Warm Water Mass Formation[1]

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ABSTRACT

Poleward heat transport by the ocean implies warm water mass formation, i.e., the retention by the tropical and subtropical ocean of some of its net radiant heat gain. Under what conditions net heat retention becomes comparable to latent heat transfer to the atmosphere depends on the relative efficiency of transfer processes across the air-sea interface, the top of the atmospheric mixed layer, and the floor of the oceanic mixed layer. A thermodynamic model of the interacting atmospheric and oceanic mixed layers, with the top of the atmospheric layer taken to be at cloud base, shows that net oceanic heat retention is significant under the following circumstances.

1) Seasonal heat storage, amplitude of order 100 W m^{-2}. This is a fairly straightforward consequence of the large heat capacity of the oceanic mixed layer and leads the seasonal forcing by about a month.

2) Massive upwelling (vertical velocity of order 10^{-5} m s^{-1}), mostly along equatorial cool tongues, with net heat retention of order 100 W m^{-2}. The upwelling cooler water is heated and transported away mainly by the divergence of surface layer flow (less by the increasing temperature in the direction of the flow).

3) Cold water advection, mostly within the subtropical gyres, net heat retention of order 30 W m^{-2}. The latitudinal variation of radiant heating, and generally equatorward surface flow in the northern portions of subtropical gyres leads to a moderate rate of warming of the water column as it moves along, i.e., to net heat retention of the above order.

A comparison of model results with observation shows that, over the subtropical gyres, observed temperature and humidity relationships can be simulated realistically only if cold water advection is taken into account. In addition, it is necessary to suppose that the transfer coefficient at the top of the atmospheric mixed layer (at cloud base) is about as large as at the sea surface, while the transfer coefficient at the oceanic mixed layer floor is negligible, except in regions of massive upwelling. The general dominance of latent heat transfer arises from the large value of a nondimensional latent heat coefficient (a material property) and from the rapid drop of saturation specific humidity with height in the atmosphere.

1. Introduction

It is now widely accepted that the ocean transports heat poleward at a rate of the order of 1 petawatt (10^{15} W), see, e.g., Hastenrath (1982). The global process is illustrated schematically in Fig. 1: in order to balance the net heat loss of the polar ocean, water at an average temperature $T_p > T_E$ is transferred poleward (either by "mean" flow or by "eddies"), at the rate M [kg s^{-1}]. From the available temperature and salinity differences and heat and freshwater transports it may be shown (Stommel and Csanady, 1980) that M/ρ is of order 10^8 m^3 s^{-1}. A similar result is obtained by direct oceanic heat transport and heat budget calculations (Bryden and Hall, 1980; Wyrtki, 1981).

At high latitudes, therefore, water is cooled at the aggregate rate of M kg s^{-1}, a process that results in "water mass formation," i.e., a major change of the temperature and salinity characteristics of the water masses involved. The mechanism of this process is fairly well understood: it involves vigorous penetrative convection, vertical mixing, frequently, but not always, a descent of the newly formed mixture to some subsurface level and its eventual escape equatorward.

A counter-process of *warm* water mass formation necessarily exists in the tropical ocean in order to regenerate water masses of higher temperature and salinity than those of water masses entering from higher latitudes. This process is less well understood. In contrast to surface cooling, heating increases stability and tends to retard vertical exchange. Thus except where the cooled water remains at the surface and flows equatorward, some other mechanism, independent of the surface heating process (and in fact counteracting the latter's stabilizing influence) must be present to expose deeper layers to the atmosphere. Only then is it possible to fuel the warm water mass formation process at the required rate and allow the global cycle sketched in Fig. 1 to be completed.

Both atmospheric and oceanic heat budgets show that the principal mechanism supplying cooler water to the surface layer is equatorial upwelling (Hastenrath, 1980, 1982; Wyrtki, 1981). Of lesser importance is coastal upwelling. Cold water advection in the surface

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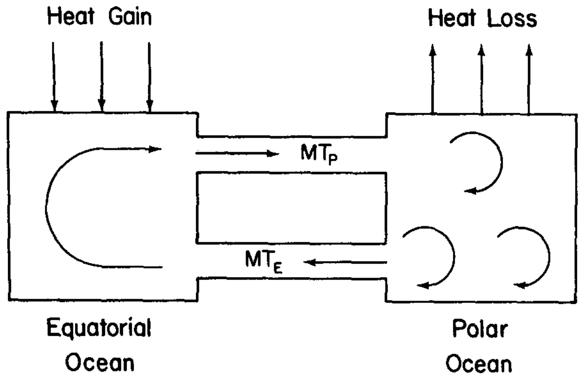

FIG. 1. Schematic of global oceanic heat transports, "cold" and "warm" water mass formation in the polar and equatorial oceans.

layer from high to low latitudes, not involving descent associated with cooling in the first place, is also significant, but contributes much less on a global scale than equatorial upwelling.

Thus the main features of the global oceanic heat transport cycle are known. It is not clear at all, however, what the controls of this cycle are. What determines the mass transfer rate M, the temperatures T_P and T_E, and therefore the net heat transport $Q_H = C_w M(T_P - T_E)$? Why does the ocean not carry an even greater fraction of the total atmospheric–oceanic poleward heat transport than it does, when solar radiation is absorbed in the first instance in the top layer of the ocean, and when the heat capacity of the ocean is so much greater than that of the atmosphere? The proximate cause of the relatively large atmospheric heat transport is evaporation from extensive areas of the tropical and subtropical ocean, and the subsequent release of latent heat to the atmosphere, a worldwide process described in detail in a remarkable essay by Malkus (1962). But why should latent heat transfer be so efficient? Evaporative cooling could conceivably depress sea surface temperatures to the point where little heat could be transferred to the atmosphere, sensible or latent.

In most of the heat budget and heat transport studies carried out so far attention was focused exclusively on either the atmosphere or the ocean. The above questions regarding poleward heat transport and its partitioning between atmosphere and ocean, however, cannot be answered if one of the partners is considered in some sense inert. The few studies in which both atmosphere and ocean were taken to be active tend to be so complex as to obscure the important physical controls (e.g., the numerical model studies of the interacting atmospheric and oceanic boundary layers by Pandolfo and his associates, Pandolfo and Jacobs, 1972; Brown *et al.*, 1982). Undoubtedly, a detailed and generally valid simulation of air–sea interaction requires this degree of complexity. However, it is likely that considerable insight can be gained from a much simpler analytical model in which some of the more complex processes are suitably parameterized. The present study is an attempt to formulate such an analytical model of the interacting tropical and subtropical atmospheric and oceanic boundary layers, explicitly taking into account advection and turbulent transfer processes only, and confining attention to shallow surface mixed layers.

In order to avoid explicit consideration of such complex thermodynamic processes as cloud formation and radiation, it is necessary to restrict the atmospheric part of the system considered to the mixed layer below cloud base. The potential temperature θ_u and specific humidity q_u "above" cloud base then become external parameters representing the end product of complex atmospheric interactions. This means that some key physical processes are at best crudely parameterized by θ_u and q_u, limiting the insight one can gain from the model. Nevertheless, the *double* mixed layer model yields more insight than a single layer one, and supplies some limited answers to the questions raised above.

