The Mean and Seasonal Circulation off Southwest Nova Scotia

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(Manuscript received 19 October 1982, in final form 7 February 1983)

ABSTRACT

Long-term measurements off southwest Nova Scotia reveal the following features of the mean circulation:

(a) a westward longshore coastal current (4–10 cm s⁻¹),
(b) an anticyclonic gyre around Browns Bank (5–15 cm s⁻¹), and
(c) an upwelling circulation off Cape Sable (1–2 cm s⁻¹ at bottom).

The gyre circulation appears permanent but the coastal current and upwelling exhibit annual variations of the same order as the means. There is a distinct annual signal in the longshore transport at Cape Sable (maximum westward in winter), whereas the mean transport (0.14 × 10⁶ m³ s⁻¹) is consistent with both geostrophic estimates and budget requirements in the Gulf of Maine. Strong seasonal cycles are also found in the salinity and density fields at Cape Sable which appear to be controlled both by buoyancy input from the coastal current and local mixing effects.

A linear diagnostic model indicates that the primary dynamical balance for the circulation is between a longshore pressure gradient and longshore mean density and stratification gradients which have summer maxima. Lesser contributions arise from longshore wind, offshore density gradient and centrifugal upwelling. Tidal rectification, deduced from coherent modulations of the semidiurnal tidal streams and low-frequency currents, supports the Browns Bank gyre circulation and drives both westward and offshore components of the near-bottom flow off Cape Sable. Thus the “centrifugal upwelling” hypothesis fails and the main driving force for Cape Sable upwelling appears to be the longshore density variations maintained by tidal mixing.

1. Introduction

The continental shelf waters off southwestern Nova Scotia are among the most productive on the east coast of North America. In this region are found the principal spawning sites for various species of demersal and pelagic fishes (Colton and Temple, 1961), the largest commercial lobster catches in Nova Scotia, and the largest nonbreeding concentrations of seabirds in the northwestern Atlantic (Sutcliffe et al., 1976). At lower trophic levels, Fournier et al. (1983) and Denman and Herman (1978) report high levels of primary production associated with the boundary region between stratified and tidally well-mixed zones.

It has been postulated that the high level of biological activity in this area is supported by persistent upwelling of rich deep waters of the Gulf of Maine and Scotian Shelf. Lauzier (1967) found evidence in his seabed–drifter results for a large-scale shoreward convergence associated with an offshore component of the surface drift. Furthermore, he could detect no seasonal variations in strength of the bottom current which ranged from 0.5 to 1.5 cm s⁻¹. Examining possible dynamic balances, Garrett and Loucks (1976) concluded that the most likely driving term for the near-bottom flow is the pressure gradient required to balance the centripetal acceleration of the tidal flow along the curving Nova Scotian coastline. They therefore dubbed the circulation “centrifugal upwelling” and estimated the magnitude of the mean onshore current at 2 cm s⁻¹.

On a broader scale, the net transport along the shelf past Cape Sable is thought to exert a strong influence on the physical characteristics and biological productivity of the Gulf of Maine as a whole. Bigelow (1927) was one of the first to recognize that the cold Nova Scotian Current brings a large volume of low salinity water from the Gulf of St. Lawrence into the Gulf of Maine and contended that this is the only source to have an appreciable effect on the temperature of the Gulf. Sutcliffe et al. (1976) pursued these ideas with correlation studies which suggest that salinity and temperature anomalies associated with freshwater runoff in the Gulf of St. Lawrence are transported by coastal currents along an “oceanic pathway” stretching from the St. Lawrence estuary through the Gulf of Maine. Recently, Hopkins and Garfield (1979) have quantified the effects on the Gulf of Maine by demonstrating that 5200 km³ yr⁻¹ (0.17 × 10⁶ m³ s⁻¹) of Scotian Shelf water is required to produce the seasonal water-mass changes they observe in hydro-
graphic data. The seasonal variations of the coastal transport are also evident in the geostrophic computations of Drinkwater et al. (1979) on the Halifax Section and Vermersch et al. (1979) on a section across the shelf at Cape Sable. However, whereas the geostrophic approximation may reasonably represent the primary dynamical balance at Halifax, the strong tides and variable topography suggest that other mechanisms such as tidal rectification (Loder 1980; Greenberg, 1983) may prevail at Cape Sable.

Another manifestation of the strong tidal streams in this area is the existence, particularly in summer, of a sizeable zone of vertically well-mixed water off Cape Sable, Nova Scotia. Garrett et al. (1978) suggest that, under uniform solar heating, the boundary between vertically stratified and mixed regions is marked by a certain ratio of the local depth ($h$) to the cube of the average tidal current ($V_T$), which measures the relative importance of surface buoyancy input to tidal dissipation. Using a numerical model of the $M_2$ tide, they then demonstrate that the $hV_T^{-3}$ criterion is consistent with observations of well-mixed zones in the Gulf of Maine, including the one off Cape Sable. With regard to the mean circulation, spatial variations in tidal mixing are capable of producing horizontal density gradients which support a baroclinic flow field. Using scale analysis, Csanady (1976) has argued that in tidally dominated regimes such as this one, the density variations are maintained by tidal eddy fluxes and are therefore independent of the mean circulation itself. This assumption, along with linearization of the bottom stress law, leads to a simple linear diagnostic model for the mean circulation which has been successfully applied off Long Island (Scott and Csanady, 1976).

To investigate the dynamics of the mean, seasonal and tidal circulations off southwest Nova Scotia, an experiment was conducted during 1979–80. Experimental methods and salient results are described in Sections 2 and 3. In Section 4 a simple diagnostic model similar to Csanady's is developed and compared to the data, while Section 5 examines evidence for tidal rectification and tests the centrifugal-upwelling hypothesis. The conclusions of this study are summarized in Section 6.

2. The Cape Sable Experiment

The mooring array (Fig. 1) for the Cape Sable Experiment consists of a line of four sites (C1–C4) across the western end of the Scotian Shelf at Cape Sable and two more sites (C5, C6) off Shelburne, NS, to monitor “upstream conditions”. Sites C1 and C5 lie on the 60-m isobath, the rest are on the 110-m isobath. The major portion of the data set was collected between April 1979 and August 1980; however, two of the nearshore moorings (C1, C2) have been maintained to the present for monitoring purposes. Each mooring carries at least three current meters at nearshore (15 m), mid-depth, and near-bottom (10 m above) locations. Also, for some deployments of the full array, bottom pressure was measured at the base of the C2 and C6 moorings. Furthermore, to carry out detailed studies of the tidal boundary layer, six additional instruments were placed on the two nearshore moorings (C1, C2) for selected periods.

In addition to the moored measurements, extensive hydrographic surveys were conducted on six primary cruises to the area. Surface-drift cards and several satellite-tracked drogues were also deployed to obtain Lagrangian estimates of the surface circulation pattern. Other auxiliary data were collected including meteorological observations at Yarmouth, Halifax and Sable Island; sea level at Yarmouth, Shelburne and Halifax; and frontal analyses of satellite-derived sea-surface temperature. An example of the latter
(Fig. 2) clearly delineates the core of the tidally-mixed region as an anomalous cold patch.

The primary moored instrument, the Aanderaa RCM-5 current meter, was set to sample at 30-min intervals. The estimated accuracies of the Aanderaa speed (±1.3% or 5 mm s⁻¹; Tang and Hartling, 1979), direction (±5°; Keenan, 1979); temperature and conductivity (±0.04°C and ±0.07 mmho cm⁻¹; Boyce, 1982) were determined by laboratory calibrations. For constant pressure, the last two figures are roughly equivalent to ±0.11% in salinity and ±0.09 kg m⁻³ in density. In some cases, it was possible to use nearby CTD profiles to improve this accuracy (to ±0.06%, ±0.05 kg m⁻³).

