the forcing mechanisms that dominate a shallow shelf area during tropical storm events.

The data for Tropical Storm Delia originated from Forristall et al. (1977), henceforth referred to as FHC. Data were recorded on the Buccaneer platform in the Gulf of Mexico. Buccaneer was equipped with three current meters, a barometer and a wind sensor. The storm center of Delia passed within a few kilometers of Buccaneer and produced a minimum pressure of 987 mb, peak wind gusts of 32 m s⁻¹ and extreme waves of 7–8 m. Fig. 1 shows the location of Buccaneer and the storm track, as well as the grid system initially used in the wind and current models. FHC used a model to simulate the wind field created by Delia. The simulated wind field was put into a hybrid numerical-analytical circulation model and the resulting hindcasts are presented.

Hurricane Anita passed through the northern Gulf of Mexico in September 1977. Current data reported by Smith (1978) were taken with two current meters moored 20 km off the Texas coast in 17 m of water (see Fig. 2). The nearest wind data were recorded at Port Aransas, Texas, 120 km from the site of the current meters. The center of the storm passed within 360 km of the current-meter site, creating maximum currents of 1 m s⁻¹. Maximum winds recorded at Port Aransas were 13 m s⁻¹. Smith did not apply models to hindcast winds or currents and his discussion was mainly qualitative in nature.

The work described in this paper is presented in seven sections. Section 2 describes the wind modeling which was necessitated by the lack of synoptic wind data for either Delia or Anita. Wind modeling provided the surface shear stress, atmospheric pressure gradient, and barometric water-level rise needed by the circulation model. Section 3 briefly describes the formulation of the circulation model used in the study. Sections 4 and 5 describe application of the circulation model to Delia and Anita, respectively. In these two sections, considerable attention is fo-
2. Wind model

Because of the lack of synoptic wind data it was necessary to use a wind model to provide the wind-forcing information needed by the circulation model. A simple wind model was used which incorporates the symmetric pressure field suggested by Harris (1958). The wind-field modeling was accomplished using relationships by Jelesnianski (1974). A complete description of the wind model is given by Pearce et al. (1979). The model requires specification of the deflection angle, radius to maximum winds, temporal variation of storm position, and central and peripheral pressure.

The shear relationship used to couple the wind model with the circulation model is taken from Wu (1969). Wind shear stresses, pressure gradients and the so-called “inverted barometer effect” were passed from the wind model to the circulation model on an hourly basis. It should be noted that in both the Delia and Anita hindcasts the wind and current modeling were performed independently. In other words, wind simulations were completed first, followed by current simulations. No attempt was made to manipulate winds so as to improve the current hindcasts.

a. Delia wind simulation

Wind data from Tropical Storm Delia are presented by FHC in two forms: the wind velocity and pressure recorded at Buccaneer, and the central pressure measured by the Air Force and Navy. Data are not available for the radius to maximum wind although FHC suggest a value of 65 km based on occurrence of maximum wind at Buccaneer and at Galveston, Texas. In the process of tuning the wind model used in this study it was found that the best comparisons at Buccaneer were obtained when the radius to maximum winds was varied between 84 and 64 km during the course of the storm. Wind-model input parameters (i.e., deflection angle, radius to maximum winds, central pressure and storm position) were varied within reasonable limits based on the available data in order to obtain the best comparison between the wind shear stresses calculated by...
Fig. 2. Grid system used in the wind and circulation modeling of Hurricane Anita.
simulation and from the observed wind data taken at the platform.

The deflection angle is often treated as a constant; however, Myers and Malkin (1961) present data which suggest that, in fact, the deflection angle displays significant temporal and spatial variability for a particular storm. Since there were no synoptic wind data for Delia, measurements of the deflection angle are not available. Hence, in the wind-field modeling of Delia, the deflection angle was treated as a tuning parameter and varied to yield the best comparison between the wind data and the model. An angle of $5^\circ$ was found to simulate the measured data best.

FHC used more sophisticated wind and shear-stress relationships than those used in this study. The models used by FHC were based on work done by Cardone et al. (1976) and Cardone (1969). Cardone's expression for the shear stress indicates that the drag coefficient varies approximately the same as Wu's in the range from 5 to 20 m s$^{-1}$. Above the latter value, however, Wu's formulas specify the drag coefficient to be constant, whereas Cardone's expression indicates that the drag coefficient increases linearly with speed (for neutral stability). Since wind speeds in Delia were never greater than about 25 m s$^{-1}$, the two shear-stress relationships behave in essentially the same manner.

Fig. 3 shows a comparison of the wind-shear stress

![Graph showing wind friction velocity squared derived from data, FHC wind model, and wind model used with Galerkin model.](image-url)
divided by water density (or friction velocity squared) at *Buccaneer* calculated in three ways: 1) using the Jeleanski model and Wu shear-stress relationship, shown as circles in the figure; 2) using actual winds recorded at *Buccaneer* and the Cardone shear-stress relationship, shown as solid circles; and 3) using FHC-modeled winds and the Cardone wind-stress relationship, shown as squares. All wind-stress calculations are referenced to a height of 10 m above still-water level (SWL).

b. Anita wind simulation

Modeling of the winds proceeded in a nearly identical manner to that used for the Delia simulation. The wind model by Jeleanski was used and the input parameters were varied to obtain identical comparisons between observed wind speeds and simulated winds at Port Aransas. Recall that no wind data were available at the site of the current meters.

The center of Anita passed more than 300 km from the site of the meters. To simulate winds at such a great distance from the storm center with the simple Jeleanski model, it was necessary to vary the deflection angle between approximately +30° and −30°. These values are somewhat extreme, and as a result the modeled winds are no doubt increasingly less representative of the actual winds as one moves farther and farther from Port Aransas. A more accurate spatial description of the winds would be desirable and could possibly be obtained using a more sophisticated wind model than the one by Jeleanski, but the additional effort was not felt to be justified in light of the lack of wind data that are available for verification.

3. Circulation model

The model (GAL) which was used to hindcast the current data, takes its name from the Galerkin numerical technique upon which the model is based. Model formulation is founded on the description of the vertical variation of the horizontal velocity by a series expansion similar to that described by Heaps (1972, 1974). A more thorough description of the model is given by Pearce and Cooper (1981), henceforth referred to as PC. Only a brief description will be given in this paper.

a. Governing equations

The model is based on the Navier-Stokes equations which, after some simplifying assumptions, can be written in the form used in the model as

\[
0 = \frac{\partial u}{\partial t} + \frac{\rho_s g \partial \eta}{\rho \partial y} - N_h (\nabla^2 u) - \frac{\partial}{\partial y} \left( N_e \frac{\partial u}{\partial z} \right) \\
- f u + \frac{1}{\rho} \frac{\partial P_a}{\partial y} + \frac{g}{\rho} \int_{-z}^{z} \frac{\partial p}{\partial y} \, dz',
\]

(1a)

where the symbols are defined as follows:

- \( t \) the time variable
- \( x, y \) the horizontal coordinates in a right-handed Cartesian coordinate system
- \( z \) the vertical coordinate, measured as positive downward from SWL
- \( u, v \) the horizontal velocity components in the \( x \) and \( y \) directions, respectively
- \( \rho_s \) the density of the fluid, where the \( s \) subscript indicates the value at the surface
- \( g \) the gravitational constant (9.8 m s\(^{-1}\))
- \( \eta \) the water height of the free surface above datum \( z = 0 \)
- \( N_h \) the horizontal eddy-viscosity coefficient
- \( N_e \) the vertical eddy-viscosity coefficient
- \( f \) the Coriolis parameter \( (\approx 2 \omega \sin \phi) \) where \( \omega \) is the angular velocity of the earth and \( \phi \) the latitude
- \( P_a \) the atmospheric pressure
- \( \nabla^2 \) the Laplacian operator \( \left[ = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right] \).

