NOTES AND CORRESPONDENCE

The Density Ratio of Frontal Intrusions in the North Rockall Trough

H. M. VAN AKEN

Instituut voor Meteorologie en Oceanografie, Rijksuniversiteit Utrecht, Utrecht, The Netherlands

2 April 1982 and 2 July 1982

ABSTRACT

The density stratification of interleaving frontal intrusions is investigated with a statistical method. For this purpose a descriptive model of temperature inversions is formulated in which the thickness of the inversion is proportional to the temperature increase in the inversion. With this model an unbiased estimate can be made of the density deficit or density excess in intrusions with respect to an undisturbed background density stratification. Analysis of temperature inversion data from the North Rockall Trough obtained during the JASIN experiment gave the following results:

1) The thickness of the temperature inversions was proportional to the inverse of the background Brunt-Väisälä frequency N^{-1}. This result disagrees with excisting theoretical models and laboratory measurements but agrees with earlier results indicating that the thickness of intrusions in the North Rockall Trough was proportional to N^{-1}.

2) The salty intrusions had a density deficit, whereas cold, fresh intrusions had a density excess. This result is supported by models in which the dynamics of the intrusions are governed by horizontal density gradients caused by the vertical mass flux at the salt-finger interface of the intrusions. Therefore the cross-frontal interleaving in the Rockall Trough seems to be driven by salt-finger convection.

1. Introduction

Interleaving intrusions are supposed to be an important mechanism in lateral and cross-frontal transport of heat and salt in the ocean (Stommel and Fedorov, 1967; Joyce, 1977). The driving forces of these intrusions are thought to be the baroclinic instability of a thermohaline front (Woods *et al.*, 1977), or the convergent eddy motion during frontogenesis (MacVean and Woods, 1980), or double-diffusive mass fluxes at the salt-finger interface of the intrusions (Stern, 1967; Ruddick and Turner, 1979; Toole and Georgi, 1981).

In laboratory experiments Turner (1978) and Ruddick and Turner (1979) showed that small horizontal intrusions can grow, deriving the energy they need for this growth from the potential energy of the background salt stratification by means of salt-finger convection. For several years now, oceanographers have known about salt-finger convection in the ocean. Williams (1975), Magnell (1976) and Molcard and Williams (1975) presented direct evidence for salt fingers in the ocean. Several other authors found indirect evidence for salt fingers as the driving agent for intrusion dynamics (e.g., Horne, 1978; Joyce *et al.*, 1978; Gregg, 1980; Toole, 1981).

The dynamics of salt-finger-driven intrusions were studied theoretically by Stern (1967) and Toole and Georgi (1981). In their models the intrusions are driven by lateral pressure gradients caused by density changes due to the vertical buoyancy flux at the salt-finger interface of the intrusions. This implies that salt-finger-driven warm and salty intrusions will show a density deficit, whereas cold and fresh intrusions will show a density excess, both with respect to an undisturbed background stratification of the density.

Thus the density excess or deficit in intrusions can be indicative of the importance of salt-finger convection for the dynamics of the intrusions. Several authors (e.g., Howe and Tait, 1972; Fedorov and Plakhin, 1975; Fedorov, 1978, Chap. 4; van Aken, 1981) have statistically analyzed the temperature-salinity (T-S) relations in temperature inversions as parts of interleaving intrusions in order to obtain information about the density deficit or excess in the intrusions.

2. The model

The statistical analysis of temperature inversions has been interpreted by Fedorov and Plakhin (1975) and Fedorov (1978) in the form of a descriptive model. These authors supposed that the inversion can be considered either as the upper part of a warm intrusion or as the lower part of a cold one. They thought that such an intrusion was embedded in a stratified water mass with an undisturbed constant vertical temperature gradient $\partial\bar{T}/\partial z$ and a salinity gradient $\partial\bar{S}/\partial z$. In Fig. 1 such an idealized warm intrusion is sketched. The temperature inversion is characterized by three parameters: the temperature

