On the β-Induced Coastal Trapping of a Baroclinic Eddy‡

MALCOLM J. BOWMAN

Marine Sciences Research Center, State University of New York, Stony Brook, NY 11794

22 October 1984 and 12 February 1985

ABSTRACT

A recurring mesoscale (diameter 100–150 km) baroclinic anticyclonic eddy lies proximate to the shelf break of the southeastern coast of the North Island of New Zealand near Cook Strait. Its appearance and persistence is interpreted in terms of a trapping mechanism suggested by Nof, which consists of an eddy-integrated dynamical balance formed between the β-induced equatorward force and a poleward directed pressure gradient attributed to an opposing southward flowing barotropic coastal current. For parameter values typical of the eddy, calculations show that alongshore coastal flows within the East Cape Current in the range of 2–10 cm s^{-1} would be sufficient to arrest and trap the eddy.

1. Introduction

The existence of an apparently persistent coastally trapped mesoscale eddy (diameter ~ 100 km) lying adjacent to the east coast of New Zealand near Cook Strait (Fig. 1) is investigated in this note. The eddy appears wedged between the coast (which has a very narrow shelf, ~6.5 km) and the southern arm of the coastal component of the poleward flowing East Cape Current which turns abruptly seaward and northeastward near the latitude of the eddy (Fig. 2).

The eddy dynamics are interpreted in terms of a balance between the equatorward β-induced eddy force and the alongshore pressure gradient attributed to the (assumed barotropic) coastal component of the East Cape Current. An application of Nof's (1984) frictionless, reduced-gravity, one-layer nonlinear analytical model gives reasonable results. He considers isolated anticyclonic eddies of lens shape whose depth is zero around their perimeter. Thus the edge represents a discontinuity in density, the radial gradient in tangential swirl velocity, potential vorticity, and perhaps the velocity itself. By integrating the equations of motion over the eddy and using a perturbation scheme in $\epsilon = \beta r_0/f_0$, the ratio of the variation of the Coriolis parameter over the radius r_0 of the eddy to its mean value f_0, he was able to compute the westward migration of both linear and nonlinear eddies provided that some basic properties are known *a priori* (viz., the maximum depth, diameter, density deficit and swirl velocity distribution). Nof finds that a thin lenslike eddy adjacent to a western boundary can remain in a fixed position if the longshore poleward current, in which it is immersed, flows at a critical speed V. This speed is shown to be proportional to β, the inclination of the coastline with respect to the east–west direction, the coefficient of (linear) bottom friction of the coastal current, and the size of the eddy.

It is assumed that the eddy has as its source the East Cape Current, since the water within the eddy core is formed of water of subtropical origin. The ring is surrounded by cooler Southland Current water originating below the subtropical convergence (Fig. 2). Sdubbundhit and Gilmour (1964), Garner (1967) and Heath (1968, 1972, 1975) found evidence of similar mesoscale eddies lying two to three degrees east of our observations. Heath (1975) also suggested that, from time to time, smaller eddies are shed from instabilities in the East Cape Current as it abruptly turns east near 42°S.

It is thought that eddies are shed from instabilities at the southern end of the East Cape Current as it turns abruptly eastward, which then drift with a westward component (McWilliams and Flierl, 1979; Meid and Lindermann, 1979; Nof, 1981) toward the New Zealand landmass. Then the β-induced eddy force initiates an alongshore translation equatorward. But as the eddy migrates along the coast, it moves into the opposing coastal component of the East Cape Current until its drift is eventually arrested. This trapping could presumably be accomplished by form drag imposed on the eddy by the underlying current, although Nof (1984) hypothesized that the eddy glides frictionlessly up the steadily increasing alongshore surface slope (which drives the current against bottom stress) until its migration is arrested.

2. Observations

The repeated appearance, in many remotely sensed thermal IR images, of a persistent warm-core ring in

‡ Contribution Number 465 of the Marine Sciences Research Center, State University of New York–Stony Brook.

