Reply

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We thank Dr. Simons for his perceptive comments on Brink and Allen (1978; BA hereafter). Although the BA analysis is essentially correct, there is an error of interpretation, and the criticism of Dr. Simons is valid. We discuss this further below after correcting some technical errors and clarifying a point about the expansion procedure in BA.

First, a typographical error occurred in Eq. (3.3a), where the third line should read

$$-iE_0^{1/2} \sum_{\substack{m=1 \\ m\neq n}} \phi_m(x) a_{nm} \alpha_{nm},$$

that is, the sign is changed and a factor of α_{nm} inserted.

Further, a numerical error has been found in the computed values of the frictional coefficients a_{nm}. Thus, for example, a_{11} should be 9.7×10^{-8} cm^{-1} rather than the reported value of 4.7×10^{-7} cm^{-1} (which should have read 10^{-8}). Corrected values for a_{nm} would modify the phase shifts shown in Fig. 1 of BA, and would make $T_F = (E_0^{1/2} c_1 a_{11})^{-1} = 2.3$ days. Recent studies, however, have shown that the value of $E_0^{1/2} = 0.12$ in BA may be too large. Thus, if we choose $E_0^{1/2} = 0.06$, then Fig. 1 of BA remains unchanged, as do T_F and the estimated value of $\theta'(x)$. This value is consistent with Allen and Smith (1981), whose results suggest that $E_0^{1/2} = 0.03$–0.08 would be reasonable [this range of values allows for some uncertainty in the bottom drag coefficient C_D (see Grant and Madsen, 1979)], giving $T_F = 3.4$–9 days. These values of T_F also are comparable with that calculated for Oregon by Brink (1982) by an entirely different approach.

In BA, a partial differential equation in three independent variables [(2.5)] was reduced to an infinite set of constant coefficient coupled differential equations in two independent variables. This set was made tractable by expanding the modal amplitude functions $Y_n(y, t)$ in powers of $E_0^{1/2}$. It should be emphasized, however, that $E_0^{1/2} \ll 1$ is not a sufficient condition for the validity of the expansion in BA, since the alongshore and time derivative terms can be small enough to make friction of lowest order important regardless of the Ekman layer depth. Thus, a better criterion is

$$E_0^{1/2} c_n a_{nn} = T_F^{-1} \ll \omega \ll f,$$

where the upper bound arises from the long-wave assumption and ω is the wave frequency. This condition is also not very exact, so that in practice one has to check for asymptotic accuracy after the fact.

The interpretational problem in BA arose from the assumption that the behavior of individual modes would be the same as the total forced solution, represented by the sum of the modes. Specifically, BA found that, for a forced response, the computed wave modes had $\theta'_{Dn}(x) > 0$, for cases where the wave frequency was both above ($n = 2$) and below ($n = 1$) the respective resonant frequency. That is, for the individual forced modes, offshore motions lag those close to the coast. Thus, BA stated categorically that motions nearshore always lead those farther offshore in time. This is not true for wind-forced problems in general. The difficulty is that the modal amplitudes A_n are complex and their phase changes substantially on passing through individual resonances.

As a demonstration of the behavior of the complete forced solution, the BA vorticity equation (2.5) was solved subject to (2.6) by a finite-difference technique for a range of frequencies. The same parameters as BA were used except $E_0^{1/2} = 0.06$. The wind stress was taken as

$$\tau_w^y = T_0 \exp[i(\omega t - ly)].$$

A similar calculation can also be found in Denbo and Allen (1982).