0022-3670/82/111318-06$05.50

increase or intensity ΔT, the salinity increase ΔS, and the thickness h. It is easy to see from Fig. 1 that

$$\Delta T = \Delta T_0 + h\frac{\partial \bar{T}}{\partial z}, \tag{1a}$$

$$\Delta S = \Delta S_0 + h\frac{\partial \bar{S}}{\partial z}, \tag{1b}$$

where ΔT_0 and ΔS_0 are the deviation of temperature and salinity from the background stratification at the level of the temperature maximum. The corresponding density deviation $\Delta\rho_0$, due to the presence of the intrusion, can be written as

$$\Delta\rho_0 = \rho_0(\beta\Delta S_0 - \alpha\Delta T_0) = (R-1)\rho_0\alpha\Delta T_0, \tag{2}$$

where $\alpha = -\rho_0{}^{-1}\partial\rho/\partial T$, $\beta = \rho_0{}^{-1}\partial\rho/\partial S$ and ρ_0 is the reference density. The density ratio for the temperature inversion R can be expressed as $\beta\Delta S_0/\alpha\Delta T_0$. If the intrusion has no density excess or deficit, $\Delta\rho_0$ will be zero and R will be equal to 1. A density excess (deficit) in warm and salty (cold and fresh) intrusions is present when $R > 1$, whereas $R < 1$ indicates a density deficit (excess) in warm and salty (cold and fresh) intrusions. Thus salt-finger convection will lower R from an initial value and whenever the intrusion started with isopycnal advection ($R = 1$) the salt finger convection will cause R to acquire values of less than 1.

Combination of (1) and (2) results in

$$\beta\Delta S = R\alpha\Delta T + h\left(\beta\frac{\partial \bar{S}}{\partial z} - R\alpha\frac{\partial \bar{T}}{\partial z}\right). \tag{3}$$

Fedorov and Plakhin (1975) and Fedorov (1978) stated that the linear regression between $\beta\Delta S$ and $\alpha\Delta T$ according to the regression equation

$$\beta\Delta S = b'\alpha\Delta T + a \tag{4}$$

will give the regression parameter b' as an estimate of R. However, it is easy to see from (3) that if h and ΔT are correlated the estimate b' for R will be biased. Statistical analysis of a large number of temperature inversions from the North Rockall Trough (van Aken, 1981) revealed that the additive constant a from (4) could not be distinguished from zero and also that h was proportional to ΔT. This implies (at least for the data set used by van Aken, which will also be used in the following section) the relation

$$h = C\alpha\Delta T. \tag{5}$$

In this expression C is a proportionality constant. Substitution of (5) into (3) yields

$$\beta\Delta S = \left[R\left(1 - C\alpha\frac{\partial \bar{T}}{\partial z}\right) + C\beta\frac{\partial \bar{S}}{\partial z}\right]\alpha\Delta T. \tag{6}$$

Then, to obtain an estimate of R, the following two regression equations are needed:

