The Parameterization of Subgrid-Scale Heat Diffusion in Eddy-Resolved Ocean Circulation Models

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ABSTRACT

The two-layer quasigeostrophic model of Holland is modified to include a parameterization of subgrid-scale heat diffusion. Results from a sequence of simple, eddy-resolved calculations illustrate the effects of increasing heat diffusivities. It is clear that even rather small diffusion coefficients (small compared to the viscosity) cause important modifications of the eddy field and of the eddy generation process. In particular, heat diffusion can be very effective at diminishing the baroclinic signal associated with mesoscale processes, making it less likely that baroclinic instability processes can exceed damping.

1. Introduction

Numerical models of ocean circulation require important choices and involve substantial compromises in carrying out calculations. These choices are both numerical and physical. The most striking example of the former is the choice of coarse versus fine horizontal resolution that presently differentiates global scale and eddy-resolved basin scale models of the kind developed by Bryan (1969). A second example, physical this time, is the choice between eddy-resolved adiabatic primitive equation (PE) models like that used by Holland and Lin (1975a,b) and the equivalent quasigeostrophic (QG) model used by Holland (1978). In this example, even with adiabatic physics in both cases, the PE model has more complete and presumably more realistic physics while the QG model is much more computationally efficient. These kinds of trade-offs are an inherent part of the modeling approach to studying and understanding large scale ocean circulation, and it is clear that an hierarchy of such models is needed for the task.

Here we would like to discuss briefly one aspect of such choices and point out the perceptions about eddy-resolved ocean circulation that result. This involves the inclusion or not of diffusive effects in the heat (density) equation, that is, whether or not we include non-adiabatic physics. The eddy-resolved general circulation models of Holland and Lin (1975) and Holland (1978) involved adiabatic physics—the heat equation describes the rate of change of density as resulting from advective flux divergence only. No parameterized subgrid scale diffusion is included in the temperature equation. Other models such as the multi-level PE model of Bryan (1969), used by Han (1975), Robinson et al. (1977), and Semtner and Mintz (1977) also to study eddy effects, inherently requires diffusive effects in the heat equation. Completely adiabatic physics is not allowed by the numerics of the model although, with enough resolution (very fine indeed), the explicit diffusion could be made quite small compared to advective effects.

The question we raise here is how small must heat diffusion be relative to viscous effects and, given some choice of heat diffusivity, how is the model result affected by the inclusion of such a process. We shall make use of QG physics in this discussion because the effect of heat diffusion becomes easily understood in this case. Since QG physics involves the assumption of thermal wind balance and the QG model is easily decomposed into baroclinic and barotropic modes, the role of heat diffusion is easily examined. Presumably these results carry over to PE model results to the extent that the thermal wind relation tends to be satisfied in such models.

2. Inclusion of heat diffusion in the Holland QG model

The two-layer QG model of Holland (1978) is modified to include a parameterization of subgrid scale horizontal heat diffusion. We shall show results making use of Laplacian lateral viscosity and a similar form for lateral heat diffusion (in the spirit of the Bryan, 1969, model). However other forms of subgrid scale diffusion could be chosen and similar analyses with such choices carried out.

Note that including horizontal heat diffusion in QG models leads to some difficulty with boundary condi-

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tions in closed basins. McWilliams (1977) discusses such incompatibilities. For the purposes here, however, we ignore such questions by allowing horizontal heat diffusion in the ocean interior but setting the heat diffusion term to zero at the boundary. Thus additional boundary conditions are not required.

The two-layer QG equations consist of vorticity equations for each layer and a temperature (density) equation at the interface between layers. The thermal wind balance relates the horizontal temperature gradients to the vertical shear of ocean currents, that is, to gradients in the streamfunctions in the two layers. Writing the equations in modal form (a barotropic and a baroclinic mode) then shows the manner in which both vorticity and thermal equation terms influence the motion field.