Note that the vertical velocity \( w \) is assumed negligible, and this simplifies the Navier-Stokes equation in the \( z \)-direction to an expression of the hydrostatic pressure. The density-gradient terms \( [\text{e.g., } (g/\rho) \times \int \frac{\partial p}{\partial x} \, dz'] \) and the tidal components were neglected in the simulations. This was felt justified given the strong wind influence readily evident in the current data. Review of the current data indicates that the tidal component is very small compared to the wind-induced signal (i.e., <10 cm s\(^{-1}\)). The effects of stratification on vertical momentum exchange can be included in the model *via* the vertical variations of \( N_v \). However, this was not done, in part because current data were taken only in shallow water (i.e., 20 m) where any stratification that might occur during normal conditions would quickly decay in the initial phases of the storm. Finally, \( N_h \) was set to zero in part because of the lack of spatial velocity data in the horizontal without which verification of the coefficient would be difficult. In essence, the model as it is applied for Anita and Delia considers only the barotropic mode.

The other governing equation used in the model formulation is the continuity equation

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = \frac{\partial \eta}{\partial t},
\]

(2)

where

- \( U \) the mass flux per unit length in the \( x \)-direction

\( \left( = \int_{-H}^{H} u \, dz \right) \).
$\dot{V}$ the mass flux per unit length in the $y$-direction

$\dot{H}$ the still water depth.

b. Auxiliary conditions

The auxiliary conditions at the surface are

$$\tau_{xx} = \left[ -\rho N_v \frac{\partial u}{\partial z} \right]_{z=0}, \quad \tau_{xy} = \left[ -\rho N_v \frac{\partial v}{\partial z} \right]_{z=0},$$

where $\tau_{xx}$ and $\tau_{xy}$ are the specified shear stresses at the surface in the $x$ and $y$ directions, respectively.

At the bottom we use a linearized friction law

$$\tau_{bx} = [\rho c_b u]_{z=H}, \quad \tau_{by} = [\rho c_b v]_{z=H},$$

where $\tau_{bx}$ and $\tau_{by}$ are the bottom shear stresses and $c_b$ is a drag coefficient.

The remaining auxiliary conditions vary somewhat according to the water body being modeled. The following auxiliary conditions were assumed on the boundaries for the simulations described in this paper:

- The mass fluxes perpendicular to a coastline were set to zero.
- The surface gradient perpendicular to a lateral ocean boundary was set to zero (a lateral boundary is defined as the boundary running from deep water to the shoreline).
- The amplitude at all open-ocean boundaries was set equal to the barometric water rise (i.e., the "inverted barometer effect").

These boundary conditions reflect long waves (Reid, 1975) and could cause unrealistic model results. However, sensitivity studies involving expansion of the grid boundaries indicate that the reflections were not important for the relatively short integration time (i.e., order of 40 h) used in the simulations, nor did the sensitivity studies indicate that the simplified open-ocean boundary condition affected results at the Buccaneer site.

c. Numerical solution technique

It is important to note that the parameters $u$, $v$, $\rho$ and $N_v$ are all functions of $(x, y, z$ and $t)$, and the parameters $N_h$, $\eta$, $c_b$ and $P_a$ are functions of horizontal space and time $(x, y$ and $t)$. Parameters which must be specified are $\rho$, $N_v$, $f$, $P_a$, $c_b$, $\tau_{xx}$, $\tau_{xy}$ and $N_h$, and the unknowns are $u$, $v$ and $\eta$.

The governing equations and boundary conditions [i.e., Eqs. (1)-(4)] are transformed using the Galerkin technique. This manipulation explicitly eliminates $z$ from the transformed equations and greatly simplifies the eventual solution process. The dependence of $u$ and $v$ on $z$ is implicitly retained in the final equations and the $u$ and $v$ velocity profiles can be regained whenever desired.

Application of the Galerkin technique begins by hypothesizing a vertical distribution of the unknown velocities, $u$ and $v$, in terms of a series expansion known as the trial functions. The function used in the model is

$$\hat{u} = \frac{\tau_{xx} z^2 (z - H)}{\rho_s H^3 N_b} + \frac{\tau_{xx}}{\rho_s \alpha} \ln \left( \frac{N_h}{N_v} \right) \sum_{l=1}^{I'} c_l \cos \left( \frac{a_l z}{H} \right),$$

where

- $\hat{u}$, $\hat{v}$ approximate $x$ and $y$ components of the velocity, respectively.
- $N_b$ vertical eddy viscosity at the bottom, $z = H$.
- $\alpha$ slope of $N_v$ in the surface layer.
- $I'$ number of terms used in the cosine series.
- $a_l$ constants given by the expression $a_l \tan(a_l) = c_b H / N_b$.
- $c_l$ the undetermined constants.

A similar function exists for $v$. The relationships for the $y$-direction will not be shown in this paper for the sake of brevity. However, the reader should remember that these equations are included in the model. Note that all parameters in (5) are specified except the undetermined coefficients $c_l$ (for the $y$-direction the undetermined coefficients are $d_l$).

The trial function consists of two constant terms and a cosine series. A good deal of flexibility exists in the choice of the two constant terms—the main limitation being that the constants in combination with the cosine term must satisfy the surface and bottom auxiliary conditions in order for the problem to be properly posed mathematically. However, as indicated by PC, the two terms must be carefully chosen if the numerical scheme is to be economical.

The trial functions are substituted into (1a, b) and, in general, there will be a residual associated with this substitution since the trial functions are not the exact solutions. The residual $\gamma$ is multiplied by a weighting factor $W$ to facilitate computation and the product is minimized by integrating over the water depth and setting the result to zero, or for the $x$-direction,

$$\int_{-\gamma}^{\gamma} \gamma W dz = \int_{-\gamma}^{\gamma} \left( \frac{\partial u}{\partial t} + \frac{\rho_s}{\rho} g \frac{\partial \eta}{\partial z} - N_h \nabla^2 \hat{u} - \hat{f} \hat{v} \right. \left. - \frac{\partial}{\partial z} \left( N_v \frac{\partial u}{\partial z} \right) + \frac{1}{\rho} \frac{\partial P_a}{\partial x} \right. \left. + \frac{g}{\rho} \int_{-\gamma}^{\gamma} \frac{\partial \xi}{\partial z} dz \right) \cos \frac{a_l z}{H} dz = 0.$$  

Again, a similar expression exists for the $y$-direction.

Before the integration in (6) can be performed, it is necessary to specify a distribution for $N_v$. This is accomplished by assuming $N_v$ to vary in a multi-linear fashion as shown in Fig. 4. Performing the integration in (6) yields a set of differential equations in which $z$ has been explicitly eliminated.
\[ \frac{\partial c_i}{\partial t} - N_k \nabla^2 c_i - \int d_i + B_i \frac{\partial \eta}{\partial x} = a_i - \sum_{j=1}^{J+1} c_j E_{ij}, \quad (7) \]

where \( A_i, B_i \) are constants which arise from the integration.

Eq. (7) and its equivalent in the \( y \)-direction represent a set of \( 2J' \) equations with \( 2J' + 1 \) unknowns (i.e., \( c_i, d_i \) and \( \eta \)). To solve for the unknowns one more equation linking \( c_i, d_i \) and \( \eta \) must be used and this is provided by substituting (5) into the continuity equation (2).

The existing version of the model uses a finite-difference scheme to discretize (7), its equivalent in the \( y \)-direction, and the transformed continuity equation. While this discretization scheme has proven satisfactory it is not limiting since other schemes such as finite elements could be used. The key in applying the Galerkin technique is in choosing the basis functions (5). In order for the model scheme to simulate economically the velocity structure, Eq. (5) must be able to converge rapidly to the velocity profile to be modeled. Eq. (5) has proven to be quite adequate in this regard. Usually only three cosine terms have been necessary for the wide variety of flow fields simulated to date. Some of these applications have included wind-induced flow which is often characterized by large velocity gradients near the surface. The computational cost of the model is about the same as for a vertically-averaged model as long as the number of cosine terms is kept within this limit. However, the computational cost of the Galerkin model rapidly increases as the number of terms increases above three, in part because of the dependence of the integration time step on the number of terms.