At each site, the near-surface and deeper instruments were placed on separate moorings (connected by a groundline) in order to minimize the influence of surface wave action on the deep current measurements. Near the surface, VACM current meters were located just above the Aanderaa instruments on several occasions to assess the degradation of the mean flow measurements. Comparisons (Smith and Lively, in preparation) indicate that mean surface speeds are augmented by the order of 10%, on average, with direction differences of ±10 deg. These figures are considerably better than those (20%, 70 deg) of Howarth (1981) for a similar tidal regime in the Irish Sea.

The current measurements were resolved into components parallel (u = offshore) and perpendicular (v = longshore) to the two mooring lines. On the

Shelburne line, the longshore direction (53° True) is very nearly aligned with local isobaths and the coastline, while on the Cape Sable line, the choice (104° T) is roughly tangent to the curving isobaths and the major axes of the semi-diurnal tidal ellipses at C1–C4. For investigating low-frequency phenomena, the records were low-passed with a Cartwright filter (half-power at 31 h) and subsampled at 6 h intervals (the “6-h data”). Finally, in the following discussion, individual records will be denoted by the mooring site and depth of instrument, e.g., C1(50 m) represents the record from 50 m on the C1 mooring.

3. The mean and seasonal circulations

Comparison of the average velocity fields from three different seasons (i.e., mooring periods, Fig. 3) reveals several distinctive features of the overall mean circulation:

1) an alongshore current to the southwest is consistently found in the nearshore zone, i.e., within the 110 m isobath,

2) the opposing currents at C3 and C4 suggest a permanent clockwise gyre around Browns Bank, and

3) a persistent difference exists between the deep cross-isobath components off Cape Sable (onshore flow) and off Shelburne (offshore flow).

This last observation clearly demonstrates the upwelling component of the circulation off Cape Sable with a magnitude of order 2 cm s⁻¹. On the other hand, the Browns Bank gyre, suggesting strong topographic control of the circulation, is confirmed by satellite-tracked drogues that tend to circuit the bank in a clockwise sense (R. Trites, personal communication, 1982). Furthermore, the variation in hydrographic properties across the Cape Sable section is consistent with a topographic gyre. For example, the salinity distributions (Fig. 4) for both the summer and late-winter seasons reveal a pocket of relatively saline (and warm) water along the inshore edge of Browns Bank, bounded by fresher water nearshore and over the offshore edge of the bank. The water mass characteristics of this anomaly, particularly at depth, resemble those of Maine Surface and Intermediate Water (Hopkins and Garfield, 1979) suggesting an origin in the Gulf of Maine. Geostrophic calculations, indicating upshelf flow in this region, reinforce this conclusion.

Some of the important seasonal variations of the circulation may be detected by comparing the two summer patterns to the winter regime in Fig. 3. For instance, the winter inflow to the Gulf of Maine at C1, C2 is stronger (6–10 cm s⁻¹) than in summer and extends to the bottom. Note that the geostrophic pressure gradient associated with the inflow would tend to produce offshore components in the bottom.
boundary layer in contrast to the observed onshore flow at C1. The low-frequency character of the inflow, as illustrated by the 6-h near-surface records at C2 (16 m) (Fig. 5), is not smooth variation but rather a succession of pulses, some of which are clearly as-

associated with the arrival of sharp fronts in the surface water mass. The salinity and temperature records at C5 and C6 indicate that similar pulses arrive several days earlier off Shelburne, but the anomalies are weaker because of the generally lower mean salinity at these sites. Furthermore, the movement of the pulses appears to be governed, at least to some extent, by wind. An examination of the coastal wind records from Yarmouth and Halifax reveals that most of the pulses are associated with westward wind events of order 6 to 10 m s\(^{-1}\) in opposition to the prevailing eastward component over the Scotian Shelf (Smith and Petrie, 1982). Nevertheless, though obscured by the variability, the seasonal mean westward (284° T) current is augmented in winter and carries a significantly fresher, colder water mass than is present in October. Also as indicated by the \(\sigma\), record, density is controlled by salinity so that the influx is also lighter.

The seasonal variation of the salinity field throughout the array is depicted by the monthly mean salinities for the first year's records (Fig. 6). A distinct annual cycle is evident at each location and at virtually all depths to 100 m. However, the annual cycle is clearly much stronger on the Cape Sable mooring line (C1–C4) than off Shelburne (C5, C6). This sug-

FIG. 3. Seasonal-mean velocity patterns near surface, bottom and at mid-depth for: (a) summer, 1979 (April–October), (b) winter, 1979–80 (October–March), (c) summer, 1980 (March–August).

FIG. 4. Salinity distributions on the Cape Sable Section during (a) 6–10 August 1979 and (b) 25 March–3 April 1980.
where

\[ X_{k\nu} = \begin{cases} \cos(k - 1) \frac{\pi \nu}{12}, & k \text{ odd} \\ \sin(k \frac{\pi \nu}{12}), & k \text{ even} \end{cases} \]

includes a constant term \((k = 1)\), the annual signal \((k = 2, 3)\), and its harmonics \((k \geq 4)\). Assuming the \(Y_{\nu}\) are normally distributed, this method has the advantage that successive \(b_k\) may be tested under the null hypothesis with \(n - r\) degrees of freedom (Fofonoff and Bryden). With a few minor exceptions, none of the coefficients of the harmonic terms \((k \geq 4)\) for the Cape Sable data was found to be significantly different from zero at the 95% confidence level. The significant estimates for all variables at each site are presented in Table 1 in terms of a mean value \((b_1)\) plus the amplitude \([\sqrt{b_2^2 + b_3^2}]^{1/2}\) and phase \([\tan^{-1}(b_2/b_3)]\), in months of the annual cycle. The seasonal signals in the velocity components, particularly the longshore component, tend to be the least stable. Nevertheless, when detectable, the annual variations in \(v\) are comparable to the mean values in the nearshore zone. At the Browns Bank sites (C3, C4), on the other hand, strong mean flows dominate.

Fig. 5. Filtered (6-h) records of cross-shore \((u)\) and longshore \((v)\) velocity components, temperature, salinity and \(\sigma\), for the near-surface instrument at C2(16 m) during winter 1979–80. Darkened lobes on the \(v\)-component represent episodes of westward flow into the Gulf of Maine.

![Velocity Components](image1)

![Temperature](image2)

![Salinity](image3)

![Sigma T](image4)

Fig. 6. Monthly mean salinity for the first full year of the Cape Sable Experiment.

![Salinity Data](image5)
in the along-isobath direction. In contrast to Lauzier's (1967) findings, the near-bottom cross-isobath currents off Cape Sable (C1, C2) appear to have a distinct seasonal signal with maximum onshore flows occurring in summer. Though return flow in the upper portion of the water column may be lost in the noise, it appears that the upwelling circulation has both a mean and an annual component. Similar but weaker cycles are found in the offshore bottom currents on the Shelburne line (C5, C6).

The temperature records show strong annual variability throughout the array with uniform amplitudes of 3–4°C in the waters off Cape Sable. As suggested by Fig. 6, the salinity variations are significantly stronger on the Cape Sable line than off Shelburne, but the phases (7 to 9 months) are consistent with winter input of fresh water. A sample of the least-square estimates for all near-surface variables at C2(16 m) is presented in Fig. 7.

