

## Statistics of Breaking Waves Observed as Whitecaps in the Open Sea

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### ABSTRACT

Conventional observations of waves carried out with a buoy in open sea conditions were supplemented with simultaneous visual observations of whitecaps to identify breaking events in the buoy records. A statistical wave-by-wave analysis of these records indicates that such seemingly obvious parameters as wave steepness or wave asymmetry cannot be used to separate breakers from nonbreakers and that breaking occurs at wave steepness values much less than the theoretically expected steepness of a limiting wave. The observed fraction of breaking waves varied from about 0.10 to about 0.16, depending on wind speed. Two-thirds of the breaking waves were breaking in one-third of the wave groups for which a  $H_{rms}$ -threshold definition was used.

### 1. Introduction

The statistical characteristics of waves breaking in open sea are important to ocean engineering, to air-sea interaction studies and to remote sensing (e.g., wave loads on structures, wave generation, aerosol production and radar backscatter). Studies of these statistical characteristics have been mostly of an analytical nature involving basic assumptions that have not in most instances been adequately verified with observations in open sea in spite of the fact that whitecap observations have been reported by various authors. We present here statistical results from observations of breaking waves in open sea conditions in an attempt to verify some of the theoretical expectations.

Whitecap observations and the quantification thereof in terms of spatial whitecap coverage have been addressed by several authors (e.g., Monahan, 1971; Ross and Cardone, 1974; Snyder et al., 1983). However, the spatial distribution of whitecaps does not provide information on wave height, length, period, steepness etc. To obtain this information, stereophotos of the sea surface could be taken (e.g., Holthuijsen, 1983) but sequential observations at one fixed location are far more convenient. Toba et al. (1971), Thorpe and Humphries (1980), Longuet-Higgins and Smith (1983) and Weissman et al. (1984) carried out such observations, but the statistical analysis of their data did not include wave height, wave period or wave steepness, except in the case of Weissman et al. (1984) who considered the amplitude and the slope of only 31 observed breakers.

Observed statistics of breaking waves are therefore still very scarce. Most of the available statistics are based on analytical studies in which a steepness criterion is used to characterize breaking waves. This criterion has been determined theoretically (e.g., Longuet-Higgins,

1975a) and empirically in the laboratory (e.g., Van Dorn and Pazan, 1975) for steep periodic waves. Ochi and Tsai (1983) established such a criterion for irregular waves in a laboratory flume. To obtain statistics on breaking waves, such a criterion is used to identify regions in the joint probability density function of wave height and wave period where the waves are assumed to be breaking. Such a joint probability density function is established either from theoretical considerations or from observations (e.g., Bretschneider, 1959; Longuet-Higgins, 1975b; Cavanié et al., 1976; Goda, 1978). Performing the proper integrations over the joint probability function yields the fraction of breaking waves or the marginal distribution of the height of breaking waves. Notable examples of this work are Battjes (1971), Nath and Ramsey (1974), Houmb and Overvik (1976) and Ochi and Tsai (1983).

The term "breaking" is generally related to some sort of instability of the sea surface, usually near the crest of an individual wave and accompanied by a whitecap. However, it is used in the literature in different contexts with widely different meanings. Two fairly objective criteria have been proposed for observations of breaking waves but these require sophisticated instrumentation (Longuet-Higgins and Smith, 1983; Weissman et al., 1984). For convenience of observation, we define in the present study a breaking wave as a wave with an active whitecap on its crest, i.e., generation of a visible aerated surface patch. This definition enables us to identify breaking waves in conventional recordings of waves with the help of simultaneous visual observations.

In the present study we consider the fraction of breakers and its dependence on wind speed, and we carry out a wave-by-wave analysis to obtain marginal and joint probability density functions of wave steepness and wave asymmetry parameters. In addition, we

investigate the occurrence and location of breakers in wave groups.

## 2. Method of observation and analysis

### a. General

The general set-up of our observations was quite simple; a waverider buoy, located in the southern North Sea, was visually observed on two occasions for a total period of about 12 hours. An observer activated a radio signal each time a whitecap was seen to pass under the buoy. Wind observations were taken from a nearby observation tower while wave directions were observed with a nearby pitch-and-roll buoy.

