NOTES AND CORRESPONDENCE

A Standard Analytic Curve of Potential Temperature versus Salinity for the Western North Atlantic[1]

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ABSTRACT

An algorithm is described for computing salinity as a continuous function of potential temperature for the western North Atlantic. The algorithm uses historical data compiled by Worthington and Melcalf (1961) for the deep western North Atlantic, and Iselin (1936) for shallow and intermediate waters of the same region.

The algorithm described here extends the concept of salinity anomaly, introduced by Helland-Hansen and Nansen (1926) to data recorded in a nearly continuous manner by profiling instruments. The algorithm allows computation of salinity (S) as a continuous function of potential temperature (θ) in the range $0.5°C < \theta < 20°C$ for typical western North Atlantic water masses. Similar algorithms could be developed as needed for other water masses. This particular algorithm has been used in diverse applications: Armi (1978) examined small scale (~20 m) anomalies of Norwegian Sea Overflow intrusions in the western North Atlantic; Roemmich (1979, 1981) calculated fluxes of salinity anomaly in the Caribbean; Bray (1980) studied the distribution of salinity anomaly along density surfaces to deduce low-frequency motions; McCartney *et al.* (1980) identified anomalous water-mass distributions at 55°W in the North Atlantic in 1977. Maillard (personal communication) has modified the algorithm to include temperatures up to 27°C for a study of Gulf Stream rings. This note documents the algorithm for future use.

Standard curves of temperature versus salinity have been used for many years to distinguish between different water-mass types. For the western North Atlantic, Iselin (1936, Fig. 53) developed such a curve for temperatures between 4 and 18°C by smoothing the then available 96 *Atlantis* stations taken between 1931 and 1933 within the Sargasso Sea. Worthington and Metcalf (1961, Fig. 2) presented a similar curve for potential temperatures below 4°C using data taken primarily during the International Geophysical Year (1957–58). These historical curves were converted to an analytic func-

TABLE 1. Conversion of T vs S from Iselin (1936, Fig. 53) to θ vs S.

T^1 (°C)	D^2 (m)	P^3 (db)	θ^4 (°C)	Observed S (‰)	Fitted S (‰)	$\sigma_\theta{}^5$
4.0	1800	1820	3.85	34.99	34.992	27.81
5.0	1250	1260	4.89	35.01	35.010	27.72
6.0	1150	1160	5.89	35.03	35.032	27.61
7.0	1100	1110	6.89	35.07	35.066	27.51
8.0	950	960	7.90	35.12	35.120	27.40
9.0	925	935	8.89	35.19	35.194	27.31
10.0	900	910	9.89	36.29	35.290	27.22
11.0	850	860	10.89	35.41	35.406	27.13
12.0	825	830	11.89	35.54	35.539	27.05
13.0	800	800	12.89	35.68	35.684	26.96
14.0	750	750	13.89	35.84	35.838	26.88
15.0	700	700	14.89	36.00	36.000	26.78
16.0	650	650	15.89	36.17	36.168	26.69
17.0	550	550	16.91	36.34	36.344	26.58
18.0	375	375	17.93	36.49	36.488	26.45
19.0	200	200	19.0	36.56	36.558	26.23
20.0	150	100	20.0	36.58	36.580	25.98

[1] T and S values from Worthington's personal reading of the Iselin curve and an unpublished curve by Fuglister that extends the Iselin curve to waters warmer than 18°C.

[2] Depth from "standard" *Atlantis* Station 5310 (Fuglister, 1960).

[3] Pressure from depth according to Saunders and Fofonoff (1976).

[4] Potential temperature according to Fofonoff (1977) and Bryden (1973).

[5] σ_θ is calculated as σ_θ (θ, fitted S).

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TABLE 2. θ vs S from Worthington and Metcalf (1961).

θ (°C)	Observed S (‰)	Fitted S[1] (‰)	σ_θ[2]	θ (°C)	Observed S (‰)	Fitted S[1] (‰)	σ_θ	θ (°C)	Observed S (‰)	Fitted S[1] (‰)	σ_θ
0.60	34.749		27.89	1.75	34.883			2.90	34.960	34.961	27.88
0.65	34.754			1.80	34.890			2.95	34.962	34.963	
0.70	34.760			1.85	34.896			3.00	34.964	34.965	
0.75	34.765			1.90	34.902	34.901		3.05	34.966		
0.80	34.771		27.90	1.95	34.906			3.10	34.968		27.87
0.85	34.776			2.00	34.911			3.15	34.970	34.969	
0.90	34.782			2.05	34.914			3.20	34.971	34.970	27.86
0.95	34.787			2.10	34.917			3.25	34.973	34.971	
1.00	34.793			2.15	34.919	34.920		3.30	34.974	34.972	
1.05	34.798			2.20	34.922	34.923	27.91	3.35	34.974	34.973	27.85
1.10	34.804			2.25	34.925			3.40	34.975	34.974	
1.15	34.810			2.30	34.928			3.45	34.975	34.976	27.84
1.20	34.815			2.35	34.931	34.930		3.50	34.975	34.977	
1.25	34.821		27.91	2.40	34.934	34.933		3.55	34.975	34.978	
1.30	34.827	34.826		2.45	34.936		27.90	3.60	34.977	34.979	27.83
1.35	34.832			2.50	34.939			3.65	34.978	34.981	
1.40	34.837	34.838		2.55	34.942			3.70	34.980	34.982	27.82
1.45	34.844			2.60	34.945			3.75	34.984		
1.50	34.850			2.65	34.947			3.80	34.987	34.986	
1.55	34.857			2.70	34.950		27.89	3.85	34.989	34.987	27.81
1.60	34.864	34.863		2.75	34.953			3.90	34.991	34.989	
1.65	34.870		27.92	2.80	34.955	34.956		3.95	34.992	34.990	
1.70	34.877			2.85	34.958			4.00	34.993	34.992	27.80