Although mixed layer models have been widely discussed, a number of pitfalls must be avoided in the formulation. Because the literature contains a number of loose statements on mixed layer balances, the problem is carefully considered in a general way before writing down the balance equations required in the argument.

2. Conservation laws with open boundaries

Consider the atmospheric and oceanic surface mixed layers in contact, bounded above and below by a cloud-base inversion layer and a diffusion floor, at height $z = Z(x, y, t)$ above sea level and depth $z = -h(x, y, t)$ below (Fig. 2). The equation of continuity and the conservation law for a conservative property in either layer will be written, using suffix notation on this one occasion:

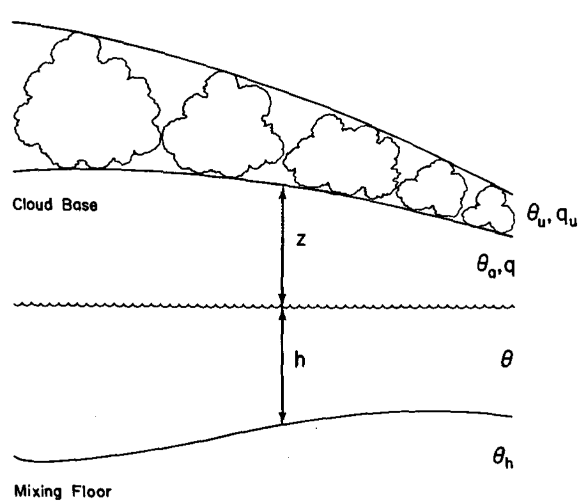

FIG. 2. Interacting atmospheric and oceanic mixed layers and variables of interest in the thermodynamic model.

$$\left.\begin{aligned}&\frac{\partial u_i}{\partial x_i}=0\\&\frac{\partial \chi}{\partial t}+\frac{\partial}{\partial x_i}(u_i\chi)=-\frac{\partial F_i}{\partial x_i}\end{aligned}\right\},\qquad(1)$$

where χ is the property in question and F_i are turbulent flux components. The inversion layer and the mixed layer floor are not fixed either in space or relative to the fluid so that Z and h are not, in general, constant, and there is, in general, a non-zero velocity of advance relative to the air or water:

$$\left.\begin{aligned}w_a&=\frac{dZ}{dt}-w(Z)\neq 0\\w_w&=\frac{dh}{dt}+w(h)\neq 0\end{aligned}\right\},\qquad(2)$$

where $w(Z)$, $w(h)$ are vertical fluid velocities at the two interfaces. It is customary to refer to w_a and w_w as "entrainment" velocities in analogy with plume and jet entrainment problems. Applied to a density interface or a separation surface defined in some other way, the term is in many respects a misnomer, because it implies the kind of one-way progress associated with the growth of jets and plumes. It is more illuminating to think of such an interface as a shock-front propagating relative to the fluid in either direction, i.e., with positive or negative w_a or w_w. However, bowing to custom, the usual terminology will be retained here, and negative values of w_a and w_w will be labeled "detrainment." Note also that when an interface is stationary ($dZ/dt = 0$) in the face of subsidence or upwelling [$w(Z) \neq 0$] entrainment exactly balances advection.

Depth-integrated balance equations are used in the discussion below. Given that the upper and lower boundaries of the systems considered are "open," i.e., Z and h variable, care is required in the integration of the conservation laws, observing relationships of the kind:

$$\int_0^Z\frac{\partial}{\partial x}(u\chi)dz=\frac{\partial}{\partial x}\int_0^Z u\chi dz-u(Z)\chi(Z)\frac{\partial Z}{\partial x}.\qquad(3)$$

Writing

$$U=\int_0^Z udz,\quad V=\int_0^Z vdz,\quad C=\int_0^Z \chi dz,\qquad(4)$$

one readily finds

$$\begin{aligned}\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}&=u(Z)\frac{\partial Z}{\partial x}+v(Z)\frac{\partial Z}{\partial y}-w(Z)\\&=w_a-\frac{\partial Z}{\partial t},\end{aligned}\qquad(5)$$

where w_a is as defined in Eq. (2). The depth integrated conservation law becomes

$$\begin{aligned}\frac{\partial C}{\partial t}+\frac{\partial}{\partial x}\left(\frac{UC}{Z}\right)+\frac{\partial}{\partial y}\left(\frac{VC}{Z}\right)\\=w_a\chi(Z)-F_z(Z)+F_z(0)+\Delta,\end{aligned}\qquad(6)$$

where Δ is the divergence of "diffusive" horizontal flux (including turbulent and "shear" diffusion):

$$\begin{aligned}\Delta=-\int_0^Z\frac{\partial F_x}{\partial x}dz-\int_0^Z\frac{\partial F_y}{\partial y}dz-\frac{\partial}{\partial x}\int_0^Z\left(u-\frac{U}{Z}\right)\\\times\left(\chi-\frac{C}{Z}\right)dz-\frac{\partial}{\partial y}\int_0^Z\left(v-\frac{V}{Z}\right)\left(\chi-\frac{C}{Z}\right)dz.\end{aligned}\qquad(7)$$

The spatial scale of boundary layer evolution will be supposed sufficiently large so that Δ may be ignored. Also, the layers will be supposed well enough mixed to write with a good approximation:

$$\chi\approx\frac{C}{Z},\quad u\approx\frac{U}{Z},\quad v\approx\frac{V}{Z}.\qquad(8)$$

This applies below the inversion layer. Above that layer the concentration is different, $\chi = \chi_u$, say. Given this discontinuity, $\chi(Z)$ in Eq. (6) is indeterminate unless one specifies the sign of w_a. For w_a positive, $\chi(Z) = \chi_u$, otherwise $\chi(Z) = \chi$:

$$\left.\begin{aligned}\chi(Z)&=\chi(Z)_+=\chi_u,\quad(w_a>0)\\\chi(Z)&=\chi(Z)_-=\chi,\quad(w_a<0)\end{aligned}\right\}.\qquad(9)$$

For the case of positive w_a, Eq. (6) now reduces to

$$Z\frac{d\chi}{dt}=w_a(\chi_u-\chi)-F_z(Z)+F_z(0),\qquad(10)$$

where

$$\frac{d\chi}{dt}=\frac{\partial\chi}{\partial t}+u\frac{\partial\chi}{\partial x}+v\frac{\partial\chi}{\partial y}$$

and $F_z(Z)$ and $F_z(0)$ are vertical turbulent fluxes across the inversion and the sea surface respectively. A similar treatment of the water side yields

$$\left.\begin{aligned}&\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=w_w-\frac{\partial h}{\partial t}\\&h\frac{d\chi}{dt}=w_w(\chi_h-\chi)-F_z(0)+F_z(-h)\end{aligned}\right\},\qquad(11)$$

where χ_h is the concentration below the mixed layer floor, again for the case of $w_w > 0$. A source term is readily added to the right hand side, e.g., to represent enthalpy gain by net radiation. In the case of detrainment, i.e., a convergent mixed layer the equations remain correct with w_a or w_w set equal to zero. Physically, the average mixed layer temperature is unaffected by a contraction of the system's boundary. It is easily possible to miss this point (e.g., see Behringer and Stommel, 1981).

