On "Oceanic Isopycnal Mixing By Coordinate Rotation"

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Redi (1982, hereafter referred to as R) has examined the problem of turbulent diffusion along and across isopycnal surfaces. That paper developed a formulation of mixing in which the Austausch or mixing tensors of temperature, salinity and radioactive tracers are transversely isotropic in isopycnal (σ) coordinates. Since such tensors must be transformed from isopycnal to geodesic coordinates for use in numerical models (Bryan, 1969), potentially important off-diagonal elements can occur. As noted by R, most general circulation models do not allow for off-diagonal elements of these tensors.

The first purpose of this communication is to point out that Solomon (1971) had previously derived the transformation for mixing along and across isentropic surfaces and had discussed the importance of off-diagonal elements in the geodesic representation of the temperature mixing tensor. In spite of the subtle physical distinction between isopycnal and isentropic coordinates, the formal mathematics of the transformation to geodesic coordinates is the same. The first part of R merely extends Solomon’s analysis to x and y coordinates in the σ surfaces.

Two applications of isopycnal mixing were suggested by R. The first was a model of Tritium distribution. In this case isopycnal mixing may indeed be appropriate since the diffusing material plays no measurable role on the density of the oceans. The other application was to the temperature T and salinity S equations in a prognostic general circulation model. The second purpose here is to point out that there is a thermodynamic constraint for isopycnal mixing of temperature and salinity. The constraint arises because the temperature, salinity and density are related through the equation of state. This fact is often ignored by advocates of isopycnal mixing. It is appropriate then to demonstrate the role of this constraint so that criteria are established for the conditions when the isopycnal mixing may not be applicable.

Since σ is a material parameter it follows that

$$\frac{d\sigma}{dt} = 0.$$  \hspace{1cm} (1)

Here $d/dt$ is the material derivative along the σ surface. Down to $10^3$ decibars $\sigma$ is only a function of $T$ and $S$. At greater pressures potential temperature can be used in place of $T$. (Presumably this is the motivation for isentropic coordinates.) From (1) one obtains

$$a \frac{dS}{dt} + b \frac{dT}{dt} = 0,$$  \hspace{1cm} (2)

where

$$a = \frac{1}{\rho} \frac{\partial \rho}{\partial S}, \quad b = \frac{1}{\rho} \frac{\partial \rho}{\partial T}$$

are, respectively, the saline contraction and thermal expansion coefficients. Note that $a > 0$ and $b < 0$.

Since $d/dt$ is evaluated at a constant $\sigma$, then $T$ and $S$ cannot vary independently as they are constrained by an equation of state. Through the latter Eq. (2) can be expressed as

$$\left( a + b \frac{\partial T}{\partial S} \right) \frac{dS}{dt} = 0$$  \hspace{1cm} (3)

or $a = -b(\partial T/\partial S)|s$. The quantity $\partial T/\partial S|s$ is the family of slopes of isopycnals on a $T$–$S$ diagram. For a substantial but not complete portion of a $T$–$S$ diagram this is constant and independent of $\rho$.

Following R the material derivatives of temperature and salinity are equated to the divergence of the respective diffusive fluxes. Thus

$$\frac{dS}{dt} = F_S,$$  \hspace{1cm} (4)

$$\frac{dT}{dt} = F_T.$$  \hspace{1cm} (5)

Inserting (4) and (5) into (2) and using (3) we obtain

$$F_T = -a \frac{\partial T}{\partial S} \mid \sigma = F_S.$$  \hspace{1cm} (6)

From (6) it is seen that if $(\partial T/\partial S)|s$ is constant the material derivative of temperature is proportional to
that of the salinity. In this case isopycnal mixing is a reasonable approximation.

However, in regions where there is significant curvature in the isopycnals on a T–S diagram, the mixing of temperature and salinity produces a fluid of greater density; i.e., cabaling. Isopycnal mixing may not be a reasonable approximation in this situation and standard geodesic model equations such as (1a) and (1b) of R are required to compute temperature, salinity, and density at each time step. An examination of Fig. 4 of R indicates that the northwestern Atlantic is a region where “isopycnal diffusion” may enhance vertical mixing. It is noted that this is also a region where cabaling occurs. Thus the isopycnal mixing of temperature and salinity may not be a viable concept here.

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REFERENCES