### b. Method of observation

For the wave observations, we used the 0.7 m diameter waverider buoy of Datawell, Holland. This is a conventional heave buoy which measures the vertical acceleration of its motion. This acceleration is integrated twice in the buoy and then transmitted to a receiver and recorded on tape. Since we are interested in parameters of individual waves, we reconstructed from the buoy data the motion of the sea surface as accurately as possible. To this end we Fourier transformed the time series of the heave signal, multiplied the result with the complex transfer function of the buoy provided by Datawell (Fig. 1) and transferred back to the time domain. The transfer function was truncated at 0.6 Hz to avoid the large amplitude amplification of high-frequency components. This cutoff frequency is about three to four times the peak frequency in our observations so that we filtered out only a few percent of the variance of the signal.

The above corrections relate mostly to the response of the buoy to a vertical surface motion. Corrections

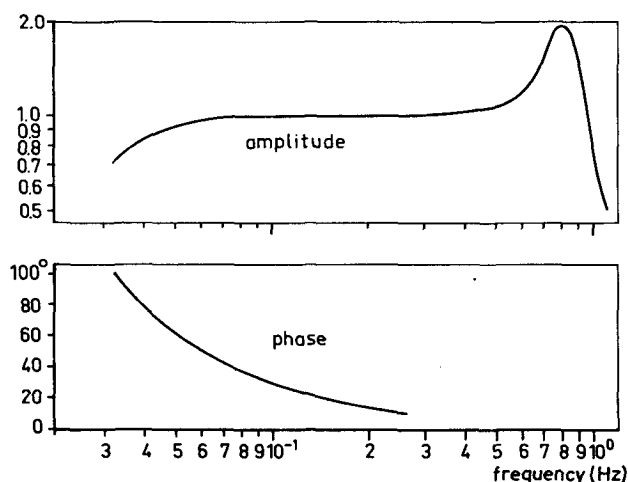


FIG. 1. Complex transfer function of Datawell waverider buoy.

for the horizontal motion of the buoy have not been carried out, as the corresponding errors were deemed acceptable within the scope of this study. Longuet-Higgins has pointed out (private communication, 1984) that observations with a free-floating buoy which follows the orbital motion at the surface of a limiting wave would overestimate the zero-crossing period by a factor of 1.27 compared with an infinitesimal wave (Longuet-Higgins, 1979). However, the wave steepness values observed in this study are only one-quarter to one-half that of a limiting wave. Considering the non-linear character of this deviation, we estimate it to be of the order of 5% at most for our observations.

The observations were analyzed in segments of 30 minutes which were sampled with a frequency of 4 Hz. The wave group analysis is based on 10-minute records.

### 1) VISUAL OBSERVATIONS

During the wave measurements, one of the authors (T.H.C.H.) watched the buoy from either a nearby observation tower ( $\sim 100$  m from the buoy) or from a nearby ship ( $\sim 50$  m from the buoy). The observer triggered a radio signal each time an active whitecap was seen to pass under the buoy. This signal was recorded synchronously with the buoy signal on one tape in an onshore station, thus identifying breaking waves with an "on-off" signal.

Implicit in the above method of observation is our operational definition of a breaking wave. As such a definition is critical in the interpretation of the results, some details of the actual visual observations merit discussion. The position of the observer (eye-level) on the tower was approximately 16 m above mean sea level and about 80–120 m downwind from the buoy. Visibility during the observations was excellent over this range. The minimum size of the whitecap noted by the observer at the location of the buoy is estimated to be about  $\frac{1}{5}$  of the diameter of the buoy (i.e., about 15 cm). Whether or not the whitecap actually passed under the buoy or just skirted the buoy was difficult to observe so that occasionally a wave may have been labeled incorrectly. But we feel that the effect of this on the results of this study is negligible. When on the ship, the observer was located at about 4 m (eye-level) above mean sea level and at a distance of about 50 m downwind from the buoy. The ship was anchored with the bow pointing upwind. These conditions are slightly less favorable than those for the tower but again the labeling errors were deemed to be negligible.