Mixed layer models described in the literature are often formulated by specifying $F_z(Z)$ to be zero (e.g., Tennekes and Driedonks, 1981; Niiler and Kraus, 1977). This is an idealization inappropriate in the present context. At cloud base, in the presence of penetrative convection, the turbulent flux $F_z(Z)$ often greatly exceeds the entrainment rate $w_a(\chi_u - \chi)$ (e.g., Betts, 1976).

Furthermore, the "entrainment flux" $w_a(\chi_u - \chi)$ is often at once identified with the turbulent flux $F_z(Z)$, a seemingly confusing step after first postulating vanishing $F_z(Z)$. "Entrainment flux" arises from the choice of a system boundary moving relative to the fluid: as more extraneous fluid is incorporated within the system, the latter's average temperature, or humidity, or whatever scalar property, changes. Rapid equalization of the property within the system implies considerable turbulent flux divergences near the interface and hence indeed large turbulent flux at some short distance inside the interface. However, this is perfectly consistent with zero flux at the chosen moving boundary: peak downward heat flux in the atmospheric mixed layer (to take a concrete example) occurs a short distance below the zero flux level (see, e.g., Ball, 1960). Confusing "entrainment flux" with turbulent flux makes it difficult to reconcile mixed layer or "slab" models with more realistic continuum models, a very undesirable outcome (Deardorff and Mahrt, 1982).

3. Balance equations

The balance equations for enthalpy in water and in air, and for water vapor in air now become, putting water temperature θ, potential air temperature θ_a or specific humidity q for χ in Eq. (10), and using standard bulk relationships for transfer rates across the three interfaces:

$$\left.\begin{aligned} h\frac{d\theta}{dt} &= (w_w + w_{**})(\theta_h - \theta) + \frac{H_r}{\rho_w c_w} \\ &\quad - c_H u_*(\theta - \theta_a)\frac{\rho_a c_{pa}}{\rho_w c_w} - Lc_q u_*(q_s - q)\frac{\rho_a}{\rho_w c_w} \\ Z\frac{d\theta_a}{dt} &= (w_a + w_*)(\theta_u - \theta_a) + c_H u_*(\theta - \theta_a) \\ Z\frac{dq}{dt} &= (w_a + w_*)(q_u - q) + c_q u_*(q_s - q) \end{aligned}\right\}. \tag{12}$$

Here w_{**} and w_* are mass transfer coefficients parameterizing turbulent fluxes at Z and $-h$ respectively, H_r is net heat gain by radiation, q_s is saturation specific humidity, c_H and c_q Stanton and Dalton numbers (Brutsaert, 1982). Over a small range of temperature the saturation specific humidity–temperature relationship may be linearized:

$$q_s = q_{s0} + \gamma\theta, \tag{13}$$

where θ is sea surface temperature excess over some specific reference temperature. For 25°C as reference, $q_{s0} = 0.02$ (kg vapor per kg moist air) and $\gamma = 1.20 \times 10^{-3}\ \mathrm{K}^{-1}$.

It is convenient to introduce the following normalized variables:

$$v_* = c_H u_* \frac{\rho_a c_{pa}}{\rho_w c_w} \quad \text{velocity scale in water,}$$

$$\alpha = \frac{w_w + w_{**}}{v_*} \quad \text{mass transfer constant in water,}$$

$$\beta = \frac{w_a + w_*}{c_H u_*} \quad \text{mass transfer constant in air,}$$

$$\mu = \frac{c_q}{c_H} = 1 \quad \text{ratio of Dalton and Stanton numbers,}$$

$$l = \frac{Lc_q\gamma}{c_{pa}c_H} \quad \text{nondimensional latent heat,}$$

$$\theta_d = \frac{q - q_{s0}}{\gamma} \quad \text{mixed layer wet bulb temperature at sea level,}$$

$$\theta_{du} = \frac{q_u - q_{s0}}{\gamma} \quad \text{wet bulb temperature above cloud base, at sea level pressure,}$$

$$\theta_r = \frac{H_r}{\rho_w c_w v_*} \quad \text{temperature scale of radiant heating.} \tag{14}$$

As indicated above, the ratio μ will be supposed unity and is not shown in the equations below. The replacement of specific humidity by wet bulb temperature implies the choice of a reference temperature, at which the saturation specific humidity is q_{s0} [Eq. (13)]. For simultaneous validity of all three Eqs. (12) it is, of course, necessary to choose the same reference temperature and consider θ, etc. departures from the reference state.

In considering steady state balance it is convenient to suppose the air temperature above cloud base constant and choose it for the reference temperature. The equations simplify somewhat by writing in such cases:

$$\theta_u = 0,$$

$$q_{s0} \equiv q_s(\theta_u). \tag{15}$$

The more general formulation, with θ_u retained as a forcing variable, is required in considering seasonal and latitudinal changes.

In terms of the normalized variables Eqs. (12) become

$$\left.\begin{aligned}\frac{h}{v_*}\frac{d\theta}{dt} &= \alpha(\theta_h - \theta) - \theta + \theta_a \\ &\quad - l(\theta - \theta_d) + \theta_r \\ \frac{Z}{c_H u_*}\frac{d\theta_a}{dt} &= \beta(\theta_u - \theta_a) + \theta - \theta_a \\ \frac{Z}{c_H u_*}\frac{d\theta_d}{dt} &= \beta(\theta_{du} - \theta_d) + \theta - \theta_d\end{aligned}\right\} . \quad (16)$$

These are inhomogeneous equations for water temperature θ, potential air temperature θ_a, and wet bulb temperature θ_d. The forcing terms are the radiant heating temperature θ_r, potential temperature θ_u and wet bulb temperature θ_{du} above cloud base, as well as water temperature θ_h below the mixed layer. With the coefficients supposed independent of θ, θ_a and θ_d the equations are linear. However, at least the mass transfer constant β is a function of buoyancy flux, i.e., of the air–sea temperature difference $\theta_a - \theta$ and of the humidity θ_d. In the following, the problem is discussed first for fixed β. Later, the dependence of β on buoyancy flux is considered. Other coefficients in (16) will be supposed constant.

