Eddy-Potential Vorticity Fluxes in the Gulf Stream Recirculation*

ELLEN BROWN**

WHOI/MIT Joint Program

W. BRECHNER OWENS AND HARRY L. BRYDEN

Woods Hole Oceanographic Institution, Woods Hole, MA 02543

(Manuscript received 17 October 1984, in final form 8 July 1985)

ABSTRACT

The local effect of the mesoscale eddy field on the mean potential vorticity distribution of the Gulf Stream recirculation region is determined from the quasi-geostrophic eddy potential vorticity flux. This flux is calculated by finite difference of current and temperature time series from the Local Dynamics Experiment. This long-term array of moorings is the only experimental data from which the complete eddy flux can be calculated. The total eddy flux is dominated by the term due to the time variation in the thickness of isopycnal layers. This thickness flux is an order of magnitude larger than the relative vorticity flux. The total flux is statistically significant and directed 217°T to the southwest with a magnitude of 1.57×10^{-5} cm s^{-2}. The direction of the eddy flux with respect to the mean large-scale potential vorticity gradient from hydrographic data indicates that eddies in this region tend to reduce the mean potential vorticity gradient.

1. Introduction

Recent analytical and numerical models have emphasized the importance of the eddy potential vorticity flux in the time-averaged potential vorticity balance. The eddy flux has been interpreted (Rhines and Holland, 1979) as a forcelike quantity allowing the mean circulation to cross planetary contours. It is composed of a relative vorticity flux which represents lateral momentum transfer and a thickness flux due to fluctuations in the thickness of isopycnal layers which causes vertical transfers of momentum. In two-layer quasi-geostrophic numerical model flows, the role of the eddy potential vorticity flux has been examined using either streamline (Holland and Rhines, 1980) or regional integrals (Harrison and Holland, 1981) of the terms in the mean potential vorticity balance. These vorticity budget analyses suggest that in certain regions of the model flows the area-integrated eddy flux can be a significant term in the mean dynamical balances.

The paucity of observational data prevents a detailed comparison of ocean measurements with theoretical predictions. The fundamental problem is the difficulty in drawing conclusions from the few point measurements that are available. Other specific inadequacies are the large uncertainties in both the mean fields and the eddy fluxes and the insufficient knowledge of the spatial distribution of these quantities. The present observational data base is nowhere sufficient for the type of analyses carried out on the numerical model flows.

Even after a decade of long-term moored array experiments, only the local Dynamics Experiment array deployed in the Gulf Stream recirculation region provides data that can be used to accurately calculate the total eddy potential vorticity flux. Estimates of eddy potential vorticity flux require both long time-series for statistical stability and adequate resolution for horizontal and vertical spatial derivatives. Part of the motivation for this study is to determine whether the fluxes can be reliably estimated from mooring data. Of the available observational data, the Local Dynamics Experiment (LDE) moored instrument array provides the most accurate spatial resolution because it was designed to sample optimally the instantaneous mesoscale eddy vorticity field. The eddy potential vorticity flux is estimated from velocity and temperature time series recorded from this array and compared to the local mean potential vorticity gradient.

Despite the severe limitations of interpreting the observational data, the measured eddy potential vorticity flux is still a useful quantity because it characterizes the effect of the eddy field on the local mean potential vorticity distribution. The eddy flux in the midthermocline of the Local Dynamics region is found to be dominated by the baroclinic term directed down the mean potential vorticity gradient. This result is consistent with both Hogg's (1983) approximate calcula-

* Woods Hole Oceanographic Institution Contribution Number 5805.

** Present affiliation: Science Applications International Corporation, La Jolla, CA 92038.

© 1986 American Meteorological Society

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

tion based on the POLYMODE Array II dataset and, in some aspects, with the predictions of numerical model results. The interpretation of the observational results is much more limited than in the numerical models because the divergence of the eddy flux in the mean potential vorticity balance cannot be calculated to sufficient accuracy. Our local measurement of the magnitude and direction of the eddy potential vorticity flux describes only the tendency of the eddy field to affect the distribution of mean potential vorticity.