The above described observation procedure relies on a subjective impression of wave breaking. It is almost identical to that used by Toba et al. (1971), the only significant difference being that those authors used a fixed wave gauge instead of a buoy. A more quantitative criterion for detecting breaking waves has been suggested by Weissman et al. (1984) based on the occurrence of high-frequency energy "bursts" near wave

crests in the time history of the surface elevation. Another quantitative criterion has been used by Thorpe and Humphries (1980). It has been formalized by Longuet-Higgins and Smith (1983). These authors used a so-called jump-meter which gives the number of events where the vertical speed of the sea surface exceeds a certain threshold level. These events are interpreted as breaking or near-breaking waves. This criterion seems to correlate well with visual observations of breaking waves in a laboratory flume but such a correlation has not been attempted at sea (Longuet-Higgins and Smith, 1983; Longuet-Higgins, private communication, 1984).

## 2) THE WIND

The wind has been observed with a cup anemometer and a wind vane at an exposed location on the observation tower at about 27.5 m above mean sea level. These measurements provided 10-minute averages of wind speed and wind direction at 30-minute intervals. The tower is fairly bulky, and the results of wind tunnel tests were used to correct the observed wind speed to free-flow wind speed (Maur, 1976). A logarithmic wind profile with a drag coefficient of  $1.5 \times 10^{-3}$  was used to estimate the wind speed at 10 m elevation from the corrected observed wind speed.

### c. Methods of analysis

#### 1) WAVE-BY-WAVE ANALYSIS

The analysis of the wave observations relates mainly to individual waves which we define as the sea surface profile between two consecutive zero-down-crossings in the time history of the surface elevation (Fig. 2). The basis for choosing a definition in terms of zero-down-crossings rather than zero-up-crossings is that the wave profiles thus obtained contain the complete forward wave face. This is physically more relevant to wave breaking than the back of the wave which would be included in a zero-up-crossing analysis. We determined for each individual wave (with a wave height larger than 0.1 m) the following parameters:

- wave height  $H$ , defined as the maximum difference in surface elevation in one wave,
- wave period  $T$ , defined as the time interval between the two zero-down-crossings of a wave, and

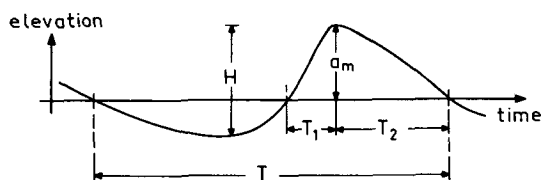


FIG. 2. Definition diagram of wave parameters.

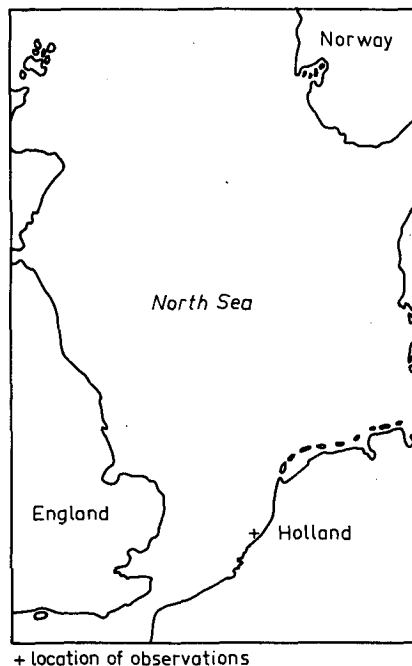


FIG. 3. Location of the observations.

- wave steepness  $s$ , defined as

$$s \equiv H / \left( \frac{g}{2\pi} T^2 \right).$$

In addition, we determined the following secondary parameters (see Fig. 2) of which  $\epsilon$  and  $\mu$  were introduced by Kjeldsen and Myrhaug (1980) (see also Myrhaug and Kjeldsen, 1984):

- crest front steepness  $\epsilon$ , defined as

$$\epsilon \equiv a_m / \left( \frac{g}{2\pi} T_1 T \right)$$

- horizontal crest asymmetry  $\kappa$ , defined as

$$\kappa \equiv \frac{T_2}{T_1 + T_2}$$

- vertical crest asymmetry  $\mu$ , defined as

$$\mu \equiv \frac{a_m}{H}$$

in which  $a_m$  is the crest elevation of the wave,  $T_1$  is the time interval between the moments of zero-up-crossing and the following cresting, and  $T_2$  is the time interval between cresting and the subsequent zero-down-crossing. The mean water level is defined as the mean of the sea surface elevation in the 30 minutes record.