4. Steady state solution

A zeroth-order, equilibrium solution is found upon setting the left-hand side of the three equations (16) equal to zero. This may be thought to apply to annual average conditions, provided that cold air and water advection is negligible. Departures from the equilibrium solution due to time and space variability will be discussed later. The equilibrium solution is

$$\left.\begin{aligned}\theta &= \frac{(1+\beta)(\alpha\theta_h + \theta_r) + \beta\theta_u + \beta l\theta_{du}}{\alpha(1+\beta) + \beta(1+l)} \\ \theta_a &= \frac{\theta + \beta\theta_u}{1+\beta} \\ \theta_d &= \frac{\theta + \beta\theta_{du}}{1+\beta}\end{aligned}\right\} . \quad (17)$$

Key parameters in these expressions are the two nondimensional mass transfer constants α and β. A significant result follows at once from the definition of α and typical characteristics of subtropical and tropical mixed layers, listed here in Table 1. The scale velocity in water, v_*, is of order 0.3×10^{-5} m s^{-1}. At the mixed layer floor, the mass transfer velocity w_{**}, which parameterizes turbulent flux, is certainly much smaller than v_*. The entrainment velocity w_w is comparable to v_* only in regions of intense upwelling, primarily near the equator. Wyrtki (1981) estimates[2]

TABLE 1. Typical parameters of trade wind region.

H_r (W m^{-2})	180
$c_H u_*$ (m s^{-1})	0.01
v_* (m s^{-1})	$0.3 \cdot 10^{-5}$
θ_r (K)	15
L (J kg^{-1})	$2.44 \cdot 10^6$
c_{pa} (J kg^{-1} K^{-1})	1030
γ (K^{-1})	$1.2 \cdot 10^{-3}$
l	2.78
θ_{du} (K)	−6

w_w to be 10^{-5} m s^{-1} at the equator, corresponding to $\alpha = 3$. Similar upward velocities are found in narrow coastal upwelling regions (Mooers *et al.*, 1976). However, over the subtropical anticyclonic oceanic gyres the surface layer is convergent, so that there is detrainment and $w_w = 0$ applies in Eqs. (12), at least on an annual average. Thus over most of the tropical and subtropical ocean $\alpha \ll 1$. In these locations terms multiplied by α may be dropped from Eqs. (17).

With the choice of reference state above cloud base [Eq. (15)] and $\alpha = 0$, the following expression is obtained for sea surface temperature:

$$\frac{(1+l)\theta}{\theta_r} = \beta^{-1} + 1 + \frac{l\theta_{du}}{\theta_r}, \quad \alpha \approx 0. \quad (18)$$

This now contains only three nondimensional parameters. One of them, $l\theta_{du}/\theta_r$, is a ratio of two forcing terms, representing respectively the temperature depressing effect of evaporation [note that with the choice of reference state according to Eq. (15) θ_{du} is negative] and radiant heating. As shown in Table 1, typical values are $l\theta_{du} \approx -17$ K and $\theta_r = 15$ K, giving a ratio somewhat greater than one, with a negative sign. The sea surface temperature is generally within 1 or 2 K of the temperature above cloud base. This implies that β cannot be small compared to unity, i.e., that $w_a + w_*$ has to be of order $c_H u_*$. Physically, the heat transfer across the cloud base inversion has to be about as efficient as across the sea surface, if measured by the respective heat transfer velocities.[3]

The effective heat transfer velocity $w_a + w_*$ was earlier seen to consist of the inversion layer velocity dZ/dt, the subsidence velocity $-w$, and the velocity w_* parameterizing turbulent exchange. Of these, the first two can be readily estimated from data summarized by Malkus (1962): dZ/dt no more than about 6×10^{-4} m s^{-1}, $-w$ about 3×10^{-4} m s^{-1}, or a total entrainment velocity w_a of about 10^{-3} m s^{-1}. The sea surface heat transfer velocity $c_H u_*$ is, on the other hand, typically 10^{-2} m s^{-1}. It follows that the mass transfer velocity parameterizing turbulent flux at cloud

[2] Wyrtki actually estimates the upwelling velocity. For fixed mixed layer depth, $dh/dt \ll w_w$, this is equivalent to the entrainment velocity; see above.

[3] This is true for negligible advective change. For the mixed layer below cloud base advective changes are in fact fairly small; see the following.

base, w_*, has to be an order of magnitude greater than w_a.

Over the Venezuela rain forest, a careful analysis of observations by Betts (1976) yielded a mass transfer velocity w_* of 0.13 m s^{-1}. For $c_H u_*$ as listed in Table 1 this would yield $\beta = 13$ and $\theta \approx \theta_a \approx 0$, $\theta_d \approx \theta_{du}$, or mass transfer so effective as to erase any property differences across the cloud base layer. However, the buoyancy flux in that situation was considerably larger than found over the sea surface. The relationship of β to the buoyancy flux over the ocean is discussed in greater detail below; here the tentative conclusion is that β is of order unity.

Consider next regions of intense upwelling where α is of order unity. The first of Eqs. (17) may be rewritten as:

$$\theta - \theta_h = \frac{\theta_r + \beta(1+\beta)^{-1} l\theta_{du} - \beta(1+\beta)^{-1}(1+l)\theta_h}{\alpha + \beta(1+\beta)^{-1}(1+l)}. \qquad (19)$$

This gives directly the temperature elevation of the mixed layer over deeper water. The three terms in the numerator are all of the same order, the middle one negative. With the typical parameters of Table 1 and $\alpha = 3$, $\beta = 1$, $\theta_h = -4$ K, one finds $\theta - \theta_h$ about 3 K. The value $\theta - \theta_h = 3$ K is exactly what was taken to be the mixed layer temperature gain in Wyrtki's (1981) box model of the equatorial oceanic heat budget. It is interesting to note that this can be accounted for without postulating horizontal temperature gradients.

Although the equilibrium solution does not simulate temperature and humidity relationships in the interacting mixed layers in a fully realistic manner, with $\beta = 1$ it reflects the partitioning of the radiant heat gain qualitatively correctly. With the left-hand side of the first Eq. (16) set equal to zero, the only term representing oceanic heat retention is the entrainment term, $\alpha(\theta_h - \theta)$. When α is negligible, the radiant heat gain is all balanced by sensible and latent heat loss; with significant α, oceanic heat retention competes effectively for some of the heat gain. In the two typical cases discussed before, the partitioning is

Heat flux	Subtropical gyre $\alpha = 0$, $\beta = 1$		Equatorial upwelling region $\alpha = 3$, $\beta = 1$	
	K	%	K	%
Radiant heat gain (θ_r)	15	100	15	100
Latent heat loss [$l(\theta - \theta_d)$]	13.24	88.3	6.82	45.5
Sensible heat loss ($\theta - \theta_a$)	1.76	11.7	−0.55	−3.7
Oceanic heat retention [$\alpha(\theta - \theta_h)$]	0	0	8.72	58.2

The major difference between the cases with or without upwelling is the large reduction of latent heat loss and its replacement by oceanic heat retention in the divergent mixed layer as the most important balance term for the radiant heat gain. This remains basically true even when storage and advection are taken into account; see later discussion.