Following Holland (1978), the equations are

$$\frac{\partial}{\partial t}\nabla^2\psi_1 = J(f+\nabla^2\psi_1, \psi_1) - (f_0/H_1)w_2 + A\nabla^4\psi_1 + H_1^{-1}\,\mathrm{curl}_z\,\tau \quad (1)$$

$$\frac{\partial}{\partial t}\nabla^2\psi_3 = J(f+\nabla^2\psi_3, \psi_3) + (f_0/H_3)w_2 + A\nabla^4\psi_3 \quad (2)$$

$$\frac{\partial}{\partial t}(\psi_1 - \psi_3) = J(\psi_1 - \psi_3, \psi_2) - (g'/f_0)w_2 + B\nabla^2(\psi_1 - \psi_3). \quad (3)$$

Here ψ_1 and ψ_3 are the streamfunctions for the upper and lower layers, and H_1 and H_3 are the mean thicknesses of these layers; H is the total depth; $g' = g\Delta\rho/\rho_0$ is "reduced gravity"; $f = f_0 + \beta y$ is the Coriolis parameter; A is the lateral viscosity; and B is the lateral heat diffusivity.

The barotropic mode can be obtained by multiplying (1) by H_1/H, (2) by H_3/H, and adding:

$$\nabla^2\phi_t = \gamma_a, \quad (4)$$

where

$$\phi = (H_1\psi_1 + H_3\psi_3)/H \quad (5)$$

$$\gamma_a = H_1J(f+\nabla^2\psi_1, \psi_1)/H + H_3J(f+\nabla^2\psi_3, \psi_3)/H + H^{-1}\,\mathrm{curl}_z\,\tau + A\nabla^4\phi. \quad (6)$$

The baroclinic mode can be obtained by subtracting (2) from (1) and using (3) to obtain