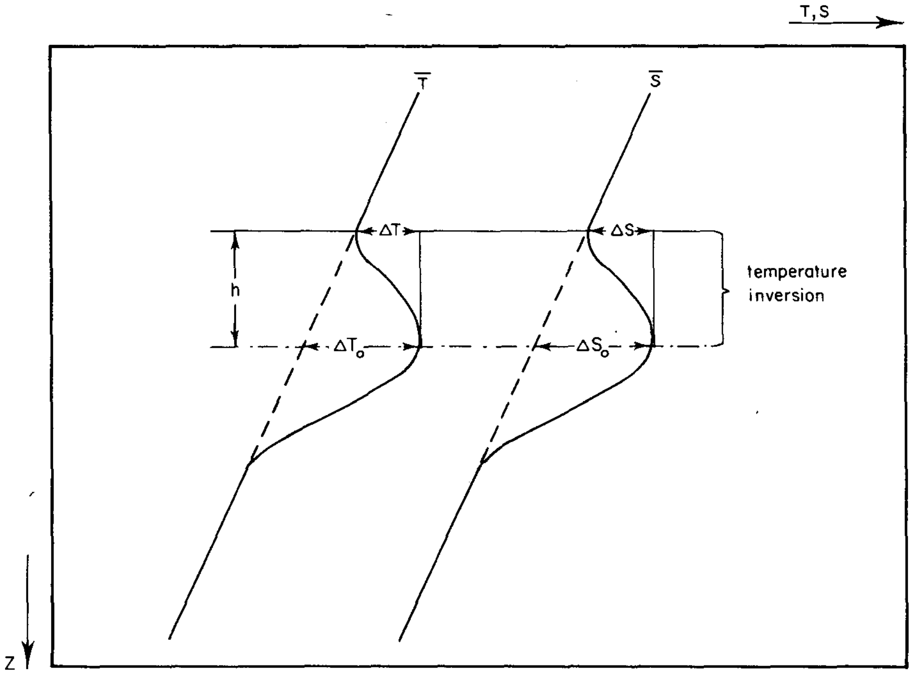

FIG. 1. Diagrammatic sketch of temperature and salinity profiles showing a warm, salty intrusion and a temperature inversion. The intensity ΔT, salinity increase ΔS, and thickness h of the temperature inversion are indicated, as well as the deviations from the background stratification (after Fedorov and Plakhin, 1975).

$$\beta \Delta S = b\alpha\Delta T, \tag{7a}$$

$$h = c\alpha\Delta T. \tag{7b}$$

In these regression equations the regression line is forced through the origin. The regression parameter c provides an estimate for the proportionality constant C. Then according to (6) the estimate for the density ratio R is given by

$$R = \frac{b - c\beta \dfrac{\partial \bar{S}}{\partial z}}{1 - c\alpha \dfrac{\partial \bar{T}}{\partial z}}. \tag{8}$$

Eq. (8) implies that, whenever the thickness of an inversion is proportional to the intensity ΔT, the regression parameters b' and b are not good estimates of the density ratio R unless the mean vertical gradients are zero. This result clearly illustrates that if a regression analysis of ΔS with ΔT in temperature inversions is to be performed to obtain R, it will be necessary, if the interpretation is to be reliable, to evaluate also the dependency of h on ΔT. If the linear relation between h and ΔT cannot be forced through the origin, (8) can easily be adapted to meet this situation.

3. Data analysis

During the JASIN experiment in the summer of 1978 (Royal Society, 1979) HRMS *Tydeman* carried out eight hydrographic surveys in the North Rockall Trough. In these surveys 160 deep CTD profiles were obtained in a digitized form. These profiles covered the water column from the sea surface to as near to the bottom as feasible. The average vertical distance between successive data cycles was 0.45 m. For a detailed description of the data handling the reader