Amplitude $|v|$ and phase θ for alongshore velocity as a function of cross-shelf distance x and frequency ω are shown in Fig. 1. Note that the phase of v is not the same as that of ψ, since the x derivative of ψ entails a derivative of the phase of ψ. The nearshore

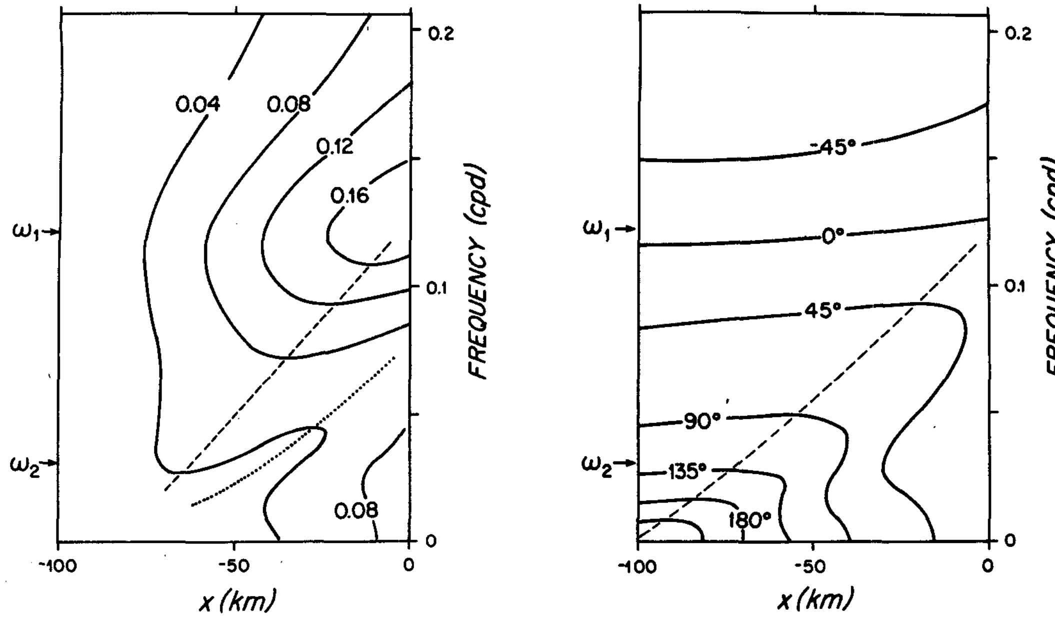

FIG. 1. Results of the forced response calculations [Eq. (2.5) of BA]. Parameters used: $H_0 = 10^2$ m, $L = 10^5$ m, $\lambda = 3 \times 10^{-5}$ m^{-1}, $f = 1.03 \times 10^{-4}$ s^{-1} (latitude 45°N), $l = 2 \times 10^{-6}$ m^{-1}, $T_0 = 0.1$ N m^{-2}. (a) Amplitude of alongshore velocity fluctuations (m s^{-1}). The dashed and dotted lines are loci of relative maximum and minimum response, respectively. (b) Phase of alongshore velocity fluctuations relative to the wind stress. The dashed line is the locus of maximum phase.

amplitude of v has maxima surrounding the frequencies of shelf wave resonance for the first two modes (ω_1 and ω_2). For frequencies greater than that of the first mode resonance, $\theta'(x) > 0$ over the entire shelf-slope region, meaning nearshore motions indeed lead those farther offshore. For $\omega < \omega_1$, there is a locus of maximum phase as a function of x, shown by the dashed line in Fig. 1b. For this example, the locus corresponds roughly to the line of maximum relative velocity amplitude (Fig. 1a). For positions offshore of the phase maximum, $\theta'(x)$ remains positive, but inshore of the line $\theta'(x) < 0$, and offshore motions *lead* those closer to shore. The persistence of positive $\theta'(x)$ offshore can be thought of as being due to higher modes ($\omega > \omega_n$) which have relatively larger v amplitudes near the outer boundary with increasing n.

In the two-dimensional ($l = 0$) case, no resonance is possible with long shelf waves. Effectively, this means $\omega_n = c_n l = 0$ so any positive nonzero frequency has $\omega > \omega_n$ for all n and nearshore fluctuations will always lead those farther offshore. This is the limit treated, for example, by Csanady (1974).

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