$$(\nabla^2 - \lambda^2)\Psi_t = \gamma_b, \quad (7)$$

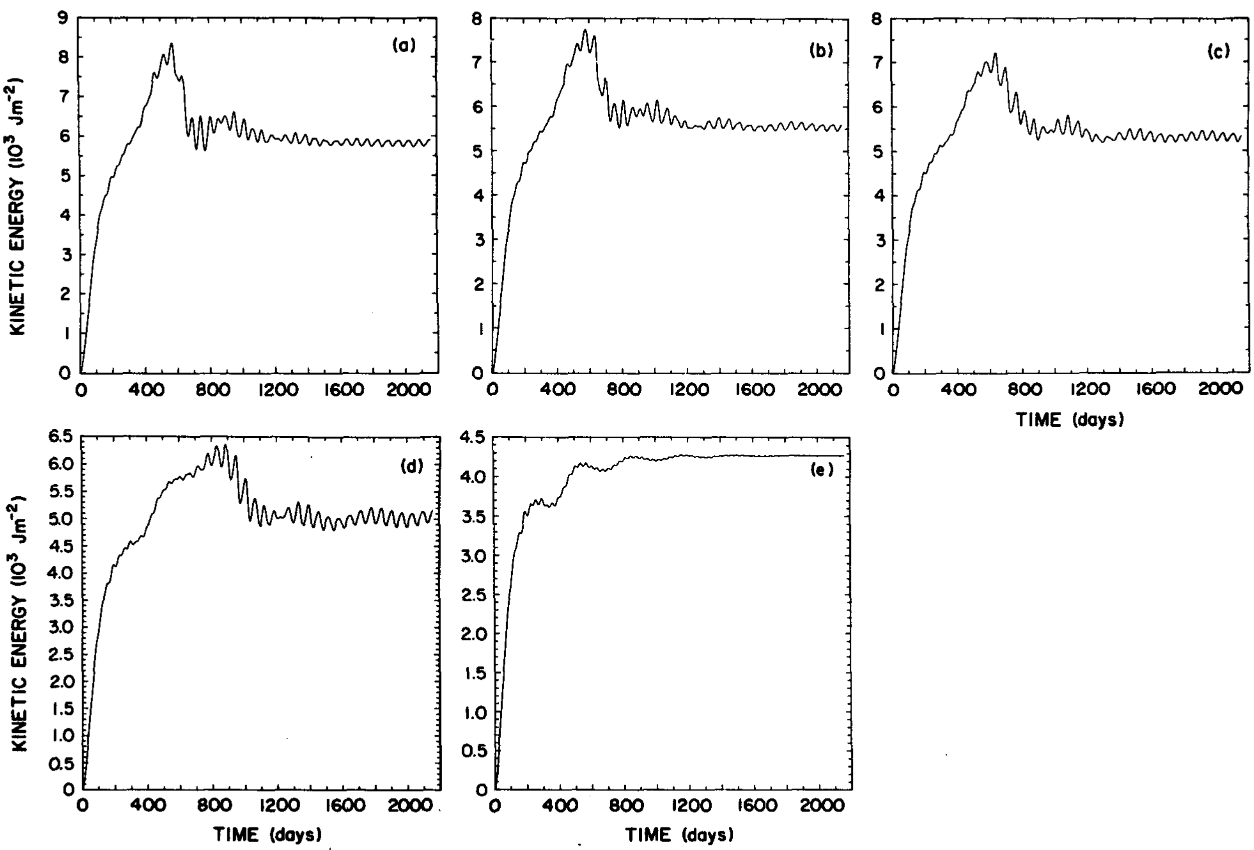

FIG. 1. The time-dependent kinetic energy per unit area for the upper layer for: (a) $B = 0$, (b) $B = 25\ \mathrm{m^2\ s^{-1}}$, (c) $B = 50\ \mathrm{m^2\ s^{-1}}$, (d) $B = 100\ \mathrm{m^2\ s^{-1}}$, (e) $B = 200\ \mathrm{m^2\ s^{-1}}$. Note the different vertical scales.

where

$$\Psi = \psi_1 - \psi_3$$

$$\gamma_b = J(f + \nabla^2\psi_1, \psi_1) - J(f + \nabla^2\psi_3, \psi_3) - \lambda^2 J(\Psi, \psi_2) - H_1^{-1}\,\mathrm{curl}_z\,\tau + A\nabla^4\Psi - \lambda^2 B\nabla^2\Psi \quad (8)$$

and $\lambda^2 = Hf_0^2/H_1H_3g'$ (Note: $R_d = \lambda^{-1}$ is the baroclinic radius of deformation). Here ψ_2 is the interfacial streamfunction, determined from a weighted average of ψ_1 and ψ_3.

Note that the heat diffusion term is present only in the baroclinic mode equation and does not directly influence the barotropic mode. An order of magnitude estimate of the relative roles of viscosity and heat diffusivity in determining the baroclinic structure can be made by assuming typical horizontal length scales. For example, choosing a field of mesoscale eddies with spatial structure $\cos(\pi y/L)$, where L is a typical eddy diameter, we find the ratio of the heat diffusivity term to the viscosity term acting upon the *baroclinic* structure of the eddy field is given by $BL^2/(AR_d^2\pi^2)$. Therefore, if A and B are chosen to have the same value, heat diffusion is equally as effective as viscosity in damping the baroclinic mode when the length scale L is equal to πR_d ($\sim$135 km, which is approximately the scale of the eddies that do arise in these experiments due to baroclinic instability). The numerical experiments discussed below suggest that only when B is 10% or less of A is heat diffusion unimportant relative to viscosity, at least for these mesoscale eddy experiments.

We note, in passing, that if a biharmonic form of heat diffusion had been chosen (as Semtner and Mintz did) instead of the Laplacian one discussed above, then the relative roles of viscosity and heat diffusion would be given by the ratio $B'/(AR_d^2)$, independent of the length scale of the eddy field. Here B' is the "biharmonic" heat diffusion coefficient. If A = 330 $m^2\ s^{-1}$ and R_d = 45 km, then $B' = 6.7 \times 10^{10}$ $m^4\ s^{-1}$ would make this ratio about 0.1 and presumably heat diffusion effects on the baroclinic structure of currents would be negligible.

What about a western boundary current length scale? The linearized steady version of (7) gives a 'modified' Munk layer for the baroclinic mode. A boundary layer analysis for the Laplacian form of heat diffusion results in

$$\beta\Psi_x + \lambda^2 B\Psi_{xx} - A\Psi_{xxxx} = 0.$$

Here the western boundary length scale is given by the decaying roots of the cubic equation