Heaps (1972, 1974) also used a cosine expansion but the constant terms in his basis function [i.e., the equivalent of the first two terms on the right-hand side of (5)] were different from those used in (5). Cooper and Pearce (1977) showed by means of a Fourier decomposition that the Heaps basis function was extremely slow to converge in the presence of large velocity gradients. They proposed the logarithmic term in (5) with the idea of "absorbing" much of the velocity gradient, leaving a moderately varying residual which could be easily matched by the cosine terms. Subsequent studies have confirmed the large improvement in convergence for a variety of flow fields in unstratified water. Davies and Owen (1979) have constructed a similar linear model and have also expanded the technique to solve the nonlinear form of the governing equations (Davies, 1979). Davies (1977) and Davies and Owen (1979) have also explored the convergence characteristics of a number of basis functions. Their work indicates the cosine expansion does not converge quite so rapidly as a Legendre or Chebyshev expansion. They found that roughly ten terms were needed to achieve reasonable accuracy for the simple, unstratified, wind-induced flow field used for their benchmarks. However, they did not use a constant term in their basis functions and this seems to be one of the major reasons why so many terms were needed. Based on the experience described by Cooper and Pearce (1977), it appears that including an appropriate constant term [such as in (5)] is more critical in insuring fast convergence than is the functional nature of the expansion, at least for the case of unstratified wind-driven flow with a slip wall boundary condition. A more thorough discussion of these computational issues is given by PC and Cooper and Pearce (1977).

4. Hindcasts of currents for Tropical Storm Delia

Preliminary application of the model to the FHC data set is described by Pearce et al. (1979). The comparisons presented in this paper are somewhat better and more thorough than those described previously. The grid size used in the final simulations presented here was 20 elements long by 32 elements wide and was basically the same as the initial grid shown in Fig. 1 except that the resolution was doubled (i.e., element size = 11.7 km).

The current data reported by FHC were obtained from three electromagnetic current meters fixed at 4, 10 and 19 m below SWL. The meters were attached to the oil production platform Buccaneer (see Fig. 1 for location).

FHC point out that there is some uncertainty associated with the directions in the current data. The meters were installed on a cable which extended from the platform superstructure to a weight on the sea floor. A check on the mooring orientation immediately after the storm revealed that the mooring rotated sometime since the previous check several weeks earlier. FHC offer several good reasons to believe that the mooring rotation did not occur during the major portion of the storm, but some uncertainty regarding the current direction must remain. The comparisons
and discussion which follow tacitly assume no rotation of the mooring during the major part of the storm. However, should the reader be skeptical of the current direction, then he may simply focus on the current magnitude which was obviously not affected by the mooring rotation. Assuming that only the magnitude is correct does not alter most of the conclusions drawn at the end of the paper, particularly those concerning the bottom shear stress and bottom-friction coefficient.

The choice of the vertical eddy-viscosity \( N_v \) and bottom-friction coefficient \( c_b \) used in the model is important and yet there is little theoretical foundation upon which to base a choice, especially in the presence of large waves. Traditionally, many numerical modelers have simply treated these coefficients as tuning parameters. The following discussion attempts to synthesize available information in order to make a rational choice of these parameters.

### a. Estimates of \( c_b \)

Estimates for \( c_b \) can be obtained by relating \( c_b \) to a quadratic bottom-friction coefficient such as Manning’s \( n \), i.e.,

\[
    c_b = n^2 u_b g / H^{1/3},
\]

where \( u_b \) is the velocity at the bottom \((z = H)\), and the units are in the MKS system. To estimate \( n \), one must estimate the sea-floor condition near **Buccaneer**. FHC indicates that the bottom is smooth clay as one might expect for the Delta region of the Gulf of Mexico. Hydraulic experiments show a range of values for \( n \) between 0.017 and 0.027 for “straight and uniform earth” (e.g., Daily and Harlean, 1966). Using a value of \( u_b = 1 \text{ m s}^{-1} \) and \( H = 20 \text{ m} \) yields from (8) a range for \( c_b \) between 0.0011 and 0.0027 m s\(^{-1} \) which compares reasonably well to the value of \( c_b = 0.0010 \text{ m s}^{-1} \) that was found to give the best overall comparison between the modeled and observed velocity profiles at **Buccaneer**. FHC also used a value of 0.0010 m s\(^{-1} \) for \( c_b \) in their simulations.

### b. Estimates of \( N_v \)

As stated in Section 3c, the Galerkin model is capable of including a vertical variation in \( N_v \). There is substantial evidence indicating that \( N_v \) will vary in the vertical direction for many flow conditions such as wind-induced flow and in the presence of stratification. PC discuss some of the evidence supporting a vertical variation of \( N_v \) for wind-induced flow. Fig. 5 shows a distribution for \( N_v \) which PC found to simulate wind-induced flows in the laboratory quite well. Csanady and Shaw (1980) have recently suggested a similar distribution. Values for \( N_v \) at the surface in the laminar sublayer are given by Csanady (1978). Immediately below the laminar sublayer, \( N_v \) is assumed to increase linearly, consistent with a “law of the wall” velocity distribution, i.e.,

\[
    N_v = \kappa W_s z,
\]

where \( \kappa \) is von Karman’s constant \((\kappa = 0.4)\). The vertical eddy viscosity is assumed eventually to reach a constant outside the surface boundary layer. An expression suggested by Townsend (1976) is used in this constant region:

\[
    N_v = W_s H / \text{Re},
\]

where \( \text{Re} \) is the flow Reynolds number and \( W_s \) the surface friction velocity, \( W_s = (\tau_{xx} + \tau_{yy})^{1/2} \). PC show that by equating (9) with (10), one can deduce that the transition point at which the linear expression for \( N_v \) changes to a constant is \( \sim 20\% \) of the water depth. Note that (10) applies all the way to the sea floor. The bottom boundary layer is accounted for in the Galerkin model using the bottom slip coefficient \( c_b \).

The distribution given in Fig. 5 served well for the laboratory studies examined by PC. Unfortunately, it is unclear whether the expressions for \( N_v \), particularly those near the surface, are applicable during the occurrence of strong breaking-wave activity so typical of storm conditions. It might at first appear that the Delia data would offer an opportunity to test the \( N_v \) suggested in Fig. 5 for storm conditions. But this is not possible since the upper meter was at 4 m below SWL (or 20% of the water depth) meaning that no data were taken within the logarithmic layer which the theory suggests is restricted to the upper 4 m of the water column.

Because of the uncertainty of applying the \( N_v \) distribution described above and because of the lack of data needed to verify any assumed distribution, it was decided simply to take \( N_v \) to be constant throughout
the water column. Sensitivity studies indicate that this assumption mainly affects predictions in the upper 4 m and not below.

If \( N_e \) is assumed to remain constant in the vertical, the problem of selecting \( N_e \) is simplified but not eliminated. Neumann and Pierson (1966) report a number of studies estimating \( N_e \). All these expressions relate \( N_e \) either to wind speed or to wind speed and latitude, and assume \( N_e \) to be constant throughout the water column. Studies by Schmidt (1908), Thorade (1914) and Neumann (1952) appear to be most appropriate for the Delia case and agree reasonably well—indicating \( N_e = 0.2 \text{ m}^2\text{s}^{-1} \) for wind speeds of 24 m s\(^{-1}\) (peak winds during Delia). However, this predicted value compares poorly to some of the other values reported by Neumann and Pierson (1966), and is nearly an order of magnitude larger than the value used by FHC to yield the best results in their modeling simulations.