The monthly-mean longshore currents may also be used to estimate variations in the transport through the Cape Sable section (Fig. 8) by assigning roughly equal segments of the cross-section to each measurement (except for the shallower C1 mooring). Despite large error bars, the seasonal cycle is clearly defined in the first year’s data with a minimum transport in late summer and a maximum in winter. The magnitude of this oscillation (order $0.3 \times 10^8 \text{ m}^3 \text{s}^{-1}$) is consistent with that found in geostrophic transport estimates on a nearly hydrographic section (Vermisch et al., 1979) and also on the Halifax Section (Drinkwater et al., 1979). Furthermore, on an annual basis, the average measured transport ($0.14 \times 10^8 \text{ m}^3 \text{s}^{-1}$) compares favorably with the Cape Sable geostrophic estimates ($0.25 \times 10^8 \text{ m}^3 \text{s}^{-1}$; Vermisch et al., 1979) and with budget requirements for Scotian Shelf water ($0.17 \times 10^8 \text{ m}^3 \text{s}^{-1}$; Hopkins and Garfield, 1979) based on seasonal water mass analysis in the Gulf of Maine.

Another interesting aspect of the monthly transport computations is that the estimates from the two inshore moorings alone closely follow the total net transport through the section. This result implies that the Browns Bank gyre is basically a closed circulation, making no significant contribution to the mass balance in the Gulf. Hence the major part of the seasonal volumetric inflow may be monitored by a few instruments in the nearshore zone off Cape Sable.

### 4. A diagnostic model

#### a. Formulation

The magnitude of the $M_2$ tidal circulation in the vicinity of Cape Sable is roughly five to ten times larger than that of the mean circulation depicted in Fig. 3. Csanady (1976) has recently formulated a simple model for the mean circulation which explicitly
recognizes the dominance of the variable ‘first-order’ flow. In addition to the hydrostatic and Boussinesq approximations, he assumes nonlinear convective terms and the divergence of horizontal momentum fluxes are small. For this study, Csanady’s analysis has been extended to include centrifugal effects by assuming that the dominant ‘first-order’ signal is a monochromatic rectilinear tide in the alongshore direction, i.e.,

\[
\begin{align*}
\mathbf{u} &= \bar{u} \\
\mathbf{v} &= V_T(z) \cos \omega t + \bar{v}
\end{align*}
\]

where overbar indicates a mean flow quantity and \(V_T(z)\) is the tidal velocity profile. As for the current data, the x-axis is directed offshore and the y-axis upshelf.

Using semidiurnal tidal-ellipse data to estimate Reynolds stresses, a scale analysis of the mean momentum equations (in curvilinear coordinates) indicates that the centrifugal forces [curvatures of order \(\kappa = (50 \ \text{km}^{-1})\)] exceed the other nonlinear terms in the cross-isobath equation by more than an order of magnitude. Under these conditions, the equations of motion are

\[
-\frac{\kappa V_T^2}{2} - f \bar{v} = -g \frac{\partial}{\partial x} (\bar{u} + D) + \frac{\partial F_x}{\partial z},
\]

Fig. 7. Least-squares fit of mean-plus-annual signal to all monthly mean data at C2(16 m). See Table 1 for amplitudes and phase appropriate to each variable.

Fig. 8. Monthly estimates of volumetric transport through the Cape Sable section during 1979–80. Positive values represent flow into the Gulf of Maine (284° True); solid curve for moorings C1–C4, dashed for C1 and C2 only. Inset shows segments of cross-sectional area represented by each instrument. Confidence intervals are based on standard errors of normal velocity components (Smith and Petrie, 1982). The annual-average transport (0.14 \times 10^6 \ \text{m}^3 \ \text{s}^{-1}) is compared to geostrophic computations and budget requirements (see text).
\[ f\vec{u} = -g \frac{\partial}{\partial y} (\vec{f} + D) + \frac{\partial F_y}{\partial z}, \quad (3) \]
\[ \frac{\partial \vec{u}}{\partial x} + \frac{\partial \vec{v}}{\partial y} + \frac{\partial \vec{w}}{\partial z} = 0, \quad (4) \]

where \((F_x, F_y)\) is the kinematic Reynolds stress, \(\vec{f}(x, y)\) is the free surface displacement, \(D = \int_0^y (\rho - \rho_0) \rho_0 dz\) is the integral of the normalized density anomaly and \(k\) is the local curvature of the tidal flow. The dominance of the 'first-order' flow allows certain simplifications of the theory such as linearizing the quadratic bottom-stress law. Parameterizing the Reynolds' stresses with a constant eddy coefficient \(A\) leads to boundary conditions of the form

\[ A\left( \frac{\partial \vec{u}}{\partial z}, \frac{\partial \vec{v}}{\partial z} \right) = (F_{wx}, F_{wy}), \quad \text{at } z = 0, \quad (5a) \]
\[ A\left( \frac{\partial \vec{u}}{\partial z}, \frac{\partial \vec{v}}{\partial z} \right) = r(\vec{u}, \vec{v}), \quad \text{at } z = -h, \quad (5b) \]

where \(r = O(C_D V_T)\) is the bottom resistance coefficient and \(C_D\) is the drag coefficient (for further detail, see Csanady, 1976, or Smith, 1979).

Another simplification of mean flow dynamics, proposed by Csanady, results from treating the density field as independent of the mean circulation itself on the assumption that spatial variations are maintained by first-order eddy fluxes and vertical mixing. The eddy salt-flux divergence estimated from mooring data at C1 and C2 exceeds the advective fluxes by almost an order of magnitude at the 50 m level while the two are comparable in the surface layer. Furthermore, hydrographic sections and sea surface temperature data indicate that the boundary between stratified and well-mixed water lies near mooring C1. Thus it is plausible that the density gradients are supported primarily by tidal exchange processes.

The implication for the mean flow model is that the density field may be treated as an external parameter of the system. Hence, expanding the density about a convenient reference value as

\[ \rho(x, y, z) = \rho_0 \left[ 1 + \sigma_x x + \sigma_y y + \sigma_z \left( z + \frac{h}{2} \right) \right. \]
\[ \left. + \sigma_{xz} \left( z + \frac{h}{2} \right) + \sigma_{xy} \left( z + \frac{h}{2} \right) + \cdots \right], \quad (6) \]

where

\[ (\sigma_x, \sigma_y) = \frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y} \right) \]

are the horizontal variations in mean density and

\[ (\sigma_{xz}, \sigma_{yx}) = \frac{1}{\rho_0} \left( \frac{\partial^2 \rho}{\partial x \partial z}, \frac{\partial^2 \rho}{\partial y \partial z} \right) \]

are the horizontal variations in stratification. Hence the momentum equations have the form (dropping bars)

\[ A \frac{\partial^2 u}{\partial z^2} + fv = g \frac{\partial \vec{f}}{\partial x} - g \sigma_x z - g \sigma_{xz} \left( z + \frac{h}{2} \right) - q, \quad (7) \]
\[ A \frac{\partial^2 v}{\partial z^2} - fu = g \frac{\partial \vec{f}}{\partial y} - g \sigma_y z - g \sigma_{yx} \left( z + \frac{h}{2} \right) - q, \quad (8) \]

where \(q(z) = \kappa V_T^2(z)/2\). Csanady also argues that, at least near the coast, the net transport should follow isobaths, i.e.,

\[ \int_{-h}^0 u dz = 0 \quad (9a) \]

and defines

\[ \int_{-h}^0 v dz = -V. \quad (9b) \]

Note that based on data in Table 1, the assumption (9a) is reasonable at sites C3 to C6 where the significant cross-isobath components nearly sum to zero. At (C1, C2), however, the directions of the vertically-averaged currents are directed offshore from the assumed 104°T by 42° and 16°, respectively. Hence some care must be exercised in interpreting longshore and cross-shore flows at these sites.