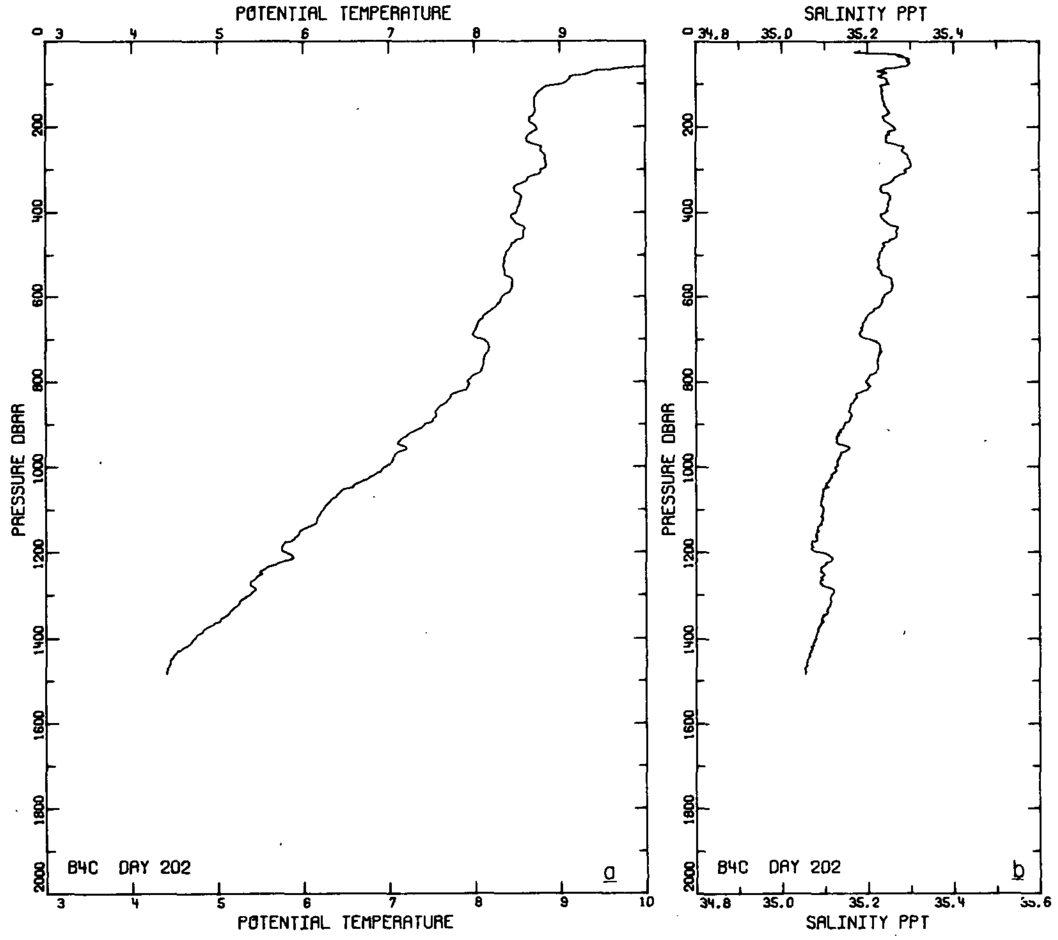

FIG. 2. Potential temperature (a) and salinity profile (b) from the North Rockall Trough, 58°48′N, 12°51′E, at day 202, 1978. Pressure is used as the vertical coordinate. These profiles show many interleaving warm, salty intrusions and many cold, fresh intrusions. The largest temperature inversion, between 200 and 300 db, has an intensity of about 0.25 K. These profiles are typical for the profiles found in the frontal zones in the North Rockall Trough during the JASIN experiment.

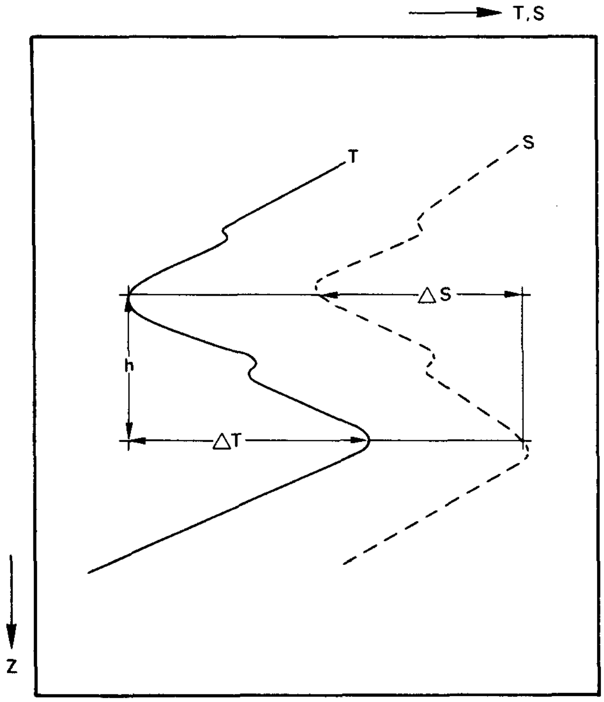

FIG. 3. Sketch of temperature (T) and salinity (S) profiles illustrating the selection of temperature inversions used for the inversion analysis. The selected inversions are considered to be a relatively large part of the intrusion and may be interrupted by smaller scale finestructure, as indicated in the sketch. The parameters ΔT, ΔS and h were calculated as the differences in T, S and z between the lower and upper boundary of the inversion, i.e., between the levels of the T extremes. By definition ΔT and h are positive.

is referred to van Aken (1981). [See the Royal Society publication (1979) for the deep CTD stations covered by the *Tydeman* surveys.]