We should emphasize that the above defined parameters are those of the "zero-crossing" wave and that our labeling of this wave as either breaking or non-

breaking is independent of the details of the generation of the observed whitecap. As one of the reviewers of this paper noted, a long wave overtaking a steep shorter wave may cause the shorter wave to break, thus causing the longer wave to be identified as a breaking wave. Also, the generation of a whitecap follows the onset of breaking so that the observed breaking wave has lost some of its energy since it started breaking. It may therefore have a steepness lower than an earlier and higher limiting steepness. Such aspects of breaking waves are not taken into account in the methods of observation and analysis used in this study.

## 2) STATISTICAL ANALYSIS

We determined the fraction of breaking waves from the wave records simply by taking the ratio of the number of breaking waves to the total number of waves. This was done for each record of 30 minutes duration.

The marginal and joint probability functions which we considered were approximated with one-dimensional and two-dimensional histograms.

## 3) WAVE-GROUP ANALYSIS

To investigate the occurrence of breakers in a wave group, we carried out a wave group analysis. We used a commonly used threshold definition: a wave group is a set of consecutive waves, each of which is higher than a certain threshold value, preceded and followed by a wave lower than this threshold value. We choose for this level the value of the root-mean-square wave height  $H_{rms}$ . This seems to be a natural level in the sense that this level gives the largest number of groups in a record (as function of the threshold level, e.g. Longuet-Higgins, 1985). The group length  $l$  is defined as the number of waves in the group. For each group we determined from the observations the number of breaking waves in the group and the sequence number of the first breaker in the group (if breakers occurred in the group).

## 3. Geophysical conditions and observations

The observations were taken near the observation tower "Noordwijk" in the southern North Sea, 10 km off the Dutch coast in 17.5 m mean water depth (Fig. 3). This is fairly deep water for the observations, for which the peak frequency of the energy spectrum was always higher than 0.15 Hz. The tidal currents were less than 0.5 m s<sup>-1</sup> during the observations.

The observations were taken on two occasions: 2–3 May and 24 May 1983. On the first occasion the weather was dominated by an atmospheric depression moving slowly eastward with its center located about 150 km NW of the area of observation. The wind correspondingly veered from about 200° to about 250° during the observations (nautical convention). The wind speed varied between 8 m s<sup>-1</sup> and 12 m s<sup>-1</sup>. On

the second occasion a fairly stationary depression located over central Europe dominated the weather. The wind direction was almost due north (10°–15°), and the wind speed was 9 m s<sup>-1</sup>.

The shape of the frequency spectra obtained with the waverider buoy was typically unimodal without noticeable secondary swell peaks. The ratio between the phase velocity at the peak frequency and the wind speed varied from about 0.7 to about 1.3, indicating that the observations covered both growing waves and "young" swell. The significant wave height varied from 1.5 to 2.0 m on the first occasion, and it was about 1.3 m on the second occasion.

Directional wave observations obtained with a pitch-and-roll buoy (a WAVEC buoy, e.g. Vlugt et al., 1981) located about 1 km west of the observation tower indicated that the main wave direction differed from the wind direction by about 40° on the first occasion and by about 10° on the second occasion (wave direction more northerly than wind direction on both occasions).

## 4. Results and discussion

### a. Wave breaking criteria

If the wave steepness parameters provide an adequate criterion to separate breakers from nonbreakers, then the joint probability density function of wave height  $H$  and wave period  $T$  of breaking waves should be well separated from that of the nonbreaking waves, probably along a line of constant steepness (e.g.,  $s = 0.14$ , Longuet-Higgins, 1975a). This, however, is not the case in our observations as shown in Fig. 4 where the two-dimensional histograms of breaking and nonbreaking waves are shown to overlap to a very large extent. On average, however, there is a marked difference between breaking and nonbreaking waves: the average wave height of the breaking waves  $\bar{H}_{br}$  is about 1.5 times the average wave height of the nonbreaking waves or 1.3 times the average wave height of all waves  $\bar{H}_{all}$  (Fig. 5). These ratios do not apply to the average crest heights due to the vertical asymmetry of the waves. It appears in our observations that the average crest height of the breaking waves  $\bar{a}_{br}$  is about 1.6 times the average crest height of all waves  $\bar{a}_{all}$  (Fig. 5).

The marginal histograms of the wave height and the wave period for the breaking and nonbreaking waves and for all waves is given in Fig. 6. No attempt has been made to fit analytical functions to these observed histograms.