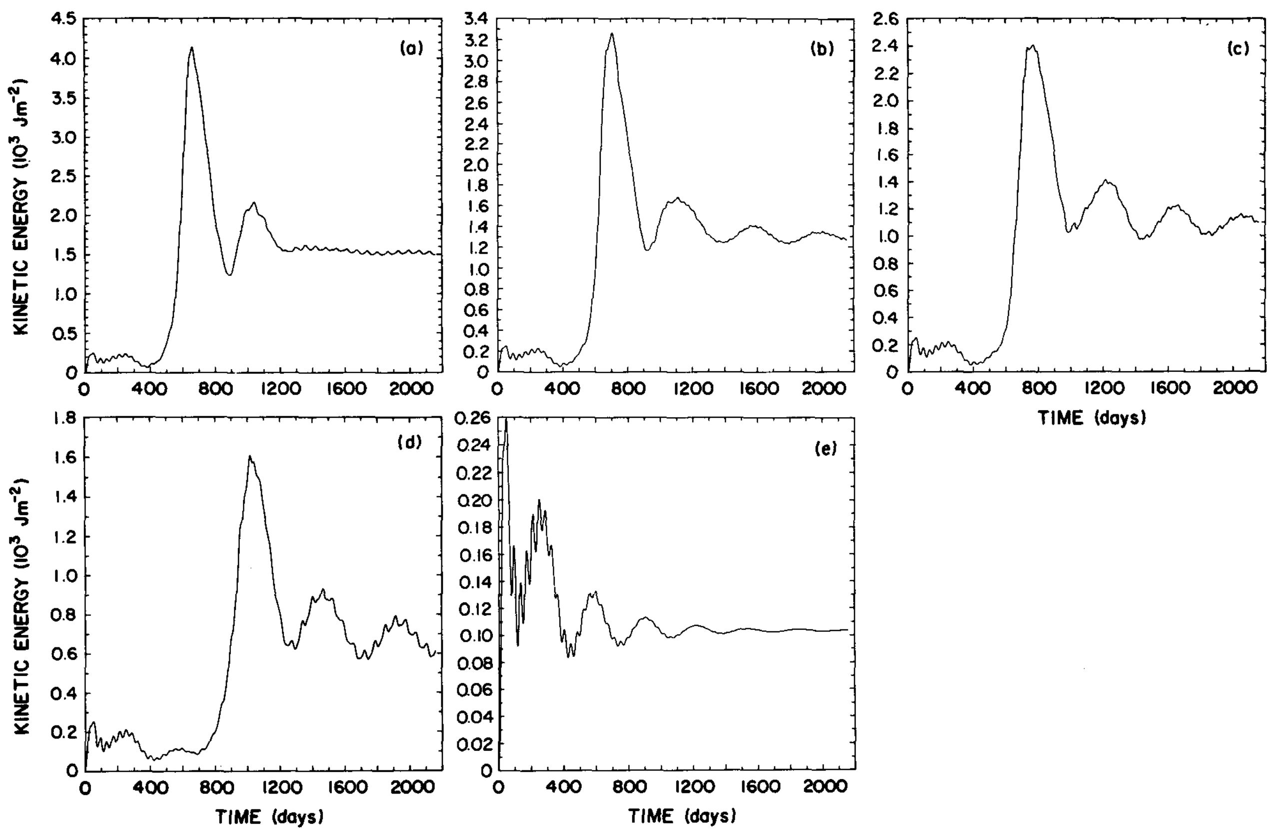

FIG. 2. As in Fig. 1 but for the lower layer.