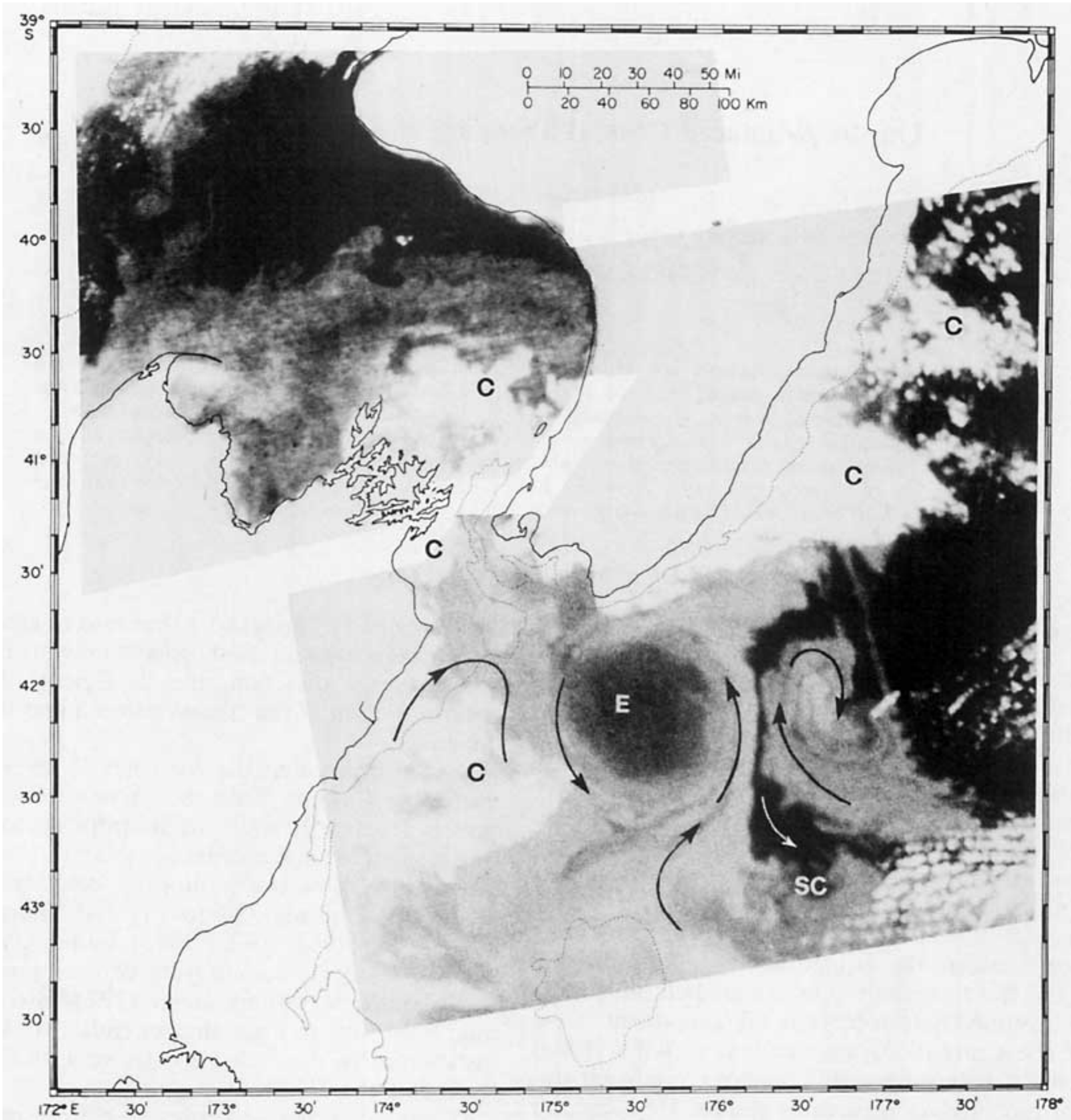

FIG. 1. NOAA-6 thermal IR image, 3 May 1980, of the anticyclonic eddy lying to the southeast of Cook Strait (which separates the two main islands of New Zealand). The arrows depict the direction of circulation around the eddy and in the intermingling warm and cold intrusions to the east at the subtropical convergence (E—eddy; C—clouds; SC—subtropical convergence).

a fixed location off the southeastern tip of the North Island (Fig. 1) suggests a trapping mechanism that is sufficiently stable to allow the eddy to remain quasi-stationary for extended periods of time (e.g., Bowman *et al.*, 1982: 60; Bowman *et al.*, 1983a: Figs. 10, 11; E. Barnes, personal communication, 1984).