5. Adjustment to equilibrium

Whether or how closely the steady state solutions discussed above are approached in a continuously varying system depends on the rate of adjustment to equilibrium. This can be determined by finding the solutions of the homogeneous equations (16), with the forcing terms set equal to zero, $\theta_h = \theta_r = \theta_{du} = 0$, and still with the reference temperature choice of Eq. (15):

$$\left.\begin{aligned} &\theta(1+\alpha+l+\epsilon^{-1}D) - \theta_a - l\theta_d = 0 \\ &\theta_a(1+\beta+D) - \theta = 0 \\ &\theta_d(1+\beta+D) - \theta = 0 \end{aligned}\right\}, \qquad (20)$$

where

$$D = \frac{Z}{c_H u_*}\frac{d}{dt}, \quad \epsilon^{-1} = \frac{h}{Z}\frac{c_H u_*}{v_*}.$$

On account of mixed layer depth variations, as well as other changes, neither ϵ nor the factor $Z/c_H u_*$ can be considered time independent on short or long time scales. However, in terms of a modified time variable

$$\tau = \int^t \frac{c_H u_*}{Z} dt',$$

the operator $D = d/d\tau$ can be treated in the usual manner. The results are then valid on a distorted time scale. Mixed layer deepening has been discussed many times in the literature and there is no need to go into the matter further.

For the typical values quoted in Table 1, $\epsilon^{-1} = 278$, so that ϵ is small compared to unity. Physically, ϵ represents the ratio of the heat capacity of a unit area column of air (height Z) to that of water, depth h. For the limiting cases considered here, an ϵ = constant approximation is adequate because its only role is to modify the effective time scale of adjustment from a short period in air to a long one in water; see below.

The determinant of Eqs. (20) must vanish for homogeneous solutions to exist. From this, one readily finds the three reciprocal times scales of adjustment:

$$\left.\begin{aligned} k_1 &= 1+\beta \\ k_2 &= 1+\beta+\epsilon\frac{1+l}{1+\beta} + O(\epsilon^2) \\ k_3 &= \epsilon\left(\alpha+\beta\frac{1+l}{1+\beta}\right) + O(\epsilon^2) \end{aligned}\right\}. \qquad (21)$$

The adjustment proceeds as $e^{-k_i t}$, $i = 1, 2, 3$. The sea surface temperature simply adjusts to its equilibrium value on the slow time scale k_3^{-1}. The air temperature and specific humidity initially also change on the fast time scales k_1^{-1} and k_2^{-1}, but at $t \gg k_1^{-1}$ they just follow the sea temperature. For $\alpha = 0$, $\beta = 1$ and data in Table 1 the value of k_3^{-1} is 0.88×10^7 s, or 102 days. The time scales k_1^{-1} and k_2^{-2} are the same to zeroth order in ϵ and do not depend on α. For $\beta = 1$ and data of Table 1 their typical value is $k_1^{-1} = 3 \times 10^4$ s or 8.3 h. Physically, changes on time scale k_1^{-1} represent the adjustment of the specific humidity in the atmospheric mixed layer, those on scale k_2^{-1} of the air temperature, while the process on time scale k_3^{-1} is clearly the adjustment of the heat content of the oceanic mixed layer.

6. Seasonal heating

The typical time scale k_3^{-1} is fortuitously close to the annual forcing frequency ω, so that the nondimensional frequency

$$\sigma = \frac{\omega Z}{c_H u_*} \sim k_1\omega \sim \epsilon k_3 \omega \quad (22)$$

is of order ϵ. Seasonal changes can be simply modeled by time variable components of the radiant heating, the upper air temperature and humidity, of amplitudes θ'_r, θ'_u and θ'_{du}, all proportional to $e^{i\omega t}$ (or, with h variable, to $e^{i\omega\tau}$). Neglecting quantities of order σ, the appropriate forced version of Eqs. (20) is now:

$$\left.\begin{aligned} \theta'_a(1+\beta) &= \theta' + \beta\theta'_u \\ \theta'_d(1+\beta) &= \theta' + \beta\theta'_{du} \\ \theta'(1+\alpha+l+\epsilon^{-1}i\sigma) - \theta'_a - l\theta'_d &= \theta'_r \end{aligned}\right\}. \quad (23)$$

The sea surface temperature is then found to vary as

$$\theta' = \frac{1}{me^{i\phi}}[(1+\beta)\theta'_r + \beta\theta'_u + \beta l\theta'_{du}] \quad (24)$$

with

$$me^{i\phi} = \beta(1+l) + \alpha(1+\beta) + i\sigma\epsilon^{-1}(1+\beta)$$

i.e.,

$$\left.\begin{aligned} \phi &= \tan^{-1}\left[\frac{\epsilon^{-1}\sigma(1+\beta)}{\beta(1+l)+\alpha(1+\beta)}\right] \\ m^2 &= \epsilon^{-2}\sigma^2(1+\beta)^2 + [\beta(1+l)+\alpha(1+\beta)]^2 \end{aligned}\right\}.$$

For annual forcing σ is typically 0.012, and the above relationships give for the typical parameters in Table 1 with $\alpha = 0$, $\beta = 1$: $\phi = 60°$, $m = 7.67$.

The typical amplitude of the annual forcing is 75 W m^{-2}, corresponding to $\theta'_r = 6.25$ K, while θ'_u is of order 2 K, θ'_{du} the same. The response amplitude θ' is then, for $\beta = 1$, 2.7 K, lagging behind the forcing by 60° or two months. The first two of Eqs. (23) yield $\theta' = \theta'_a = 2.35$ K, both in phase with the sea surface temperature.

A quantity of some practical interest is the seasonal heat storage in the oceanic mixed layer, represented in Eq. (16) by the term

$$S = \frac{h}{v_*}\frac{\partial\theta}{\partial t}. \quad (25)$$

In the above periodic solution this varies as $i\sigma\theta'$, i.e., it *leads* the forcing by $(\pi/2 - \phi)$, or typically 30°, corresponding to a month. Furthermore, at this (annual) frequency the storage term is only of order unity in the first of Eqs. (16), i.e., in the heat balance of the oceanic mixed layer, not in the heat or vapor balance of the atmospheric layer. With the typical amplitudes used above, the storage term amplitude $\epsilon^{-1}\sigma\theta'$ corresponds to a dimensional value of about 90 W m^{-2}. The seasonal cycle of temperature changes and heat storage is clearly of significant amplitude. Recall here that significant seasonal variations in mixed layer depth distort the time axis, i.e., change the phase relationships found here, as well as the dimensional amplitudes, but these are only matters of detail of no further interest in the present context.

