# Data Assimilation Tests with an Oceanic Mixed-Layer Model

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## ABSTRACT

A data assimilation technique using a one-dimensional, ocean mixed-layer model to advance the thermal structure observations to the analysis time is tested. The effects of insertions of erroneous temperature profiles in such a model are studied for winter and summer periods. Sets of 5, 15 or 30 model-generated profiles at random times during the 15 days prior to analysis time are modified by adding errors in layer depth and temperature and in the thermocline temperature gradient. Each of the modified profiles is advanced by the mixed-layer model to the initial forecast time. This produces a series of temperature profiles that are consistent with the prediction model and with the atmospheric forcing at that time. The profiles are combined by a simple level-by-level averaging. A prediction from this averaged profile, or a similar profile after screening for extreme temperatures, is compared to a prediction from the last available observation in the set. A control run of the model provides complete 3 h verification data.

For the winter season tests, the errors in the predictions of layer depth from the last available profile are 3–5 times larger than the errors in the averaged and screened-averaged cases. During the summer, the larger errors are in surface temperature. Predictions from the averaged initial profile have typical errors of 0.25°C, whereas those from the last available profile are 3–4 times larger. Having 15 or 30 observations during a 15-day period appears to provide a relatively small improvement over a set of only five observations. It is thus concluded from these model-generated tests that the data assimilation technique will permit the long-term retention of the ocean thermal structure observations in the data base. Using the ocean mixed-layer model to combine these observations appears to be useful in the diagnosis and prediction of upper ocean thermal structure.

## 1. Introduction

One of the problems hindering development of an ocean prediction capability similar to that achieved in meteorology (Elsberry and Garwood, 1980) is the paucity of ocean thermal structure observations. Numerical ocean prediction is an initial value problem, and one must provide the best possible specification of the initial ocean thermal structure. It has been recognized [see reviews by McPherson (1975) and Bengtsson (1975)] in meteorology that the initial thermal structure should also be consistent with the numerical model structure and characteristics. The procedure by which the forecast model itself is used to produce the best specification on the initial conditions is generally termed data assimilation.

The longer time scales of large-scale oceanographic phenomena provide a physical justification for retaining the observations in the analysis data base for longer times than is the case in meteorology. However, the retention time varies in different analysis schemes. In daily operational analyses, the ocean temperature profiles may be retained for less than five days. An analysis system designed by White and Bernstein (1979) for the North Pacific Experiment (NORPAX) anomalies uses all observations collected during the month. How much information content regarding the future state of the ocean is present in a single thermal structure observation? In open-ocean regimes away from major current systems, much of the upper ocean thermal structure variability on time scales of less than about 15 days appears to be related to the atmospheric forcing. We propose to use a one-dimensional, mixed-layer model to account for the variability introduced by the atmospheric forcing during the interval since the observation was taken. We thus may use the one-dimensional ocean model as a tool to "diagnose" the present thermal structure of the ocean by using all temperature profiles observed during the previous 15 days. One of the ultimate objectives of real data assimilation tests will be to determine selection criteria for the retention time scale in different ocean regimes. Because the ocean model is used to specify the conditions at the analysis time, the technique may be thought of as data assimilation. In a stricter sense, however, the procedure is not analogous to data assimilation in the meteorological models, because the thermal structure information is not spread horizontally in a one-dimensional model.

The primary objective of this work is to demon-

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strate an assimilation method for combining recent temperature profiles for use in a one-dimensional ocean prediction model. We first examine the response of the model to erroneous temperature profile insertions. The effects of having a larger number of temperature profiles are tested. Both the "historical" and verification data are derived from model-generated time series. Based on meteorological assimilation experiments (McPherson, 1975), we expect that assimilation of real data would be a more robust test of the assimilation technique. Another important limitation of these assimilation tests is the use of "perfect" atmospheric forcing. That is, the same atmospheric forcing is used in the model predictions from the simulated profiles as was used to generate the verification data. Therefore, if the correct temperature profile is re-inserted, a perfect prediction will result, because there are no horizontal dispersive effects. The advantage of deriving the historical profiles from model-generated data and the use of perfect atmospheric forcing is that the single effect of the uncertainty due to the temperature profile information can be specified. In the real world, uncertainty is introduced by virtue of imperfect initial data, imperfect models, imperfect assimilation techniques and imperfect verification data. These tests provide a lower bound on the expected errors.

## 2. Prediction model

An ocean prediction model is desired that is relatively simple and yet produces realistic variations of upper ocean thermal structure. The one-dimensional, oceanic mixed-layer model of Garwood (1977) meets both needs. This vertically integrated or bulk model utilizes an entrainment hypothesis that is dependent on the horizontal and vertical components of turbulent energy. Part of the wind-generated turbulent kinetic energy that increases potential energy by deepening the mixed layer is dependent on stability. This results in modulation of the entrainment rate by the diurnal heating/cooling cycle. Another feature of the Garwood model is the planetary influence on the dissipation time scale for turbulence, which enhances dissipation for deeper mixed layers. The model-generated thermal structure profiles used in this work are extracted from a series of annual simulations of the model, which were kindly provided by Bill Garwood and David Adamec. These simulations contain much of the diurnal, synoptic and seasonal variability that was observed at ocean weather ship PAPA (50°N, 145°W).

The Garwood bulk model is relatively easy to initialize. The temperature and depth of the well-mixed layer must be provided along with the remainder of the vertical temperature profile. No salinity information is available for the annual runs, so a constant value is specified. Likewise, no observations of ocean currents are available. Thus, a possible turbulence generation by shear of the mean currents at the base of the mixed layer is omitted. The temperature profile is resolved to 200 m at 1 m intervals. A resolution of 5 m was also tested. The primary differences due to the 5 m interval occurred during shallowing regimes and during the summer when the mixing depth was restricted by the minimum vertical resolution.

The atmospheric forcing for the model is calculated from 3 h observations of wind speed, cloud cover, sea surface temperature, air temperature and dew point. These 3 h values of the surface fluxes of buoyancy and momentum are interpolated to the 1 h time step of the ocean model. Because the atmospheric forcing is calculated from observed values, there is no additional error that would be induced by using an atmospheric prediction model to drive the ocean model. As mentioned above, the same atmospheric forcing that is used in the control run is also used in the predictions from the assimilated profiles.

