Seasonal Heat Transport in a Forced Equatorial Baroclinic Model

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ABSTRACT

Seasonal heat transport is examined in a simple, linear, shallow-water model on the equatorial beta plane. It is found in this model that meridional transport by the seasonally varying western boundary current is of the same magnitude but opposite phase to the seasonally varying interior transport and therefore tends to cancel.

1. Introduction

In a previous paper (Cane and Sarachik, 1981) we solved for the response of the linear shallow water equations to a periodic zonal forcing:

$$i\omega u - yv = -h_x + F(y)\exp(i\omega t), \qquad (1a)$$

$$yu = -h_y, \qquad (1b)$$

$$i\omega h + u_x + v_y = 0. \qquad (1c)$$

These equations have lengths scaled by the equatorial radius of deformation $(C/\beta)^{1/2}$ and time by $(C\beta)^{-1/2}$, where C is the Kelvin wave speed. The equations were solved in the long-wave approximation [hence the absence of $i\omega v$ in Eq. (1b)] on an equatorial beta plane basin bounded by meridians at $x = 0$ and $x = X_E$ and unbounded in the y direction. The forcing was chosen, for analytic convenience, to be independent of x, symmetric in y and Gaussian: $F(y) = \exp[-(1/2)\mu y^2]$. The boundary condition at the eastern boundary of the basin was $u = 0$ but at the western boundary the boundary condition had to implicitly include the effect of the western boundary current which, being composed of short Rossby waves, is not described by Eq. (1). The proper boundary condition on the interior motions is $\int_{-\infty}^{\infty} u dy = 0$ [see Cane and Sarachik (1977, 1981) for more details].

It turned out that the only internal parameter in the calculation is the ratio of the basin scale to the distance a Kelvin wave travels in time ω^{-1}, i.e., $\phi = \omega X_E C^{-1}$ (see also Kindle, 1979). For annual forcing in a basin the width of the Atlantic, $\phi = 0.54$ for the first baroclinic mode. Calculation of the annual depth variations for this value of ϕ showed some close (perhaps fortuitous) agreement with the observed annual thermocline variations in the Atlantic (Merle, 1980). In particular, the interface in the calculation exhibited a minimum amplitude somewhere near one-third of the basin length from the western boundary rather than at the center. This implies that mass is redistributed meridionally as well as zonally so that zonal strips of ocean, extending zonally from $x = 0$ to $x = X_E$ and meridionally from $y = y_S$ to $y = y_N$, will exhibit annual variations in total mass content above the interface.

It is the purpose of this note to examine the *mechanism* for annual mass content changes in our model. Since the thickness of the layer above the interface is a proxy for the total heat content, the terms mass and heat will be used interchangeably in the following.

2. Mechanisms for heat content variations

We may define the total rate of storage of heat in a zonal strip across the basin to be proportional to

$$S(y_N, y_S) = i\omega \int_{y_S}^{y_N} \int_0^{X_E} h(x, y) dx dy, \qquad (2)$$

where the integral term is the heat content of the strip and the $i\omega$ gives the rate of change in this periodic problem.

Because we allow no heat fluxes across the ocean surface, the variations in heat content are caused solely by convergences and divergences of mass. Integrating the continuity equation (1c) across the zonal strip yields

$$S(y_N, y_S) = \int_{y_S}^{y_N} u(x = 0) dy - \int_0^{X_E} [v(y_N) - v(y_S)] dx. \qquad (3)$$

The second term, denoted $I(y_N, y_S)$, is the net flow of mass into the zonal strip by interior meridional motions and is positive when $v(y_N)$ is southward or $v(y_S)$ northward. The first term of (3), denoted $B(y_N, y_S)$ is the influx of mass into the zonal strip by convergence due to the western boundary layer.