Given the similarity between the fundamental governing equations used in the model and the FHC model, a value of \( N_e = 0.03 \text{ m}^2\text{s}^{-1} \) was first used in the model along with \( c_b = 0.001 \text{ m s}^{-1} \) as suggested in the previous section. The model gave currents of the right order of magnitude in the vicinity of Buccaneer, but the currents oscillated significantly, and currents in the deeper grid elements were in excess of 7 m s\(^{-1}\), a value considered unrealistically high. The reason for this is apparently linked to the fact that depths in the model grid ranged from 7 to almost 200 m. For depth variations of this order, a constant \( N_e \) is physically unrealistic since, in shallower waters, the vertical eddy size and hence \( N_e \) is restricted by the water depth. Thus there is good reason to believe that \( N_e \) should be a function of at least the water depth.

Eq. (10) suggested by Townsend (1976) takes into account the effect of water depth and was applied in subsequent modeling. It is interesting to note that the friction velocity \( W_z \) in (10) is proportional to the wind speed and can be used to vary \( N_e \) in both time and space, i.e., \( N_e(x, y, t) \). However, it was discovered that in both simulations, variation of \( N_e \) with time did not substantially improve the results obtained by simply taking \( W_z \) to be constant (i.e., \( N_e \) is a function of \( x, y \) only). The values of \( W_z \) used were those corresponding to the time of peak \( W_z \) at the site.

c. Modeling results

Fig. 6 shows a comparison of the FHC data (solid curve) to various simulations. The figure shows temporal variations of the currents at 4, 10 and 19 m below SWL at Buccaneer. Also shown in the lower portion of the figure are the winds at Buccaneer. All data are broken into cross-shelf and alongshore components. A positive alongshore component would be pointed in roughly a northeast direction in Fig. 1. Onshore is indicated as a positive cross-shelf component. The data were originally plotted by FHC in north–south and east–west components. An adjustment of about 30° is needed to convert to cross-shelf and alongshore components and this was done in Fig. 6 since it facilitated physical interpretation of the data.

As one can see from Fig. 6, the FHC model (dotted curve) simulates the alongshore component of the velocities nicely, although the comparison deteriorates after 1600 hours. The cross-shelf component is not simulated as well by the FHC model, there being a considerable discrepancy beginning at about 1400 hours.

Case I (dashed curve) in Fig. 6 indicates the results from the Galerkin model using \( c_b = 0.001 \text{ m s}^{-1} \). The value for \( Re = 13 \) suggested by Townsend (1976) for open channel flow was used in (10). The water depth \( H \) used in (10) was the local grid element water depth \( H \), which varied in the model grid between 0.56 m\(^2\) s\(^{-1}\) at the deepest element \( (H = 183 \text{ m}) \) to 0.022 m\(^2\) s\(^{-1}\) at the shallowest element \( (H = 7 \text{ m}) \).

As can be observed from the figure, Case I compares quite well to the data, being within 0.20 m s\(^{-1}\) in magnitude, although the peak currents in the simulation precede the observed peak by about 1–3 h. Discrepancies of the order observed lie well within the range attributable to various uncertainties in the modeling process such as the wind-field simulations (see lower part of Fig. 6 for comparison of winds) or unknown tidal and baroclinic velocity components. The comparison is particularly good in light of the fact that a general method of deriving \( N_e \) and \( c_b \) was used.

Fig. 6 shows the simulations using the Galerkin model to be somewhat better than those achieved by FHC, particularly for the cross-shelf component. This outcome is somewhat puzzling if one considers the fact that the Galerkin model as implemented in the Delia simulations uses the same fundamental equations as the FHC model. Discussions with Forristall (personal communication, 1980–81) revealed the following differences: (1) numerical schemes, (2) spatial variation of \( N_e \), (3) lateral boundary conditions, and (4) wind field models. The Galerkin model uses the weighted residual scheme in the vertical, quite a different approach from that used by FHC. However, to prove that the technique is numerically superior to the extent needed to explain the discrepancy in the results would be difficult, and certainly not worth examining until all the other more simple explanations are explored.

In reference to item 2, recall that the Galerkin model used a distribution for \( N_e \) relating it to the local depth. FHC on the other hand simply assumed \( N_e \) to be spatially constant. To investigate this possible explanation a bit further, more simulations were made with the Galerkin model. It was not possible
to use a constant $N_v$ throughout the grid because of the oscillations mentioned earlier. However, $N_v$ was set equal to two values, i.e.,

$$N_v = \begin{cases} 
0.07 \text{ m}^2 \text{s}^{-1}, & H < 50 \text{ m} \\
0.28 \text{ m}^2 \text{s}^{-1}, & H \geq 50 \text{ m}.
\end{cases}$$

**Fig. 6.** Comparison of currents and winds for Delia derived from: data, FHC and Galerkin model (Case 1).
The value \( N_0 = 0.07 \) corresponds to the value used at \textit{Buccaneer} for Case I. Currents from the second run compared to within a few percent of Case I, strongly implying that the spatial variation of \( N_0 \) was not the cause of the discrepancy between Case I and FHCS's results.

Item 3 was also investigated. FHCS specified the velocity gradient normal to the lateral boundary as zero. However, for the Case I simulations described above, the surface-slope gradient was specified as zero. Both boundary conditions are similar in that they do not radiate longer-period oscillations. Case I was rerun with the same lateral boundary conditions as those of FHCS. Currents from this run compared to within a few percent of Case I in the \textit{Buccaneer} area, indicating that the boundary conditions were not the reason for the difference.

Item 4 might at first seem unlikely in light of the close comparison between the wind simulations and data as evidenced in Fig. 3. However, reconfiguring the information shown in Fig. 3 into cross-shelf/alongshore components (see lower portion of Fig. 6) reveals that the winds used in the Galerkin model had a considerably stronger onshore component than either the data or FHCS wind simulations beginning at 1300 hours. Since the Galerkin model used winds with a stronger onshore component than the FHCS model one might reasonably expect the simulated currents from the Galerkin model to have a stronger onshore component than the FHCS current simulations starting at sometime after 1300 hours. A quick look at Fig. 6 seems to confirm this expectation.

To investigate further whether discrepancies in the wind field on the order observed could affect the simulated velocities, another wind simulation was made using the Jelesianski wind model but this time it was tuned so as to yield nearly exact winds at \textit{Buccaneer}. This required the use of somewhat large values for the deflection angle and less strict adherence to using measured central pressure. It is expected that the wind simulations from this later run were less accurate than from the previous run when one moved away from \textit{Buccaneer}, but this was probably unavoidable given the inherent simplicity of the wind model. The Galerkin circulation model was rerun using the modified wind field and it produced the currents shown as Case II in Fig. 7. Note the decrease from Case I of the onshore currents beginning at about 1500 hours. The Case II cross-shelf currents follow the FHCS simulations much more closely. However, discrepancies between the alongshore currents increase. This latter result tends to cloud the issue a bit, but it is apparent that currents in the area are highly sensitive to the local wind field, and that differences between the wind fields used in the Galerkin model and the FHCS model are probably the main reason for the differences in the simulated currents.

5. Hindcasts of currents for Hurricane Anita

The current data described by Smith (1978) were recorded by two ENDECOT-type-105 recording current meters located at 2 and 10 m above the bottom in approximately 17 m of water. Wind velocity was recorded 120 km away from the site at Port Aransas. Winds were modeled as described in Section 2 and the wind data were put into the Galerkin model. Preliminary application of the model to Anita was described by Cooper et al. (1981).

a. Estimates of \( N_0 \) and \( c_b \)

As in the case of Tropical Storm Delia, Eq. (10) was used to calculate \( N_0 \) with a value of \( Re = 13 \). Eq. (8) was used to estimate \( c_b \). Smith (personal communication, 1980) reports that diver inspection of the site showed the bottom to be flat, soft and composed of muddy clay-sand much like that reported by FHCS at \textit{Buccaneer}. Therefore the same value of \( n \) used for Delia was used in (10) for Anita. Taking a mean value of \( u_b = 0.5 \) m s\(^{-1}\) gives a range for \( c_b \) between 0.0005 and 0.0014 m s\(^{-1}\). This compares to a value of \( c_b = 0.00025 \) m s\(^{-1}\) which gave the best overall comparisons.