A very prominent feature of the temperature and salinity profiles was the presence of numerous temperature inversions. These inversions were generally not unstable since the temperature inversions coincided with a salinity increase which overcompensated for the temperature effect on density. An example of such a profile is given in Fig. 2. The inversions occurred mainly in and close to frontal zones separating different water masses and are thought to be part of interleaving intrusions. The thickness of the intrusions was found to be proportional to the intensity ΔT, the proportionality constant being inversely proportional to the Brunt-Väisälä frequency (van Aken, 1981). This fact supports the use of (5) and (7b), provided that the background stability is more or less constant for the inversions analyzed.

Inversions were selected in such a way that they were clearly the upper part of a warm intrusion or the lower part of a cold one. This is illustrated in Fig. 3 and corresponds to the way in which Howe and Tait (1972) selected inversions. The inversions thus selected are relatively large-scale features and may be interrupted by smaller-scale finestructure.

The water masses in the Rockall Trough show a three-layer structure (Ellett and Martin, 1973). Since the distribution of inversion properties in the North Rockall Trough also showed this three-layer structure (van Aken, 1981), the inversions were analyzed for each layer separately. The three layers are the upper layer between the potential-density surfaces with $\sigma_\theta = 27.35$ and $\sigma_\theta = 27.43$ (depth from about 140 to 600 m), the intermediate layer between $\sigma_\theta = 27.43$ and $\sigma_\theta = 27.64$ (600 to 1100 m) and the deep layer with $\sigma_\theta > 27.64$ (below 1100 m). Here σ_θ is the potential-density anomaly. Inversions at levels above $\sigma_\theta = 27.35$ were not analyzed because at those levels there were too large changes in the background stratification due to the presence of the seasonal thermocline. Statistics on the inversions in the three layers are given in Table 1.

For each layer the regression coefficients b and c were calculated by a least-squares method and the standard deviations of b and c were also computed (Pollard, 1977). The α and β values used were the actual values for the given temperature, salinity and pressure, derived from the equation of state given by Fofonoff (1962). The density ratio R as well its standard deviation were calculated, according to (8), by means of the mean background gradients $\partial\bar{T}/\partial z$ and $\partial\bar{S}/\partial z$, averaged over all profiles, and the regression coefficients b and c. The results of this analysis are given in Table 2.

4. Discussion

In Fig. 4 the regression parameter c is plotted versus the reciprocal of the Brunt-Väisälä frequency, N^{-1}, or the buoyancy period. As can be seen from that figure, c appears to be proportional to N^{-1}. Thus the

TABLE 1. Mean values and standard deviations (in parentheses) of temperature-inversion properties of 160 deep CTD profiles in the North Rockall Trough.

	Inversion intensity (°C)	Salinity increase (‰)	Thickness (m)	Number of inversions
Upper layer	0.051 (0.047)	0.013 (0.012)	17.8 (19.0)	295
Intermediate layer	0.037 (0.036)	0.011 (0.011)	8.6 (7.3)	234
Deep layer	0.043 (0.041)	0.011 (0.011)	7.8 (6.1)	90

TABLE 2. Regression coefficients, mean gradients and estimates of the density ratio R. Estimated standard deviations are given in parentheses. For the definition of the parameters, see text.