## 3. Effects of data errors

A new or different temperature profile may be inserted into a one-dimensional model without "shock". That is, insertion of a new vertical temperature profile in a one-dimensional model does not affect adjacent temperatures through inertio-gravity motions or diffusive processes that are present in multi-dimensional models. The only adjustments in this model arise from changes in thermal structure that affect vertical mixing processes, such as a new mixed-layer depth (MLD) or temperature jump at the base of the layer. This complete assimilation into the one-dimensional model means the effect of a new profile—whether accurate or not—may be retained for some time.

The effect of data errors on the subsequent solution is tested by interrupting the annual integration of the mixed-layer model at a particular day. The temperature profile is then replaced with an erroneous profile, and the model integration is continued. Extreme variations are used here for emphasis. The primary variables are the mixed-layer depth and temperature, and the buoyancy (temperature) jump at the base of the layer. The mixed-layer depth in the model is defined to be the maximum depth at which the temperature is equal to the surface temperature. Each of the variables is separately changed for a winter (Fig. 1a) and a summer (Fig. 2a) date. The thermocline temperature structure is also idealized by a linear variation between the temperature at the base of the layer and the 200 m temperature of the control.

Insertion of a profile with a too shallow layer depth for the atmospheric forcing during winter (Fig. 1b) results in a persistent deepening of the layer toward

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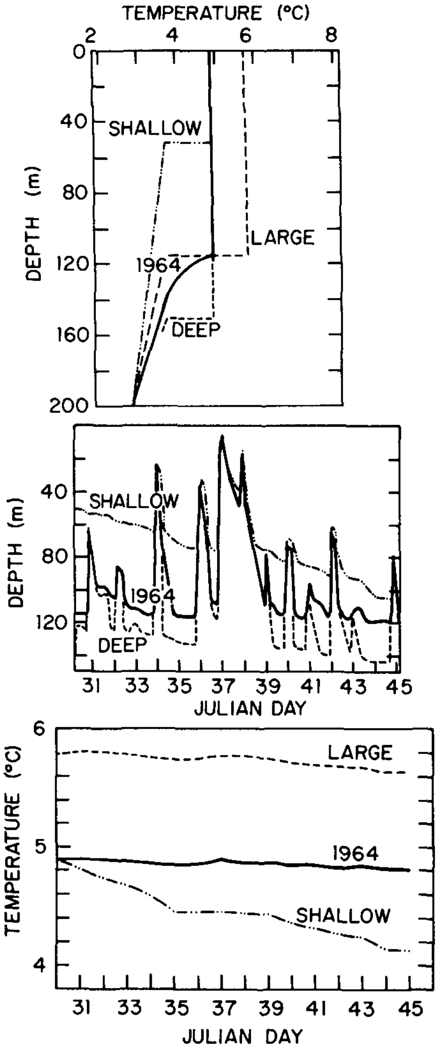

FIG. 1. (a) Temperature profile from Day 30 of the 1964 control run (solid) and the modified profiles inserted to test the response to data errors; (b) evolutions of the layer depth predicted from the control run (solid) and the modified temperature profiles shown in (a); and (c) corresponding mixed-layer temperature predictions. The prediction from the deep profile in (a) is essentially identical to the 1964 control run.

the control run result. When the layer in the control reforms at a smaller depth, for example on Day 34, the model run with the altered profiles also reforms. Because this process is almost entirely related to the atmospheric forcing, which is identical in each case, the new depths are very similar. Resumption of the deepening subsequently uncovers the different thermal structure below the shallow layer. If the atmospheric forcing is just strong enough to maintain the layer depth in the control run, or if it is a shallowing regime, then insertion of a profile that has a deeper mixed layer than the control will result in an immediate reformation at the control depth. However, the temperature jump at the base of the new layer is very small. As the strength of the atmospheric forcing subsequently increases, there is little impediment to deepening toward the inserted layer depth.

The predicted temperature response (Fig. 1c) to the insertion of a profile with a shallow layer depth may be explained in terms of the depth changes. Although the initial temperature is the same as the control, the entrainment deepening results in temperature decreases relative to the control. Given the large differences in initial layer depth and upper thermocline temperatures in this example, the decrease in temperature by Day 45 is quite large. Insertion of a profile with a higher mixed-layer temperature results in a persistent bias in temperature (Fig. 1c). Because the atmospheric forcing is being distributed over nearly the same layer depth (not shown in Fig. 1b) as in the control, the time tendencies in the two cases are essentially identical.

The predicted response to insertion of erroneous profiles during the summer may be interpreted as in the winter case. Here the dependence on the atmospheric forcing is even more striking for the depth evolutions (Fig. 2b). An initial layer depth that ex-

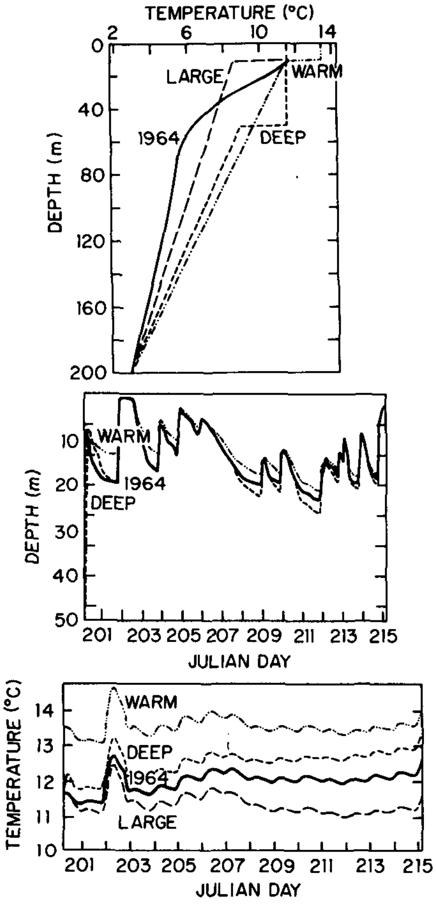

FIG. 2. As in Fig. 1, except for Day 200 of the 1964 control run.