Figure 3 presents 200 m horizontal temperature contours (which are also assumed to follow geostrophic streamlines) derived from a hydrographic survey made in February and March 1969 (Heath, 1975). This diagram was chosen for its similarity to the surface temperature patterns suggested in the satellite image of Fig. 1 (for which there are no contemporaneous hydrographic data). The East Cape Current exhibits considerable meandering as a series of large anticyclonic eddies along the North Island east coast. It would appear that there is a smaller anticyclonic eddy off the southeastern tip of the North Island (centered at station D 856) in the same location as the one depicted in Fig. 1. A vertical section was constructed through the 1969 eddy and the southern arm of the East Cape Current to investigate the vertical extent of these features (Fig. 4). The vertical axis of the Cook Strait eddy appears tilted near the surface and may have significant structure down to at least 400 m.

3. Dynamics

Nof (1981) investigated the nonlinear β-induced translation of a thin isolated baroclinic eddy set in a

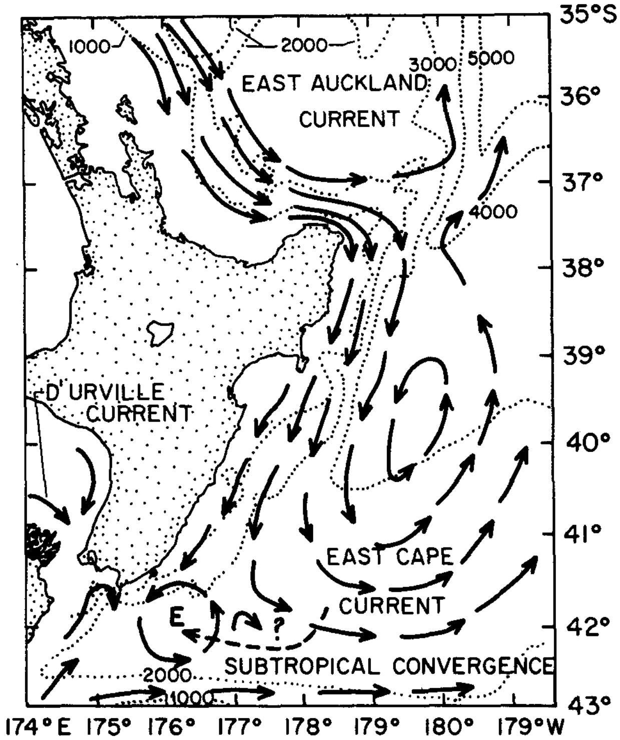

FIG. 2. Schematic diagram of the circulation to the east of New Zealand, showing the assumed path of various coastal currents, based on historical hydrographic data (e.g., Brodie, 1969; Garner, 1969; Heath, 1972, 1975). It is thought that the Cook Strait eddy is shed from the East Cape Current at ~42°S, drifts westward to the edge of the continental shelf and then northeastwards until it encounters and is arrested by the coastal component of the East Cape Current.

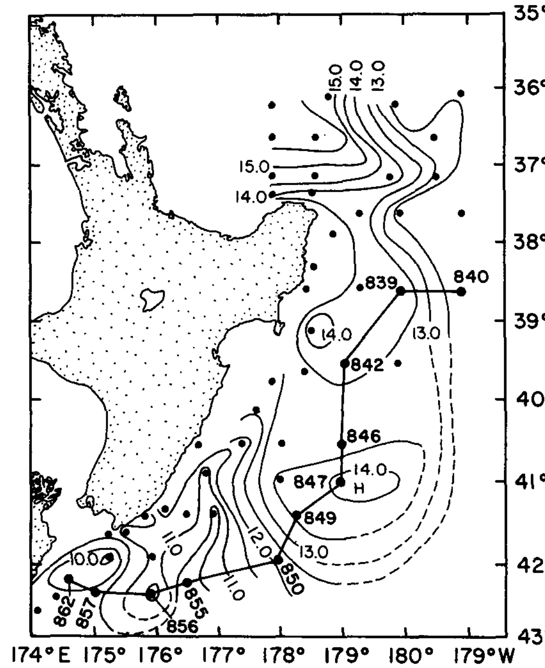

FIG. 3. Isotherms (°C) at 200 m, February–March 1969, assumed to be geostrophic streamlines. The crooked line joining stations D862–D840 denotes the extent of the vertical section shown in Fig. 4 (after Heath, 1975). These contours should be compared for similarities with surface-temperature gradients illustrated in Fig. 1.

uniform, motionless ocean. Eddies that satisfy $(\beta r_0/f_0)^2 \ll \mathrm{Ro}$ (where Ro, the Rossby number, is defined later) will drift westward at a velocity C, given by

$$C = -\beta \int_0^{r_0} \psi(r) r dr \Big/ \left(f_0 \int_0^{r_0} h(r) r dr \right) \tag{1}$$

where

$$\psi(r) = \int_{r_0}^{r} v(r) h(r) dr \tag{2}$$

is the transport streamfunction. Here $\psi = 0$ around the perimeter of the eddy of radius r_0, where the thickness $h(r_0) = 0$.