The failure of the wave steepness  $s$  to separate breaking waves from nonbreaking waves is also evident in the histograms of the observed values of  $s$  (Fig. 6). The considerable overlap of the histograms of breaking and nonbreaking waves indicates that the steepness  $s$  cannot be used to distinguish between breaking and nonbreaking waves. Moreover, breaking occurs at values of  $s$  which are only a small fraction of the theoret-

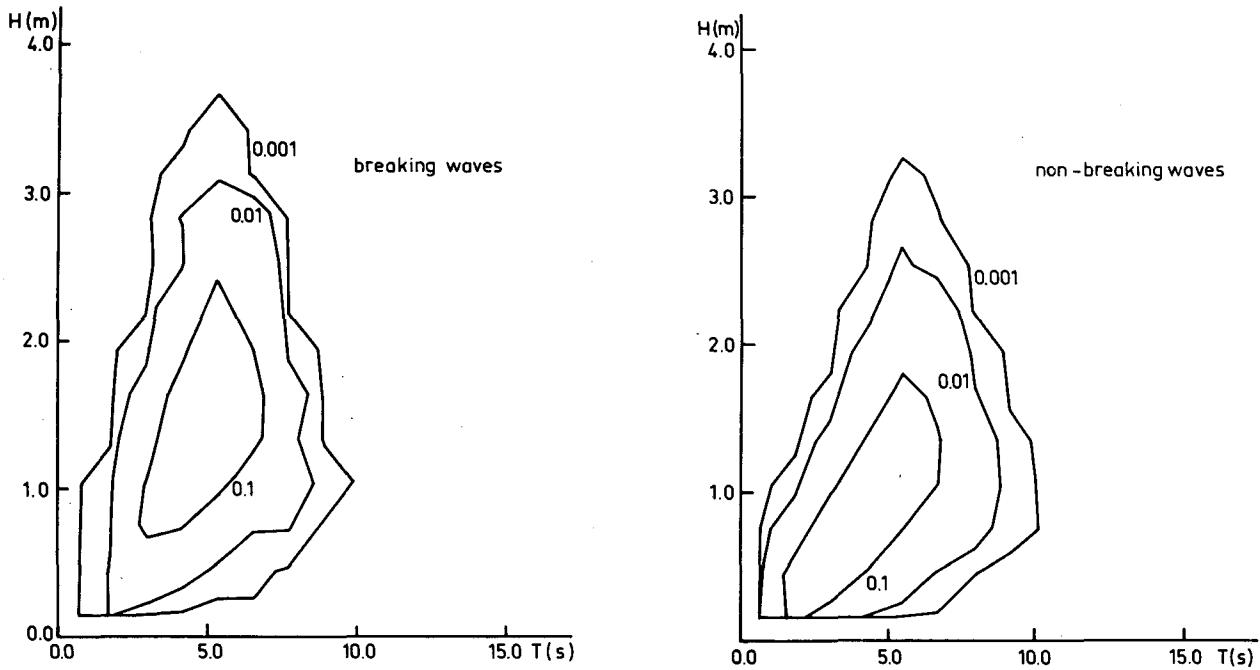


FIG. 4. Observed joint probability density function of wave height  $H$  and wave period  $T$  for breaking waves and nonbreaking waves.

ically expected steepness of a limiting wave ( $s = 0.14$ ). Our observations of wave steepness  $s$  can be roughly compared with the observations of Weissman et al. (1984) who defined a wave slope  $r$ :

$$r = a_m(2\pi f_m^2)/g$$

in which  $f_m$  is the frequency at the peak of the wave energy spectrum. The numerical value of  $r$  is roughly a factor  $\pi$  larger than that of  $s$  if  $a_m \approx \frac{1}{2}H$  and  $f_m \approx T^{-1}$ . The average value of  $r$  for the 31 breakers observed by Weissman et al. (1984) is 0.103 with a stan-

dard deviation of 0.05 and a maximum of 0.223. The equivalent values for  $s$  would be 0.033, 0.016 and 0.071, respectively, which agrees fairly well with our observations of  $s$  for the breakers (Fig. 6).