[1] If no value for fitted S is given, the fitted value agrees with observed S within less than 0.001‰.

[2] σ_θ is calculated as $\sigma_\theta(\theta$, fitted $S)$. σ_θ is given only when it differs from the previous value.

tion of S vs θ using a cubic spline fit. The cubic spline is a standard regression fit over an arbitrary number of sections which are joined by points called knots. Over each section the data are fit to a third-order polynomial, with the constraint that the value and the first derivative of the fitted function be continuous across the knots (de Boor and Rice, 1968). The International Mathematical and Statistical Library supported subroutine for fitting to a cubic spline (ICSFKU) was used (IMSL, 1980). Before fitting the Iselin data, the temperature (T) values were converted to potential temperature using a typical hy-

TABLE 3. Knots and coefficients for salinity algorithm.

θ_i (°C)	C_{0i} (‰)	C_{1i} (‰ $°C^{-1}$)	C_{2i} (‰ $°C^{-2}$)	C_{3i} (‰ $°C^{-3}$)
0	34.738063	0	0	
0.5	34.738053	0.107290	0.584849E-2	−0.253429E-2
1.20	34.815152	0.111753	0.523726E-3	0.582151E-1
1.50	34.850297	0.127785	0.529320E-1	−0.135379
1.75	34.883436	0.128868	−0.485828E-1	−0.129913
2.0	34.910587	0.802174E-1	−0.146093	0.228920
2.25	34.925087	0.500936E-1	0.255484E-1	−0.267382E-1
2.50	34.938790	0.578544E-1	0.552526E-2	−0.359945E-1
2.75	34.953036	0.538681E-1	−0.214953E-1	−0.374594E-1
3.00	34.964575	0.360969E-1	−0.495364E-1	0.509274E-1
3.20	34.970220	0.223936E-1	−0.189292E-1	0.580683E-1
3.40	34.974406	0.217901E-1	0.157868E-1	0.479730E-2
3.60	34.979434	0.286805E-1	0.185975E-1	−0.294172E-1
3.80	34.985679	0.325895E-1	0.102958E-2	−0.279688E-1
4.00	34.992014	0.296450E-1	−0.157123E-1	0.643397E-2
5.00	35.01238	0.175223E-1	0.357759E-2	0.114377E-2
7.00	35.07089	0.455579E-1	0.104386E-1	0.865592E-5
10.00	35.30174	0.108423	0.105172E-1	−0.763343E-3
13.00	35.70106	0.150916	0.364790E-2	0.310805E-4
16.00	36.18748	0.173643	0.392926E-2	−0.689782E-2
19.00	36.55753	0.109775E-1	−0.581443E-1	0.696380E-1
21.00	36.9040118	0	0	0