First, consider a nonlinear eddy whose swirl velocity $v_\theta(r)$ increases linearly with distance r from the center (i.e., solid body rotation; a "type I" eddy):

$$v_\theta(r) = -\mathrm{Ro} f_0 r. \tag{3}$$

Thus, Ro is here defined as

$$\mathrm{Ro} = -v_\theta(r_0)/f_0 r_0,$$

where $v_\theta(r_0)$ is the maximum value of $v_\theta(r)$ located at the boundary of the eddy. By integrating the gradient current equation

$$v_\theta^2/r + f_0 v_\theta = g' dh/dr$$

with $v_\theta(r)$ substituted from Eq. (3), it follows that (Nof, 1981):

$$h(r) = h(0) + \mathrm{Ro} f_0^2 r^2 (\mathrm{Ro} - 1)/2g', \tag{4}$$

$$r_0 = [2g'h(0)/\mathrm{Ro}(1 - \mathrm{Ro})]^{1/2}/f_0. \tag{5}$$

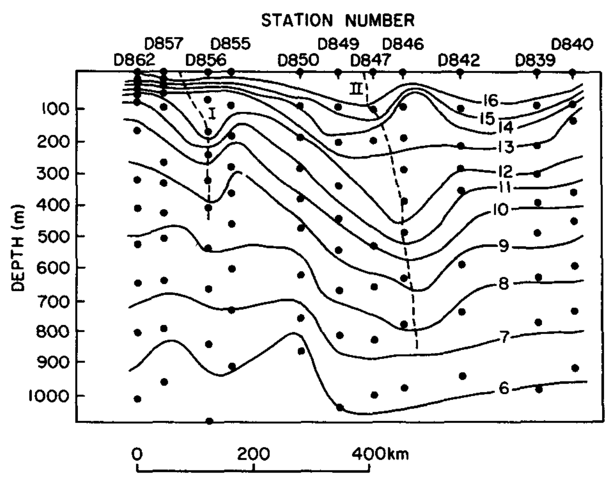

FIG. 4. Vertical temperature section (°C) for stations D862–D840, February–March 1969, showing the Cook Strait eddy I and the larger anticyclonic eddy II centered near station D847 (see Fig. 3). Data from Heath (1975).

Substituting $h(r)$ and r_0 from Eqs. (4) and (5) into (1) and (2), we get

$$C = -\frac{1}{3}\beta R_d^2/(1 - \text{Ro}), \quad 0 < \text{Ro} < 0.5 \tag{6}$$

where the internal Rossby radius $R_d = [g'h(0)]^{1/2}/f_0$. For a "type II" eddy with a perhaps more realistic parabolic velocity distribution the swirl velocity $v_\theta(r)$ is given by:

$$v_\theta(r) = 2\,\text{Ro}f_0 r(r/r_0 - 1), \quad 0 < \text{Ro} < 0.25. \tag{7}$$

Here Ro is defined by

$$\text{Ro} = 2v_\theta(r_0/2)/f_0 r_0.$$

Then, as before,

$$h(r) = h(0) + \text{Ro}f_0^2 r^2(2\,\text{Ro} - 1)/g' + 2\,\text{Ro}f_0^2 r^3 \frac{1 - 4\,\text{Ro}}{3g'r_0} + \text{Ro}^2 \frac{f_0^2 r^4}{g' r_0^2}, \tag{8}$$

$$r_0 = \left[\frac{3g'h(0)}{\text{Ro}(1 - \text{Ro})}\right]^{1/2} \Big/ f_0, \tag{9}$$

$$C = -(0.285 + 0.306\,\text{Ro})\beta R_d^2, \quad 0 < \text{Ro} < 0.25. \tag{10}$$

Other velocity and depth profiles can be derived as, for example, in eddies of constant potential vorticity (e.g., Csanady, 1979; Flierl 1979) with similar results. Killworth (1983) further developed Nof's (1981) theory and discovered that *only* westward propagation for realistic eddies can occur with the drift rate less than ⅔ that of the long Rossby wave speed $-\beta R_d^2$, and that the (unperturbed) eddy is stable on a f-plane to small disturbances. Hence, this potentially contributes to a long eddy life.