The crest front steepness parameter  $\epsilon$  and the asymmetry parameters  $\kappa$  and  $\mu$  also cannot be used to separate breaking waves from nonbreaking waves. We inspected the marginal probability density functions of  $\epsilon$ ,  $\kappa$  and  $\mu$  and also the joint probability density functions of  $(a_m, T_1)$  and  $(a_m, \epsilon)$  and  $(H, s)$  for both breaking and nonbreaking waves, but all pairs of probability

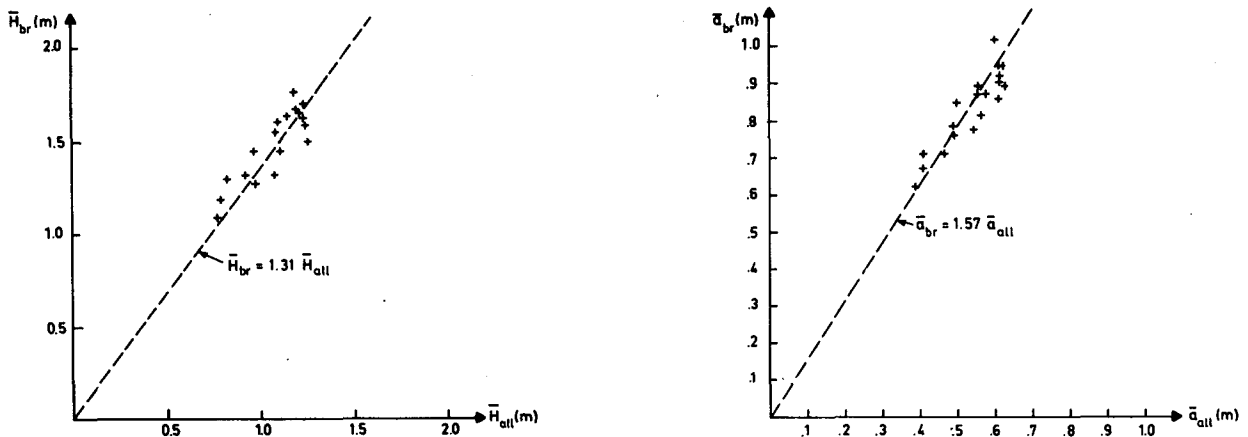


FIG. 5. Observed average wave (or crest) height of breaking waves ( $\bar{H}_{br}$  and  $\bar{a}_{br}$ ) as a function of observed average wave (or crest) height of all waves ( $\bar{H}_{all}$  and  $\bar{a}_{all}$ ). The crosses represent the average values from each of the 30-min wave records. The dashed line represents the average values from all records.

