NOTES AND CORRESPONDENCE

Topographically Controlled Flow Around a Deep Trough Transecting the Shelf off Kodiak Island, Alaska[1]

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ABSTRACT

Current measurements from the axis of a deep trough normal to the coast and from the adjacent shelf show that the mean flow is barotropic and follows depth contours, conserving potential vorticity, to form a cyclonic vortex or meander over the trough. The data are interpreted as an example of an inertial Taylor-Proudman column on the continental shelf. The scale of the topographic variations dominates the potential vorticity balance and a simple steady-state numerical model is in good agreement with the data when the vorticity balance approaches the limit $\mathbf{U}\cdot\nabla H = 0$. Streamlines following isobaths converge over steeper topography and current speeds are nearly proportional to the local topographic gradient. Estimates from the data support this behavior and indicate that the $\sim$20 cm s^{-1} mean current around the trough is driven by a typical cross-shelf-averaged velocity scale of $\sim$5 cm s^{-1}.

1. Introduction

The continental shelf adjoining the southeastern shore of Kodiak Island is complex, with alternating shallow banks (50–100 m depths) and deep troughs (150–200 m depths) cutting across the shelf. Depth variations nearly as large as the depth of the shelf break occur with alongshelf length scale equal to or less than the shelf width (Fig. 1). Several recent studies on the impact of alongshelf topography have dealt with the wind-driven circulation and upwelling (e.g. Hsueh, 1980; Han *et al.*, 1980; Killworth, 1978; Peffley and O'Brien, 1976). Hsueh (1980) and Galt (1980) also examined the effect of topography on a steady alongshelf flow with no wind forcing.

These latter two models made use of a vorticity relation in which cross-isobath flow was balanced by the curl of the bottom stress. Inertial terms, which lead to potential vorticity conservation, were neglected. Such a balance requires scaling where the square root of the (vertical) Ekman number is much greater than the Rossby number (Pedlosky, 1979, p. 201). With qualitatively valid results, Hsueh (1980) applied his model to the Hudson Shelf Valley, in the New York Bight region, and Galt (1980) applied his to areas in the northern Gulf of Alaska, including Kodiak Island. Nevertheless, there is evidence that inertial effects can be important on deeper shelves with large vertical-scale bathymetric features of short length scales. For example, Eide (1979) described an anticyclonic vortex over the Haltan Bank, Norway, which was well described by the stratified, potential vorticity model of Hogg (1973). Butman *et al.* (1982) have observed a definite anticyclonic circulation around the shallow Georges Bank region on the east shelf of North America. In both cases, the anticyclonic flow around shallow banks indicates a tendency toward potential vorticity conservation whereby relative vorticity becomes more anticyclonic along streamlines where flow is toward shallower water.

The present study focuses on the Kiliuda Trough–Middle Albatross Bank region of the Kodiak shelf, where the deep canyon should have a significant signature on the mean circulation owing to the tendency to conserve potential vorticity. Direct current measurements and results from a simplified barotropic, potential vorticity model presented here indicate that a steady, cyclonic, topographically trapped meander or vortex resides over Kiliuda Trough. This circulation, mentioned by Muench and Schumacher (1980), is somewhat evident in the results of Galt (1980), and is given more direct attention here.

Kodiak Island is situated in the northwest Gulf of Alaska in the perimeter of the cyclonic Pacific Subarctic Gyre. As the flow in the gyre turns southwestward it intensifies to form a western boundary current, the Alaskan Stream (50–100 cm s^{-1}), which is evident over the continental slope off Kodiak (Favorite and Ingraham, 1977; Reed *et al.*, 1980) (Fig. 1). A density-driven low-salinity coastal current parallels the oceanic circulation around much of the Gulf (Royer, 1981) and appears to pass north of Kodiak through Shelikof Strait (Schumacher and Reed, 1980).