That the first term has a similar form to the second may be seen by applying the western boundary condition in its original form $u^b(x = 0) + u(x = 0) = 0$ to the definition of $B(y_N, y_S)$ (super b here represents boundary layer quantities). Then

$$B(y_N, y_S) = -\int_{y_S}^{y_N} u^b(x = 0)dy \quad (4a)$$

$$= -\int_0^{\infty} [v^b(y_N) - v^b(y_S)]dx, \quad (4b)$$

where (4b) follows by simply integrating the continuity equations $u_x^b + v_y^b = 0$. (The upper limit ∞ is the usual boundary layer terminology for integrating over the entire layer, which is here of dimensional scale ω/β, or non-dimensional scale $\omega \ll 1 \ll X_E$.

We see that the storage rate in a zonal strip is simply the net meridional mass flow into the strip, part in the thin boundary layer [Eq. (4b)] and part in the interior.

3. Calculations

In order to evaluate the storage rate using only interior quantities, we must know $u(x, y)$ and $v(x, y)$. The solutions for $u(x, y)$ and $h(x, y)$ were previously given in Cane and Sarachik (1981); $v(x, y)$ may be calculated from the solutions for u and h given in that paper using the momentum equation (1a) or may be constructed directly using the methods of that paper. In either case, the result is

$$v = y \exp(i\omega t)\left\{\frac{\rho(i\phi)}{\sqrt{1+\mu}} \frac{\exp[(1/2)y^2\eta(0, \phi\xi)]}{t^3(0, \phi\xi)} + \frac{\mu \exp[(1/2)y^2\eta(\mu, \phi\xi)]}{t^3(\mu, \phi\xi)} + i\int_0^{\phi\xi} \frac{\exp[(1/2)y^2\eta(\mu, \zeta)]}{t^3(\mu, \zeta)} d\zeta\right\}, \quad (5)$$

where

$$\xi = (x - X_E)/X_E,$$

$$\rho(i\phi) = (1 + \mu)^{1/2}(1 - \mu^2)^{-1}(i \sin 2\phi)^{-1/2} \times \left[\mu^{3/2} - \mu q(\mu, -\phi) + i\int_0^{\phi} q(\mu, -\phi')d\phi'\right],$$

$$q(\mu, \phi) = (\mu \cos 2\phi - i \sin 2\phi)^{1/2},$$

$$t(\mu, \phi) = (\cos 2\phi - \mu i \sin 2\phi)^{1/2},$$

$$\eta(\mu, \phi) = q^2(\mu, \phi)/t^2(\mu, \phi).$$

TABLE 1. Amplitude and phase (radians, positive phases lead the forcing) of the storage rate S, the western boundary layer contribution B, and the interior contribution I: $S = B + I$. Case corresponds to first baroclinic mode Atlantic annual response, $\phi = 0.54$. The scale of zonal forcing is 10° of latitude.

y_N, y_S	$B(y_N, y_S)$	$I(y_N, y_S)$	$S(y_N, y_S)$
1, 0	0.252, 0.761	0.256, −2.67	0.0740, 2.24
2, 1	0.485, −1.28	0.472, 1.85	0.0140, −0.888
3, 2	0.449, 2.20	0.474, −1.10	0.0741, −2.25
4, 3	0.00943, −0.530	0.038, −2.98	0.0314, −2.79
5, 4	0.166, −1.79	0.167, 1.36	0.00173, 2.50
6, 5	0.157, 1.53	0.154, −1.55	0.00944, 0.2769

For a symmetric[1] zonal wind of form $F(y) = \exp[-(1/2)\mu y^2]$ having a scale of about 10° latitude, $\mu = 0.2$. The zonal strips were taken to be of nondimensional width 1 (~3° latitude). The two terms on the right-hand side of (3) were calculated numerically and the results of the complex integrations were checked by verifying the sum with Eq. (2) calculated directly. The results are shown in Table 1. Note that results for thicker zonal strips can be calculated directly from those given here using

$$S(y_N, y_1) + S(y_1, y_S) = S(y_N, y_S),$$

where $y_N < y_1 < y_S$. Similar relations result for $I(y_N, y_S)$ and $B(y_N, y_S)$.