The depth-averaged momentum equations are given by

\[ F_{wx} - ru_b - fV = gh \frac{\partial \vec{f}}{\partial x} + \frac{gh^3 \sigma_x}{12} - Q_h, \quad (10) \]
\[ F_{wy} - rv_b = gh \frac{\partial \vec{f}}{\partial y} + \frac{gh^3 \sigma_y}{12}, \quad (11) \]

where quantities with subscript \(b\) are evaluated at the bottom and \(Qh = \int_{-h}^0 q(z) dz\). These equations provide useful expressions for the bottom velocity components.

In these equations, the longshore pressure gradient, \(g\partial \vec{f}/\partial x\), as well as the density gradients and centrifugal term are treated as external parameters in the diagnostic approach. The general solution for the complex velocity function is

\[ W = u + iv = \frac{g}{f} \left[ \vec{W}_p + a e^{(1+i)(\bar{z}+1)} + \beta e^{(1+i)h_k} \right], \quad (12) \]

where \(\bar{z} = z/h\), \(\lambda = h/h_k\), and \(h_k = (2A/f)^{1/2}\) is the Ekman depth. With a quadratic form for the centrifugal term,

\[ q(\bar{z}) = gB(1 + b_1 \bar{z} + b_2 \bar{z}^2) = g\bar{q}, \quad (13) \]

the particular solution is given by

\[ \vec{W}_p = -\frac{\partial \bar{q}}{\partial \bar{y}} + \sigma_y \bar{z} + \frac{1}{2} \sigma_{yz} \bar{z} + \frac{1}{2} \sigma_{zx} (\bar{z} + h) - \frac{\sigma_{xz} h^2}{2 \lambda^2} - \frac{B \bar{b}_2}{\lambda^2} \]
\[ - i \left[ \bar{q} - \frac{\partial \bar{q}}{\partial \bar{x}} + \sigma_x \bar{z} + \frac{1}{2} \sigma_{zx} (\bar{z} + h) + \frac{\sigma_{xz} h^2}{2 \lambda^2} \right]. \quad (14) \]
The system (7)–(9), sufficient to determine the six unknowns \( \partial \xi / \partial x, V \), and the complex constants \( \alpha \) and \( \beta \), has been solved numerically using a Gaussian elimination technique. Details of the derivation are presented in the Appendix for the centrifugal-upwelling component of the circulation. Since the equations are linear, the components corresponding to each of the driving terms may be summed to give the total flow field.

For cases where the depth is much greater than the Ekman depth, \( h_E \), useful analytical expressions may be obtained from the asymptotic forms of the equations for \( \lambda \gg 1 \) (see Appendix). In this limit, the complete solutions for the offshore sea-surface slope and longshore transport are:

\[
(1 + \lambda R) \frac{\partial \xi}{\partial x} = -[2\lambda^2 R^2 + \lambda R(2 - R) + 1] R^{-1} \frac{\partial \xi}{\partial y} + \left[2\lambda(1 + \lambda R)(1 - b_1 + b_2) + (1 + 2\lambda R)(b_1 - 2b_2 - 2Rb_2) \right] \frac{h_E B}{2h} + \left[2\lambda R - 2\lambda(1 + \lambda R) \right] \frac{h_E \sigma_x}{2} + \frac{h_E \sigma_y}{2R} + \frac{h_E \omega y}{ghR} - \frac{h_E \omega z}{2} - \frac{h_E \omega z}{2} - 2\lambda^3 R \left[1 + \lambda R + \lambda^2 - 3\lambda R - 6R \right] \frac{h_E \sigma_x}{12R}, \tag{15}
\]

and

\[
(1 + \lambda R) \frac{fV}{gh} = (1 + \lambda R) \left[ \left(1 - \frac{b_1}{2} + \frac{b_2}{3} \right) B - \frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y} \right] + (1 - \lambda) R \frac{\partial \xi}{\partial y} + \frac{RF_w}{gh_E} + \frac{h_E \sigma_x}{2} - \lambda^2 - 2\lambda + 1 \frac{R h_E \sigma_y}{2} \right]

- \frac{h_E \sigma_z}{12} - \frac{(\lambda - 2) h_E \sigma_z}{12} - \frac{(\lambda^2 + 3) R h_E}{\sigma_y}, \tag{16}
\]

where \( R = r/fh \). Then for a given set of external parameters, the approximate form of the solution, (12), yields the following characteristic values of the velocity field:

**Surface (\( z = 0 \))**

\[
u_s = \frac{g}{f} \left[ \frac{\partial \xi}{\partial y} + (1 - 2b_2) \frac{h_E^2 B}{2h^2} + \frac{h_E (\sigma_x - \sigma_y)}{2} \right] + \frac{\left( F_{w_x} + F_{w_y} \right)}{gh_E} + \frac{h_E \sigma_x - \sigma_y}{4} - \frac{h_E^2 \sigma_z}{2}, \tag{17a}
\]

\[
v_s = \frac{g}{f} \left[ \frac{\partial \xi}{\partial x} - (1 - b_1) B + \frac{h_E (\sigma_x + \sigma_y)}{2} + \frac{\left( F_{w_y} - F_{w_x} \right)}{gh_E} \right] + \frac{h_E \sigma_x + \sigma_y}{4} - \frac{h_E^2 \sigma_y}{2}. \tag{17b}
\]

**Mid Depth (\( z = -h/2 \))**

\[
u_m = -\frac{g}{f} \left[ \frac{\partial \xi}{\partial y} + \frac{h_E^2 B}{2h^2} + \frac{h_E \sigma_x}{2} \right] + \frac{h^2 \sigma_z}{8} - \frac{h_E^2 \sigma_z}{2}, \tag{17c}
\]

\[
v_m = \frac{g}{f} \left[ \frac{\partial \xi}{\partial x} - (1 - b_1 + b_2 + 4b_2) B + \frac{h_E \sigma_x}{2} + \frac{h^2 \sigma_z}{8} - \frac{h_E^2 \sigma_y}{2} \right]. \tag{17d}
\]

**Bottom (\( z = -h \))**

\[
u_b = -\frac{g}{f} \left[ \frac{\partial \xi}{\partial y} + \frac{fV}{gh} - \left(1 - \frac{b_1}{2} + \frac{b_2}{3}\right) B \right] + \frac{h_E \sigma_x}{2} - \frac{F_{w_x}}{gh_E} + \frac{h^2 \sigma_z}{12}, \tag{17e}
\]

\[
v_b = \frac{g}{f} \left[ \frac{\partial \xi}{\partial y} + \frac{h_E \sigma_y}{2} - \frac{F_{w_y}}{gh_E} + \frac{h^2 \sigma_z}{12} \right]. \tag{17f}
\]

Note that \( u_s, u_m \) and \( v_b \) depend only on certain external parameters while the others depend on all parameters via \( \partial \xi / \partial x \) and \( V \).