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TABLE 1. Summary of insertion of erroneous temperature profiles into the oceanic mixed-layer depth (MLD) model during two oceanic regimes at OWS PAPA in 1964.

| Changes inserted | Shallowing | Deepening |
|---|---|---|
| Initial MLD > control | When forcing is insufficient to maintain greater depth, the value coincides with the smaller depth in the control. | Will tend to reach greater depths due to lower stability below the initial MLD of the control. |
| Initial MLD < control | Depth returns to the control value, if the depth of the newly formed stable layer is less than the initial MLD. | Deepening impeded, resulting in shallower layer. Conservation of buoyancy causes layer to cool. |
| Initial $T$ large | No effect on newly formed stable layer near surface. | Stability impedes deepening, resulting in shallower layer. Layer cools due to conservation of buoyancy, especially in summer. |
| Initial $T$ small | No effect on newly formed stable layer near surface. | Decreased stability at the base of the layer permits more rapid deepening and slightly larger maximum depth. |
| Initial $T$ control | Parallels control $T$ but retains high (low) bias. | Parallels control $T$ while retaining high (low) bias. High $T$ increases stability and impedes deepening. Lower $T$ allows increased deepening. |

ceeds that for the control is immediately adjusted to that of the control. Subsequent deepening uncovers the nearly isothermal profile below, and the predicted MLD will be greater. However, the depth differences are not large, because of the increasing stability near the surface due to the net downward heat flux during this season. The temperature tendencies (Fig. 2c) are remarkably similar, although there is a persistent bias in each relative to the control. A high initial temperature results in a persistent high bias throughout the 15-day period. A low temperature bias occurs for the initial profile with the large temperature jump at the base of the layer. The effect of the entrainment mixing of the very cold upper thermocline water in this case is magnified because the cooling occurs over a rather shallow layer. Finally, the higher temperatures for the case with an initial deep layer is a result of a special feature in the Garwood model. When a new mixed layer is formed near the surface, the layer temperature is adjusted to conserve heat and potential energy in the column. This adjustment is normally small. However, conserving the potential energy of the very erroneous profile introduces a warm bias which is maintained throughout the forecast period.

A summary of the effects of erroneous data insertions is given in Table 1. There is an implicit assumption in the one-dimensional model that the vertical mixing processes determine the oceanic thermal structure evolution. Because of the importance of the atmospheric forcing in this model, the response to erroneous data insertions is different for shallowing or deepening regimes. Although the depth predictions may appear very similar during shallowing periods, the erroneous layer depths are uncovered during deepening periods. The effects of erroneous temperature jumps at the layer base appear only during deepening regimes. An important feature of the erroneous temperature profile cases is that there is essentially no adjustment toward the correct solution. A temperature bias will continue until a new observation is inserted. This emphasizes the importance of a quality control procedure for examining new observations prior to insertion in the one-dimensional model.

## 4. Data assimilation tests

A proposed method for assimilating data taken at the same point but at random times will be demonstrated using model-generated data. The general approach in these tests is illustrated in Fig. 3. A 15-day "history window" is defined prior to the initial time. Simulated temperature profiles are available from the model for each 3 h interval during this period. Similarly, complete verification data are available each 3 h interval of the 15-day "forecast window." A random number generator is used to select the times at which separate sets of 5, 15 and 30 profiles are extracted during the history window. The last profile in each of these sets is labeled for use as the most recently available observation prior to the initial time. Comparisons of the model runs with these sets will be used to examine the impact of more observations during the history window. If the one-dimensional model is re-started with any one of the extracted temperature profiles, and the same forcing is used, the control run will be duplicated. Random errors are added to these profiles to simulate real profiles, including possible observational errors.

Special consideration must be given to the model characteristics in simulating temperature observations from the model-generated profiles. The accu-

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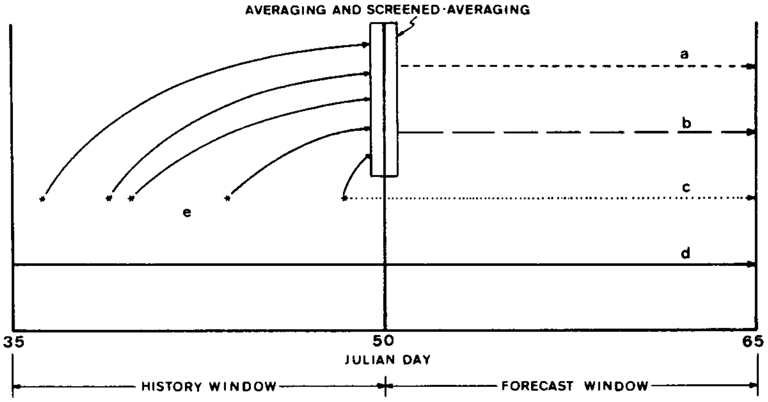

FIG. 3. Forecast paths for a winter regime study using five simulated temperature profiles, at the times represented by asterisks (e). Forecasts are based on the screened-average of observations (a), the average of all observations (b), the last available profile only (c), and the control run (d).

racy of present observations does not permit definition of the isothermal depth used in the model. Therefore, all of the simulated observations will be isothermal to the top of the thermocline and thus do not include the highly transient, near-surface isothermal layers that will appear in the verification data. Variations in the depth of the thermocline due to tidal and inertial influences, as well as advective processes, are commonly observed. However, these processes are not simulated by the model, and thus are not included in the verification data used here. Finally, the model-generated profiles include discrete temperature jumps in 1 m increments below the mixed layer. The magnitude of the jump in the first meter below the mixed layer determines the amount of deepening for a given entrainment heat flux. Normally available observations do not contain such vertical resolution. For these tests the thermocline temperatures are first represented by a logarithmic function

$$T = T_{200} + a \ln Z,$$

where $T_{200}$ is the temperature at 200 m. The slope ($a$) is determined from the model-generated profile using the temperature at a point 10 m below isothermal layer depth. The top of the thermocline (ZT) is then found by setting $T$ equal to the mixed-layer temperature. This "potential" mixed layer would ideally represent a conservative estimate of the isothermal layer if no near-surface layers are present.