4. Results and discussion

A glance at Table 1 or Fig. 1 immediately shows the main result of this note: the boundary layer contribution to the storage rate is of the same magnitude as the interior contribution but almost exactly out of phase (i.e., π radians). Therefore the boundary and interior terms tend to cancel and, indeed, the total storage rate is at least a factor of 4 smaller than either of the constituents.

Although neither B nor I have ever been observed in the Atlantic, Merle (1980) has calculated, from monthly climatological data, the annual variations in the heat content of zonal strips in the tropical Atlantic. He shows that (his Fig. 9c) the time rate of change of heat content from 6°S to 6°N across the Atlantic reaches its maximum in October. If from Table 1 we calculate the corresponding quantity, $S(2, 0)$, we find its phase to be 2.23 so that we expect the storage rate to lead the wind forcing by about four months. The zonally averaged easterlies across the Atlantic are weakest in February–March (Katz *et al.*, 1977), so that the annual westerly (positive) anomaly

[1] An antisymmetric zonal wind does not contribute appreciably to heat content changes in near-equatorial zonal strips [see Cane and Sarachik (1981) and Schopf (1980)] so that only the symmetric part of the wind need be considered.

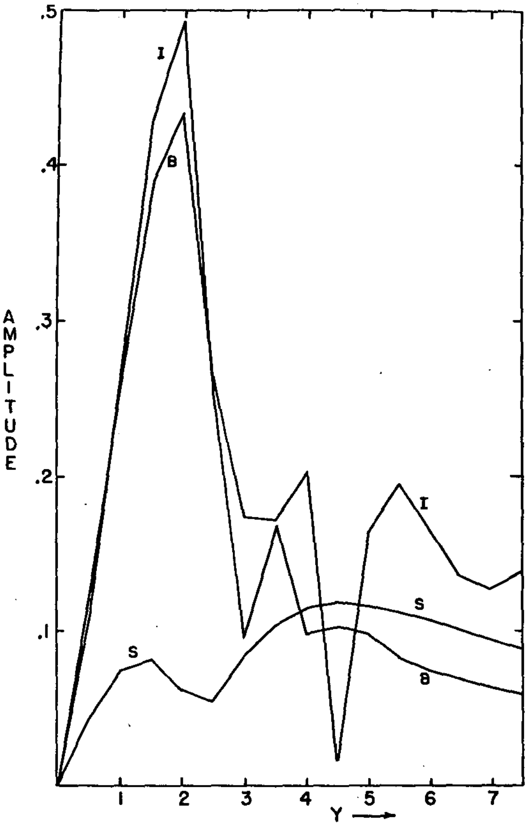

FIG. 1. Amplitude of the total storage rate $S(y, 0)$, the western boundary layer contribution $B(y, 0)$, and the interior contribution $I(y, 0)$. Case corresponds to first baroclinic mode Atlantic annual response, $\phi = 0.54$. The scale of zonal forcing is 10° of latitude. $y = 1$ corresponds to about 3° of latitude.

is largest during these same months. Thus our simple theory predicts the storage rate to be maximum in October–November, in agreement with Merle's results. Merle also shows that the amount of heat exchanged seasonally between the eastern and western halves of the equatorial Atlantic is much greater than the amount exchanged with higher latitudes, consistent with our results [see also Fig. 5c of Cane and Sarachik (1981)].

A detailed comparison of linear theory with observation requires a calculation with realistic winds and surface heating. We anticipate that our most striking result, the near cancellation of the boundary layer and interior contributions to the storage rate, will continue to hold. Moreover, we have reason to expect that this result will carry over to realistic nonlinear numerical models: in the interior an integrated quantity like heat content tends to be well predicted by linear theory, while the usual boundary layer arguments imply that the western boundary current transport is determined by the flux at the coast associated with the interior solution regardless of the detailed dynamics of the boundary layer. Of course, this should be tested in nonlinear models. Our results also have important implications for attempts to calculate heat transports in the tropics from hydrographic data (e.g., Behringer and Stommel, 1981). In particular, both the interior and western boundary current transports must be known to great accuracies, accuracies that we believe are unobtainable with presently available data.

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