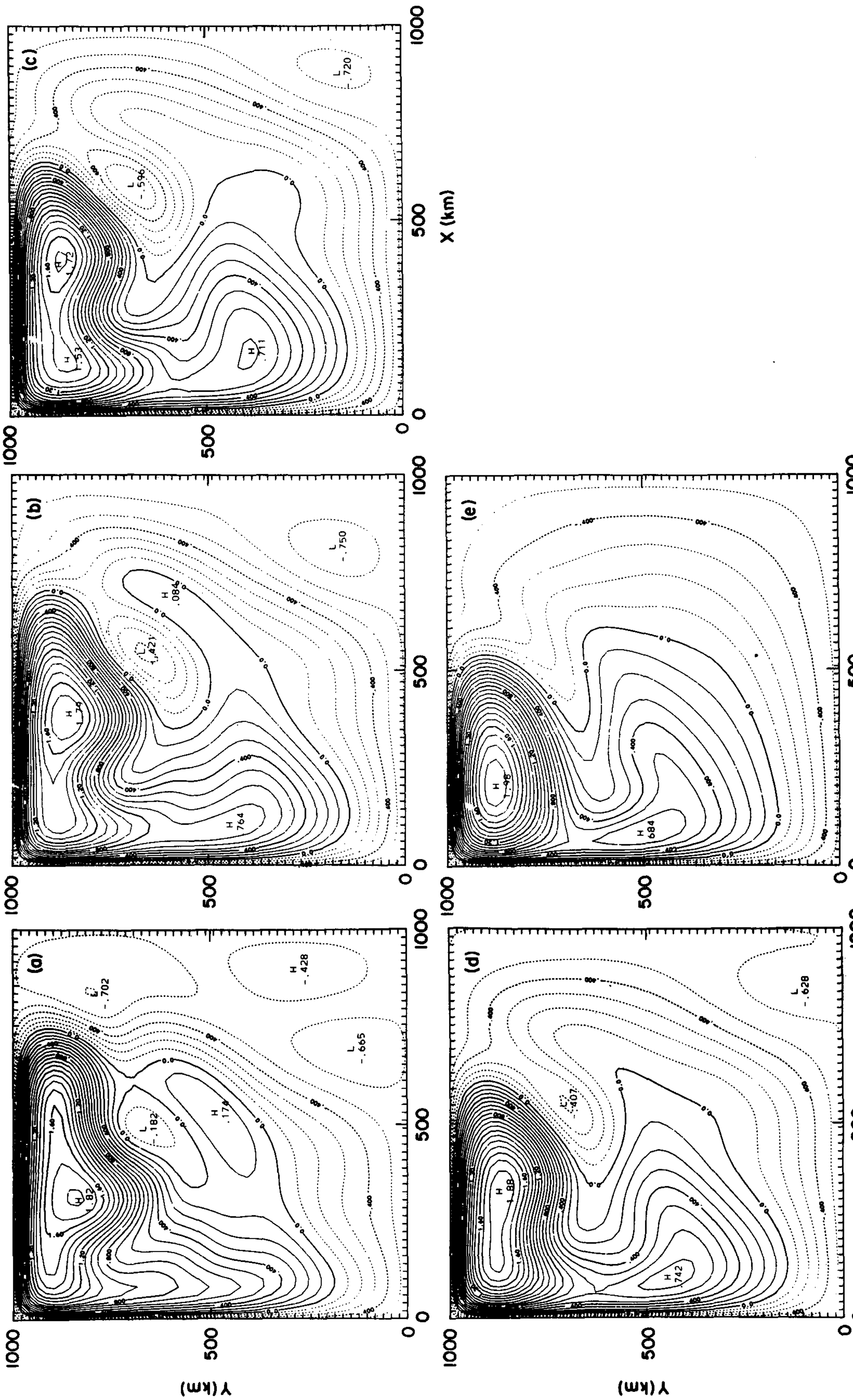

FIG. 3. Instantaneous maps of the upper layer streamfunction (ψ_1) for: (a) $B = 0$, (b) $B = 25\ m^2\ s^{-1}$, (c) $B = 50\ m^2\ s^{-1}$, (d) $B = 100\ m^2\ s^{-1}$, (e) $B = 200\ m^2\ s^{-1}$. The contour interval is 0.1 in each figure.