	b	$10^{-6}\,c$ (m)	$10^7\,\alpha \frac{\partial \bar{T}}{\partial z}$ (m^{-1})	$10^7\,\beta \frac{\partial \bar{S}}{\partial z}$ (m^{-1})	R
Upper layer	1.15 (0.01)	1.86 (0.08)	−2.21	−0.67	0.90 (0.01)
Intermediate layer	1.26 (0.03)	0.95 (0.06)	−6.48	−1.54	0.87 (0.02)
Deep layer	1.31 (0.04)	1.05 (0.06)	−6.54	−1.93	0.90 (0.06)

thickness of the temperature inversions is proportional to N^{-1}. This $h \approx N^{-1}$ relation was also found between the thickness of intrusions and N in the Rockall Trough area (van Aken, 1981). This relation disagrees with the laboratory experiments performed by Ruddick and Turner (1979) who found a $h \approx N^{-2}$ relation for salt-finger-driven intrusions and it disagrees also with the $h \approx N$ relation for the fastest-growing intrusions in the linear intrusion model of Toole and Georgi (1981).

Table 2 shows that the regression coefficient b differs significantly for the different layers, and for all three layers b has values clearly larger than 1. This can only be interpreted as an indication of the stability of the temperature inversions, not as an indication of the density excess in the warm intrusion, which was the interpretation given by Fedorov and Plakhin (1975) and by Fedorov (1978).

The density ratio, estimated according to (8), has values clearly below 1. In all three layers, R is nearly equal, with an average of 0.89. This low value of R in all three layers agrees with spectral estimates of R in the vertical wavenumber space, based on a selection of the profiles analyzed here; these spectral estimates gave R-values between 0.75 and 0.90 (van Aken, 1981). The values of $R < 1$ indicate that there is a density deficit in the warm, salty intrusions and density excess in the cold fresh ones, although the temperature inversions are still stably stratified (b values > 1). This is in agreement with the intrusion models of Stern (1967), Ruddick and Turner (1979) and Toole and Georgi (1981) and supports their proposition that salt-finger convection is a driving mechanism for frontal interleaving in the ocean and thus for cross-frontal transport of heat and salt. However these models do not succeed in predicting the observed relation between the background Brunt-Väisälä frequency and the thickness of the intrusions. This clearly indicates that further work is needed to understand the behavior of finite-amplitude ocean intrusions.

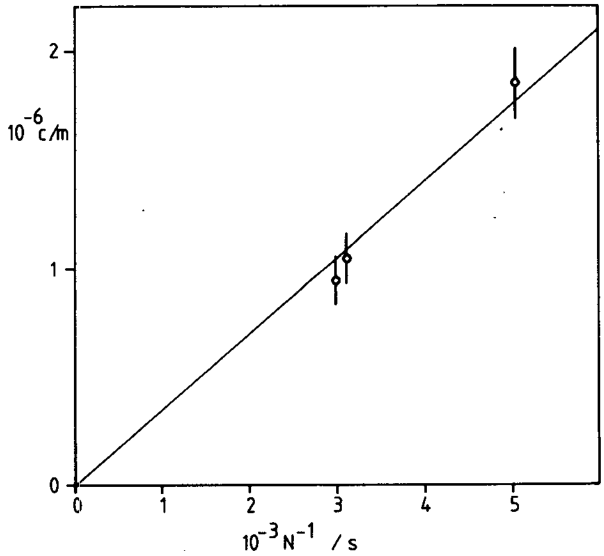

FIG. 4. Regression coefficient c for the three different layers plotted against the mean buoyancy periods for these layers, or N^{-1}, where N is the mean Brunt-Väisälä frequency (cps). The straight line indicates perfect proportionality between c and N^{-1}. The 90% confidence interval is indicated.

Acknowledgments. I am grateful to Gé Prangsma and Pieter Kruseman of the Royal Netherlands Meteorological Institute for giving me the opportunity to take part in the Dutch contribution to the JASIN experiment.

REFERENCES

Ellett, D. J., and J. H. A. Martin, 1973: The physical and chemical oceanography at the Rockall Channel. *Deep-Sea Res.*, **20**, 585–625.

Fedorov, K. N., 1978: *The Thermohaline Finestructure of the Ocean.* Oxford, Pergamon Press, 170 pp.

——, and Ye. A. Plakhin, 1975: Thermohaline disturbances accompanying the intrusion process in the ocean. *Oceanology*, **15** (English edition), 286–290.