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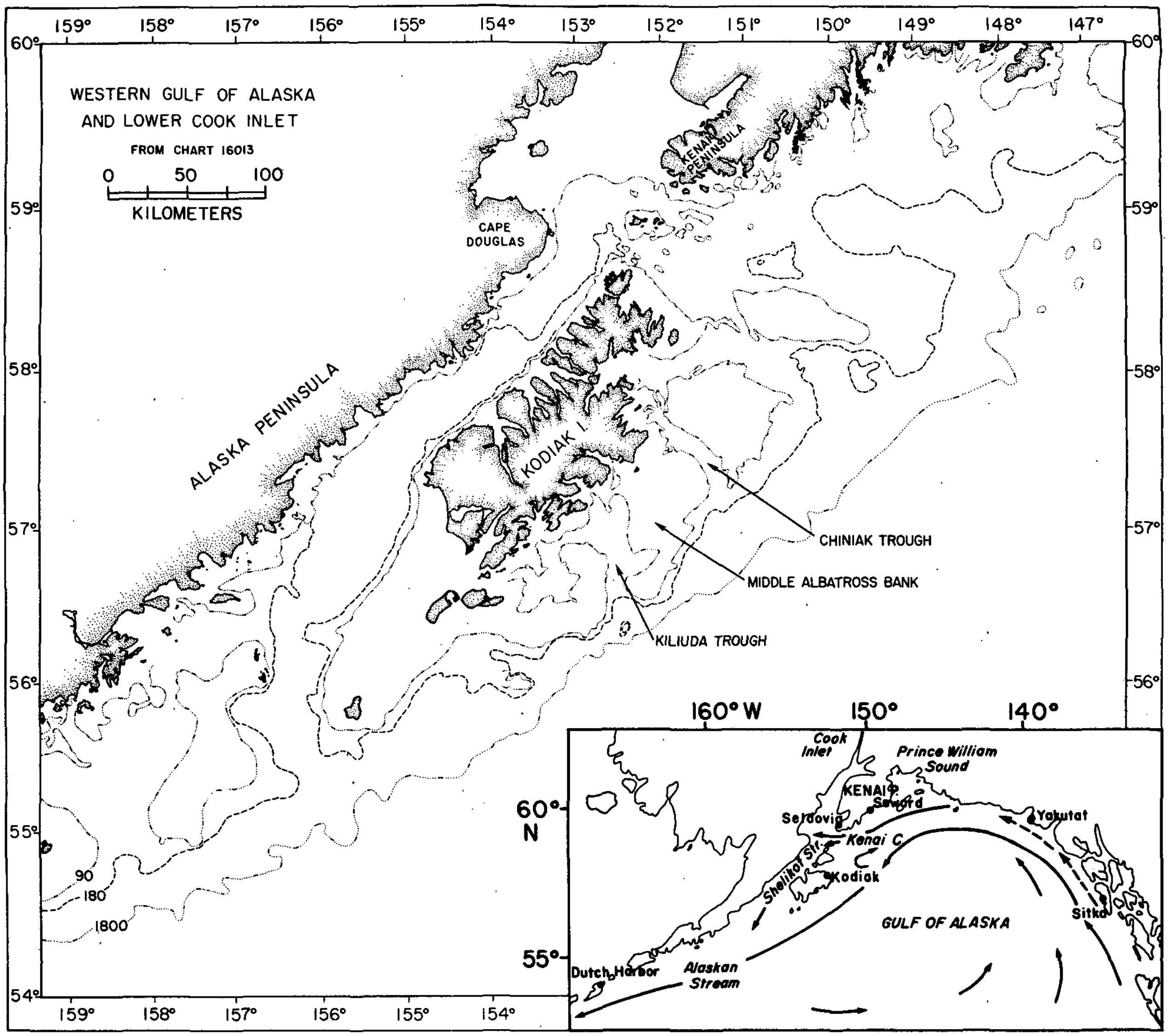

FIG. 1. Topography of the shelf off Kodiak Island showing Kiliuda and Chiniak troughs separated by Middle Albatross Bank. Depth is contoured in meters. Inset: The northern Gulf of Alaska and regional ocean circulation.

During periods of high runoff (September and October), some of this flow may pass over the southeast Kodiak shelf or runoff from Kodiak Island may generate a similar feature locally. It is not certain to what extent flow over the Kodiak shelf is driven by local conditions, by upstream conditions, or by conditions at the shelf break. Nevertheless, data show that the spatial and temporal mean flow over the shelf parallels the Alaskan Stream, but at much reduced intensity (Muench and Schumacher, 1980).