**b. Seasonal parameters**

The external parameters to be used in the model computations are listed in Table 2. Since the purpose here is to investigate the dominant force balances in the Cape Sable circulation rather than to provide a 'best fit' to the data, the parameter selections represent realistic but uncertain estimates of the driving terms that are intended to demonstrate the character and relative magnitude of the various components of the circulation. The rationale for these selections is as follows:
### Table 2. External parameters for mean flow model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\frac{\delta \psi}{\delta y})</th>
<th>(\sigma_x)</th>
<th>(\sigma_y)</th>
<th>(\sigma_{xz})</th>
<th>(\sigma_{yz})</th>
<th>(F_{wx})</th>
<th>(F_{wy})</th>
<th>(gB)</th>
<th>((b_1, b_2))</th>
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<td></td>
<td>((\times 10^3))</td>
<td>((\times 10^6 \text{ m}^{-3}))</td>
<td>((\times 10^3 \text{ m}^{-3}))</td>
<td>((\times 10^2 \text{ m}^2 \text{ s}^{-2}))</td>
<td>((\times 10^2 \text{ m}^2 \text{ s}^{-2}))</td>
<td>((\times 10^4 \text{ m}^2 \text{ s}^{-2}))</td>
<td>((\times 10^4 \text{ m}^2 \text{ s}^{-2}))</td>
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<td>1.4</td>
<td>0.32</td>
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<td>-3.36</td>
<td>-0.110</td>
<td>0.08</td>
<td>0.83</td>
<td>(0.45, -0.30)</td>
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<td>0, 0</td>
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<td>0.17</td>
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<td>0.17</td>
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<tr>
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<td>-1.21</td>
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<td>0.06</td>
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<td>(0, 0)</td>
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<td>0.12</td>
<td>0</td>
<td>(0, 0)</td>
<td>110</td>
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</tbody>
</table>

1) **Longshore Pressure Gradient, \(g \frac{\delta \psi}{\delta y}\)**

Using correlations of carefully selected longshore wind and current data, Scott and Csanady (1976) found indirect evidence for an alongshore surface slope of \(1.4 \times 10^{-7}\) in the Mid-Atlantic Bight off Long Island. Csanady (1978) then discussed possible origins of the longshore pressure gradient and concluded that the most likely source is a large-scale pressure field impressed by the deep ocean. Beardsley and Winant (1979) support this hypothesis with calculations from a numerical model of the North Atlantic circulation but suggest that the estimate may be high because of neglect of the acceleration terms in the proposed balance. On the other hand, Smith (1979) has used a long-term mid-depth observation of onshore velocity (2.2 cm s\(^{-1}\)) at the Scotian Shelf break to infer (geostrophically) an alongshore gradient of order \(2 \times 10^{-7}\). As for seasonal variations, N. Pettigrew (personal communication, 1983) has detected a sea-level variation of order 5 cm between Halifax (high in winter) and several Gulf of Maine gauges. These observations, consistent with the passage of fresh Gulf of St. Lawrence water along the Scotian Shelf in winter, indicate a seasonal variation in the longshore surface slope of order \(1 \times 10^{-7}\). Hence for these calculations, the shelf-break estimate (\(2 \times 10^{-7}\)) will be used in winter and the Scott and Csanady value (\(1.4 \times 10^{-7}\)) in summer.

2) **Density Gradients (\(\sigma_x, \sigma_y\), (\(\sigma_{xz}, \sigma_{yz}\))**

To estimate the seasonal-mean density and stratification gradients suggested by the monthly-mean salinity variations (Fig. 6), the horizontal differences in the 6-h \(\sigma\) records were calculated and averaged over the mooring periods. Two density-difference records from the surface layer (Fig. 9) reveal a broad minimum in the longshore density contrast (C5–C1) and peaks in the offshore density contrast (C3–C2) associated with the arrival of the fresh water pulse in November. To derive seasonal values of \(\sigma_x\) and \(\sigma_y\), the net contrasts were averaged over the common portions of the water columns at adjacent moorings (Fig. 10) and divided by the distance between them. Similarly, the horizontal differences in vertical density contrast were used to derive \(\sigma_{xz}\) and \(\sigma_{yz}\). Since the Shelburne moorings (C5, C6) are somewhat removed from the regime of strong tidal mixing, it might be reasonable to assume that the longshore density gradients at these sites would be smaller than those at C1, C2. However, this conclusion is not borne out by the hydrographic data which, despite strong aliasing, indicate that the density field varies relatively smoothly between Cape Sable and Shelburne. Therefore, the full density and stratification gradients between the two lines were applied to the calculations on each of them. Offshore gradients were computed as central differences where possible.

3) **Wind Stress (\(F_{wx}, F_{wy}\))**

The seasonal-average wind stresses at Yarmouth (Fig. 10) were computed using a variable drag coefficient (Smith and Banke, 1975) and resolved locally into components parallel (\(F_{wx}\)) and normal (\(F_{wy}\)) to the two mooring lines.

4) **Centrifugal Term**

\[
q(z) = \kappa V_r^2(z)/2 = gB \left(1 + \frac{b_1 z^2}{h^2} + \frac{b_2 z^2}{h^2}\right).
\]

Greenberg's (1979) two-dimensional numerical model of the \(M_2\) tide in the Gulf of Maine was used.
to estimate the curvature parameter in the centrifugal driving term. According to his results (Fig. 11a), curvature of the tidal flow (order of 50 km⁻¹), as indicated by the rotation of the major axes of the ellipses, is significant mainly at C1 and C2, where the rectilinear tidal streams are maximal and directed alongshore as described by Eq. (1). When the vertical stratification of the water column is a minimum (February/March), the moored data indicate that the vertical variation in the squared major-axis component of the semi-diurnal tide is roughly parabolic at C1, where the primary upwelling is expected, and, to a lesser extent, at C2 (Fig. 11b). The appropriate constants in the centrifugal term, based on a 50 km radius of curvature, are listed in Table 2 along with the rest of the external parameters.

5) Friction

To complete the model specification, frictional effects must be parameterized by the vertical eddy viscosity $A$ and the bottom resistance coefficient $r$. For the rectilinear tidal flow (1), the resistance coefficient is related to the bottom drag coefficient by

$$r = \frac{2C_d V_T(-h)}{\pi},$$

where $V_T(-h)$ is the tidal stream amplitude just above the constant stress layer at the bottom boundary. Similarly the mean eddy viscosity coefficient may be calculated from an empirical formula (Csanady, 1976) as

$$A = \frac{u_*^2}{200f} = \frac{C_d V_T^2(-h)}{400f}$$

provided $h > h^* = 0.1u_*/f$. Noble et al. (1983) have used direct measurements to estimate bottom drag coefficients on the continental shelf in the range: $4 \leq 10^3 \ C_d \leq 8$, where the maximum values apply to energetic tidal regimes on Georges Bank. Adopting the median value of these estimates ($10^4 \ C_d = 6$) and using the major-axis component of the $M_2$ tidal stream measured 10 m off the bottom during the "unstratified" season (February/March) for $V_T(-h)$ leads to the friction parameters (adjusted slightly to give integral values for $\lambda$) in Table 2. The maximum eddy viscosity at C1 gives an Ekman depth ($h_E = 30$ m) equal to half the total depth, qualitatively consistent with the strongly sheared tidal-velocity profile (Fig. 11b). Furthermore, the resistance coefficients are in the range estimated by Noble et al. (0.07 to 0.20 $\times 10^{-2}$ m s⁻¹) with the maxima again found in the strong tidal regimes. The effects of seasonal mean stratification on the vertical eddy viscosity, as estimated by Garrett and Loder (1981), would be to reduce $A$ by 25 and 17% in summer and winter, respectively. However, variations of this order in both friction parameters produce no qualitative changes in the model results.

c. Results

Various components of the model circulation at Cape Sable (C1, C2) for winter 1979/80 are depicted in Fig. 12. The dominant elements are the longshore
driving longshore flows of 10 to 20 cm s\(^{-1}\) up the Scotian Shelf and an onshore component of 1.4 cm s\(^{-1}\) near the bottom at C1. On the other hand, the wind stress and longshore pressure and offshore density gradients are reduced in summer while centrifugal forces are unchanged.

By linearly superimposing the various elemental circulations in each season, the total model flow may pressure gradient, which drives westward flow toward the Gulf of Maine, and the longshore density and stratification gradients, which create an opposing flow up the Scotian Shelf. The westward flow is supported by weaker contributions from the centrifugal term and offshore density gradient and opposed by the longshore wind. The offshore wind and stratification gradient components are negligible (<1 cm s\(^{-1}\)). The flows driven by the three dominant terms and the offshore density gradient increase in the offshore direction, whereas the centrifugal circulation is reduced along with the forcing term. In the cross-isobath direction, the density gradients and longshore wind create roughly equivalent upwelling circulations of order 1 cm s\(^{-1}\), whereas the longshore pressure gradient causes downwelling at C1.