2. Methodology: The eddy potential vorticity flux

The mean potential vorticity equation (Pedlosky, 1979) contains the total effect of the eddy field on the mean circulation because it incorporates the heat equation into the vorticity equation. For a continuously stratified inviscid fluid the quasi-geostrophic potential vorticity equation is given by

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right]\left[\zeta + f + f_0\frac{\partial}{\partial z}\left(\frac{\rho}{\rho_z{}^s}\right)\right] = S - D \quad (1)$$

where

- $\zeta = v_x - u_y$ the vertical component of relative vorticity
- ρ the perturbation density $\rho_{\text{total}} = \rho_0 + \rho^s(z) + \rho(x, y, z, t)$
- ρ^s the density in an unperturbed reference state
- $f = f_0 + \beta y$ the Coriolis parameter
- (u, v) the east and north velocity components of the horizontal velocity vector $\mathbf{u}$
- $q = \zeta + f + f_0\dfrac{\partial}{\partial z}\left(\dfrac{\rho}{\rho_z{}^s}\right)$ the potential vorticity.

The source term S has been included to represent the wind input of vorticity at the surface. It takes the form $S = fw_{\text{Ek}}\delta(z)$ where w_{Ek} is the Ekman velocity at the base of the mixed layer. Smaller-scale dissipation by internal waves and other processes is subsumed into the term D. The equation for the mean potential vorticity is derived by separating the variables into time-mean (denoted by an overbar) and time-dependent components (primed variables), and averaging the equation to obtain

$$\bar{\mathbf{u}}\cdot\nabla\bar{q} = -\nabla\cdot\overline{\mathbf{u}'q'} + \bar{S} - \bar{D} \quad (2)$$

where the mean potential vorticity is defined as

$$\bar{q} = \underbrace{\beta y}_{\text{planetary vorticity}} + \underbrace{\bar{v}_x - \bar{u}_y}_{\text{relative vorticity}} + \underbrace{f_0\overline{\frac{\partial}{\partial z}\left(\frac{\rho}{\rho_z{}^s}\right)}}_{\text{vortex stretching}} \quad (3)$$

and the total eddy-potential-vorticity flux vector is given by

$$\overline{\mathbf{u}'q'} = \overline{\mathbf{u}'(v_x' - u_y')} + f_0\overline{\mathbf{u}'\frac{\partial}{\partial z}\left(\frac{\rho'}{\rho_z{}^s}\right)}. \quad (4)$$

The mean potential vorticity balance in Eq. (2) equates the advection of the mean potential vorticity by the mean circulation with the horizontal divergence of the eddy potential vorticity flux and with other sources and sinks. The eddy potential vorticity flux divergence acts like a force in the basin interior which drives the mean flow across contours of mean potential vorticity.

The total eddy potential vorticy flux contains two components—the relative vorticity flux and the thickness flux which represent, respectively, the lateral and vertical transfers of mean potential vorticity. The eddy relative vorticity flux $\overline{u'\zeta'}$ is defined to be

$$\overline{\mathbf{u}'\zeta'} \equiv [\overline{u'(v_x' - u_y')}, \overline{v'(v_x' - u_y')}], \quad (5)$$

while the eddy thickness flux $\overline{u'\eta'}$ is given by

$$\overline{\mathbf{u}'\eta'} \equiv \left[\overline{f_0u'\frac{\partial}{\partial z}\left(\frac{\rho'}{\rho_z{}^s}\right)}, \overline{f_0v'\frac{\partial}{\partial z}\left(\frac{\rho'}{\rho_z{}^s}\right)}\right]. \quad (6)$$

If the Rossby number for the eddy field is small, the eddy velocity is nearly horizontally nondivergent and the components of Eq. (5) can be rewritten in terms of derivatives of the quadratic eddy-velocity components $\overline{u'^2}$, $\overline{v'^2}$, and $\overline{u'v'}$:

$$\overline{u'\zeta'} = \frac{\partial}{\partial x}\overline{u'v'} + \frac{\partial}{\partial y}\frac{\overline{v'^2 - u'^2}}{2} \quad (7)$$

$$\overline{v'\zeta'} = -\frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial x}\frac{\overline{v'^2 - u'^2}}{2}. \quad (8)$$

In the numerical model experiment (Holland and Rhines, 1980) the eddy relative vorticity flux is larger than the thickness flux in the western boundary current and jet-separation regions, but the thickness flux dominates vertical transfers of potential vorticity in the gyre interior.