2. Data description

The current-meter mooring locations (K5–K10) and mean currents at depths indicated are given in Fig. 2. Details of mooring design and data statistics were presented by Muench and Schumacher (1980). Moorings K6–K10 were occupied for a four-month period during the winter of 1977–78 and again for four months during the summer 1978. Mooring K5 was occupied for the same winter period a year earlier (1976–77) and is included as a supportive observation. The arrays consisted of Aanderra RCM-4 current meters in a taut-wire configuration with subsurface flotation at about 20 m. Data in this analysis were processed with a 35 h low-pass filter to remove tidal and inertial signals.

The error bars fixed to the 25 m depth vectors in Fig. 2 denote one standard deviation (sub-tidal) in the major and minor axes. The flow at stations K5–K8 was quite stable and followed the trend of the bathymetry to within one standard deviation. The

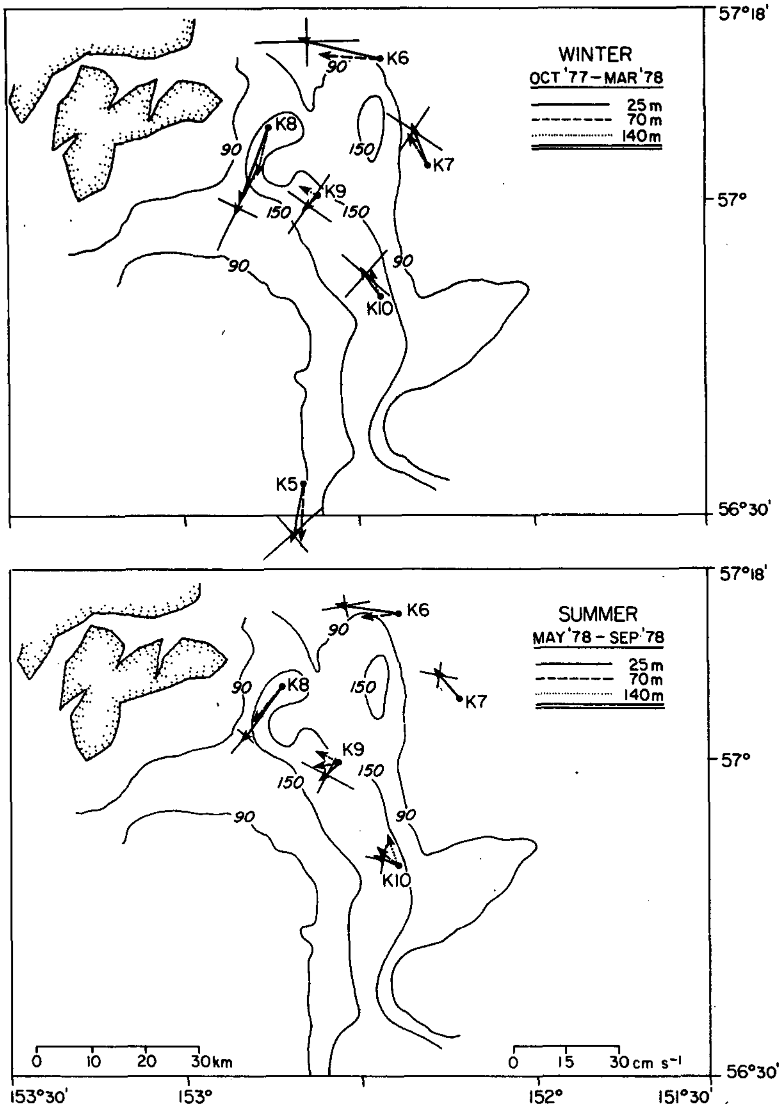

FIG. 2. Mean (~4 month) currents near Kiliuda Trough during winter (top) and summer (bottom). Error crosses represent one standard deviation (35 h low-pass) in the major and minor axes of variance for currents at 25 m depth. Mooring K5 was in place during the same winter period one year earlier and is not concurrent with the others. Depth is contoured in meters.

vertical shear, whether due to viscosity or stratification, does not indicate a significant deflection of along-isobath flow. Since the flow pattern was similar between winter and summer, the data from moorings K6–K8 indicate that they were on the perimeter of a steady cyclonic feature. It appears that moorings K9 and K10 were near the middle of the feature. Mean currents there were insignificant at less than one standard deviation. Vertical shear in the mean-flow direction was apparent but may have been significant only during the summer.