As in the case of Delia, (8) suggests a value for \( c_b \) of the right order of magnitude but a value which is a bit on the high side. Of more concern is the fact that the best fit \( c_b \) from the Anata simulation is about four times smaller than for the Delia simulations (i.e., \( c_b = 0.00025 \) m s\(^{-1}\) for Anita vs \( c_b = 0.001 \) m s\(^{-1}\) for Delia). This difference has significant implications regarding the use of the model for predictive purposes, reasons for which are discussed more thoroughly in Section 6b.

b. Modeling results

Fig. 8 shows a comparison of the Smith data (solid curve) to model hindcasts. The figure includes temporal variations of the currents at 7 and 15 m below SWL, and the simulated winds at the site. All data are broken into cross-shelf and alongshore components. Positive alongshore and cross-shelf components are pointed in approximately northeast and northwest directions, respectively, in Fig. 2. Measured winds at Port Aransas are shown in the lower portion of the figure.

As in the case of Delia the currents in Fig. 8 display a strong alongshore component in the direction of the wind. The cross-shelf component is much smaller and primarily onshore. Unlike Delia, Anita did not create a rapid and distinct peak. This characteristic caused a minor problem in the simulations, because in order to minimize computer expenditures it was necessary to limit Fig. 8 to the 36 h period in which the peak currents occurred. Unfortunately, actual
winds at 2100 hours on 31 August were still substantial so if the circulation model were started from rest at 2100 hours the figure would show a substantial spin-up period. To avoid this, the simulations actually started 12 h earlier at 0900 hours to allow for the necessary model spin-up.

Case I (dashed curve) in Fig. 8 shows the results from the Galarkin model using \( c_b = 0.00025 \) m s\(^{-1}\). The value for \( N_v \) was calculated utilizing (10) with a constant \( \beta \), based on the maximum simulated wind speed at the site, and a value of \( Re = 13 \) was used in (10) as suggested by Townsend (1976) and the previous simulations of Delia. The water depth \( H \) used in (10) was the local grid element depth, so \( N_v \) varied in the model grid between 0.26 m\(^2\) s\(^{-1}\) at the deepest element (\( H = 200 \) m) to 0.009 m\(^2\) s\(^{-1}\) at the shallowest element (\( H = 10 \) m). As one can see from the figure, Case I compares quite well to the data during all but the last few hours of the 36 h period. It should be noted that the data as well as the simulations show rapid 2–4 h oscillations on the order of 0.20 m s\(^{-1}\) and these were removed by averaging over three consecutive 1 h intervals.

6. Discussion of hindcast results

Both data sets have some significant similarities, including:

1) Water depths and distance to the coastline were of the same order of magnitude (i.e., 20 m and 40 km, respectively).
2) The two sites were located on the Texas Shelf approximately 150 km apart.
3) The bottom at each site consisted of smooth muddy sand without sand waves.
4) Winds were predominantly in an alongshore direction during both storms.

There were also some differences, the major one being that the maximum wind stress associated with Delia was a factor of 5 larger than the maximum wind stress associated with Anita. These similarities and differ-
ences allow for some interesting observations, some of which are included below.

a. Sensitivity of currents to changes in $N_o$ and $c_b$

In part because of the uncertainty in selecting the empirical parameters $N_o$ and $c_b$, it is useful to investigate the influence of these parameters on the velocity hindcasts. Fig. 9 shows hodographs for Delia at two representative sites—Buccaneer, and a relatively deep site at grid element (13,2) in Fig. 1 (depth of 180 m). The depths of the points at which currents are plotted are indicated beside the nodes of each curve. Currents were taken at 1700 hours, about the time currents peaked at Buccaneer. Fig. 10 shows similar hodographs for Anita at 1100 hours on 1 September, about the time of peak currents at the current-meter site. Currents at a representative deep-water site, grid element (14,5) in Fig. 2, are shown on the left side of Fig. 10, and currents at the meter site are shown on the right side of the figure. Each hodograph shows three cases. Table I summarizes these cases as well as all simulations reported in this paper.

Examination of Case V in the two figures gives some insight into the sensitivity of currents to $c_b$. Case V is the same as Case I reported in earlier sections except that $c_b$ for Case V is double that of Case I. Increasing $c_b$ is found to have a substantial effect on the velocity in shallow water as indicated in the figures which show velocities are approximately inversely proportional to $c_b$ at the two sites where data were taken (depth on the order of 20 m). At the deeper sites, the effect of $c_b$ is not so large. For Anita, doubling of $c_b$ does not result in a significant change in velocities at the deep-water site. For Delia, a change in $c_b$ only changes the velocity by about 10–20 cm

FIG. 8. Comparison of currents for Anita derived from: data, Case I and Case II.
s$^{-1}$, although the percentage change is somewhat more substantial.

The effects of $c_b$ observed in the simulations are consistent with what one would expect based on a review of the Ekman depth of frictional influence at both sites. The depth of frictional influence $D$ can be written as

$$D = \pi (2N_e f)^{1/2}.$$ Substituting the values for $N_e$ used in Case I yields values for $D$ of 90 m for (14,11) Delia, 275 m for (13,2) Delia, 60 m for (21,19) Anita, and 200 m for (21,19) Anita. In the case of the two shallow-water sites (i.e., elements 14,11 for Delia and 21,19 for
Anita) the local water depth is considerably smaller than the depth of frictional influence and hence one would expect local currents to be sensitive to changes in $c_b$ as observed in Figs. 9 and 10. Similar reasoning can be used to explain the insensitivity to $c_b$ observed at the two deeper stations.

Insight into the sensitivity of currents to changes in $N_e$ can be gained by comparing Case VI with Case I. Parameters used in Case VI were the same as Case I except that $N_e$ for VI was one-half that of I. Review of Figs. 9 and 10 shows that the vertical shear stress is highly sensitive to changes in $N_e$, as indicated by the comparatively large separation between nodal points in the hodographs for Case VI. Velocities are also sensitive to $N_e$, somewhat more so for Anita than for Delia. The main reason for the particular sensitivity of the Anita currents to $N_e$ is probably the damping effect of $N_e$ on the amplitude of the velocity oscillations described earlier in Section 5. These oscillations grow in magnitude as $N_e$ is decreased, and hence velocities at any given point in time could deviate significantly from Case I to Case VI, depending at which point of the oscillation the recorded current occurs.

In summary, velocities at the two sites where data were taken are dependent on both $N_e$ and $c_b$. For the latter parameter, velocities are approximately inversely proportional to $c_b$. Locations with water depths on the order of 100 m are not very sensitive to changes in $c_b$ but are sensitive to changes in $N_e$. The shear developed in the water column is particularly sensitive to $N_e$.

### b. Influence of surface gradient on bottom shear stress

Both Smith and FHC calculated bottom shear stresses from the observed velocity profiles and they consistently report bottom stresses considerably in excess of the surface-wind shear stress. For instance, the Delia data set indicates a maximum bottom shear stress of 200 dyn cm$^{-2}$ during the course of the storm, a value which is about a factor of 12 greater than the maximum surface shear stress of 16 dyn cm$^{-2}$. Similarly, Smith (1978) reports a maximum bottom stress of 100 dyn cm$^{-2}$, a factor of 30 greater than the maximum surface shear stress. The large bottom friction reported by Smith and FHC could be due to two causes: (1) the method of calculating the bottom shear stress and (2) the presence of an additional forcing mechanism other than the local wind stress.