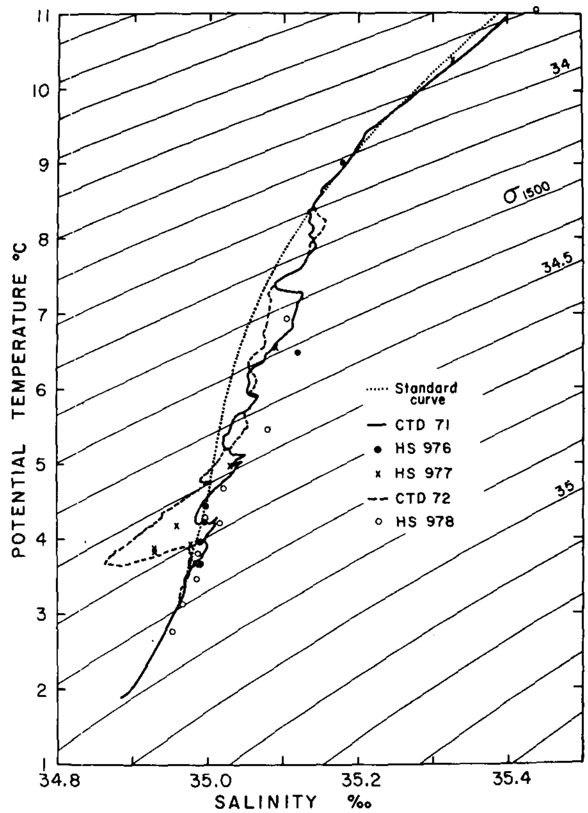

FIG. 1. The deviation of selected *Knorr* Cruise 66 (July 1977) CTD and hydro (HS) stations from the standard curve of potential temperature versus salinity. Potential temperature and the density (σ) isopleths shown are referenced to 1500 db. Nominal station positions along 55°W for the stations are 36°32'N (CTD 71), 36°46'N (HS 976), 37°15'N (HS 977), 37°27'N (CTD 72), 37°40'N (HS 978).

drographic station from the IGY section at 32°N, in the Sargasso Sea (Fuglister, 1960). *Atlantis* Station 5310, located at 33°42'N, 67°24'W (13 June 1955), was used to determine the depth corresponding to selected temperatures from the Iselin T vs S curve, by interpolation (see Table 1). The depths were converted to pressure using the algorithm of Saunders and Fofonoff (1976). Finally, the temperatures were converted to potential temperature using the formula from Fofonoff (1977), and Bryden's (1973) formula for the adiabatic temperature gradient. The converted Iselin data are listed in Table 1, which, together with selected points from the Worthington and Metcalf curve (Table 2), were fitted to the cubic spline. The two data sets used overlap at potential temperature 3.85°C, where the historical salinities agree within 0.001‰.

Salinity may then be computed as

$$S(\theta) = C_{0i} + C_{1i}\Delta\theta_i + C_{2i}\Delta\theta_i^2 + C_{3i}\Delta\theta_i^3,$$

with

$$\Delta\theta_i = \theta - \theta_i .$$

Here θ is the observed potential temperature and θ_i is the knot corresponding to the ith interval of θ, such that $\theta_i \leq \theta < \theta_{i+1}$. The coefficients and knots are listed in Table 3. The difference between the original curves and the cubic spline (Tables 1 and 2) does not exceed 0.003‰ for values of $\theta < 4.0$°C, nor 0.004‰ between 4 and 20°C. Potential density in the form σ_θ is included in Tables 1 and 2.

An example of the use of this algorithm and the concept of salinity anomaly is seen in the figures. Fig. 1, taken from McCartney *et al.* (1980, Fig. 7), shows the deviation of *Knorr* CTD Station 72 and four adjacent stations from the standard curve of potential temperature versus salinity. McCartney *et al.* have chosen 1500 db as the reference pressure for potential temperature and have referenced the standard curve also to this pressure. Station 72 was taken in a pronounced cell of negative salinity anomaly representing Labrador Sea Water. The vertical extent and structure of this anomaly are shown in Fig. 2. We plotted salinity anomaly as a function of pressure for the anomalous CTD Station 72 and the adjacent CTD station 71 shown in Fig. 1. The upper portion (~1400 db) of the cell is sharp, in contrast to the deeper portion (between 1500 and 2200 db) which approaches the background standard curve smoothly.

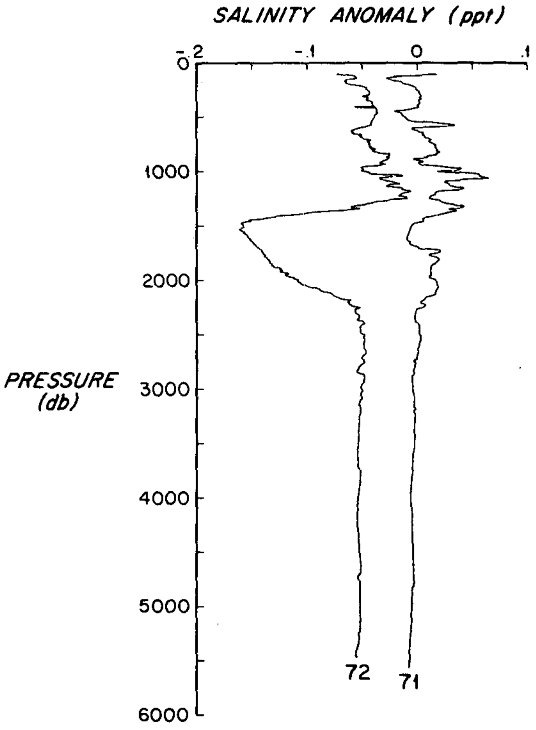

FIG. 2. Salinity anomaly versus pressure for *Knorr* Cruise 66 CTD Stations 71 and 72 of Fig. 1. Salinity anomaly is computed as $S_{\text{obs}} - S(\theta)$. The curve for Station 72 is offset from that for 71 by −0.05‰, for clarity.

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