Fofonoff, N. P., 1962: Physical properties of sea water. *The Sea*, Vol. 1, M. N. Hill, Ed, Interscience, 3–30.

Gregg, M. C., 1980: The three-dimensional mapping of a small thermohaline intrusion. *J. Phys. Oceanogr.*, **10**, 1468–1492.

Horne, E. P. W., 1978: Interleaving at the sub-surface front in the slope water off Nova Scotia. *J. Geophys. Res.*, **83**, 3659–3671.

Howe, M. R., and R. I. Tait, 1972: The role of temperature inversions in the mixing processes of the deep ocean. *Deep-Sea Res.*, **19**, 781–791.

Joyce, T. M., 1977: A note on lateral mixing of water masses. *J. Phys. Oceanogr.*, **7**, 626–629.

——, W. Zenk and J. M. Toole, 1978: The anatomy of the Antarctic Polar Front in the Drake Passage. *J. Geophys. Res.*, **83**, 6093–6114.

Magnell, B., 1976: Salt fingers observed in the Mediterranean outflow region (34°N, 11°W) using a towed sensor. *J. Phys. Oceanogr.*, **6,** 511–523.

MacVean, M. K., and J. D. Woods, 1980: Redistribution of scalars during upper ocean frontogenesis: a numerical model. *Quart. J. Roy. Meteor. Soc.*, **106,** 293–312.

Molcard, R., and A. J. Williams III, 1975: Deep stepped structure in the Thyrrenian Sea. *Mem. Soc. Roy. Sci. Liège*, **6,** No. 7, 191–210.

Pollard, J. H., 1977: *A Handbook of Numerical and Statistical Techniques.* Cambridge University Press, 349 pp.

Royal Society, 1979: *Air-Sea Interaction Project: Summary of the 1978 Field Experiment, JASIN 1978.* London, 139 pp.

Ruddick, B. R., and J. S. Turner, 1979: The vertical length scale of double-diffusive intrusions. *Deep-Sea Res.*, **22,** 509–515.

Stern, M. E., 1967: Lateral mixing of water masses. *Deep-Sea Res.*, **14,** 747–753.

Stommel, H., and K. N. Fedorov, 1967: Small-scale structure in temperature and salinity near Timor and Mindanao. *Tellus*, **19,** 306–325.

Toole, J. M., 1981: Intrusion characteristics in the Antarctic Polar Front. *J. Phys. Oceanogr.*, **11,** 780–793.

——, and D. T. Georgi, 1981: On the dynamics and effects of double-diffusively driven intrusions. *Prog. Oceanogr.*, **10,** 123–145.

Turner, J. S., 1978: Double-diffusive intrusions into a density gradient. *J. Geophys. Res.*, **83,** 2887–2901.

Williams, A. J., 1975: Images of ocean microstructure. *Deep-Sea Res.*, **22,** 811–829.

Van Aken, H. M., 1981: The thermohaline finestructure in the North Rockall Trough. Thesis, Rijksuniversiteit Utrecht.

Woods, J. D., R. L. Wiley and M. G. Briscoe, 1977: Vertical circulation at fronts in the upper ocean. *A Voyage of Discovery*, supplement to *Deep-Sea Res.*, M. Angel, Ed., Pergamon Press, 253–275.

U.S. POSTAL SERVICE

STATEMENT OF OWNERSHIP, MANAGEMENT AND CIRCULATION

(Required by 39 U.S.C. 3685)

1. TITLE OF PUBLICATION: JOURNAL OF PHYSICAL OCEANOGRAPHY

A PUBLICATION NO: 0 0 2 2 3 6 7 0

2. DATE OF FILING: 9/27/82

3. FREQUENCY OF ISSUE: MONTHLY

A. NO. OF ISSUES PUBLISHED ANNUALLY: 12

B. ANNUAL SUBSCRIPTION PRICE: $120. & $30. members

4. COMPLETE MAILING ADDRESS OF KNOWN OFFICE OF PUBLICATION *(Street, City, County, State and ZIP Code) (Not printers)*: 45 Beacon St., Boston, Suffolk, Mass. 02108