These equations [Eqs. (6)–(8)] are used to compute the eddy thickness flux and the eddy relative vorticity flux from time series of temperature and velocity in the LDE mooring array. The mean potential vorticity gradient is estimated from both the point measurements and from hydrographic data. By comparing the magnitude and direction of the eddy potential vorticity flux to the mean potential vorticity gradient, we can deduce the total effect of the eddy field on the mean potential vorticity balance in the LDE region.

3. Data and measurements

The POLYMODE Local Dynamics Array was deployed by the Woods Hole Oceanographic Institution

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

Buoy Group at 31°N, 69°30'W, in the Gulf Stream recirculation region from May 1978 until July 1979. The maximum observational record length is fifteen months, although for this analysis the failure of certain instruments reduces this usable data record to ~225 days. The array (Fig. 1) consists of nine moorings arranged in two crosses at spacings of approximately 25–40 km with current and temperature recorders in the thermocline and with a central mooring instrumented throughout the water column. The configuration was specifically designed using objective mapping principles for calculating optimal estimates of mesoscale dynamics (McWilliams et al., 1986). A more complete description of the low-frequency and mean-flow components of this dataset is given by Mills et al. (1981) and by Owens et al. (1982).

The potential vorticity fluxes are calculated from the time series of velocity and temperature and from hydrographic data. A mean hydrographic profile for the vertical temperature gradient was obtained by averaging 13 CTD profiles taken during an intensive survey of the Local Dynamics area in July–August 1978. The time series of velocity and temperature from the moored array were low-pass filtered with a 24-hour half-width Gaussian window. The temperature records were corrected for vertical mooring motion by using the pressure time series to estimate the excursions of the instruments from nominal depth (Mills et al., 1981).