With the logarithmic representation, each extracted temperature profile may be represented by only three parameters: mixed-layer temperature, $a$ and ZT. The random error added to the mixed-layer temperature is proportional to the standard deviation during the history window, plus an observational error of 0.25°C. The latter value dominates the variation during the winter case, whereas the standard deviation is about 0.85°C for the summer case. Random errors in ZT are 7 and 3 m for the winter and summer cases, respectively. Corresponding values of the slope random errors are 0.025 and 0.02. A representative sample of adjusted profiles for a winter case is illustrated in Fig. 4. The signs of the temperature and depth random errors are selected to

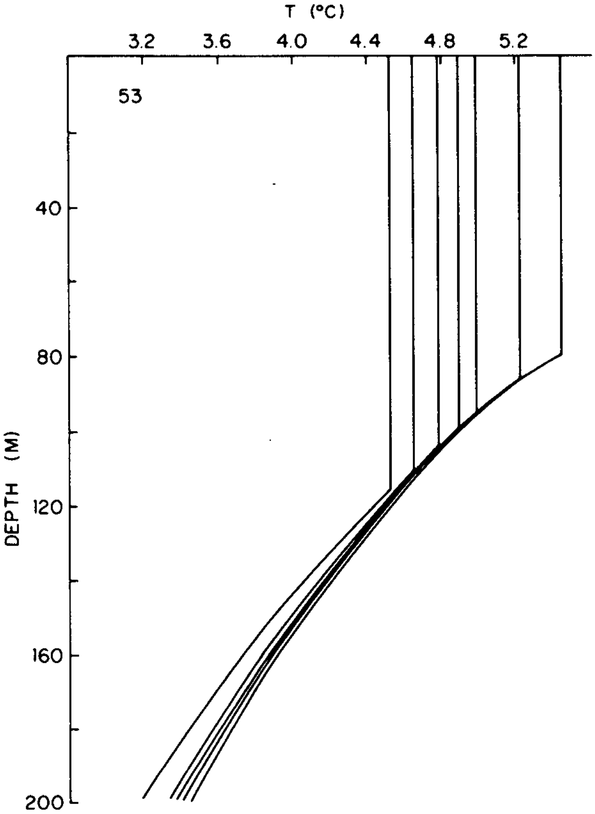

FIG. 4. Examples of temperature profiles generated by addition of random errors to the mixed-layer depth and temperature and the thermocline slope.

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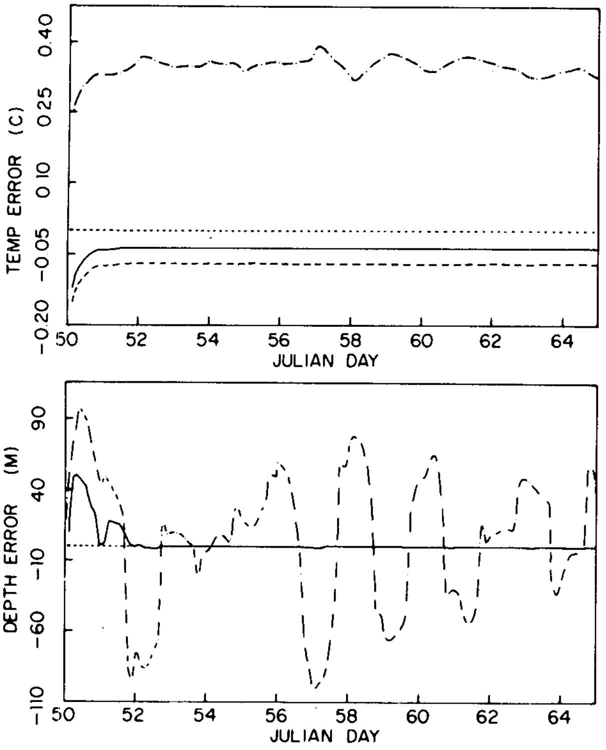

FIG. 5. Errors relative to the 1959 control run for (a) mixed-layer temperature and (b) mixed-layer depth. Predictions are initialized from averaged (dashed), screened-averaged (solid), and the last available (dashed-dotted) of the 30 profiles at Day 50. The averaged and screened-averaged depths coincide during much of the period.

produce either warm and shallow or cold and deep layers. Likewise, the sign for the slope error is taken to minimize the spread in the 200 m temperatures.

When the profiles represented on the left-hand side of Fig. 3 are modified, the predictions will no longer reproduce the control run. The errors in the predictions may be thought of as combinations of those discussed in the previous section. If each of these profiles is used and the model is integrated to the initial time, the resulting profiles are separate representations of the actual initial profile. However, each profile will tend to be in equilibrium with the atmospheric forcing at the initial time, as well as be consistent with the model used in the forecast window. It is in this sense that this procedure is termed data assimilation as used in meteorology.

The combination or blending of these profiles into a single initial profile is a simple average of the temperatures. The isothermal depths from the different representations are also averaged, and the initial profile is made homogeneous to that depth. Another initial profile, hereafter termed the "screened-average," is constructed in the same manner using only those representations in which the mixed-layer temperature was within 1.5 standard deviations of the mean temperature during the history window.

In summary, the data assimilation experiment compares the control predictions with those based on the last of *n* profiles, the average of *n*, and the screened average of *n* profiles. The value of *n* varies among 5, 15 and 30. The initial time is selected as 0000 GMT on Day 50 or Day 190 of 14 different annual simulations at ocean weather ship PAPA. In addition to the results from the individual years, an overall verification versus prediction time is produced.

## 5. Assimilation tests—individual cases

A variety of initial ocean thermal structure characterizations and atmospheric forcing conditions is desirable for testing the assimilation scheme. Nevertheless, the individual cases should be generally similar in character for meaningful ensemble averages. The control runs for all 14 years are started on 1 January. To provide time for any adjustments in the initial profiles, the first test period is centered on Day 50. The mixed-layer depths are typically about 80–140 m during this period in winter. Deepening beyond about 120 m would actually be constrained by the salinity profile in the pycnocline. Because salinity is not included here, the variations in MLD in the model are probably greater than in nature. However, the primary objective is to use the model evolution as a control for determining the effect of different initial profiles, rather than for model verification studies.