7. Cold water and air advection

Advective changes may be treated much as local ones, by supposing that the forcing terms θ_u, θ_{du} and θ_r are functions of location, and replacing time by space derivatives:

$$\left.\begin{aligned} D &= \frac{Z}{c_H u_*}\frac{d}{dt} \equiv \frac{ZV}{c_H u_*}\frac{d}{dy} \equiv \frac{d}{dY} \\ \kappa D &= \frac{h}{v_*}\frac{d}{dt} \equiv \frac{hv}{v_*}\frac{d}{dy} \equiv \kappa\frac{d}{dY} \\ \kappa &= \frac{hvc_H u_*}{ZVv_*}, \quad Y = \frac{c_H u_* y}{ZV} \end{aligned}\right\}. \quad (26)$$

Here the gradients have for simplicity been supposed to point along the y axis, which is the direction of positive advection velocities V and v, in air and water respectively. The forcing terms in Eqs. (16), θ_r, θ_u and θ_{du}, will all be supposed functions of the coordinate y alone:

$$\left.\begin{aligned} \theta(1+l+\kappa D) - \theta_a - l\theta_d &= \theta_r(y) \\ \theta_a(1+\beta+D) - \theta &= \beta\theta_u(y) \\ \theta_d(1+\beta+D) - \theta &= \beta\theta_{du}(y), \quad \alpha \approx 0 \end{aligned}\right\}. \quad (27)$$

The case of negligible upwelling ($\alpha = 0$) has been assumed for simplicity, because later discussion of advection will deal mainly with the subtropical gyres. The typical value of the parameter $\kappa \equiv (v/V)\epsilon^{-1}$ is now not large, but of order unity, on account of the small typical value of the water:air advection ratio v/V. Usual advection velocities are $v = 0.05$ m s^{-1} and $V = 5$ m s^{-1}, giving $\kappa = 2.78$. The nondimensional distance Y is scaled by $ZV/(c_H u_*)$, which is typically 300 km.

Upon eliminating θ_a and θ_d from Eqs. (27), one finds the single equation for sea surface temperature

$$\kappa D^2\theta + [1 + l + \kappa(1 + \beta)]D\theta + \beta(1 + l)\theta = \phi(Y), \quad (28)$$

where $\phi(Y)$ is the effective forcing function for sea surface temperature

$$\phi(Y) = (1 + \beta + D)\theta_r + \beta\theta_u + \beta l\theta_{du}.$$

The characteristic equation derived from (28) has roots r_1, r_2, given by

$$\left.\begin{aligned} r_1 + r_2 &= -\left[\frac{1 + l}{\kappa} + 1 + \beta\right] \\ r_1 r_2 &= \frac{(1 + l)}{\kappa} \end{aligned}\right\}. \quad (29)$$

The solution of (28) is

$$\theta = \frac{1}{\kappa(r_2 - r_1)}\int_0^\infty (e^{-r_1\eta} - e^{-r_2\eta})\phi(Y - \eta)d\eta. \quad (30)$$

The result shows that the forcing terms over a "backward" ($Y' < Y$) sector of the y axis, from where the advection comes, influence the local temperature. With r_1, r_2 of order unity, the width of the influence zone is the scale of the Y variable, earlier seen to be typically 300 km. This is small on a global scale and it is therefore realistic to replace the forcing function $\phi(Y)$ by its linear expansion. Explicit solutions for θ, θ_a and θ_d may then be written, but they do not prove useful because the gradients of the forcing terms are not well known.

In general such calculations arrive at the equilibrium solution (calculated earlier) plus terms proportional to the gradients of the forcing functions. In case the latter are not known very well, the perturbations can also be expressed directly from Eqs. (27) in terms of the gradients of the dependent variables themselves:

$$\left.\begin{aligned} \theta &= \bar{\theta} - \theta' \\ \theta_a &= \bar{\theta}_a - \frac{D\theta_a + \theta'}{1 + \beta} \\ \theta_d &= \bar{\theta}_d - \frac{D\theta_d + \theta'}{1 + \beta} \end{aligned}\right\}, \quad (31)$$

where θ' is the sea surface temperature depression

$$\theta' = \frac{\kappa(1 + \beta)D\theta + D\theta_a + lD\theta_d}{\beta(1 + l)},$$

and the overbars denote the earlier determined equilibrium solutions.

The variation of sea surface temperature in the direction of the likely surface advection over at least part of the North Atlantic subtropical gyre is known. Over the northern half of this gyre (in the Gulf Stream "recirculation" region) surface drift is more or less along and "up" the surface temperature gradient. According to the charts of Böhnecke (1938) that gradient has a typical magnitude of 2 K/1000 km (2×10^{-6} K m^{-1}), so that using earlier estimates

$$\frac{d\theta}{dY} = 0.6 \text{ K}.$$

The air–sea temperature difference and the dry bulb minus wet bulb difference do not change very much, and one may suppose in a first approximation

$$\frac{d\theta_a}{dY} \approx \frac{d\theta_d}{dY} \approx \frac{d\theta}{dY}. \quad (32)$$

The typical magnitudes of the advection terms in Eqs. (27) then become

$$\left.\begin{aligned} \kappa D\theta &= 1.67 \text{ K} \\ D\theta_a = D\theta_d &= 0.6 \text{ K} \end{aligned}\right\}.$$

With the aid of Eqs. (31) these give the following corrections to the equilibrium temperature

$$\left.\begin{aligned} \theta' &= -1.51 \text{ K} \\ \theta'_a = \theta'_d &= -1.05 \text{ K} \end{aligned}\right\},$$

showing that cold water and cold air advection tends to reduce the sea–air temperature difference because it causes a greater temperature depression in the water than in the air.

8. Radiation heat loss of the atmospheric mixed layer

One observation is difficult to reconcile with the heat balance equation for the atmospheric mixed layer as formulated so far [second of Eqs. (16) or (27)]. This is the fact that θ_a is almost always less than either θ or θ_u. Given efficient heat transfer across both interfaces, the right hand side of the second Eq. (16) is typically greater by 1 or 2 K than can be explained by cold air advection. One is led to consider heat loss by gas radiation H_{ar}, another potentially important forcing term in Eq. (12). In a normalized form appropriate for inclusion in the second of Eq. (16), this term is:

$$\theta_{ar} = \frac{H_{ar}}{\rho_a c_{pa} c_H u_*}. \quad (33)$$

The magnitude of θ_{ar} may be estimated from data summarized by Fleagle and Businger (1963), according to which the air at low levels cools by radiation typically at the rate of 3 K day^{-1} or $d\theta/dt = 3 \times 10^{-5}$ K s^{-1}. The corresponding heat loss term in Eq. (12) would be

$$\frac{H_{ar}}{\rho_a c_{pa}} = Z\frac{d\theta}{dt} = 1.8 \times 10^{-2} \text{ K m s}^{-1}.$$

With $c_H u_* = 0.01$ m s^{-1} this gives a typical value for the normalized variable θ_{ar} of -1.8 K or more or less the magnitude required to balance the second Eq. (16).

The modification of the equilibrium solution [Eq. (17)] on account of the θ_{ar} term consists of the inclusion of θ_{ar} in two of the three numerators, giving correction terms of

$$\theta' = \frac{\theta_{ar}}{\beta(1+l)}, \quad \theta'_a = \frac{\theta_{ar}}{1+\beta}. \tag{34}$$

Typical values are -0.9 K for θ'_a, -0.5 K for θ', comparable to the effect of cold water and air advection, and contributing further to the sea surface temperature depression, while also modifying the sea–air temperature difference, this time in a positive direction.