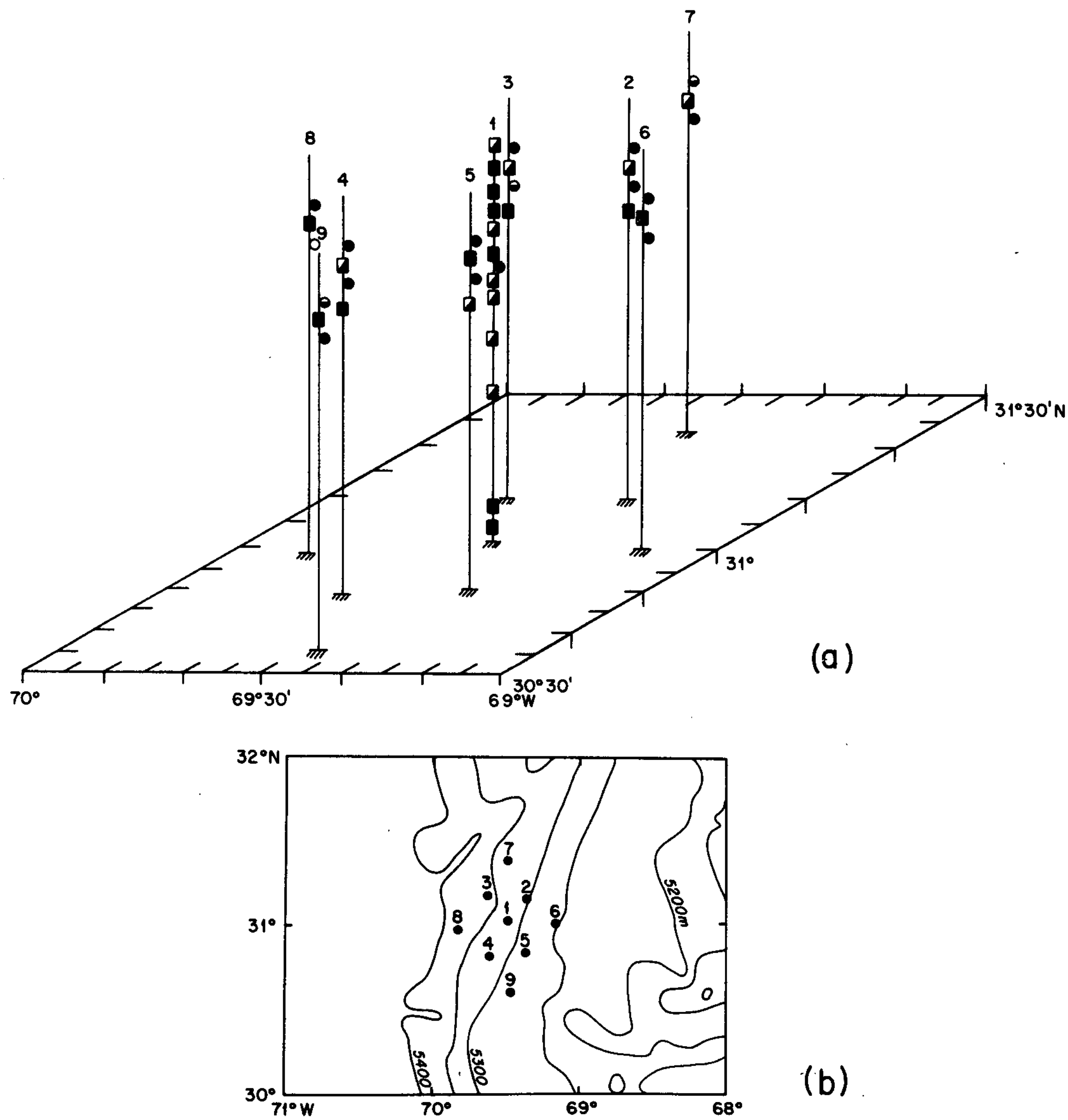

FIG. 1. Local dynamics mooring array configuration from Mills et al. (1981) in perspective (a) and plan (b) views. Squares represent pressure and temperature records and the circles are vector averaging current meters (VACMs).

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

This correction is necessary to avoid spurious correlations between velocity and temperature caused by mooring motion. The terms in the eddy potential vorticity fluxes were calculated by finite difference from the velocity component and temperature correlations. As shown in section 2, the eddy potential vorticity flux is the sum of two components—the relative vorticity flux and the thickness vorticity flux, which have different sampling and measurement requirements.

The relative vorticity flux calculation from Eq. (7) requires velocity measurements at a single depth from an array of at least three moorings. From Eqs. (5) and (7) and (8) the flux can be calculated by two methods which should be equivalent for an eddy field with small Rossby number. For an eddy-resolving array of current meters, time series of vorticity can be computed. The correlation of these vorticity time series with the velocity components is the relative vorticity flux. Alternatively, if the array cannot resolve eddy vorticity, then Eqs. (7) and (8) can be used to compute the relative vorticity flux vector components from horizontal derivatives of the horizontal velocity component correlations. Since the Local Dynamics Experiment array contains nine moorings which spatially resolve the eddy field, these data also permit accurate estimates of the sampling and measurement errors.

4. The measured eddy potential vorticity flux

The first method for calculating the relative vorticity flux is from differencing the horizontal velocity correlations according to Eqs. (7) and (8). The uniform velocity data record in the midthermocline for the nine moorings covers 225 days between 1 May and 13 December 1978. The horizontal velocity correlations (Table 1) at the nine moorings were first adjusted to a common depth (637 m) by using the average vertical gradients of these quantities computed at moorings 1, 2 and 3. The measurement error in these velocity correlations was estimated by Bryden (1982) from the sum of the residuals to a least-squares planar regression through the nine values. The error in $\overline{u'v'}$ and $(v'^2 - u'^2)/2$ are 3.21 $cm^2\,s^{-2}$ and 4.27 $cm^2\,s^{-2}$, respectively. The relative vorticity flux can then be calculated by a least-squares regression to a planar fit through the nine values of the velocity correlations. As shown in Table 1, the individual gradients are comparable in magnitude within the statistical error and tend to cancel in the expression for the relative vorticity flux.