A summer season test is generated by centering the assimilation test on Day 190 in each of the 14 years. This is expected to be a period of maximum temperature increases within a relatively shallow layer. Two types of presentations are given for each season. In this section, some individual cases are presented to indicate the differences between winter and summer season assimilations. In the following section, the overall statistics will be summarized to compare the effect of data insertion frequency during winter and summer.

### *a. Winter cases*

There are generally three kinds of results in the winter period assimilations. An example of one type (four of the 14 cases) is shown in Fig. 5. This type is characterized by the large departures from the control depth during the first 1–2 days, and then an essentially perfect depth forecast for the averaged and screened-averaged initial profiles. During the initial adjustment period, the predicted depths are as much as 100 m greater than in the control run. The stability at the base of the layer and the atmospheric forcing in the control are favorable for relatively shallow layer depths. The initial layer depth based on the 15 assimilated profiles is only a few meters greater than the control, because each profile is consistent with the atmospheric forcing at

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Day 50. However, the averaging process has reduced the already small stability at the base of the layer. Thus, the same atmospheric forcing can eliminate the transient shallow layer and the layer becomes well-mixed to much greater depths. Subsequent periods with diminished atmospheric forcing and the associated formation of a shallow layer can follow the control depths very closely. This is similar to the case with the deep profile in Fig. 1.

Simply using the last of 15 (L-15) profiles (Fig. 5b) may result in an initial layer depth that is quite different from the control, because of the prescribed initial error in MLD. The predicted depth in this case is also too large relative to the control during the first day. However, the layer is too shallow by nearly 100 m during the second day. Whereas the layer depth during local nighttime in the control run is quite large, the prediction from the L-15 profile remains shallow. The L-15 depth prediction is quite close to the control during the next two days. However, the L-15 depths are alternatively too deep and then too shallow during the last 9 days of the test. Such a degradation in the predictions would clearly be unacceptable.

The corresponding temperature errors (Fig. 5a) relative to the control are generally small during the winter period. In this particular case, the L-15 profile is initially about 0.25°C higher, and the other two profiles are about 0.2°C too low. The screened-averaged profile is a little closer to the control, because one or more extreme layer temperature profiles have been omitted by the screening process. After the initial adjustment period, the predicted temperature evolutions for the A-15 and S-15 case closely parallel the control run. When the layer depth predictions in these cases are essentially perfect, one would expect the surface heat fluxes to be distributed correctly, and the temperature tendencies would be identical. The temperature in the L-15 case also generally parallels the control run, except during periods of extreme changes in layer depth. These changes are consistent with the discussion in Section 3.

A second type of result of the winter season assimilations applies to six of the 14 years. The A-15 and S-15 layer depth predictions (not shown) are generally excellent throughout the 15-day period, with only a few days of significant deviations from the control run. The L-15 depth errors are smaller than in Fig. 5, but there are alternatively positive and negative errors. As expected, the corresponding A-15 and S-15 temperature predictions are quite good when the layer depth changes are correctly predicted.

Finally, four of the 14 winter assimilations may be grouped into a third type, as illustrated by Fig. 6. As before, there is generally an initial adjustment period of one day in the A-15 and S-15 depth predictions. However, these depth traces relative to the control continue to have significant positive (too deep) errors throughout the period. This result is quite similar to the deep trace in Fig. 1. The L-15 depth predictions again have 1–2 day periods of negative errors, which the A-15 and S-15 predictions do not have. The explanation for the A-15 and S-15 predictions being too deep lies in the stability at the base of the layer and the character of the atmospheric forcing. During periods of general layer deepening due to strong atmospheric forcing, the assimilated profiles have a smaller density jump to oppose the deepening tendency compared to the control run. The smaller density jump arises from averaging some profiles that are isothermal to depths greater than the average depth with other profiles that have large temperature gradients in this zone, because the layer depth is smaller than the average depth. Even though the individual profiles may have approximately the correct density (temperature) gradient at the base of the layer, a simple level-by-level averaging of the temperatures will diminish the density gradient in the assimilated profile. Given sufficiently strong atmospheric forcing to deepen the layer, the assimilated profile will permit greater deepening than the control profile which has not been averaged. After the initial adjustment period, the A-15 and S-15 temperature evolutions in Fig. 6a show increasingly negative temperature errors relative to the control run. This is consistent with the positive depth errors for these cases. The L-15 temperature predictions also have a slightly negative trend, but there are superposed short-period temperature changes that are associated with the L-15 depth changes. It might also be noted that the screening process is effective in producing the smallest temperature errors in this assimilation test case.

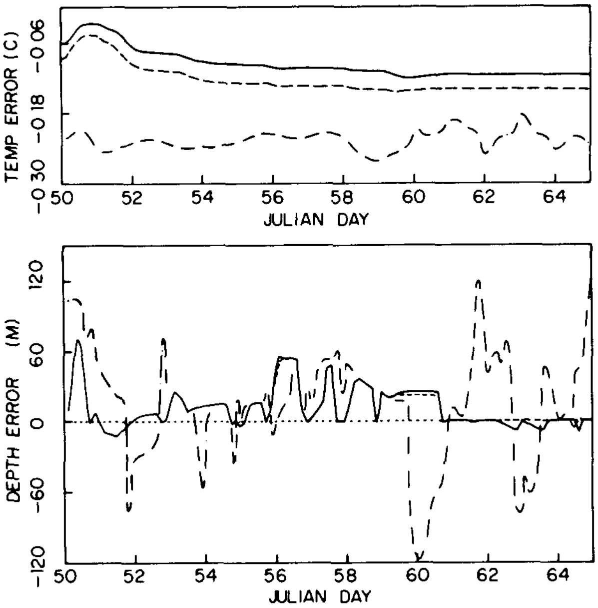

FIG. 6. As in Fig. 5, except for 1961 and the set of 15 profiles.