Additional evidence for the steady, cyclonic circulation pattern trapped over the Kiliuda Trough can be found in the series of three infrared satellite images in Fig. 3. These were taken on 22, 23 and 27 February 1979. Note the pattern of warm water extending shoreward over the eastern edge of the trough in the first frame. With time this water mass appears to have been advected around the head of the trough and then seaward over the western trough edge. Also note that Chiniak Trough, to the east of Kiliuda, had a similar resident of warm water. Several of the other satellite images that were examined did not have quite the clarity and contrast of these, but did indicate warm water extending across the shelf in the vicinity of Kiliuda and Chiniak Troughs.

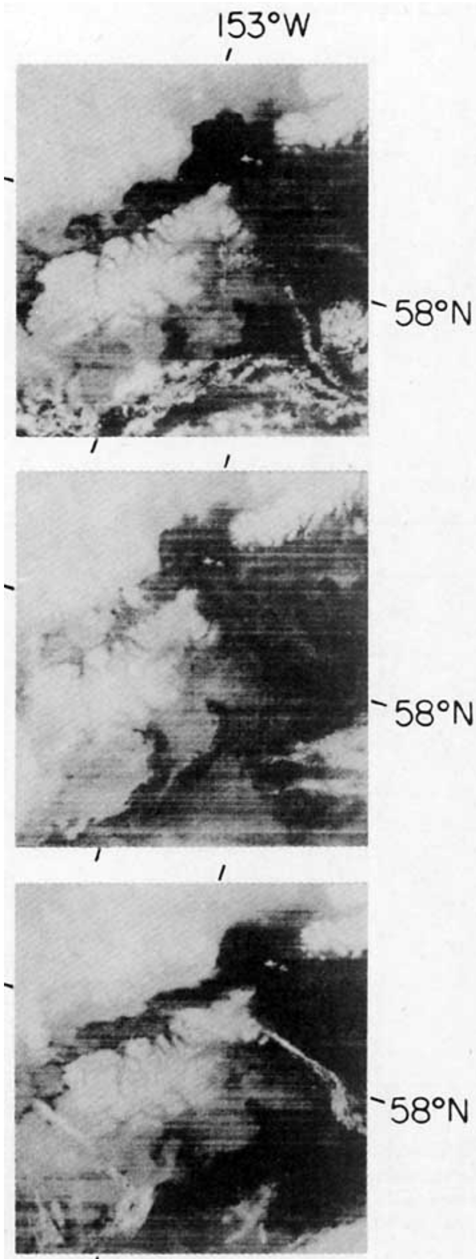

FIG. 3. Sequence of infrared images taken on 22, 23 and 26 February 1979 by the Nimbus-7 Coastal Zone Color Scanner. Kodiak Island is the whiter area in the center. White areas indicate cloud or snow cover. Warm water is indicated by darker shades and cool water by lighter shades.

3. Hypothesis

The data constitute an example of the Taylor-Proudman effect, which is likely to occur in the ocean under appropriate conditions (e.g., Hogg, 1973; Huppert, 1975). This theory holds that steady flow in a rotating, constant-density fluid does not vary along the axis of rotation or, equivalently, that barotropic, geostrophic flow follows isobaths. The effect is modified by nonlinearity, stratification and viscosity. Eide (1979) found that the data from Halten Bank were in good agreement with a stratified Taylor-Proudman column model. One would expect flow encountering a depression or trough to show analogous behavior.

In describing the initiation of stratified Taylor-Proudman columns, the investigators mentioned above have utilized several scaling parameters, which are here applied to the Kodiak shelf to assess the relevance of the theory. Table 1 lists the appropriate scales and scaling parameters. Most important are the Rossby number (ϵ) and the Ekman numbers (E_H, E_v), all of which must be very small for the lowest-order steady flow to be geostrophic.