9. Heat and mass transfer through penetrative convection

In order to complete the analytical argument, it is necessary to consider the physical processes responsible for determining the value of the mass transfer coefficient β, and show that the order one typical values supposed above are plausible. The problem of mixed layer deepening (in the atmospheric or oceanic application) has an extensive literature (e.g., Niiler and Kraus, 1977; Tennekes and Driedonks, 1981) but it is almost exclusively aimed at density interfaces across which the turbulent flux is negligible, and entrainment is the only mechanism of heat and mass transfer. As pointed out above, this is not the case at cloud base, and a different approach must be adopted.

Given the intense penetrative convection at cloud base, it is reasonable to suppose that the vertical motions that "vent" the mixed layer have the velocity scale of free convection (Deardorff, 1974)

$$w_c = (BZ)^{1/3}, \tag{35}$$

where B is the sea surface buoyancy flux, due to the transfer of sensible heat and also of water vapor (Brutsaert, 1982)

$$B = g\left[c_H u_* \frac{\theta - \theta_a}{T} + 0.61 c_q u_* (q_S - q)\right], \tag{36}$$

with T the reference absolute temperature.

The simplest parameterization scheme for the net mass transfer coefficient at cloud base is

$$w_a + w_* = \lambda w_c, \tag{37}$$

where λ = constant. This certainly does not do full justice to the physics of penetrative convection, the net mass transfer being in addition dependent on the distance Z from the lower boundary, the buoyancy change b at the top of the mixed layer, and possibly some other factors, i.e.,

$$\lambda = \mathrm{func}\left(\frac{w_c^2}{bZ}, \cdots\right). \tag{38}$$

Typically, however, the cloud base inversion is weak and it is not too unreasonable to suppose that turbulent transfer processes in its neighborhood are determined by the parameters characterizing the free convection regime, a supposition that leads directly to Eq. (37). This amounts to postulating bZ/w_c^2 to be suitably small in the parameter range of interest.

In the case analyzed by Betts (1976) in detail the characteristic convection velocity was $w_c = 1.85$ m s^{-1}, while the net mass transfer coefficient was empirically determined to be $w_a + w_* = 0.13$ m s^{-1}. This gives $\lambda = 0.07$, a value that will be adopted in the following.

Combining Eqs. (35) to (37), expressing them in terms of the variables of Eq. (14), and substituting the equilibrium relationships (17), one finds the following expression for the coefficient β (reference temperature choice according to Eq. (15)):

$$\beta^2\theta_* = \theta_a + 0.61\gamma T\left(\theta_a - \frac{\theta_{du}}{1+\beta}\right), \tag{39}$$

where θ_* is a temperature scale:

$$\theta_* = \frac{c_H^2 u_*^2 T}{\lambda^3 g Z}, \tag{40}$$

and θ_a is given by (17).

With the above chosen value of $\lambda = 0.07$, $T = 300$ K, and the typical quantities of Table 1, one calculates

$$\theta_* = 0.167 \text{ K},$$

while $0.61\gamma T = 0.22$. The value of β may now be determined (supposing steady state conditions) from Eq. (39) and (17), as a function of the coefficient α, and the forcing parameters θ_r, θ_{du} and θ_h.

With the typical quantities of Table 1, and for $\alpha = 0$ one finds $\beta = 2.3$. Physically, such a relatively low value of β (compared to the over-land case of Betts, 1976) is due to the fact that most of the heat is transferred as latent heat, which causes only little buoyancy flux.

For $\alpha = 4.02$, β drops to zero: at this water entrainment rate, with $\theta_h = -4$ K, there is zero buoyancy flux, and by the hypothesis of Eq. (35), vanishing entrainment of air from above the atmospheric mixed layer. This asymptotic case is characterized also by

$$\theta = \theta_a = \theta_d, \tag{41}$$

or an atmospheric mixed layer of the same temperature as the sea surface, saturated with water vapor, clearly a somewhat unrealistic result. A value of $\beta = 1$ is reached at $\alpha = 2.57$. The precise numerical values are not significant in virtue of the uncertainty of λ, which was deduced from a single case observed over land. However, the calculations show that an order one value of β is compatible with realistically assumed properties of penetrative convection over the ocean. The value of β is kept modest because the buoyancy flux is low at the usual relatively high rate of evaporation.

10. Comparison with observation

To illustrate quantitatively the influence of cold air and water advection, and radiation heat loss, consider a typical subtropical gyre characterized by

$$\left.\begin{aligned} \theta_{du} &= -9\ \mathrm{K} \\ \beta &= 1 \\ c_H u_* &= 0.01\ \mathrm{m\ s^{-1}} \\ H_r &= 180\ \mathrm{W\ m^{-2}} \end{aligned}\right\}.$$

One calculates the following equilibrium temperatures from Eq. (17):

$$\left.\begin{aligned} \theta &= 1.32\ \mathrm{K} \\ \theta_a &= 0.66\ \mathrm{K} \\ \theta_d &= -3.84\ \mathrm{K} \end{aligned}\right\}.$$

These conflict with observation: air below cloud base is generally slightly colder than above, i.e., θ_a is negative, as is θ, although by a smaller margin. Correcting for radiation loss [Eq. (34)] one finds the slightly more realistic results:

$$\left.\begin{aligned} \theta &= 0.84\ \mathrm{K} \\ \theta_a &= -0.48\ \mathrm{K} \\ \theta_d &= -4.08\ \mathrm{K} \end{aligned}\right\}.$$

The calculated sea surface temperature is still too high. However, after allowing for cold water and air advection according to Eq. (32), one arrives at

$$\left.\begin{aligned} \theta &= -0.67\ \mathrm{K} \\ \theta_a &= -1.54\ \mathrm{K} \\ \theta_d &= -5.15\ \mathrm{K} \end{aligned}\right\}.$$

The sea–air temperature difference is now within its typical range of 0.5–1.0 K; the dry bulb minus wet bulb temperature difference is 3.6 K, which is somewhat low but not atypical, and the cloud base inversion strength is 1.54 K or pretty much as generally observed. Better agreement can hardly be expected.

The main point is that without allowing for atmospheric radiation loss *and* cold water and air advection, the long-term mean temperature–humidity relationships cannot be realistically simulated. At the same time, the corrections to the equilibrium solution remain of order 1 K, i.e., small compared to θ_r.

It is of interest to consider also the partitioning of the total heat gain θ_r of the subtropical oceanic mixed layer into sensible and latent heat loss and net heat retained (the latter owing to cold water advection). Assuming that the just calculated values realistically represent annual average conditions, one finds the following:

	K	%
Heat gain, θ_r	15	100
Latent heat loss, $l(\theta - \theta_d)$	12.45	83
Sensible heat loss, $\theta - \theta_a$	0.87	6
Cold water advection, $vh/v_* d\theta/dy$	1.67	11

As remarked earlier, these differ little from what one deduces from the equilibrium solution. The Bowen ratio

$$\frac{(\theta - \theta_a)}{l(\theta - \theta_d)}$$

is 7%, or a typical value. The dominance of the latent heat loss is seen to be caused by the high value of the material constant l and the high absolute value of θ_{du} (which, with β of order unity, translates into a large dry bulb minus wet bulb difference in the mixed layer). The high absolute value of θ_{du} presumably derives from low saturation specific humidity at the height of the trade inversion and is, in a sense, another property of the water substance, although it is also an outcome of boundary layer processes determining the thickness of the cloud layer.