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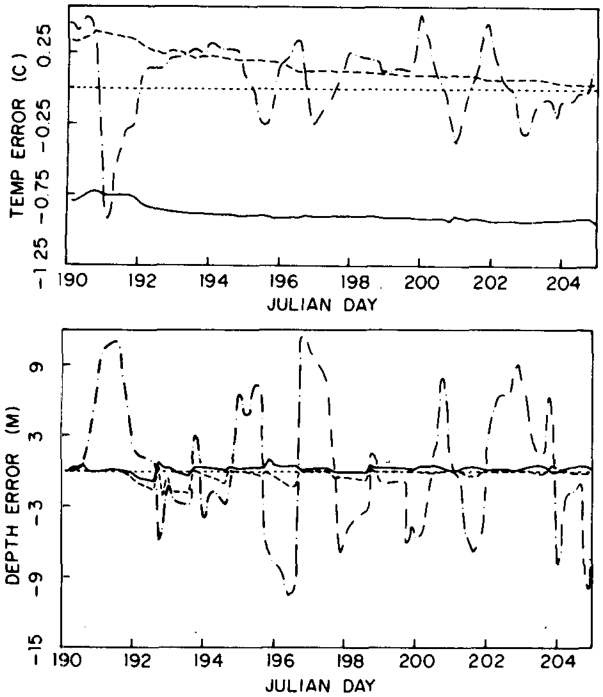

FIG. 7. As in Fig. 6, except for 1956 at Day 190.

In summary, the most sensitive variable is the mixing depth, because of its large variation between stormy and non-stormy periods. The balance between maintenance of a deep well-mixed layer and the tendency to form shallow layers nearer the surface can be rather delicate. If the stability of the shallow layers is not correctly represented, the layer depth predictions can have large errors. After an initial adjustment period of 1–2 days, the averaged and screened-averaged assimilated profiles generally resulted in the correct layer depth predictions. Using the last available profile often resulted in shallow layers when the control run had a deep layer, and vice versa.

### *b. Summer cases*

It is only necessary to show one example (see Fig. 7) of a summer period assimilation. The primary difference from the winter period is that the layer depths are quite small and they are closely tied to the atmospheric forcing. Averaged or screened-averaged profiles generally produce initial layer depths that are near the control. As shown in Fig. 7b, the depth predictions from these profiles have errors of less than 2–3 m. Consequently, the corresponding temperature evolutions tend to parallel the control run. The temperature error is primarily due to the initial error, which is similar to the erroneous data insertion cases in Fig. 2. The initial screened-averaged temperature was nearly always too low relative to the control case. All 14 screened-averaged predictions had mean temperature and depth errors that were too cold and too low, whereas the predictions from the last available profile had nearly equal numbers of too warm versus too cold. The cause of the low temperature bias in the screened-averaged cases is that screening based on the averaged mixed-layer temperature during the history window results in the exclusion of the warmest (and most shallow) profiles. The surface layers are being warmed rapidly during this season. The increase in temperature from the history window to the forecast window averaged 1.69°C in this 14-year sample. Given such large warming rates, the surface temperatures can have a large range during a 15-day period. One must allow a variation greater than the 1.5 times the standard deviation of temperatures in the history window that was used here as the screening criteria.

The L-15 depth predictions (see Fig. 7b) show both positive and negative errors. An initial L-15 depth error of zero indicates an equilibrium with the atmospheric forcing at that time. However, the increase in atmospheric forcing on Day 191 eliminates the transient layer depth and exposes a deeper seasonal thermocline. A similar event occurs on Day 197. Each of the periods with larger layer depths can be associated with a temperature decrease relative to the control. On Day 191, the decrease is over 1.0°C, whereas the decreases are smaller during the other deepening events. It is of interest to note that the L-15 profile starts with the same temperature error in the A-15 case, and generally follows the same trend, except during the anomalous periods of deepening and shallowing in the L-15 prediction.

Unlike the winter case, the most sensitive variable in the summer assimilation tests is clearly the temperature rather than the layer depth. The averaged assimilated profiles result in predictions with depth errors that are nearly always less than 3 m, and with temperature errors that are almost entirely due to the initial error. As indicated above, one must be very careful in screening the profiles so that profiles with high surface temperatures (and shallow layer depths) are not systematically excluded. The temperature predictions based on the last available profile can be quite misleading at some times.

## 6. Assimilation tests—overall statistics

Root-mean-square errors (RMSE) are calculated for the 14 values of layer depth and temperature that are available at each 3 h period of the integrations. In addition, the overall means and RMSE for the entire 15-day integrations are calculated to compare quantitatively the differences between the assimilation methods and the number of available profiles during the history window.

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### *a. Winter cases*

The RMSE of layer depth and temperature are shown in Fig. 8 for the tests with the set of 15 initial profiles. Similar results are generally found when only five or 30 profiles are available in the history window. The differences due to the number of available profiles will be discussed below. As indicated in the description of the individual cases, the A-15 and S-15 depth predictions have large errors during the first day of integration. However, the errors are markedly reduced during the next few days. The diurnal oscillation in depth RMSE is larger than the daily mean RMSE, which is approximately equal to the observational error in depth (10 m) that was randomly added to the history profiles. The screened-averaged depth predictions are only slightly better than the averaged predictions. Using the last available profile results in rather large depth errors during the first few hours of these 14 integrations. There is a strong diurnal oscillation in the depth RMSE for the remainder of the period. Minimum depth errors occur at 1500 GMT (0500 local) and maximum errors occur at 0000 GMT (1400 local). This suggests that predictions based on the last available profile frequently fail during the time of transient shallow layers. As noted above, there are both positive (transient layer not predicted) and negative (transient layer incorrectly predicted) errors.