Hogg (1973) and Huppert (1975) investigated inertial Taylor-Proudman columns where $E_v^{1/2} \ll \epsilon \ll 1$. That is, advection of the relative vorticity exceeds the Ekman suction. On the Kodiak shelf it appears that $E_v^{1/2} \approx \epsilon$ (Table 1), so that both advection and Ekman suction should be of similar importance. The same scales should be valid for the areas investigated by Hsueh (1980) and Galt (1980) with models that neglected the inertial terms. The data from the Kodiak shelf indicate a cyclonic, or positive, vortex over a deep trough and anticyclonic flow around a shallow bank; a pattern consistent with the conservation of potential vorticity. For this reason this study uses an inviscid potential vorticity model. Owing to the scaling, such a model should be no less valid than one which includes Ekman suction but not the inertial terms.

The stratification parameter $S^{1/2}$ measures the baroclinic shear. For $\epsilon < S^{1/2} < 1$, a topographically induced disturbance decays vertically with an e-fold-

TABLE 1. Scales and scaling parameters appropriate to the Kiliuda Trough region of the Kodiak Shelf.

$U \approx$ 5–20 cm s^{-1}	velocity scale
$L \approx$ 20 km	length scale (trough width)
$f \approx 1.2 \times 10^{-4}$ s^{-1}	Coriolis parameter
$A_v \approx 10^2$ cm^2 s^{-1}	vertical eddy viscosity
$A_H \approx 10^5$ cm^2 s^{-1}	horizontal eddy viscosity
$D \approx$ 200 m	depth scale (trough axis)
* * * * * * *	
$\epsilon = \dfrac{U}{fL} \approx 0.02$–$0.08$	Rossby number
$\tfrac{1}{2}E_v = \dfrac{A_v}{fD^2} \approx 2 \times 10^{-3}$; $E_v^{1/2} \approx 0.06$	vertical Ekman number
$\tfrac{1}{2}E_H = \dfrac{A_H}{fL^2} \approx 2 \times 10^{-4}$	horizontal Ekman number
$N^2 = \dfrac{g}{\rho}\dfrac{\partial \rho}{\partial z} \approx 3.5 \times 10^{-3}$ to 9.5×10^{-3} s^{-1}	Brunt-Väisälä frequency
$S^{1/2} = \dfrac{ND}{fL} \approx 0.29$–$0.80$	stratification parameter
$\lambda^{-1} = \dfrac{fL}{N} \approx 250$–$700$ m	vertical e-folding scale

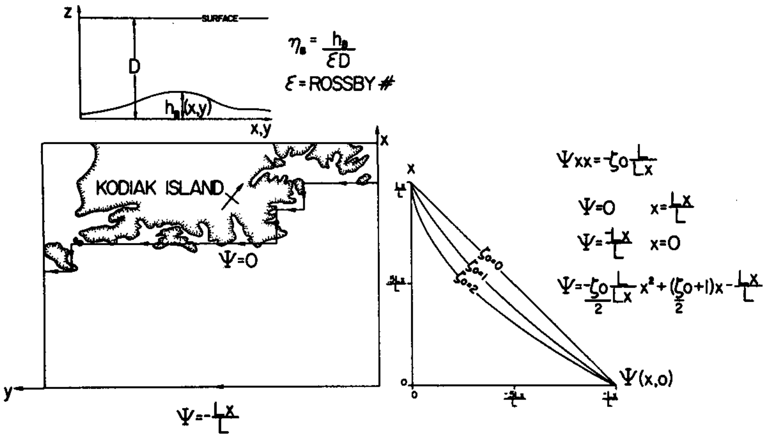

FIG. 4. Schematic scaled bathymetry and model boundary conditions.

ing scale, $\lambda^{-1} = D/S^{1/2}$, which has been identified for steady (Huppert, 1975) and unsteady (e.g., Rhines, 1977) stratified flows interacting with topography. Therefore $S^{1/2}$ is the ratio of the depth scale to the e-folding scale, and if $\lambda^{-1} \gg D$, then the baroclinic shear is small.

Table 1 shows a range of $S^{1/2}$ computations based on the range of Brunt-Väisälä frequencies representing the seasonal extremes of stratification noted from CTD measurements made over the axis of Kiliuda Trough. Only during periods of peak stratification is λ^{-1} on the order of the depth of the trough axis; usually it is much greater. The current-meter data above show that flow at all depths was uniform in direction except possibly near the middle of the trough vortex where the depth was greatest and the mean flow weakest. The stratification seems sufficiently weak overall that a barotropic model will adequately describe the topographic effects in this case.