The sampling error in the relative vorticity flux is the combined error in the slopes for least-squares planar regressions through the velocity correlations at the nine moorings. The confidence limits for the slope of a planar regression through the nine values is determined from Student's t distribution with six degrees of freedom as described by Guttman et al. (1971). According to this method, the relative vorticity flux components for the 225 day records are determined to be

TABLE 1. Eddy velocity correlation gradients in the Local Dynamics Experiment.

(a) 225-day record from moorings 1–9

$$\frac{\partial}{\partial x}\overline{u'v'} = -1.38 \times 10^{-6} \pm 1.21 \times 10^{-6}\ [\mathrm{cm\ s^{-2}}]$$

$$\frac{\partial}{\partial y}\overline{u'v'} = 0.78 \times 10^{-6} \pm 0.85 \times 10^{-6}$$

$$\frac{\partial}{\partial x}\overline{\frac{v'^2 - u'^2}{2}} = 2.73 \times 10^{-6} \pm 1.29 \times 10^{-6}$$

$$\frac{\partial}{\partial y}\overline{\frac{v'^2 - u'^2}{2}} = -0.19 \times 10^{-6} \pm 0.91 \times 10^{-6}$$

$$\overline{u'\zeta'} = -1.57 \times 10^{-6} \pm 1.51 \times 10^{-6}\ [\mathrm{cm\ s^{-2}}]$$

$$\overline{v'\zeta'} = 1.95 \times 10^{-6} \pm 1.54 \times 10^{-6}\ [\mathrm{cm\ s^{-2}}]$$

(b) 434-day record from moorings 1, 6, 8 and 9

$$\frac{\partial}{\partial x}\overline{u'v'} = 0.53 \times 10^{-6}\ [\mathrm{cm\ s^{-2}}]$$

$$\frac{\partial}{\partial y}\overline{u'v'} = 0.36 \times 10^{-6}$$

$$\frac{\partial}{\partial x}\overline{\frac{v'^2 - u'^2}{2}} = 3.05 \times 10^{-6}$$

$$\frac{\partial}{\partial y}\overline{\frac{v'^2 - u'^2}{2}} = 0.33 \times 10^{-6}$$

$$\overline{u'\zeta'} = 0.86 \times 10^{-6}\ \mathrm{cm\ s^{-2}}$$

$$\overline{v'\zeta'} = 2.69 \times 10^{-6}\ \mathrm{cm\ s^{-2}}$$

$$\left.\begin{aligned}\overline{u'\zeta'} &= -1.57 \times 10^{-6} \pm 1.51 \times 10^{-6}\\ \overline{v'\zeta'} &= 1.95 \times 10^{-6} \pm 1.54 \times 10^{-6}\end{aligned}\right\}\ [\mathrm{cm\ s^{-2}}] \qquad (9)$$

where the 90-percent error estimate gives a statistically significant result but with large uncertainties. Table 1b shows the effect of longer averaging time on the stability of these estimates. A 434-day record length from moorings 1, 6, 8, 9 can be used to calculate finite difference derivatives of the momentum fluxes. For the longer record length there is a significant increase in the zonal component of the relative vorticity flux. The meridional component also increases but still remains within the estimated error for the 225-day record.

Since the horizontal nondivergence of the eddy velocity field has been used to obtain the relative vorticity flux in terms of the divergence of the momentum fluxes, the momentum flux derivatives tend to cancel in the relative vorticity flux expression Eqs. (7) and (8). The Rossby number for the LDE mesoscale eddy field is $\mathrm{Ro} = U/fL = O(0.01)$ assuming a length scale of 100 km and a scaling velocity of 7 $cm\ s^{-1}$. Thus, the assumption of horizontal nondivergence should be satisfactory. Since the LDE array spatially resolves the eddy variability, this relative vorticity flux can also be