Another test of the simple model developed here is a comparison of results calculated for annual harmonic forcing with the observed variation of monthly mean temperatures. For this comparison a value $c_H u_* = 0.008\ \mathrm{m\ s^{-1}}$ will be taken, deduced from the annual average temperature–flux relationships appropriate to a case reported by Bunker (1976), Fig. 3. Also, $\beta = 1$ will be used. The amplitude of the forcing function is estimated at $H_r' = 75\ \mathrm{W\ m^{-2}}$, directly from Fig. 3. Less certain estimates are mixed layer depths of $z = 600$ m and $h = 50$ m, and seasonal fluctuation amplitudes of dry and wet bulb temperature above cloud base, $\theta_u' = \theta_{du}' = 2$ K. The nondimensional annual frequency is with the above value of $c_H u_*$ equal to $\sigma = 0.015$. From Eq. (24) one finds

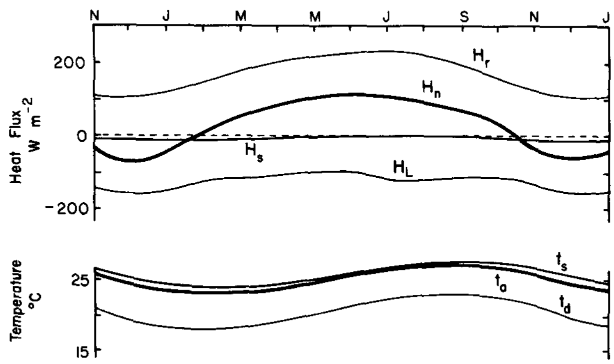

FIG. 3. Annual cycle of surface heat fluxes and temperatures: H_r net radiant heat gain by the ocean; H_s sensible, H_L latent heat transfer to the air; $H_n = H_r - H_s - H_L$ net oceanic heat retention; t_s sea surface, t_a air, dry bulb, t_d wet bulb temperature. From Bunker (1976), based on more than 12 000 observations in the trade wind region (23°N, 52°W).

$$\left.\begin{aligned} \phi &= 65.62 \\ m &= 9.16 \\ \phi' &= 2.46\ \mathrm{K} \\ \epsilon^{-1}\sigma\theta' &= 10.27\ \mathrm{K} \end{aligned}\right\}.$$

This represents a phase lag of a little over two months, a sea surface temperature response amplitude of 2.5 K and a heat storage amplitude of 100 W m^{-2}. The latter is to be compared with the amplitude of oceanic heat retention, H_n, in Fig. 3. The storage should lead the heating rate by about one month. The model predictions are very close to the observed results, considering the crudeness of some of the estimates that went into the calculations, not to speak of the neglect of mixed layer depth variations. All of the evidence confirms that $\beta = 1$ is a reasonable estimate of the cloud base mass transfer coefficient. The order of magnitude of this constant may therefore be taken as firmly established.

11. Conclusions

What does the above analysis imply about warm water mass formation? The process takes place when and where the ocean retains a portion of its radiant heat gain, i.e., when there is net heat retention H_n in excess of what is transferred to the atmospheric mixed layer:

$$H_n = H_r - c_H u_* \rho_a c_{pa}(\theta - \theta_a) - Lc_q u_* \rho_a (q_s - q). \quad (41)$$

According to Eq. (12) the net heat retention is also

$$\frac{H_n}{\rho_w c_w} = h\left(\frac{\partial\theta}{\partial t} + \mathbf{u}\cdot\nabla_1\theta\right) + (w_w + w_{**})(\theta - \theta_h). \quad (42)$$

The right-hand side of this equation shows that heat retention by the oceanic mixed layer may manifest itself in three different ways: 1) as local heating, i.e., storage, $h\partial\theta/\partial t$; 2) as horizontal cold water advection, or heating the fluid as it moves along, $h\mathbf{u}\cdot\nabla_1\theta$; 3) as vertical advection, i.e., heating of fluid entrained from below, accommodated in the divergent surface layer. In the previous section storage and cold water advection were considered in detail in the North Atlantic subtropical gyre.

Figure 3 shows net annual heat retention of order 30 W m^{-2}. According to the charts of Bunker and Worthington (1976) or of Hastenrath (1980), a similar net gain characterizes large areas of the trade wind belt, i.e., the subtropical oceanic gyres. However, these values of H_n were estimated from meteorological and satellite data as a difference between positive and negative terms in an energy budget like Eq. (12), with H_r further split into incoming and outgoing radiation. It is readily shown that the typical error of such calculations is of the same order as the calculated net heat gain, i.e., 30 W m^{-2} (Weare *et al.*, 1981). It is therefore important to ask whether the oceanic evidence supports the meteorological estimates.

The comparison of the boundary layer model with observation in the last section showed that the temperature–flux relationships in the subtropical gyres can only be fully understood if cold water advection is taken into account, at about the rate corresponding to the meteorological evidence. To this extent the present calculations support the radiation balance estimates.

In the area of equatorial upwelling an important heat retention mechanism is the warming of water entering the mixed layer from below, which is accommodated in the divergent surface layer. In Eq. (12) the corresponding heat gain was expressed as $w_w(\theta - \theta_h)$, the turbulent transfer velocity w_{**} being presumably negligible. Wyrtki (1981) has examined the available evidence and concluded that upwelling velocities at the equator are of order 10^{-5} m s^{-1}. The equilibrium solution discussed briefly above with $\alpha = 3$, $\beta = 1$ corresponds roughly to this case and results in $|\theta_h - \theta| = 3$ K. This is a realistic value of the temperature difference between the surface mixed layer and the layer immediately below. Applied to Wyrtki's box model, it shows that most of the net oceanic heat gain can be accounted for by the warming of the upwelling water mass, without postulating significant cold water advection, i.e., relatively large horizontal temperature gradients.

Even more interesting are the meteorological implications of the result that the equilibrium solution gives a good zeroth order estimate of heat flux partitioning. According to this solution, when the equatorial easterlies turn southerly to the point that upwelling becomes insignificant ($\alpha \approx 0$) net oceanic heat retention ceases and latent heat transfer takes its place. A large amount of extra heat is then transferred to the atmosphere at the equator, and not at some higher latitude, where the oceans usually lose heat. The increased sea surface temperature is a relatively feeble symptom, rather than a cause, of this massive increase of air–sea heat transfer. The proximate cause is the ocean's inability to absorb and transport away large amounts of heat without a supply of cooling water. The resulting increased evaporation may be supposed to intensify the convergence of moisture in the near-equatorial convergence zone (NECZ), and hence the "fuel" supply (in the words of Malkus, 1962) for the atmospheric Hadley circulation. This is in accord with the hypothesis of Bjerknes (1966) and also with a recent empirical study of equatorial Pacific latent heat flux and rainfall rate due to Khalsa (1983).

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