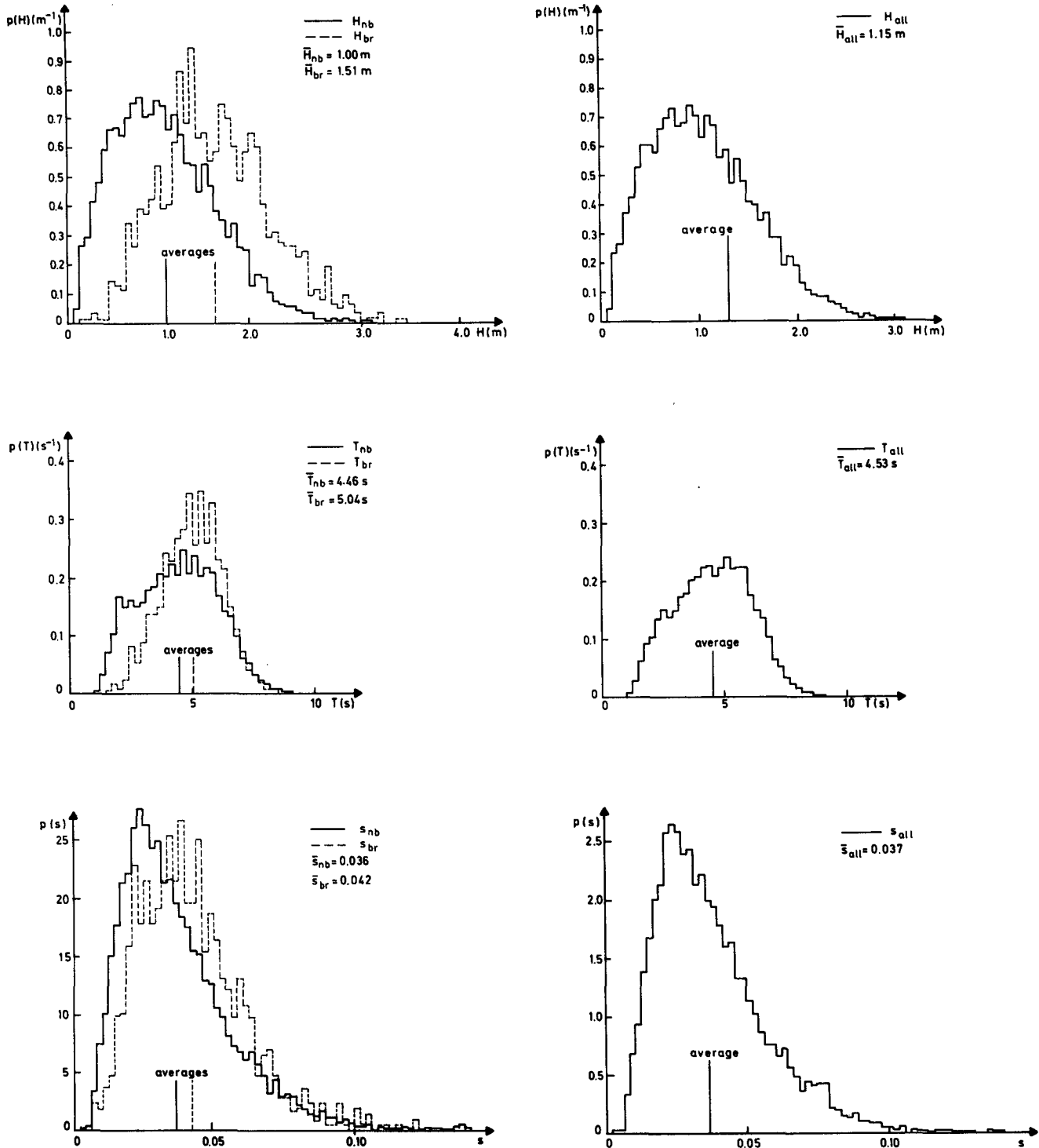


FIG. 6. Observed marginal distribution functions of wave height  $H$ , wave period  $T$  and wave steepness  $s$  for breaking waves (br), nonbreaking waves (nb) and all waves (all).

density functions investigated (of breaking and non-breaking waves) overlapped to such an extent that breaking waves were not separated from nonbreaking waves.

It should be noted that the typical values for  $\epsilon$  and  $\mu$  obtained for breakers by Kjeldsen (1983) in a laboratory tank are higher than in our observations. Myrhaug and Kjeldsen (1984) have also observed wave

steepness and wave asymmetry of ocean waves but they do not consider the absolute values of their parameters (amongst which are  $s$ ,  $\epsilon$  and  $\mu$ ). We have therefore not attempted to compare our data with those of Myrhaug and Kjeldsen (1984).

### b. Fraction of breaking waves

We observed a total of 8717 waves of which 1027 waves were breaking. The fraction of breakers is thus 0.12. As a function of wind speed the fraction of breakers per wave record is shown for our observations in Fig. 7. This fraction varies from about 0.10 at wind speed  $8 \text{ m s}^{-1}$  to about 0.16 at wind speed  $12 \text{ m s}^{-1}$ . These observed fractions are generally somewhat lower (for a given wind speed) than those of Toba et al. (1971), but there is a fair overlap of the two data sets (Fig. 7). The one observation of Weissman et al. (1984) is consistent with those data. The observations with a jumpmeter of Thorpe and Humphries (1980) and of Longuet-Higgins and Smith (1984) give fractions of breaking waves that are much smaller (by almost one order of magnitude). This quantitative difference may plausibly be ascribed to the difference in the definition of a breaking wave and the related observation technique.