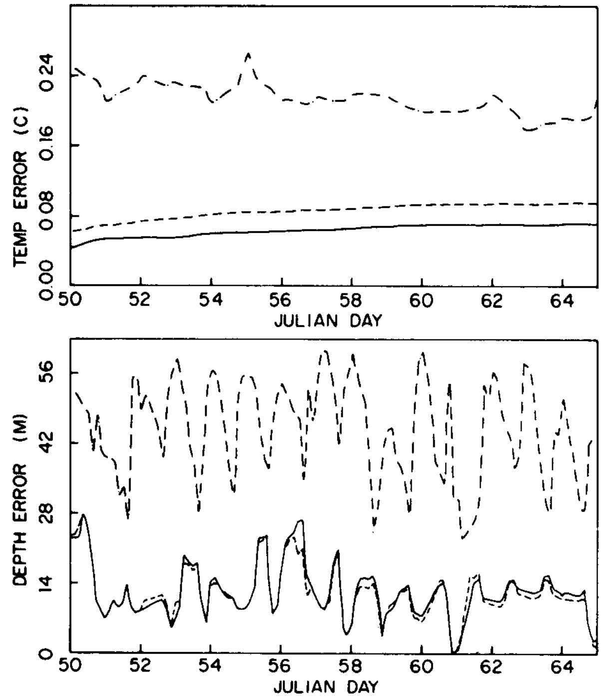

FIG. 8. Root-mean-square-errors relative to the control runs for a 14-year sample of predictions of (a) mixed-layer temperature and (b) mixed-layer depth initialized from averaged (dashed), screened-averaged (solid) and the last available (dashed-dotted) of the set of 15 profiles at Day 50.

TABLE 2. Root-mean-square-errors (RMSE) and overall average error (bias) in 3 h depth (m) predictions during the winter season (sample size = 1680) for different sets of input profiles in the history window. L, A and S indicate predictions made from the last available, the averaged and the screened-averaged initial profiles.

| | L | A | S |
|---|---|---|---|
| | | Set of 5 | |
| RMSE $h$ | 45.7 | 12.5 | 13.4 |
| $\bar{h}$ | 3.3 | 5.2 | 6.3 |
| | | Set of 15 | |
| RMSE $h$ | 45.8 | 13.1 | 13.5 |
| $\bar{h}$ | 3.3 | 5.6 | 6.8 |
| | | Set of 30 | |
| RMSE $h$ | 46.7 | 13.4 | 13.6 |
| $\bar{h}$ | 4.9 | 5.7 | 5.8 |

The RMSE in temperature for the A-15 and S-15 predictions are less than 0.10°C throughout the 15-day integration. There is a trend for increasing temperature error with time for the A-15 cases, whereas the S-15 cases have a decreasing trend. It is not surprising that the typical RMSE in temperature for the 14 cases with the L-15 profiles is nearly equal to the superposed observational error of 0.25°C. There is some reduction in error with time, but the improvement is small.

The overall RMSE in layer depth for the 15-day integrations presented in Table 2 are useful to specify the differences between the assimilation types. All of the averaged and screened-averaged cases have similar errors. The predictions from the last available profiles have layer depth errors that are more than three times as large. Each set has positive overall mean layer depth errors. This too deep bias is obvious in the A-15 and S-15 individual cases. Even though the corresponding L-15 cases have negative depth errors, there is still a bias toward overly deep layers. Another feature of interest in Table 2 is that the RMSE in layer depth are slightly smaller for the sets of five than for sets of 15 or 30 input profiles. The overall RMSE of temperature (not shown) shows an opposite tendency. Minimum temperature errors are found for the sets of 30 and maximum errors in each method are found for the sets of five input profiles. However, the differences due to the number of profiles are less than 0.04°C.

We conclude from these winter tests that the averaged and screened-averaged assimilation methods produce layer depth and temperature errors that are a significant improvement over the use of the last available profile. As few as five observations during

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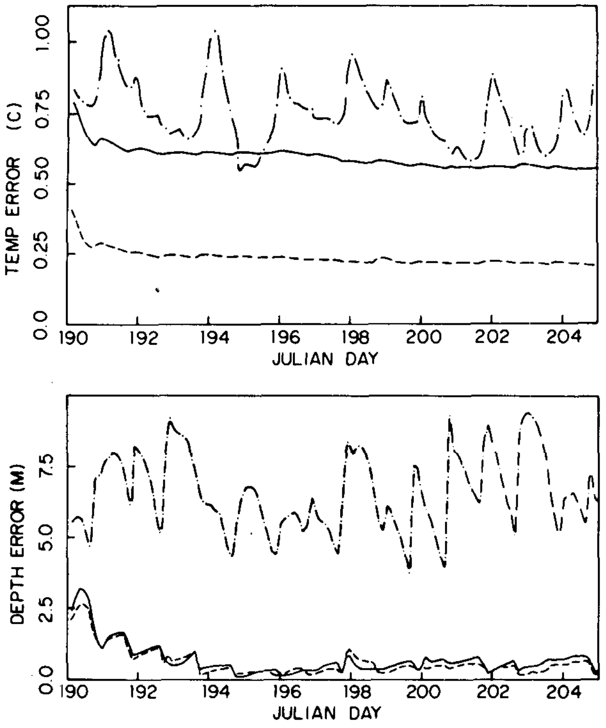

FIG. 9. As in Fig. 8, except for Day 190.

the 15-day history window are sufficient to reduce the uncertainty in the initial profiles, if these profiles are advanced in time by the mixed-layer model to the beginning of the forecast. After an initial adjustment period with large layer depth errors, the predictions appear to be useful for at least 15 days. Each of these conclusions is dependent on the credibility of the model used to advance the observations and on the availability of atmospheric forcing data. In practice, the atmospheric forcing would be well known during the history window.

### *b. Summer cases*

A similar presentation of the time variation in RMSE of depth and temperature for the summer cases is given in Fig. 9. The striking feature in the layer depth predictions is the reduction in magnitude from the winter cases. Both the averaged and screened-averaged depth predictions have initial errors of only 3 m. These errors decrease in time until a minimum of less than 1 m occurs about 6 days into the prediction. Even with the slow increase in layer depth errors during the remainder of the integrations, the values are less than the observational error of 3 m that is inserted in the history profiles. As was the case in the winter tests, the predictions from the last available profile have much larger depth errors. There is a pronounced diurnal oscillation in these errors. Maximum and minimum errors tend to occur around 1100 local and 0500 local, respectively. This suggests that the timing and magnitude of the almost daily transition to a shallow layer during the summer is not well predicted from the last available profile.