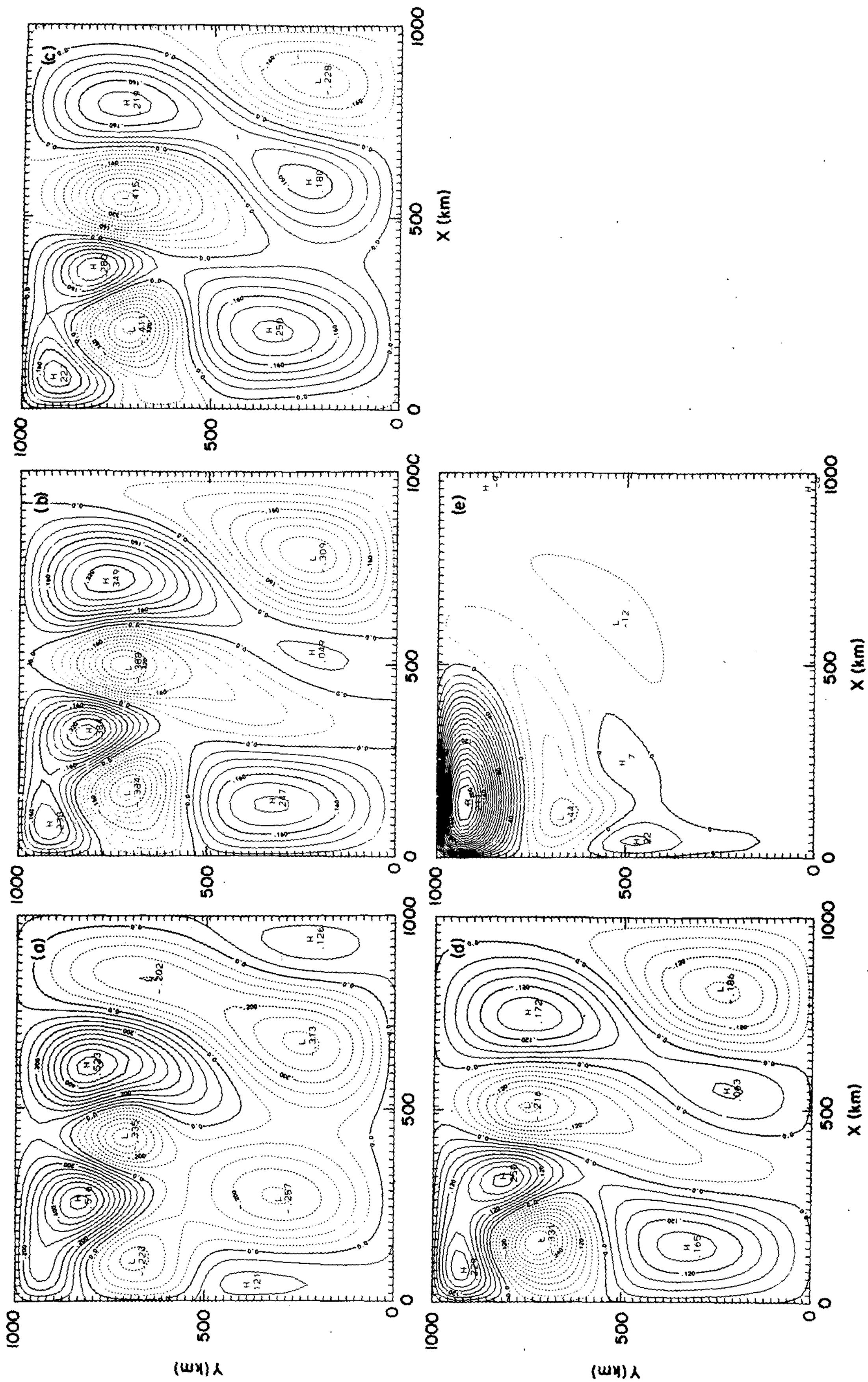

FIG. 4. As in Fig. 3 but for the lower layer. The contour interval is 0.5E-1 for (a), 0.4E-1 for (b)–(d), and 0.1E-1 for (e).

$$m^3 - \left(\frac{\lambda^2 B}{A}\right)m - \beta/A = 0.$$

If $W_m = (A/\beta)^{1/3}$ is the unmodified Munk width (this *would be* the boundary current width for the linear barotropic mode), then the diffusion term becomes important when $BW_m^2/(AR_d^2)$ is order one. If A and B are chosen to have the same value, heat diffusion can be unimportant only if the Munk width is much less than the radius of deformation. For typical eddy-resolving experiments like those used for illustration here, W_m = 25 kilometers (A = 330 m^2 s^{-1}), R_d = 40 to 50 kilometers, and again heat diffusion will be important unless $B \ll A$.

3. Results

Here we show results from a sequence of simple, eddy-resolved calculations to illustrate the effects of increasing heat diffusivities. Following Case 1 of Holland (1978), we use the parameters $f_0 = 8.3 \times 10^{-5}$ s^{-1}, $\beta = 2 \times 10^{-11}$ m^{-1} s^{-1}, $g' = 0.02$ m s^{-2}, H_1 = 1 km, H_3 = 4 km; in addition we use slip boundary conditions. The wind stress amplitude is 10^{-4} m^2 s^{-2}, the pattern is single-gyred, and the basin is 1000 km on a side. The Laplacian lateral viscosity A is 330 m^2 s^{-1}.