Both Smith and FHC calculated the bottom shear stress by fitting a logarithmic profile to the current measurements. In order for this technique to be strictly valid, the current measurements have to be in the bottom logarithmic boundary layer. Recall that the near-bottom meters were located at roughly 1 and 10 m above the bottom for the FHC study and 2 and 10 m above for the Smith (1978) study. The log layer is probably dominated by oscillations due to surface-wave activity, and FHC estimated that it extended less than a meter above the bottom for the Delia case [a similar value is appropriate for the Smith (1978) data]. Thus none of the meters would appear to be within the log layer.

However, not all is lost. Intuitively one would expect some relationship between the lower meters and the bottom shear. Using the log law in effect assumes the bottom shear is proportional to the square of the difference in the observed velocity data at the two lower levels, not an unreasonable estimate.

The error involved in applying the log relationship can be estimated by comparing the shear calculated using a log layer to the actual shear in an elementary current system. We proceed as follows: consider a situation where the flow can be modeled using the simple Ekman drift and slope components in a finite-depth sea [i.e., Eqs. (8.51) and (8.59) of Neumann and Pierson (1966, pp. 210–211)]. Note that these equations give a reasonable approximation of the vertical shear during Delia and Anita. Now let us take parameters consistent with the physical situation during Delia: $W_g = 0.04$ m s$^{-1}$, $N_e = 0.062$ m$^2$ s$^{-1}$, $H = 20$ m, and an alongshore surface slope of $1 \times 10^{-3}$. Substituting these parameters into the Ekman equations and combining the drift and slope components one calculates a bottom-friction velocity of 0.05 m s$^{-1}$ using 8.51 and 8.59—a value identical to that calculated using the Ekman velocity at 10 and 1 m above the bottom in conjunction with the log law. It should be noted that the alongshore surface slope was estimated from Case I of the model results. If this is doubled for the sake of argument and all other inputs are kept constant, then the analytical bottom friction velocity is 0.07 m s$^{-1}$ which is slightly lower than the 0.08 m s$^{-1}$ estimated from the log law.

Other observational evidence tends to support the use of the log law to estimate the bottom shear stress for the two data sets. For instance, both data sets

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<th>$c_b$ (m s$^{-1}$)</th>
<th>Re</th>
<th>$W_g$ (cm s$^{-1}$)</th>
<th>$\partial n/\partial x$, $\partial n/\partial y$</th>
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* Not calculated because of lack of relevance to discussion.
indicate the bottom shear stress is roughly equal to the surface shear stress before and after the storms. A review of currents implies that non-wind forcing is not very important and hence equality of the surface-wind shear stress and bottom shear stress before and after the storm is reasonable. Another bit of qualitative support comes from a review of the temporal variation of the calculated bottom shear during Delia. The time series shows a clear increase as the storm peaks, followed by a rapid drop as the storm subsides—an intuitively reasonable result. In summary, the use of the log law to estimate the bottom shear is not exact but it does not appear to be inaccurate to the degree needed to explain the large bottom shear calculated from the data.

The second and most likely reason for the large bottom shear stress reported by Smith (1978) and FHC is the presence of an additional forcing mechanism other than the local wind. Csanady (personal communication, 1979) and Gordon (1982) have suggested that the additional mechanism is the surface gradient. Smith suggested the possibility but concluded that the data were inadequate to explain the large observed shear. Cooper and Pearce (1980) explored the likelihood of this explanation for the Delia data set and concluded that the surface gradient was not the mechanism causing the observed difference between bottom and surface shear stress. The study of Cooper and Pearce (henceforth referred to as CP) is summarized below. The techniques applied by CP for the Delia data set are likewise applied to the Anita data set and the results also presented below.

As noted by CP, the importance of the surface gradient component can be investigated using the Galerkin model with the surface gradient terms removed. Terms yielding a surface gradient are $\delta \eta / \delta x$, $\delta \eta / \delta y$, $H_0$ (inverted barometer effect), atmospheric pressure gradients and bathymetric variations. To eliminate the surface gradient, one can eliminate the above terms from the model, though the exact method in which this should be done is debatable. For example, to eliminate the effect of bathymetric variations, one might assume a constant depth throughout the grid. However, since the water depths in the grids for both storms vary by an order of magnitude, this method obviously distorts other forcing mechanisms such as frictional effects.

Because it is not evident which is the best method of eliminating the surface gradient component of velocities, several approaches were tried. For the results reported here $\partial \eta / \partial x$ and $\partial \eta / \partial y$ were set to zero throughout the grid. Variation of the water depth was retained, including the no-flow boundary condition at the shoreline. Other methods were tried, such as eliminating the no-flow boundary at the coastline by setting all water depths in the grid to the greater of 20 m or the actual depth taken from charts. Thus the grid was essentially an unbounded sea of finite and variable depth. Surface gradients were generated but these were much less than for the case with a coastline. Flows for the no-coast case were essentially identical to those calculated by setting $\partial \eta / \partial x = \partial \eta / \partial y = 0$ with a coastline.

### 1) TROPICAL STORM DELIA

Recall that the simulated velocities shown as Case I in Fig. 6 compare well to the velocity data, but it is also of interest to compare the bottom shear stresses. Estimates of the bottom friction velocity $W_{fb}$ were made by FHC by fitting the observed velocity profile to the well-known logarithmic bottom boundary layer [note that $W_{fb} = r_{fa} + r_{fb}$]. Fig. 11 gives the temporal variation of $W_{fb}$ based on data from the two current meters nearest the bottom (CM = 2) and three current meters (CM = 3). The simulated $W_{fb}$ for Case I was calculated in the same manner and is shown in the figure. Also shown is the surface friction velocity $W_s$.

A comparison between the observed and simulated (Case I) $W_{fb}$ indicates a considerable discrepancy. It can be argued that $W_{fb}$ calculated from Case I seems reasonable since it closely follows the observed $W_{fb}$. Equality of the surface and bottom shear stresses would strongly indicate a flow pattern analogous to 1-D flow in a channel, and, in fact, the observed currents can be nicely modeled with a simple 1-D model during much of the storm.

However, the disparity between observed and simulated $W_{fb}$'s could also be explained by an improper choice of $N_c$ and $c_b$ used in Case I. Therefore, further simulations were made until a combination of $c_b$ and $N_c$ was found that yielded a reasonable fit of both velocity and bottom shear stress. This simulation is labeled as Case III in Figs. 7 and 11. Table 1 summarizes the differences between the various cases.
Case III represents a reasonable simulation of the data and at this point is possible to explore the importance of the pressure-gradient terms. Case IV is the same as Case III except that the surface gradients were set to zero throughout the grid. Variation of the water depth was retained as was the no-flow boundary condition along the shoreline.

By comparing the respective curves for Cases III and IV in Fig. 11, one can see that the bottom friction velocity is decreased by excluding the pressure gradient but \( W_{sh} \) for Case IV still consistently remains well above \( W_s \). These results would strongly imply that another forcing mechanism is the major cause for the difference in \( W_s \) and \( W_{sh} \). In lieu of other likely alternatives, it would appear that the difference is due to wave activity. Further support for this argument is given in Section 6c and from the Anita simulations described immediately below.