The summer circulation (not shown) is characterized by generally enhanced longshore density and stratification gradients which alone are capable of

**Fig. 10.** Seasonal vertically averaged mean density and stratification (bracketed) contrasts (kg m\(^{-3}\)) and Yarmouth wind stress (Pa) during (a) summer 1979 and (b) winter 1979–80. Mean density contrasts are calculated by averaging all seasonal horizontal \(\sigma\) differences to the depth of the shallower mooring. Stratification contrasts represent the horizontal variation in vertical density difference over the maximum common depth range (i.e. 15–50 or 15–100 m). Arrows indicate directions of increasing \(\sigma\).

**Fig. 11.** (a) Model result (Greenberg, 1979) for \(M_2\) tidal ellipses off southwest Nova Scotia. (b) Vertical profiles of squared major-axis component at C1 and C2 during February/March 1980. Continuous curves are least-squares parabolic regressions of the form: \(V_z^2 = a(1 + b_1 z + b_2 z^2)\), where \((a, b_1, b_2) = (0.83, 0.37, -0.25 \text{ m}^2 \text{ s}^{-2})\) and \((0.34, -0.43, -0.62 \text{ m}^2 \text{ s}^{-2})\) for C1 and C2, respectively.
**Fig. 12.** Linear components of model mean flow for winter 1979–80 (see Table 2 for external parameter specifications). Velocity vectors at surface(s), mid-depth (m) and bottom (b) are shown for two water depths corresponding to C1(60 m) and C2(110 m) moorings, with component breakdown for C1 only. The Coriolis and gravitational constants are $f = 1 \times 10^{-4} \text{s}^{-1}$ and $g = 10 \text{ m s}^{-1}$. 
be compared to the observed circulation (Fig. 13). The model flow in summer, driven eastward primarily by the longshore density variations, is basically inconsistent with the observations which indicate a weak westward flow with the exception of C3 and possibly C2. Qualitatively it appears that this comparison could be improved by arbitrarily decreasing the magnitude of the longshore density gradient and/or increasing the longshore pressure gradient, but this avenue will not be pursued. The winter comparison, on the other hand, is much more satisfying since, in general, the observed vertical relationships of the currents on a given mooring are modeled correctly in both magnitude and direction. In addition, the observed reversal of the offshore components of bottom current between C1 and C2 is reproduced by the model. In fact the only major flaw in the comparison is found at C3.

One likely source of the wintertime discrepancy at C3 is the neglect of the tidal rectification process (e.g., Loder, 1980) on the steeply sloping sides of Browns Bank. Greenberg (1983) has recently used a nonlinear two-dimensional model to investigate the $M_2$ residual circulation in the Gulf of Maine/Bay of Fundy system. Fig. 14 shows the observed depth-averaged annual mean currents at the Cape Sable moorings superimposed on a model result in which mean sea level is flat on the seaward boundary and linearly set up against the coast on the Scotian and New England Shelf boundaries to produce a realistic longshore flow to the west. In the absence of coastal set-up, longshore current vanishes on the Scotian Shelf east of Shelburne, but the westward jet at Cape Sable and the Browns Bank gyre remain. These results suggest that tidal rectification makes significant contributions to the influxes to the Gulf of Maine at C1 and C4 and the upshelf currents at C3. The tidally-driven components of the mean circulation will be explored further in the next section.

5. Tidal rectification

A simple technique for investigating the effects of tidal rectification in long-term current records is to examine coherent modulations in the low-frequency components and the amplitude of the dominant ($M_2$) tidal signal (Tee, 1975; Butman et al., 1983). The progressive vector diagram (Fig. 15) for near-bottom summer currents at C1 (50 m) reveals such modulations as periods of onshore (∼ northward) flow, roughly normal to the major axis of the tidal ellipse, punctuated by short episodes of longshore flow. This behavior is consistent with the ideas of centrifugal upwelling driven strongly during spring tides but giving way to other forces (e.g., longshore pressure gradient) during neaps.

In order to test this hypothesis and the rectification processes which underlie the circulation in Fig. 14,
the current components in the direction of the major axis of the semidiurnal constituents ($M_2$, $N_2$, and $S_2$ ellipses typically have the same orientation $\pm 3^\circ$) were complex demodulated at semi-diurnal frequency ($f_{CD} = 0.0800$ cph) and compared to the low-passed current components. The complex demodulation technique acts like a bandpass filter at the specified frequency (half-power points at $\pm 0.004$ cph about $f_{CD}$) and produces an amplitude, $V_{CD}$, which measures the average magnitude of the semidiurnal constituents, and phase, which drifts at a rate proportional to the frequency difference between $f_{CD}$ and the dominant semidiurnal signal (i.e. $M_2 - f_{CD} = 0.0005$ cph). The demodulation amplitude for the C1(50 m) record (Fig. 16) exhibits both monthly ($\approx 28$ day) and fortnightly ($\approx 14$ day) modulations, which reflect the beating of $M_2$ with the $N_2$ and $S_2$ constituents respectively.

The corresponding low-frequency current components ($u$, $v$), created by low-passing (half-power point at 7 day period) the 6-h data, are compared to $V_{CD}$ in Fig. 17. It is clear that distinctly similar monthly and fortnightly oscillations are present in the low-frequency currents, and, following Tee (1975), it may also be demonstrated that these variations exceed the natural astronomical constituents by at least an order of magnitude. Note also that the relationship with $V_{CD}$ is inverse (scale reversed) for the $u$-component but direct for the $v$-component, indicating that strong tides drive both westward longshore and offshore residual flows. That is, the cross-shore behavior is opposite to that suggested by centrifugal upwelling!

In order to quantify these results, two methods for estimating the tidally-rectified flow were intercompared. First, linear regressions of ($u$, $v$) on $V_{CD}$ were performed, e.g.,

$$u = \alpha + \beta V_{CD}.$$  \hspace{4cm} (18)

Note that although simple analytical theories of rec-
linear dependence (18) is adopted. When the correlation coefficient, $r^2_*$, is significant, the product of $\beta$ times the average semidiurnal amplitude, $V_{CD}$, may be interpreted as the total tidally-rectified flow, $u^*$, consistent with the long-period modulations of the tidal forcing, i.e.,

$$u^* = (\beta \pm t_{95}S_{\beta})V_{CD}, \quad (19a)$$

whereas the constant term

$$\alpha \pm t_{95}S_{\alpha} \quad (19b)$$

represents a flow bias produced by other forces. The 95% confidence intervals (Students' $t$) in these expressions are based on the standard errors ($S_{\beta}$, $S_{\alpha}$) of the coefficients.