A sequence of five calculations were made with heat diffusivities of B = 0, 25, 50, 100 and 200 m^2 s^{-1}, respectively. The calculation with the heat diffusivity set to zero corresponds to Case 1 of Holland (1978), and is used as a standard case against which the effects of increasing heat diffusivities can be compared. Figures 1 and 2 show the time dependent kinetic energies for the upper and lower layers, respectively, while Figs. 3 and 4 show the streamfunction fields for the upper and lower layers, respectively, at a particular instant at the end of the six-year runs. As shown by comparing the various figures, the model has nearly adiabatic physics for $B \leqslant 50$, but for $B \geqslant 100$, the results indicate increasing diffusive effects.

This is most easily seen by examining the kinetic energy curves in Figs. 1 and 2. In the upper layer the amplitude and time dependent nature of the total kinetic energy is only slightly changed in proceeding from B = 0 to B = 50 m^2 s^{-1} (Fig. 1a to Fig. 1c). Fig. 1d (B = 100 m^2 s^{-1}) however shows that not only is the spinup of the ocean changed importantly but the final equilibrium has considerably less total energy. Finally Fig. 1e shows that the final equilibrium is a nearly steady state one; the instability processes have been almost entirely eliminated.

The lower layer energetics (Fig. 2) more or less show the same sequence except that, since the deep ocean is entirely eddy driven, there is greater sensitivity to increasing B. The final total equilibrium energy for the case with B = 25 m^2 s^{-1} is 15% less than for the case with B = 0; when B = 100 m^2 s^{-1} it is less than half as large and when B = 200 m^2 s^{-1} it is only a tiny fraction.

The streamfunction plots in Figs. 3 and 4 support these findings. Note particularly how the meandering diminishes as B increases (it is entirely absent when B = 200 m^2 s^{-1}) and how the amplitude of the eddies (shown best in the deep ocean) decreases with increasing B.

It is clear that even rather small diffusion coefficients (small compared to the viscosity A) cause important modifications of the eddy field and of the eddy generation processes, substantiating the simple analyses of the preceding section. Thus the need for overdriving the PE model of Robinson et al. (1977) (they used a wind stress several times too large) and for choosing a biharmonic form for heat diffusion by Semtner and Mintz (1977) is clarified. Heat diffusion can be very effective at diminishing the baroclinic signal associated with mesoscale eddies, making it less likely that baroclinic instability processes can exceed damping.

4. Discussion

As demonstrated by these results, heat diffusion can be quite effective at modifying the physical behavior found in mesoscale eddy experiments when viscosity and heat diffusivity are the same order of size. This is because these two damping terms have different orders of differentiation in the baroclinic mode equation. Scale analysis suggests and numerical experiments confirm that, for the mesoscale, B must be an order of magnitude smaller than A for heat diffusion terms to be small. In fact, when both are included at the same small values ($\sim$300 m^2 s^{-1}), the eddies are completely damped and the solution becomes a steady state one.

Note that even the coarse resolution, noneddying models of Bryan (1969) and others could have substantial effects due to lateral heat diffusion modifying the vertical structure of currents (i.e., the baroclinic signal). Sensitivity experiments are necessary to establish whether this is true in any particular parametric circumstances. Nevertheless, given the tendency in a rotating, stratified fluid for thermal wind balance to occur, regions of large horizontal temperature gradient, such as in intense western boundary currents, could tend to have the baroclinic signal damped by heat diffusion effects while the barotropic signal is unaffected. Thus the relative damping by lateral friction and lateral heat diffusion is an important parameter of the problem and can strongly affect the barotropic/baroclinic ratio of resulting currents.

In conclusion, we reemphasize that since every numerical experiment involves compromises, it is incumbent upon the modeler to clarify as much as possible how such choices might influence his conclusions and to make this a basic part of his presentation of results. In essence, this is similar to the need for the observational oceanographer to make estimates of the errors in his observations to ascertain how firm are the conclusions drawn from those observations. Without such an assessment, the results of numerical experimentation and observational analysis are incomplete.

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