It should be pointed out at this juncture, that in the quasi-geostrophic numerical models of McWilliams and Flierl (1979) and Meid and Lindemann (1979), there is a predicted meridional (equatorward) component of the drift in addition to the mainly westward migration. This equatorward drift, absent in the isolated Nof eddies, is a function of both eddy intensity $Q = V/\beta l^2$ (where V is the scale of swirl velocity, and l the pressure e-folding radius) and $\gamma = l/R_d$. The maximum limit to the westward drift is the Rossby long wave speed $-\beta R_d^2$, and the equatorward drift $-\beta R_d^2/4$.

Killworth also showed that the minimum radius for isolated eddies is $2\sqrt{2}R_d$ [the radii of the Cook Strait eddies discussed in this paper range from $\sim 2\sqrt{2}R_d$ (Fig. 1) to $\sim 1.5(2\sqrt{2}R_d)$; Fig. 4].

Nof (1984) hypothesized that the eddy translates westward until it encounters a western boundary; then it drifts along the coast with an equatorward component. He showed the critical arresting velocity U of the opposing coastal current to be

$$U = \beta f_0 R_d^2[1 + 2(R_d/r_0)^2]\sin\alpha/3K \tag{11}$$

for a type I eddy, and

$$U = \beta f_0 R_d^2[0.285 + 0.918(R_d/r_0)^2]\sin\alpha/3K \tag{12}$$

for a type II eddy. The α is the inclination of the coastline (measured clockwise with respect to the east–west direction) and $K \sim 2 \times 10^{-6}\ \text{s}^{-1}$ is a linearized bottom friction coefficient associated with the barotropic coastal current.

4. Comparison with observations

Based on characteristics for the observed Cook Strait eddies depicted in Figs. 1 and 4, we set

$$f_0 = -9.73 \times 10^{-5}\ \text{s}^{-1}$$
$$\beta = 1.69 \times 10^{-11}\ \text{m}^{-1}\ \text{s}^{-1}$$
$$\alpha = 145° \quad \text{(Fig. 1)}$$
$$K = 2 \times 10^{-6}\ \text{s}^{-1} \quad \text{(Veronis, 1981).}$$
$$g' = 1.17 \times 10^{-2}\ \text{m s}^{-2} \quad \text{(Fig. 4)}$$
$$h(0) = 400\ \text{m} \quad \text{(Fig. 4)}$$
$$r_0 = 95\ \text{km} \quad \text{(Fig. 1).}$$

For a type I eddy, from Eqs. (4), (3), (6) and (11), we get:

$$\text{Ro} = 0.12$$
$$v_\theta(r_0) = -1.3\ \text{m s}^{-1}$$
$$C = -0.30\ \text{cm s}^{-1},$$

and

$$U = 9\ \text{cm s}^{-1};$$

and for a type II eddy, from Eqs. (8), (7), (10) and (12):

$$\text{Ro} = 0.21$$
$$v_\theta(r_0/2) = -0.97\ \text{m s}^{-1}$$

(i.e., the maximum tangential velocity)

$$C = -0.30\ \text{cm s}^{-1},$$

and

$$U = 2.6\ \text{cm s}^{-1}.$$

5. Discussion

The above analysis suggests that a poleward coastal current in the range of 2–10 cm s^{-1} is sufficient to arrest and trap eddies with the characteristics of those observed east of Cook Strait. This is well within the range of currents reported for the East Cape Current (B. G. Sanderson, personal communication, 1984; further direct current measurements within the East Auckland Current are presently being gathered; A. C. Kibblewhite, personal communication, 1984).

The predicted westward drift rate is quite small (approximately -0.3 cm s^{-1}). This suggests that if the eddies are to become trapped against the coast, then the shedding must occur near to, or in, the coastal stagnation region just south of the point of separation of the East Cape Current from the North Island coast ($\sim$41°S). Otherwise they would likely be swept away by the zonal circulation in the subtropical convergence.