An equivalent technique for identifying coherent modulations in the tides and low-frequency currents is based on spectral analysis (Butman et al., 1983). In this case, the tidal modulations in either the monthly ($\sigma = \sigma_m$) or fortnightly ($\sigma = \sigma_f$) band are calculated as

$$M_\sigma(\sigma) = (2P_{cc}(\sigma) \cdot bw)^{1/2}, \quad (20)$$

where $P_{cc}$ is the power spectral estimate for $V_{CD}$ and $bw$ is the bandwidth. The strength of the coherent current modulations is then determined by

$$M_d(\sigma) = (2 \cdot \gamma_{uc}^2(\sigma) \cdot P_{uu}(\sigma) \cdot bw)^{1/2}, \quad (21)$$

where $\gamma_{uc}$ is the coherence between $u$ and $V_{CD}$ and $P_{uu}$ is the spectrum of $u$. When the coherence is sig-

---

**FIG. 15.** Progressive-vector diagram for near-bottom summer currents at CI(50 m).

---

**FIG. 16.** Complex demodulation at $f_{CD} = 0.08$ cph of semidiurnal major-axis component (104° T) at CI(50 m) during summer 1979. The tidal amplitude, $V_{CD}$, exhibits both monthly (M) and fortnightly (F) modulations and the phase drifts at a rate proportional to $M_2 - f_{CD}$ = 0.0005 cph.
significant, this method produces both an estimate of the tidally-rectified current component

$$u' = (\frac{M_u}{M_\nu}) \cdot V_{CD}$$  \hspace{1cm} (22)$$

and the phase relationship between it and the forcing. At 95% confidence, the coherence (Fig. 18) between the tidal amplitude and the $u$-component of low-frequency current at C1(50 m) is significant in the monthly band, whereas for the $v$-component estimates in both the monthly and fortnightly bands are significant. As expected from the regression results, the $u$- and $v$-components are roughly in and out of phase with $V_{CD}$, respectively.

Attempts to extend these analyses to other seasons and locations within the Cape Sable array have been largely unsuccessful. For instance, the overall correlation coefficients between the winter near-bottom currents and tidal forcing at C1, do not exceed 0.15. Close inspection of these data reveals a coherent modulation near the start of the record which breaks down in early November with the arrival of the fresh water pulse and later because of the dominance of the wind-driven circulation. Similarly, summertime correlations at other mooring sites are generally weak with the exception of mid-depth records at C3. For this case, a comparison of the low-pass currents and semi-diurnal amplitude (Fig. 19) suggests a strong positive correlation between $V_{CD}$ and both $u$ and $v$. These results are borne out by the statistical estimates.

A summary of the tidally-rectified current estimates by both regression and spectral techniques is compared to the observations in Table 3. At C1(50 m) the overall correlation coefficients ($r_u^2, r_v^2$) are significant at the (0.01, 0.001) levels based on 28 'effective' degrees of freedom (Bayley and Hammersley, 1946) and the coherence method produces one and two independent estimates of the $u$- and $v$-components, respectively. The results suggest rectified currents in the range: $0.13 \leq u \leq 0.20$ m s$^{-1}$, $0.09 \leq v \leq 0.11$ m s$^{-1}$; but the observations indicate that
the actual flow is biased in both the onshore and upshelf directions! At C3(44 m), the predicted currents also exceed the observed, but here the bias is downshelf (westward) and strongly onshore.

It is interesting at this point to compare the measurable rectified currents to the difference between the observed mean and diagnostic model currents (Fig. 20). At C1(50 m), the rectified current is larger than the summertime observation - model discrepancy and rotated offshore by 40°-50°. Much of the directional difference could be removed by reorienting the model “longshore” axis to the direction of the observed depth-averaged current at C1 but an amplitude disparity would persist. At C3(44 m), the tidally-coherent component is also clearly larger and directed onto Browns Bank. Although the uncertainties involved in these calculations do not allow a truly meaningful comparison, the results suggest tidal rectification does have a significant impact on the circulation dynamics in the vicinity of Cape Sable.

6. Summary and conclusions

Long-term measurements off southwest Nova Scotia reveal several distinctive features of the circulation:

1) a longshore coastal current to the west,
2) an anticyclonic gyre around Browns Bank, and
3) an upwelling circulation off Cape Sable.

The gyre circulation appears to be permanent but distinct and annual signals are found in the coastal current and upwelling. Strong seasonal cycles are also found in the salinity and density fields off Cape Sable, which are governed jointly by an annual cycle in the longshore transport and by local effects (e.g., tidal mixing).

Table 3. Tidal rectification off Cape Sable.

<table>
<thead>
<tr>
<th>Record duration</th>
<th>Observations</th>
<th>Regression analysis</th>
<th>Long-period coherence estimates</th>
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<tr>
<td></td>
<td>$\vec{v}_{CD}$</td>
<td>$(u, v)$</td>
<td>$\gamma_{se}$, $\gamma_{ce}$</td>
</tr>
<tr>
<td></td>
<td>(m s$^{-1}$)</td>
<td>(m s$^{-1}$)</td>
<td>(m s$^{-1}$)</td>
</tr>
<tr>
<td>C1(50 m) Apr/Sep 1979</td>
<td>0.563</td>
<td>(−0.027, −0.015)</td>
<td>(0.52, −0.63) 0.131, −0.087</td>
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<td>(28, 28)</td>
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<tr>
<td>C3(44 m) Aug/Oct 1979</td>
<td>0.590</td>
<td>(0.026, 0.143)</td>
<td>(0.80, 0.60) 0.275, 0.290</td>
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<td></td>
<td></td>
<td>(19, 6)</td>
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<tr>
<td>Band</td>
<td>$M_{e}$</td>
<td></td>
<td>$\gamma_{se}$, $\gamma_{ce}$</td>
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<tr>
<td></td>
<td>$\vec{v}_{CD}$</td>
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<td></td>
</tr>
<tr>
<td>M:</td>
<td>0.18</td>
<td>(0.86, 0.93)</td>
<td>(0.018, −0.094)</td>
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<tr>
<td>F:</td>
<td>0.10</td>
<td>(0.27, 0.77)</td>
<td>(−0.108)</td>
</tr>
<tr>
<td>M:</td>
<td>0.17</td>
<td>(0.94, 0.84)</td>
<td>(0.274, 0.300)</td>
</tr>
<tr>
<td>F:</td>
<td>0.10</td>
<td>(0.73, 0.94)</td>
<td>(0.225, 0.251)</td>
</tr>
</tbody>
</table>

$\dagger$ $n^*$ is the number of “effective” degrees of freedom for the regression analysis (Bayley and Hammersley, 1946).
A simple diagnostic model incorporating a longshore pressure gradient, horizontal density and stratification gradients, wind stress, and centrifugal forces has been developed to investigate the dynamics of the circulation. For realistic estimates of the external parameters the flow is driven primarily by the longshore pressure gradient, which dominates to give westward flow in winter, and the longshore density and stratification gradient, which drive eastward currents in summer. The centrifugal effects produce an upwelling current of only 0.5 cm s⁻¹, which is considerably smaller than the scale estimate by Garrett and Loucks (1976), essentially because they neglect the depth-averaged Coriolis force on the longshore current. While the model flow field is highly uncertain, it does provide a framework for future modeling efforts.

By examining coherent modulations in the semi-diurnal tidal streams and low-frequency current components, estimates of the tidally-rectified component of the circulation are derived at C1(50 m), near bottom off Cape Sable, and at C3(44 m), mid-depth on the inshore side of Browns Bank. The results indicate that these nonlinear processes drive large mean currents which crudely account for the difference between the observed and diagnostic-model mean flows. At C3, rectification supports the Browns Bank gyre with eastward current but produces a much larger "on bank" component than is measured. At C1, on the other hand, tidal nonlinearities produce westward flow into the Gulf of Maine, as expected, but the estimated cross-isobath component is offshore, i.e., opposite to that associated with centrifugal upwelling. Nevertheless, the observed flow is onshore and apparently driven by longshore density contrasts associated with spatial variations in tidal mixing. This result is also consistent with the observed seasonality in the upwelling current.

Acknowledgment. The author would like to acknowledge the assistance and support of members of the Coastal Oceanography Division of Bedford Institute, particularly R. R. Lively, T. R. Foote and J. W. Pritchard. Dr. C. J. R. Garrett is responsible for suggesting a means of incorporating centrifugal upwelling dynamics into the diagnostic model and Dr. D. A. Greenberg kindly supplied results of his numerical model. Thanks are also due to Drs. J. Loder, B. Petrie, K.-T. Tee, and D. Wright for helpful comments and suggestions on the manuscript.