2) Hurricane Anita

The techniques applied to Delia were also applied to the Anita data set. Before any comparisons could be made it was necessary to calculate \( W_{sh} \) from the data. In his paper, Smith (1978) mentioned that he had calculated \( W_{sh} \) using the same method as FHC. Smith further mentions that his calculations show: "The friction velocity increases from values of less than 2 cm s\(^{-1}\) before Anita forms to values that are quite variable but generally between 8 and 10 cm s\(^{-1}\) after the storm had attained hurricane strength." Since the maximum surface friction velocity at the site was on the order of 2 cm s\(^{-1}\) this means that the bottom shear was always greater than the surface value even before Anita formed. Intuitively one would expect the flow at the site to be predominately driven by local wind, given the lack of other important forcing mechanisms such as tides. Smith’s (1978) reported values also imply that the maximum ratio of the bottom friction velocity to the surface value is about 5 yet under more severe wave conditions FHC observed a maximum ratio of 3.

The reason for the high ratios reported by Smith can be traced to an error in his calculation of \( W_{sh} \). Smith (personal communication, 1980) determined \( W_{sh} \) graphically by fitting the two current-meter readings to a logarithmic velocity profile. However, Smith failed to multiply his \( W_{sh} \) calculations by von Kármán’s constant (0.4) and thus his reported values should be reduced by a factor of 2.5. Thus \( W_{sh}/W_s \) at 2.5 the peak of the storm, a value which is slightly less than for Delia. It also means that \( W_{sh} \) follows \( W_s \) much more closely before the storm arrives at the site.

Fig. 12 shows the correct values of \( W_{sh} \) derived from the data. A 3 h average has been used to remove much of the noise in the data.

As in the case of Delia, it was found that the Case I simulation of the Anita currents did not adequately simulate \( W_{sh} \). Therefore \( N_x \) and \( c_b \) were varied until a combination was found that gave a reasonable match of both \( W_{sh} \) and the velocity at the current-meter site. This simulation is labeled as Case III in Table 1 and Figs. 8 and 12. Case IV shown in Fig. 12 is simply Case III but without the surface gradients. The figure clearly shows that removal of the pressure gradients does reduce \( W_{sh} \) particularly at about 0800 hours on 1 September when a strong shift in the cross-shelf wind occurred. But during most of the storm \( W_s \) for Case IV remains above \( W_{sh} \), implying the presence of another forcing mechanism. As with Delia, it would appear likely that wave activity is this other forcing mechanism.

![Fig. 12. Comparison of bottom friction velocity for Anita derived from: data, Case I, Case III and Case IV.](image-url)
c. Variability of bottom friction coefficient

The bottom friction coefficients $c_b$ found to yield the best current hindcasts for Anita and Delia differed by a factor of 4. Recall that Section 6a showed the velocity magnitude to be inversely proportional to the bottom friction coefficient, for much of the modeled area. Hence if the model is to be used in a predictive mode it is essential to gain some explanation of the difference in $c_b$ observed between Anita and Delia. To illustrate this point a bit further, consider the case where only hindcasts of Delia had been performed. Recall that the hindcasts implied $c_b = 0.001$ m s$^{-1}$. Now assume that one wished to forecast currents for the Anita data set. Based on similarities in bottom structure it would be reasonable to assume $c_b = 0.001$ m s$^{-1}$ for Anita, as well. However, if this were done, calculated currents for Anita would be approximately one-fourth those reported at the current-meter site by Smith (1978). Therefore errors of a factor of four would be likely if the model were used in a predictive mode for Anita. A similar error could be expected for all circulation models using a slip bottom-friction law.

Three possible explanations for the difference in $c_b$ for the two simulations are as follows: (i) the bottom roughness at the Anita site was smaller than at the Delia site; (ii) the bottom boundary layer might best be modeled by a quadratic friction law rather than the linear law used in the model; and (iii) smaller surface waves were present for Anita than for Delia. The first explanation seems unlikely. Both Smith and FHC indicate that inspection of the respective sites showed the bottom to be flat, soft, and composed of muddy clay-sand with no major sand ridges.

Regarding the second possibility Winant and Beardsley (1979) noted that it is not clear from data which bottom friction law is more appropriate—a linear or a quadratic law. If it is assumed that a quadratic law is more appropriate for Anita and Delia, then it would not be surprising to see a discrepancy between the linear bottom-friction coefficients used in the two simulations.

For the sake of argument, assume the quadratic law is correct. The linear coefficient $c_b$ can be written in terms of a quadratic coefficient $f$ as $c_b = f u_b$, or more conveniently in terms of a ratio
\[ r = c_b^D/c_b^A = f^P u_b^P/(f^A u_b^A), \tag{11} \]
where the superscripts A and D indicate values from Anita and Delia, respectively.

If a quadratic law is valid, then $f$ should not differ substantially between the Delia and Anita sites. Accordingly, $f^A$ and $f^D$ cancel in (11) leaving
\[ r = u_b^P/u_b^A. \]
The ratio $r$ can be estimated by taking $u_b$ from the bottom current-meter data for each storm. It is perhaps most appropriate to take time-averaged values of $u_b$ but then the issue of selecting an averaging internal must be addressed. To avoid this dilemma, $r$ is estimated by simply taking the maximum $u_b$ which occurred in each storm record. A review of the data suggests $u_b^P = 1.9$ m s$^{-1}$ and $u_b^A = 0.8$ m s$^{-1}$, giving $r = 2.4$. This ratio is only approximately one-half the ratio observed from the simulations (i.e., $r = c_b^D/c_b^A = 0.001/0.00025 = 4$).

In conclusion, even if a quadratic law is correct it can only explain about a factor of 2 difference between $c_b^D$ and $c_b^A$. A factor of 2 still remains unexplained.

The third possible reason for the difference between $c_b^D$ and $c_b^A$ is suggested by examining the characteristic surface-wave activity during the two storms. FHC report that waves during the peak winds at Bucaneer reached 7–8 m in height. Smith (personal communication, 1980) estimates waves at the current-meter site during Anita reached 3–4 m. Thus there appears to be a significant difference between wave activity during the two storms suggesting, at least qualitatively, a reason for the difference between $c_b$.

Grant and Madsen (1980, henceforth referred to as GM) have suggested a simple boundary-layer model which calculates the effective bottom shear stress due to the combined effect of waves and steady current. In order to apply the model, it is necessary to estimate the wave height, wave period and the steady current. For Delia, these parameters are adequately estimated from the data provided by FHC. However, for Anita, wave data were not taken and only the qualitative estimates of Smith (1978) were available. In order to refine those estimates a bit further, the parametric wave model by Ross (1977) was used. The model is easily applied, requiring only specification of wind speed and radius to maximum winds, yet the model has been shown to yield excellent results in several hindcasts. Fig. 13 shows the wave hindcast results for Anita. The figure indicates the temporal variation of the significant wave height and period at the current-meter site. During the 48 h of interest, the significant wave height varies from 2.5 to 4 m, corresponding reasonably well to Smith's (1978) estimates.

To use the GM model, one must also estimate the magnitude of the steady current $W_b$, where $W_b = (u_b^2 + v_b^2)^{1/2}$. The direction of $W_b$ with respect to the wave motion is considered in the GM model but for this application it is assumed that the directions of the two currents coincide. The GM model requires specification of one further parameter, $k_b$, the Nikuradse equivalent sand-grain roughness. Based on conversations with Forristall, GM estimated that $k_b = 2$ cm near the Bucaneer platform. This value for $k_b$ was used in the calculations described here, both for Delia and Anita.

All the parameters used in the GM model vary temporally, so a decision must be made as to when to choose the parameters. Ideally one would simply
show the time variation of the calculated bottom shear stress from the GM model but this would require extensive calculations. Therefore, values were taken during the peak of each storm. For the case of Delia the peak occurred around 1500–1700 hours on 4 September. In the case of Anita, the peak occurred over a broader period, roughly 0800–1800 hours on 1 September. The values used for the significant wave height and period (H_s and T_m) and the steady bottom current (W_b) are given in Table 2 for each storm.

Based on the numbers shown in Table 2, the GM model yields values for the bottom friction velocity W_b of 13.5 cm s⁻¹ and 4 cm s⁻¹ for Delia and Anita, respectively. The value for Delia corresponds well to the values of 11–13.5 cm s⁻¹ calculated by FFHC from the current data during 1500–1700 hours.