### c. Breakers in wave groups

The total number of wave groups (a sequence of waves higher than  $H_{\text{rms}}$ , section 2c) in our observations is 1577, of which 590 contained at least one breaking wave so that only about one-third of the groups contained one or more breakers. The probability of a wave group having at least one breaker is obviously higher as the length of the wave group is larger (Fig. 8). The fraction of breakers which were breaking in a wave group ( $H_{\text{rms}}$  threshold level) is 0.69 of all breaking waves. In other words, about two-thirds of the breakers are concentrated in about one-third of the wave groups whereas the remaining two-thirds of the wave groups

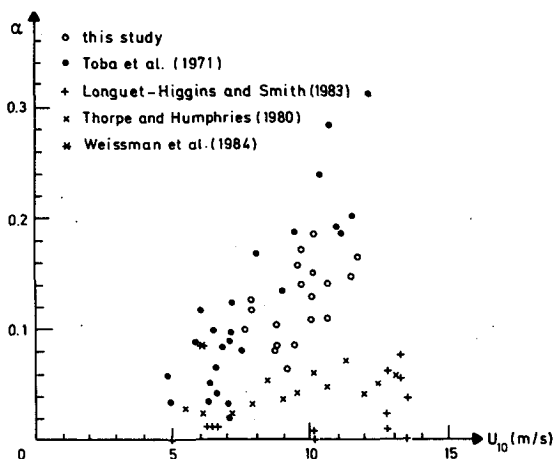


FIG. 7. Observed fraction of breaking waves ( $\alpha$ ) as a function of wind speed ( $U_{10}$ ).

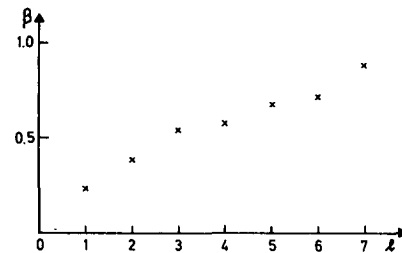


FIG. 8. Observed fraction  $\beta$  of wave groups ( $H_{\text{rms}}$  threshold level) with given length  $l$  containing breaking waves as a function of group length ( $l$ ).

are breaker free (if the threshold level for wave groups is chosen at  $H_{\text{rms}}$ ).

The position of the first breaker in a wave group (if there is a breaker in the group) appears to be slightly ahead of the center of the group as shown in Fig. 9. This is consistent with the observations of Donelan et al. (1972) who showed that waves primarily break in the center of wave groups (of unspecified definition). About 20% of the groups with breakers contained more than one breaker. This indicates that in our observations the breakers have little tendency to occur in pairs. This seems to contrast with the observations of Thorpe and Humphries (1980) who found that 58% of the groups with breakers contained more than one breaker. However, Thorpe and Humphries (1980) do not give their definition of wave group so it may well be different from ours.

## 5. Conclusions

Our buoy observations of breaking waves (defined as waves with an active whitecap at the crest) indicate that breaking waves cannot be separated from non-breaking waves with such seemingly obvious parameters as wave steepness or wave asymmetry of the zero-crossing wave. This empirical finding undermines the relevance of many theoretical studies in which such parameters are used to identify breaking waves in open sea conditions.

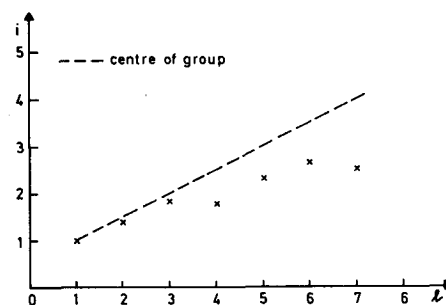


FIG. 9. Observed average position (sequence number  $i$ ) of first breaker in wave group ( $H_{\text{rms}}$  threshold level) as a function of group length ( $l$ ).

Associated with the failure of wave steepness to separate breakers from nonbreakers is the fact that waves break at steepness values much less than the conventionally expected theoretical value. It remains to be explained why this is the case. This finding is consistent with the observations of Weissman et al. (1984).

Our observations of the fraction of breaking waves as a function of wind speed overlap to a large extent those of Toba et al. (1971). These observed values are much larger than those of Thorpe and Humphries (1980) and of Longuet-Higgins and Smith (1983), who used a different operational definition of a breaking wave.

Waves in open sea tend to break inside wave groups (with threshold level  $H_{rms}$ ) as two-thirds of the observed breaking waves occurred in one-third of the wave groups. This implies that two-thirds of the breaking waves are higher than  $H_{rms}$ . Breaking generally occurs in the center of such a group. We observed hardly any tendency of breakers to occur as pairs in a group (in the time domain) in contrast to what Thorpe and Humphries (1980) observed. These conclusions on the occurrence of breakers in groups support, to some extent, the model of ocean wave breaking suggested by Donelan et al. (1972) in which all waves break near the center of a wave group.

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