APPENDIX

Diagnostic Model for Centrifugal Upwelling

To derive the elemental circulation driven solely by the centrifugal term in (7), consider the following reduced forms of (7), (8), (10) and (11):

\[
A \frac{\partial^2 u}{\partial z^2} + fv = g \frac{\partial \zeta}{\partial x} - q, \quad q(z) = \frac{kV^2}{2},
\]

\[
A \frac{\partial^2 v}{\partial z^2} - fu = 0,
\]

\[
-r_{ub} - fv = gh \frac{\partial \zeta}{\partial x} - Qh, \quad Q = \int_{-h}^{0} q(z) dz,
\]

\[
-r_{vb} = 0.
\]

Normalize variables as

\[
u, v = \frac{g}{f} (\tilde{u}, \tilde{v}), \quad z = h \tilde{z}, \quad S_x = -\frac{\partial \zeta}{\partial x},
\]

\[
\tau = \frac{fV}{gh}, \quad (q, Q) = g(\tilde{q}, \tilde{Q}),
\]

with

\[
R = r/fh, \quad E = \frac{2A}{fh^2} = \lambda^{-2}, \quad \lambda = h/h_E,
\]

\[
h_E = \left( \frac{2A}{f} \right)^{1/2} = \text{Ekman depth}.
\]
Dropping the hats, the scaled equations are
\[
\frac{E}{2} \frac{\partial^2 u}{\partial z^2} + v = -S_x - q, \quad \text{(A6)}
\]
\[
\frac{E}{2} \frac{\partial^2 v}{\partial z^2} - u = 0, \quad \text{(A7)}
\]
with boundary conditions
\[
\left( \frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = 0, \quad \text{at } z = 0, \quad \text{(A8)}
\]
\[
\left( \frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = 2\lambda^2 R(u, v), \quad \text{at } z = -1. \quad \text{(A9)}
\]
The integrated forms of (A6)–(A7) are
\[
Ru_b = S_x + Q - \tau, \quad \text{(A10)}
\]
\[
Rv_b = 0, \quad \text{(A11)}
\]
where the transport conditions are
\[
\int_{-1}^{0} u dz = 0, \quad \text{(A12)}
\]
\[
\int_{-1}^{0} v dz = -\tau. \quad \text{(A13)}
\]
In the vicinity of Cape Sable, the measured profile of "centrifugal momentum" may be reasonably fitted to a quadratic function (Fig. 11) of the form:
\[
q(z) = gB(1 + b_1 \hat{z} + b_2 \hat{z}^2) = g\hat{q}; (\hat{z} = z/h) \quad \text{(A14)}
\]
so that
\[
Q = gB \left( 1 - \frac{b_1}{2} + \frac{b_2}{3} \right) = g\hat{Q}. \quad \text{(A15)}
\]
For this case, the general solution for the complex velocity function is
\[
W = u + iv = -i(S_x + q) - \frac{Bb_2}{\lambda^2} + \alpha e^{(1+i)Dz} + \beta e^{(1+i)D^{\alpha}z}. \quad \text{(A16)}
\]
Note that, in general, the system (A6)–(A13) overdetermines the six unknowns \(S_x, \tau, \alpha, \) and the complex constants \(\alpha, \) and \(\beta. \) Thus (A10) and (A11), for example, are redundant but convenient forms for expressing the bottom-current components.
Substituting (A16) into the surface boundary condition (A8) gives
\[
\beta = \frac{(1 + i)Bb_1}{2\lambda} + \alpha D, \quad \text{(A17)}
\]
where \(D = D_x + ID_x = e^{(1+i)D_{\alpha}}. \) Then the bottom boundary condition (A9) and transport conditions, (A12)–(A13), yield the following real system:
\[
[1 + 2\lambda R - (1 - 2\lambda R)\Delta_x + \Delta_x]\alpha_x - [1 - \Delta_x - \Delta_x] \frac{q_x}{\lambda} + Rq'' \frac{R}{\lambda}, \quad \text{(A18)}
\]
\[
[1 - \Delta_x - (1 - 2\lambda R)\Delta_x + \Delta_x] \alpha_x + [1 + 2\lambda R] - (1 - 2\lambda R)\Delta_x + \Delta_x] \alpha_x - 2\lambda \Delta R S_x \frac{q_x}{\lambda}, \quad \text{(A19)}
\]
\[
= 2\lambda Rq_b - \frac{q_b}{\lambda} + [D_x - \lambda R(D_x + D_x)] \frac{q_x}{\lambda}, \quad \text{(A20)}
\]
\[
= Q - (D_x - D_x) \frac{Rq_i}{\lambda} + Rq'' \frac{R}{\lambda}, \quad \text{(A21)}
\]
\[
\Delta_x \alpha_x + (1 + \Delta_x) \alpha_x - S_x = q_b + (D_x + D_x) \frac{q_x}{\lambda}, \quad \text{(A22)}
\]
where \(\alpha_x, \alpha_x \) are the real and imaginary parts of \(\alpha, \Delta_x = D_x + D_x, \) \(\Delta_x = 2D_x D_x, \) \(q_b = \beta(1 - b_1 + b_2), \) \(q'_b = Bb_1, \) \(q''_b = B(b_1 - b_2), \) and \(q'' = 2BBb. \) In general, (A18) may be solved numerically for \(\alpha_x, \alpha_x, S_x, \) and \(\tau \) using a Gaussian elimination technique. Substitution into (A17) then yields \(\beta \) and hence the solution (A16). However, when the friction is weak, manageable analytical forms may be obtained.
In the limit of small Ekman number (\(E = \lambda^{-2} \ll 1), \) the terms in (A18) containing \(e^{-\lambda z} \) vanish and manipulation of the reduced equations leads to
\[
S_x = -q_b - \left( \frac{1 + 2\lambda R}{1 + \lambda R} \right) \frac{q_b}{2\lambda} + \frac{Rq''}{2\lambda(1 + \lambda R)}, \quad \text{(A19)}
\]
\[
\tau = S_x + Q + \frac{Rq_b}{2\lambda(1 + \lambda R)} + \frac{Rq''}{2\lambda^2(1 + \lambda R)}, \quad \text{(A20)}
\]
which may be used to compute characteristic values of the dimensionless velocity field, e.g.,
Surface (\(z = 0\))
\[
\begin{align*}
\left\{ \begin{array}{l}
W_T = \frac{q_x}{2\lambda} - \frac{q''}{2\lambda^2} \\
W_T = -(S_x + B) + \frac{q'_b}{2\lambda}
\end{array} \right\}
\end{align*}
\]
Mid Depth (\(\hat{z} = -\frac{1}{2}\))
\[
\begin{align*}
\left\{ \begin{array}{l}
W_m = -\frac{q''}{2\lambda^2} \\
v_m = -(S_x + q_m); \quad q_m = B \left( 1 - \frac{b_1}{2} + \frac{b_2}{4} \right)
\end{array} \right\}
\end{align*}
\]
Bottom (\(\hat{z} = -1\))
\[
\begin{align*}
\left\{ \begin{array}{l}
u_b = R^{-1}(S_x + Q - \tau) \\
v_b = 0
\end{array} \right\}
\end{align*}
\]
Note that since \(q(z) \) is positive definite and decreases
with depth, the solution represents a sheared longshore current toward negative $y$, which reduces to zero at the bottom where the upwelling velocity is required to balance the sum of the offshore pressure gradient and depth-averaged Coriolis and centrifugal forces. In addition to physical insight, these forms provide a useful check on the numerical method.

REFERENCES