A steady, nondimensional, barotropic, inviscid, quasi-geostrophic potential vorticity equation in an f-plane can be derived by expansion of the momentum and continuity equations in powers of ϵ to yield[2]

$$\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\left(\nabla^2\psi + \eta_B\right) = 0, \qquad (1)$$

where ψ is the geostrophic streamfunction, or nondimensional sea level, defined by $u = -\partial\psi/\partial y$, $v = \partial\psi/\partial x$; and η_B is given by $\eta_B = (h_B/\epsilon D)$, where h_B is the height of the bottom above the reference depth D, as shown schematically in Fig. 4.

This expression was derived by assuming h_B/D is small, or $O(\epsilon)$. As such it is a valid approximation for the lowest-order flow since it approaches

$$\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\eta_B = 0$$

for $h_B/D \rightarrow O(1)$ (Pedlosky, 1979, p. 91). This is the well-known result that when the topographic relief is large, geostrophic flow follows depth contours. The expression for potential vorticity, $\nabla^2\psi + \eta_B$, however, is valid only for small h_B/D. Therefore, Eq. (1) is only an approximation for potential vorticity conservation (in dimensional variables with ζ as relative vorticity and H as depth):

$$\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right)\left(\frac{f+\zeta}{H}\right) = 0. \qquad (2)$$

If this expression is non-dimensionalized, then after some algebra, it is found that (1) retains certain $O(\epsilon)$ terms while neglecting others when $h_B/D \rightarrow 1$. Nevertheless, in that limit, the potential vorticity is so dominated by the topographic term that the effect would hardly be noticeable. The main advantage of using (1) over (2) is that it avoids the singularity of $H \rightarrow 0$ near the coast and facilitates the numerical computation while still adequately approximating the lowest-order flow.

Eq. (1) states that the potential vorticity $(\nabla^2\psi + \eta_B)$ is conserved along a streamline. An equivalent expression is

$$\nabla^2\psi + \eta_B = K(\psi), \qquad (3)$$

where $K(\psi)$ is a value that must be determined for each streamline.

Eq. (3) was solved numerically using a finite-differencing scheme. The layout of the model is shown in Fig. 4. The presence of the Alaskan Stream was partially taken into account by making the seaward boundary (y axis) a streamline. However, the velocity scales were chosen to be typical of currents over the shelf, which are substantially weaker than in the

[2] See, for example, Pedlosky (1979, Chap. 3), from which this notation has been borrowed.