The other striking feature from the summer cases is the increase in temperature errors compared to the winter cases. This is not unexpected, because of the small layer depths and the large downward heat flux. The best results are clearly obtained with the averaged assimilation technique. After the first day, there is a slight trend toward smaller errors. Since the random error that is added to the initial profiles is more than 1°C, the predicted temperature errors of about 0.25°C are quite good. The screened-averaged assimilation has larger temperature errors because the screening erroneously removed the profiles that had the highest surface temperatures. The resulting cold bias in the initial temperature is demonstrated by the parallel behavior of the averaged and screened-averaged temperature errors with time. Finally, the largest and most variable errors occur in the predictions from the last available profile. Values exceeding 1.0°C are found after 1 day of integration. There is some improvement during later periods, but these predictions are clearly less useful.

The overall RMSE and mean temperature errors in Table 3 tend to confirm these interpretations regarding the different assimilation methods. The effect of numbers of available profiles on the accuracy of the temperature predictions during summer is somewhat variable. For the averaged profiles which produce the best predictions, the smallest errors occur if 30 profiles are available. For the other cases with poorer predictions, the smallest errors occur when only five initial profiles are available. These

TABLE 3. Root-mean-square-errors (RMSE) and overall average error (bias) in 3 h temperature (°C) predictions during the summer season for different sets of input profiles in the history window. L, A and S indicate predictions made from the last available, the averaged and the screened-averaged initial profiles.

| | L | A | S |
|---|---|---|---|
| | | Set of 5 | |
| RMSE $T$ | 0.49 | 0.23 | 0.50 |
| $\bar{T}$ | 0.04 | −0.02 | −0.37 |
| | | Set of 15 | |
| RMSE $T$ | 0.76 | 0.24 | 0.59 |
| $\bar{T}$ | 0.12 | −0.10 | −0.53 |
| | | Set of 30 | |
| RMSE $T$ | 0.82 | 0.17 | 0.75 |
| $\bar{T}$ | 0.14 | −0.06 | −0.59 |

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tests suggest that a relatively small number of profiles is necessary to minimize the effect of observational error and natural variability, if the observations are properly advanced in time.

## 7. Conclusions

The first purpose of this work is to demonstrate an assimilation technique that is appropriate for use with a one-dimensional ocean mixed-layer model. Such a model exhibits perfect assimilation. That is, insertion of a new thermal structure observation at a point has no blending, diffusive or other effects at adjacent points. The only shock to the model prediction due to the insertion of a new profile arises from a change in thermal structure parameters that affect the vertical mixing processes. Any inconsistencies among the new mixing depth, the new density jump at the base of the mixing layer, and the strength of the atmospheric forcing will produce rapid changes early in the prediction. The effects of the erroneous temperatures tend to persist throughout the forecast interval. Depth errors tend to be small during shallowing periods, and large during deepening events. This is because the depth of a shallowing mixed layer is determined primarily by the atmospheric forcing, whereas the depth of a deepening mixed layer is also affected by the internal ocean structure.

For the winter season tests, the most sensitive variable is the mixing depth. There is an initial adjustment period of 1–2 days during which the depth errors are large with all methods. The predictions from the averaged and the screened-averaged initial profiles typically have root-mean-square errors of about 10–15 m. These depth errors grow slowly during the 15-day predictions. The errors in the predictions from the last available profile are 3–5 times larger than the errors in the averaged and screened-averaged cases. The average magnitude and marked diurnal variation in these errors greatly reduce the usefulness of the depth predictions from the last available profile. Errors in surface temperature predictions from the averaged and screened-averaged initial profiles are about 0.07°C during the winter period. Corresponding values for the predictions from the last available profile are about 0.24°C, which is close to the random observational error added to the initial profiles.

During the summer, the mixed-layer temperature errors are a more sensitive indicator of the usefulness of the assimilation technique. The predictions from the averaged initial profile have typical errors of 0.25°C, whereas those from the last available profiles are 3–4 times larger. Similarly large errors are found for the screened-averaged initial profiles cases. However, the temperature errors in these cases are almost entirely due to a low temperature bias introduced initially because the screening process removed the profiles with highest surface temperatures. An initial adjustment in the layer depth occurs during the first day of the predictions. Following this adjustment period, the depth predictions from the averaged and screened-averaged profiles are essentially equal to the vertical depth increment of 1 m. Depth errors ranging from 5–9 m are found for the predictions from the last available profiles.

The conclusion from these model-generated data assimilation tests is that the ocean prediction model can be used to advance recent observations to the initial forecast time. If as few as five observations are available during the 15-day history for blending the profiles to reduce the random errors, useful predictions of mixed-layer depth and temperature are produced for as long as 15 days. Having 15 or 30 observations during the history window produces a relatively small improvement in the most sensitive variable and may even result in some deterioration. Therefore, it is concluded that the relatively few ocean thermal structure observations could be retained in the analysis for extended periods if they are suitably advanced in time.

There are several limitations in these tests. First, the one-dimensional mixed-layer model does not contain such physical processes as advection or buoyancy changes due to salinity effects. The thermal variability induced by these processes during the advancement of the history profiles or during the prediction is not included in these tests. Second, real observations may contain more variability than the model-generated profiles. The observations may include systematic as well as random errors. Furthermore, the initial profiles are generated from the same model that is used in the prediction. Finally, the atmospheric forcing used in these tests is essentially perfect, in the sense that both the predictions and the control run use the same forcing. While the atmospheric forcing will be available for advancing the observations to the initial time, the ocean predictions will be limited by the accuracy and predictability of the atmospheric forcing. Thus, these tests only provide an indication of the best predictive limit due to random errors in the observations.

Some of the limitations in these tests will be removed in future work. As improved ocean prediction models are developed, they can be used in the data assimilation. Actual temperature profiles from ocean weather ships can be used, although this means that there will be incomplete and noisy verification data sets. More realistic tests can be run by also including the effects of errors in the atmospheric forcing. Based on these model-generated data tests, it is anticipated that a data assimilation technique similar to the one demonstrated here will be useful in the diagnosis and prediction of upper ocean thermal structure.

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