For comparison, we can compare our results to those of McWilliams and Flierl (1979). Our eddies correspond to $Q \sim 10$ and $\gamma^2 \sim 12$. This is in the large asymptotic range of γ; their numerical calculations predict a zonal propagation rate of $(-1/\gamma^2)(\beta l^2) = -\beta R_d^2 \sim -0.8$ cm s^{-1} and a meridional rate of $\frac{1}{4}\gamma^2(\beta l^2) = \beta R_d^2/4 \sim 0.2$ cm s^{-1}. Thus the eddies propagate three times faster than Nof's isolated eddies, and in a direction 14° north of west. This would require a coastal current in the range of 6–30 cm s^{-1} for trapping, still not unreasonable.

One aspect of the problem not addressed in the theory of relatively thin eddies is the possible influence of the Chatham Rise, a zonally oriented ridge abruptly rising to about 500 m at about 42°30′S. While the ridge may not directly affect the eddies, it certainly has an important influence on the location of the subtropical convergence which controls the southern extent of the East Cape Current.

The analysis presented in this note is unable to distinguish whether McWilliams and Flierl's (1979) or Nof's (1981) theories better describe the observations, since they both predict a similar westward drift for the appropiate eddy parameter ranges. One has to doubt on the basis of Fig. 4 whether these eddies are in fact "isolated" and "thin." It might be more appropriate to consider the eddies as being trapped by a balance between the β-induced force and the form drag imposed by shear between the eddies and the underlying water. However, it is difficult to arrive at a realistic estimate of what this might be. The equipment requirements to make a meaningful field experiment to establish the correct dynamics are enormous. First order calculations, however, indicate that the form drag is likely to be only a few percent of the effect due to the sea surface topography of the opposing frictionally balanced coastal current. The magnitude of the arresting coastal current is also proportional to the linear bottom friction coefficient K which is poorly known for this area. We can conclude from this study that β-induced trapping is a viable candidate mechanism for the remarkable persistence of the Cook Strait eddy.

An apparently similar oceanic regime exists at a corresponding latitude in the Northern Hemisphere to the east of Tsugaru Strait between the Japanese islands of Honshu and Hokkaido. The Tsugaru Strait possesses comparable dimensions, sill depth, water mass characteristics and direction of net flow to Cook Strait. Conlon (1982) has identified two outflow modes of the Tsugaru warm current. The summer and fall gyre mode is characterized by the presence of a mesoscale anticyclonic eddy adjacent to the eastern mouth of the strait, while the winter and spring outflow from the strait tends to flow equatorward as a narrow coastal current along the eastern shore of Honshu. Although surface currents of warm Tasman Sea water have been mapped intruding into Cook Strait from the west during gale conditions (Bowman *et al.*, 1983a,b), it is thought that intense tidal mixing within the narrows of Cook Strait limits their penetration and precludes any significant contribution of Tasman Sea water to the eddy.

Vastano and Bernstein (1984a,b) have also reported on the development and dissipation of frequently observed anticyclonic eddies formed from the interaction of Oyashio intrusions and the Kuroshio extension east of Tsugaru Strait. They suggested a 20 day cycle for the degeneration of the Oyashio/Kuroshio front associated with the production of eddies and horizontal movement of existing eddies. In contrast, Heath (1975) suggested a 50–70 day periodicity in the East Cape Current eddy shedding sequence, linked across the Tasman Sea to an associated periodicity in the East Australian Current (Hamon and Kerr, 1968; Hamon *et al.*, 1975; Bennett, 1983).

Clarification of exactly how the Cook Strait eddies are created, migrate and dissipate, must await further studies. Future investigations might also focus on whether the eddies are reabsorbed back into the East Cape Current in much the same way as warm core rings migrating down the eastern slope waters of the Middle Atlantic Bight are reabsorbed back into the Gulf Stream near Cape Hatteras.

Acknowledgments. This work was supported by ONR Grant N00014-84-C-0178, NSF Grant OCE 81-8283 and the Maui Development Environmental Study (MDES). The MDES was commissioned by Shell BP Todd Oil Services acting on behalf of Maui Development, Ltd, and was coordinated by Prof. A. C. Kibblewhite at the University of Auckland. I wish to thank Mr E. C. Lewis and the staff of the Scripps Remote Sensing Facility, La Jolla, for assistance in processing the satellite imagery, and to Drs A. C. Kibblewhite, D. Nof and G. Holloway for helpful discussions and comments on the manuscript. The research program was facilitated by a Memorandum of Understanding between the State University of New York and the University of Auckland.