Estimating a value for W_b from the Anita data is a bit more difficult. Recall that Fig. 12 showed the bottom friction velocity based on the current data to vary substantially from hour to hour. Nevertheless a reasonable average over the period from 0800–1800 hours would be about 3 cm s⁻¹, which compares well to the value of 4 cm s⁻¹ calculated from the GM model.

In summary, the bottom-boundary-layer model of GM has been used to estimate the “effective” bottom shear stress that occurred during the period of peak currents for the two storms. By effective it is meant that the shear stress calculations reflect the increase in stress due to the interaction of waves and a steady current. The effective value is considerably higher than the value that one might expect from only a steady current. The estimates of W_b calculated from the GM model are reasonably corroborated by the data.

Now that a means of estimating W_b has been found it is possible to back out values of c_b and c_b^P. Recall that c_b is related to W_b via (4). Rewriting (4) and solving for c_b yields

$$c_b = W_b^2 / W_b.$$

Taking this a bit further, one can calculate the ratio of c_b^P to c_b, i.e.,

$$r = c_b^P / c_b^P = (W_b^P / W_b^P)^2 W_b^P / W_b^P.$$

Substituting the values for W_b^P, W_b^P, W_b^P, W_b^P of 0.135, 0.040, 0.8 and 1.9, respectively, gives r = 4.8. Recall that the simulations indicated r = 4 which compares well to the ratio predicted by the GM model.

Based on the good correlation between the ratio from (13) and the ratio calculated from the simulations, it would appear that the difference between c_b^P and c_b^P is primarily due to the difference in wave activity at the two sites. Examination of additional data sets is needed to make a firm conclusion.

Eq. (13) together with the GM model and a simple wave-forecasting model offer some interesting options to those modeling storm-induced currents in shallow

<table>
<thead>
<tr>
<th>Storm</th>
<th>Time (CDT)</th>
<th>H_s (m)</th>
<th>T_s (s)</th>
<th>k_b (cm)</th>
<th>W_b (m s⁻¹)</th>
<th>W_b (cm s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Delia</td>
<td>1700</td>
<td>7.5</td>
<td>10</td>
<td>2.0</td>
<td>1.9</td>
<td>14.0</td>
</tr>
<tr>
<td></td>
<td>4 Sep.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Anita</td>
<td>0800–1800</td>
<td>3.0</td>
<td>9</td>
<td>2.0</td>
<td>0.8</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>1 Sep.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
water. For instance, suppose a hindcast has been made in a certain area and it is desired to make a forecast in the same area or a similar area. Eq. (13) in conjunction with the GM model and wave-prediction model can be used to estimate \( c_b \) for the forecast of a different storm. In this way, the \( c_b \) used in the forecast will reflect the differences in wave climate between the two situations. Another possibility would be to use (12) and the GM and wave models periodically to update the \( c_b \) used in the circulation modeling. Based on the findings above, these techniques would appear to improve current forecasting capabilities. The improvements could be large, particularly in the case where a relatively small storm is hindcasted to tune a current model for forecasting of more extreme "design" storms.

7. Summary and conclusions

The data from both Delia and Anita indicate that the bottom shear stress is larger than the surface shear stress. The large reported bottom shear could be due to 1) inappropriate use of the log law to calculate the bottom shear or 2) the presence of other forcing besides the local wind.

Use of the log law to calculate the bottom shear is not entirely appropriate because at least some data were probably taken outside the log layer. The error involved in using the log law is difficult to estimate and little substantial work seems to have been done in previous studies. However, a crude estimate of the error involved was made by comparing the bottom shear for the elementary current system by Ekman to the bottom shear calculated by using the log law. Parameters used in the Ekman equations were typical of the Delia and Anita data sets, and the comparisons between the two approaches indicated that the log-law method gave a reasonable estimate of the actual bottom shear stress for the elementary current system. In addition, the bottom shear stress calculated from the data appears to be reasonable in terms of its temporal behavior during the storm and in terms of balance-of-force considerations before and after the storm.

It would therefore appear that the large shear indicated from the data sets is due to the presence of other forcing mechanisms besides the local wind. Two likely candidates are an alongshore pressure gradient and wind waves.

The importance of the pressure gradient in determining the bottom shear in the data reported by FHC and Smith (1978) is investigated by applying a linear Galerkin circulation model which considers the barotropic mode only. The model is first applied in an effort to simulate the observed currents. All pressure-gradient terms are included. Good comparisons between the model and measured velocities are achieved using generalized expressions for the two empirical coefficients used in the model (i.e., \( N \) and \( c_b \)). These simulations (Case I) yielded a bottom shear stress that was roughly equal to the observed surface shear stress and was considerably less than the bottom shear indicated by the data. This implies that an additional forcing mechanism influenced the data and that this mechanism was not included in the model. That additional mechanism is probably waves since the model includes all other major forcing mechanisms. It could be argued, however, that the smaller \( W_{cb} \) in the simulations was due to an improper combination of \( N \) and \( c_b \) in the modeling. Therefore, a second combination of \( N \) and \( c_b \) was found, which produced a reasonable match of both currents and bottom shear stress.

The pressure terms were removed from the model and the model was then rerun and compared to the otherwise identical earlier run. These runs indicate that the pressure gradient increases the bottom shear stress by a factor of 2. However, the data indicate that the bottom shear stresses are greater than the surface shear stresses by a factor of roughly 10. Thus it seems apparent that the pressure-gradient terms are not the major reason for the large ratio of bottom to surface shear stress observed in the data. It is hypothesized that waves are the reason for the large ratio. Since \( c_b \) used in the model is a measure of the bottom shear stress one would expect it to be influenced by wave activity. Furthermore if the model is applied to two storms with different wave characteristics one would expect the \( c_b \) used in the model to be different for the two storms. The model simulations of Anita and Delia show that \( c_b \) is approximately one-fourth of \( c_{bp} \). Examination of the wave climate for the two storms shows that the waves were considerably more severe for Delia, thus at least qualitatively explaining why \( c_{bp} \) is much greater than \( c_b \).

A more quantitative test was made by using a bottom-boundary-layer model by GM to estimate the "effective bottom shear stress" (i.e., the total shear due to the steady current and waves). Application of the GM model yields effective bottom shear stresses that compare closely to those observed during the peak of the storms, enforcing the hypothesis that waves are the major reason for the high bottom to surface shear ratio observed in the data. The results of the Grant and Madsen (1980) models are used to show that wave effects would cause \( c_{bp} \) to be 4.8 times \( c_b \) during the peak of the storm, comparing nicely to the average ratio of 4 found from the model simulations.

From the standpoint of estimating the accuracy of modeling storm-induced currents, the comparisons between the Galerkin numerical model and the observed velocity field for the storms show that the model is capable of hindcasting currents with accuracy of the lesser of 0.2 m s\(^{-1}\) or 10% of the current magnitude. This accuracy may not be achievable in
other applications where baroclinicity, tides, etc., may be important and are not included in the modeling. The model simulations were obtained by utilizing generalized expressions for determining the two empirical coefficients (i.e., $c_b$ and $N_0$) used in the model. The major inadequacy of the expression used to estimate $c_b$ is related to the effects of waves on the parameter. Given the sensitivity of simulated currents in shallow water to $c_b$, it is clear that this inadequacy can introduce errors of greater than a factor of 2 into predictions of storm-induced currents using any model which incorporates a linear or quadratic bottom friction law. The bottom-boundary-layer model by GM was successfully used to explain the differences observed between the $c_b$'s used for the two storms as summarized above. Models such as the GM model clearly offer some hope in improving velocity predictions in shallow water, but a final assessment demands further verification including comparisons such as those described in this paper.

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