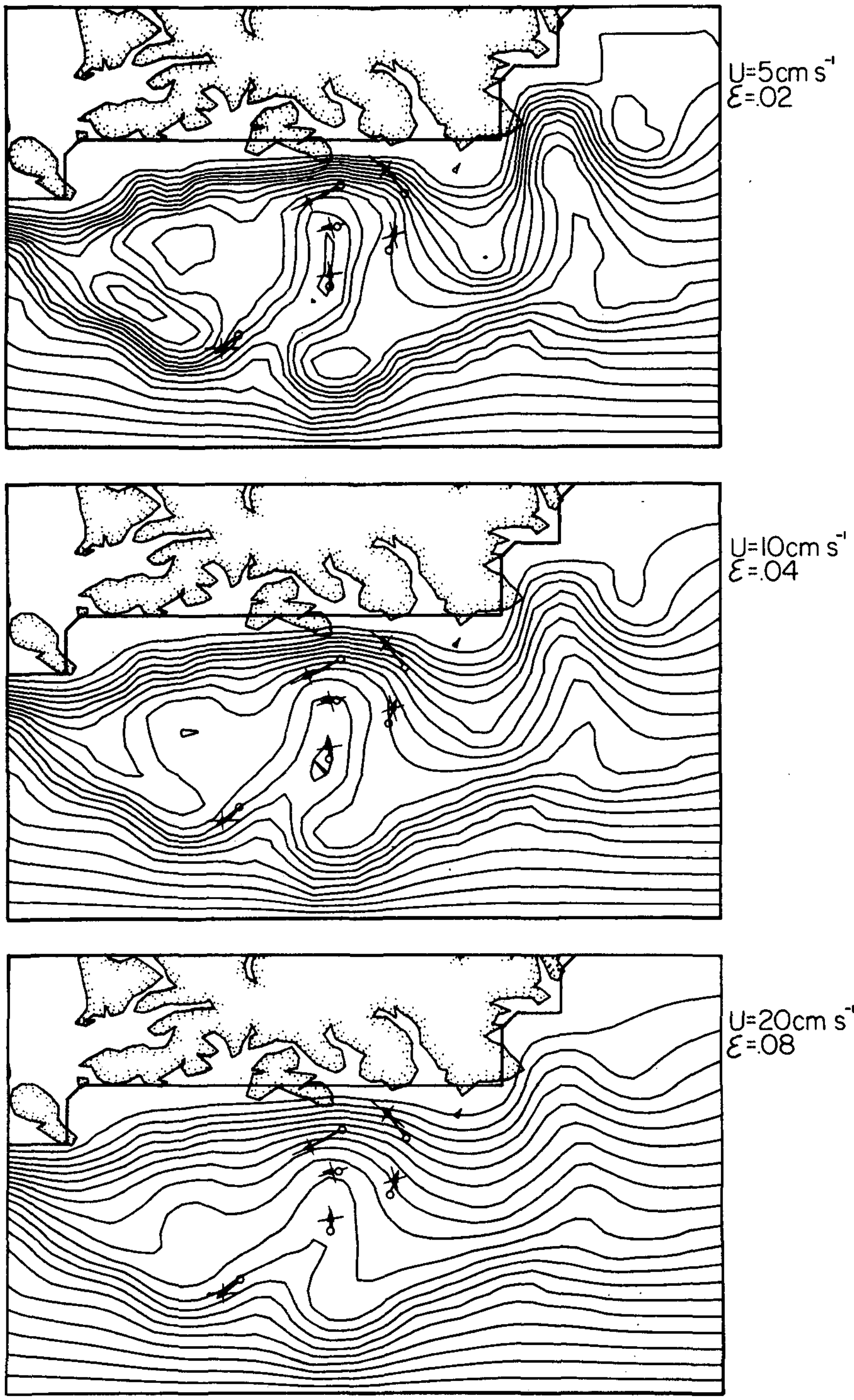

FIG. 5. Streamlines for a range of Rossby numbers determined from $U = 5, 10, 20$ cm s^{-1} ($\epsilon = 0.02, 0.04, 0.08$, respectively); $\zeta_0 = 1$. Streamline spacing midway along the right-hand boundary represents the velocity scale. Mean 25 m currents from Fig. 2a are superimposed.

Stream. The sea-level decrease from the coast seaward and consequent geostrophic flow entering from the right was presumed from the regional data and is likely an effect of the larger scale circulation. The x axis is the upstream (right-hand) boundary where a cross-shelf distribution of ψ was imposed so the function $K(\psi)$ could be determined. For simplicity, the stronger flow of the Alaskan Stream, which remains seaward of the shelf break, was not included in this boundary condition on ψ. A parabolic function was chosen assuming $\partial^2\psi/\partial y^2 = 0$ and $\partial^2\psi/\partial x^2 = -\zeta_0 L/L_x$, where L is the length scale in Table 1 and L_x is the shelf width along the x axis. With $\psi = 0$ being the shoreline and $\psi = -L_x/L$ being the seaward streamline, ζ_0 could range from 0 to 2 without any negative flow imposed at the boundary. It was found that the value of ζ_0 was not qualitatively important. The nondimensional velocity is unity midway along the x axis, meaning that the velocity scale U represents the cross-shelf-averaged, along-shelf velocity at the upstream boundary. At the downstream boundary it was assumed that $\partial^2\psi/\partial y^2 = 0$.

4. Discussion

The Rossby number ϵ was determined for the model by choice of a velocity scale once the length scale was fixed at ~20 km (Kiliuda Trough width). Only small Rossby numbers validate the quasi-geostrophic theory and, as expected, the model predicts a strong tendency for flow along isobaths when velocity scales of $U = 5$, 10 and 20 cm s^{-1} ($\epsilon = 0.02$, 0.04, 0.08) were employed (Fig. 5). These speeds are typical of direct current observations (Fig. 2). As the theory would predict, the smaller Rossby numbers show the greatest topographic effect and the flow tends to form a cyclonic vortex over the trough.