REFERENCES

Bennett, A. F., 1983: The South Pacific including the East Australian Current. *Eddies in Marine Science,* A. R. Robinson, Ed., Springer-Verlag, 609 pp.

Bowman, M. J., D. A. Jones and A. C. Kibblewhite, 1982: Tidal studies. *Maui Development Environment Study, Report on Phase 2.* A. C. Kibblewhite *et al.*, Eds. University of Auckland.

——, A. C. Kibblewhite, R. A. Murtagh, S. M. Chiswell and B. G. Sanderson, 1983a: Circulation and mixing in greater Cook Strait. *Oceanol. Acta,* **6,** 383–391.

——, ——, S. M. Chiswell and R. A. Murtagh, 1983b: Shelf fronts and tidal stirring in greater Cook Strait, New Zealand. *Oceanol. Acta,* **6,** 119–391.

Brodie, J. W., 1960: Coastal surface currents around New Zealand. *N.Z. J. Geol. Geophys.,* **3,** 235–252.

Conlon, D. M., 1982: On the outflow modes of the Tsugaru warm current. *La Mer,* **20,** 60–64.

Csanady, G. T., 1979: The birth and death of a warm core ring. *J. Geophys. Res.,* **84,** 777–780.

Flierl, G. R., 1979: A simple model for the structure of warm and cold core rings. *J. Geophys. Res.,* **84,** 781–785.

Garner, D. M., 1967: Hydrology of the Hikurangi Trench region. *Mem. N.Z. Oceanogr. Inst.* 39, Wellington, 177 pp.

——, 1969: The geopotential topography of the ocean surface around New Zealand. *N.Z. J. Mar. Freshwater Res.,* **3,** 209–219.

Hamon, B. V., and J. D. Kerr, 1968: Time and space scales of variations in the East Australian Current from merchant ship data. *Aust. J. Mar. Freshwater Res.,* **19,** 101–106.

——, J. S. Godfrey and M. A. Greig, 1975: Relation of mean sea level, current and wind stress on the east coast of Australia. *Aust. J. Mar. Freshwater Res.,* **26,** 389–403.

Heath, R. A., 1968: Geostrophic currents derived from oceanic density measurements north and south of the subtropical convergence east of New Zealand. *N.Z. J. Mar. Freshwater Res.,* **2,** 659–677.

——, 1972: The Southland Current. *N.Z. J. Mar. Freshwater Res.,* **6,** 497–533.

——, 1975: Oceanic circulation off the east coast of New Zealand. *New Zealand Oceanogr. Inst. Mem.,* No. 55, Wellington, 80 pp.

Killworth, P. D., 1983: On the motion of isolated lenses on a β-plane. *J. Phys. Oceanogr.,* **13,** 368–376.

McWilliams, J. C., and G. R. Flierl, 1979: On the evolution of isolated, nonlinear vortices. *J. Phys. Oceanogr.,* **9,** 1155–1182.

Mied, R. P., and Lindemann, 1979: The propagation and evolution of cyclonic Gulf Stream Rings. *J. Phys. Oceanogr.,* **9,** 1183–1206.

Nof, D., 1981: On the β-induced movement of isolated baroclinic eddies. *J. Phys. Oceanogr.,* **11,** 1662–1672.

——, 1984: On the interaction between thin isolated eddies and longshore currents. *J. Phys. Oceanogr.,* **14,** 125–137.

Sdubbundhit, C. E., and A. E. Gilmour, 1964: Geostrophic currents derived from oceanic density over the Hikurangi Trench. *N.Z. J. Geol. Geophys.,* **7,** 217–278.

Vastano, A. C., and R. L. Bernstein, 1984a: Mesoscale features along the first Oyashio intrusion. *J. Phys. Oceanogr.,* **89,** 587–596.

——, and ——, 1984b: Correction to "Mesoscale features along the first Oyashio intrusion." *J. Phys. Oceanogr.,* **89,** 3733–3734.

Veronis, G., 1981: Dynamics of large scale ocean circulation. *Evolution of Physical Oceanography,* B. Warren and C. Wunsch, Eds., The MIT Press, 623 pp.