5. COMPLETE MAILING ADDRESS OF THE HEADQUARTERS OR GENERAL BUSINESS OFFICES OF THE PUBLISHERS *(Not printers)*: 45 Beacon St., Boston, Mass. 02108

6. FULL NAMES AND COMPLETE MAILING ADDRESS OF PUBLISHER, EDITOR, AND MANAGING EDITOR *(This item MUST NOT be blank)*

PUBLISHER *(Name and Complete Mailing Address)*: American Meteorological Society, 45 Beacon St., Boston, Mass. 02108

EDITOR *(Name and Complete Mailing Address)*: Dr. William R. Holland, NCAR, Boulder, Co. 80307 and Dr. Bruce Warren, Woods Hole Oceanographic Institute, Woods Hole, Ma. 02543

MANAGING EDITOR *(Name and Complete Mailing Address)*: Dr. Kenneth C. Spengler, American Meteorological Society, 45 Beacon St., Boston, Ma. 02108

7. OWNER *(If owned by a corporation, its name and address must be stated and also immediately thereunder the names and addresses of stockholders owning or holding 1 percent or more of total amount of stock. If not owned by a corporation, the names and addresses of the individual owners must be given. If owned by a partnership or other unincorporated firm, its name and address, as well as that of each individual must be given. If the publication is published by a nonprofit organization, its name and address must be stated.) (Item must be completed)*

FULL NAME	COMPLETE MAILING ADDRESS
American Meteorological Society	45 Beacon St., Boston, Ma. 02108

8. KNOWN BONDHOLDERS, MORTGAGEES, AND OTHER SECURITY HOLDERS OWNING OR HOLDING 1 PERCENT OR MORE OF TOTAL AMOUNT OF BONDS, MORTGAGES OR OTHER SECURITIES *(If there are none, so state)*

FULL NAME	COMPLETE MAILING ADDRESS
None	

9. FOR COMPLETION BY NONPROFIT ORGANIZATIONS AUTHORIZED TO MAIL AT SPECIAL RATES *(Section 411.3, DMM only)*
The purpose, function, and nonprofit status of this organization and the exempt status for Federal income tax purposes *(Check one)*

☒ *(1) HAS NOT CHANGED DURING PRECEDING 12 MONTHS*

☐ *(2) HAS CHANGED DURING PRECEDING 12 MONTHS*

(If changed, publisher must submit explanation of change with this statement.)

10. EXTENT AND NATURE OF CIRCULATION	AVERAGE NO. COPIES EACH ISSUE DURING PRECEDING 12 MONTHS	ACTUAL NO. COPIES OF SINGLE ISSUE PUBLISHED NEAREST TO FILING DATE
A. TOTAL NO. COPIES *(Net Press Run)*	1,994	1,961
B. PAID CIRCULATION 1. SALES THROUGH DEALERS AND CARRIERS, STREET VENDORS AND COUNTER SALES	18	5
2. MAIL SUBSCRIPTION	1,448	1,460
C. TOTAL PAID CIRCULATION *(Sum of 10B1 and 10B2)*	1,466	1,465
D. FREE DISTRIBUTION BY MAIL, CARRIER OR OTHER MEANS SAMPLES, COMPLIMENTARY, AND OTHER FREE COPIES	14	2
E. TOTAL DISTRIBUTION *(Sum of C and D)*	1,480	1,467
F. COPIES NOT DISTRIBUTED 1. OFFICE USE, LEFT OVER, UNACCOUNTED, SPOILED AFTER PRINTING	514	494
2. RETURN FROM NEWS AGENTS	0	0
G. TOTAL *(Sum of E, F1 and 2 - should equal net press run shown in A)*	1,994	1,961

11. **I certify that the statements made by me above are correct and complete**

SIGNATURE AND TITLE OF EDITOR, PUBLISHER, BUSINESS MANAGER, OR OWNER: Kenneth C. Spengler Executive Director

PS Form 3526 July 1981 *(See instruction on reverse)* *(Page 1)*