With decreasing ϵ the flow also tends to become more of a jet-like current confined to the region of greatest topographic relief. This result owes to the small-Rossby-number limit of streamlines closely following isobaths. Where isobaths converge, so will streamlines, causing stronger flow over steeper topography.

In dimensional terms, the lowest-order balance is nearly

$$\frac{\partial\Psi}{\partial x}\frac{\partial h_B}{\partial y} - \frac{\partial\Psi}{\partial y}\frac{\partial h_B}{\partial x} = 0, \qquad (4)$$

where Ψ, x and y are now dimensional variables. This means that $\Psi = \Psi(h_B)$ and

$$u = -\frac{\partial\Psi}{\partial y} = -\frac{\partial\Psi}{\partial h_B}\frac{\partial h_B}{\partial y} \quad \text{and} \quad v = \frac{\partial\Psi}{\partial x} = \frac{\partial\Psi}{\partial h_B}\frac{\partial h_B}{\partial x}.$$

In vector notation

$$\mathbf{v} = \frac{\partial\Psi}{\partial h_B}\mathbf{k} \times \nabla h_B,$$

$$|\mathbf{v}| = \frac{\partial\Psi}{\partial h_B}|\nabla h_B|.$$

Then along a streamline, the current magnitude should be proportional to the local gradient of depth. For moorings K6, K7 and K8 the values of $|\nabla h_B|$ were estimated over intervals of 5, 10 and 15 km and averaged (Table 2). The values of $|\mathbf{v}|/|\nabla h_B|$ were then computed from the mean currents and are shown to be quite close, ranging between 3.0 and 4.2 × 10^3 cm s^{-1}. Averaging these values indicates that $\partial\Psi/\partial h_B \approx 3.5 \times 10^3$ cm s^{-1}.

TABLE 2. Values of local topographic gradient averaged over 5, 10 and 15 km and their ratios with mean current at stations K6, K7 and K8.

	Current-meter stations		
	K6	K7	K8
	Values of $\nabla h_B \times 10^3$		
Distance (km)			
5	10.0	0.8	4.8
10	7.0	2.2	6.5
15	4.7	4.5	11.3
Average	7.2	2.5	7.5
$\|\mathbf{v}\|$ (cm s^{-1})	21.5	10.6	25.1
$\|\mathbf{v}\|/\|\nabla h_B\| \times 10^{-3}$ (cm s^{-1})	3.0	4.2	3.3
$\|\mathbf{v}\|/\|\nabla h_B\|$*	3.5 × 10^3 cm s^{-1}		

* Average for all three stations.

Since

$$\frac{\Delta\Psi}{\Delta x} \approx \frac{\Delta\Psi}{\Delta h_B}\frac{\Delta h_B}{\Delta x},$$

the cross-shelf average of the along-shelf flow at the upstream boundary of the model can be estimated using $\Delta h_B \approx 200$ m and $\Delta x \approx 150$ km to yield $\Delta\Psi/\Delta x \approx 4.7$ cm s^{-1}. This implies that ~5 cm s^{-1} is an appropriate velocity scale for the shelf flow. As evidenced in Fig. 5, the model and data are in good agreement with this scaling. Of the three velocity scales shown, the streamline pattern matched the observed currents best for the $U = 5$ cm s^{-1} case. Currents of appropriate speeds around the trough were reproduced by the model as a result of the tendency of the streamlines to be concentrated by the topography. This result implies that a very weak cross-shelf-averaged along-shelf flow (~5 cm s^{-1}) will support a stronger along-isobath current around the trough.

5. Summary

The Kodiak Island shelf is characterized by steep topographic relief in the alongshelf direction with a mean alongshelf flow toward the southwest, and is bounded on the seaward side by a strong western boundary current. Over the shelf the geostrophic flow is steered by the topography and forms a steady cyclonic meander or vortex over the Kiliuda Trough. A barotropic model is in good agreement with observations and predicts that the flow is strongest where the topographic relief is strongest when the Rossby number is appropriately small. This effect arises because streamlines are constrained to follow isobaths and are therefore concentrated over steeper topography. An appropriate velocity scale for the shelf appears to be ~5 cm s^{-1} (cross-shelf-averaged) and stronger currents $\sim$20 cm s^{-1} occur around the trough owing to the local topographic gradients.

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