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

calculated directly from the correlations between time series of velocity and vorticity. Time series of vorticity and spatially averaged velocity are calculated from each of the triangles of three moorings—an inner cross composed of triangles 123, 134, 145 and 152 and an outer cross composed of triangles 167, 178, 189 and 196 where the digits represent the moorings shown in Fig. 1. The relative vorticity flux for the 225-day record averaged over the eight triangles is $-1.37 \times 10^{-6} \pm 3.05 \times 10^{-6}$ cm s^{-2} for the zonal component and $3.86 \times 10^{-6} \pm 3.83 \times 10^{-6}$ cm s^{-2} for the meridional component. The estimated 90-percent confident limits are based on the standard deviation of the correlation between velocity and vorticity assuming a 13-day time scale for independent data values. The 13-day time scale is the average of the integral time scales for the zonal (10 day) and meridional (15 day) velocity components. Because of the tendency for terms to cancel in Eqs. (7) and (8), the direct method Eq. (5) for computing the relative vorticity flux gives values larger than but not significantly different from the previous result in Table 1.

Estimates of the thickness flux require velocity and temperature records from only a single mooring. Since there is a tight correlation between temperature and salinity in the midthermocline of the western North Atlantic, the thickness flux is evaluated using temperature in place of density in Eq. (6). The LDE thickness fluxes (Table 2) are calculated at midthermocline depths from velocity records at 567–728 m and from temperature measurements at 469–625 and 665–839 m divided by the mean vertical temperature gradient from the averaged CTD profile. The average instrument depths for the six moorings are 525 and 743 m for temperature and 623 m for velocity so the array-averaged thickness flux is calculated at nearly the same level (637 m) as the relative vorticity flux.

In order to make statistically reliable estimates of the thickness flux, the data record must be long compared to the time scale of the eddy field. The average integral correlation times for the nine instruments at 600 m depth are 10 days for u and 15 days for both v and t. Six of the LDE moorings—1, 2, 4, 5, 6 and 9—have 225-day records (No. 4 is short with only 178 days) while a longer 434-day record is available at moorings 1, 5, 6 and 9. The temperature/pressure sensors on moorings 7 and 8 (Mills et al., 1981) failed, so these moorings cannot be used in this calculation. Table 2 shows that doubling the record length tends to decrease the absolute value of the thickness-flux components at a given mooring although the changes in the average values are not statistically significant. At least in this dataset from the recirculation region, a 225-day record seems sufficient to estimate the thickness term. The estimated error is the 90-percent confidence level computed from the sample variance and a Student's t distribution assuming a 15- and 13-day time scale for the meridional and zonal velocity components. The eddy thickness flux components averaged over the six moorings with 225-day records are determined to be

$$\left.\begin{aligned}\overline{u'\eta'} &= -0.79 \times 10^{-5} \pm 0.53 \times 10^{-5}\\ \overline{v'\eta'} &= -1.45 \times 10^{-5} \pm 0.71 \times 10^{-5}\end{aligned}\right\} [\text{cm s}^{-2}]. \quad (10)$$

The thickness flux is significant according to our error estimate and it is directed to the southwest.

The total eddy potential vorticity flux (Table 4) in the midthermocline LDE region is dominated by the thickness flux which is an order of magnitude larger than the relative vorticity flux. In addition, the two vectors shown in Fig. 3 differ in direction. The magnitude of the thickness-flux vector ($\overline{u'\eta'}$) is 1.65×10^{-5} cm s^{-2} with a range due to the errors of between 0.78 $\times 10^{-5}$ cm s^{-2} and 2.53×10^{-5} cm s^{-2}. The direction of this vector is to the southwest at 209°T. From the possible errors in the components this direction could lie between 187°T and 241°T. The relative-vorticity flux ($\overline{u'\zeta'}$) is an order-of-magnitude smaller with a greater percentage of uncertainty. The magnitude of the relative-vorticity flux vector is 0.25×10^{-5} cm s^{-2} with a range due to the errors of between 0.04×10^{-5} cm s^{-2} and 0.47×10^{-5} cm s^{-2}. The direction of this vector is to the northwest at 321°T with a possible range of 278° to 359°T. The sum of these two vectors, the total eddy potential vorticity flux, has components:

$$\left.\begin{aligned}\overline{u'q'} &= -0.95 \times 10^{-5} \pm 0.55 \times 10^{-5}\\ \overline{v'q'} &= -1.25 \times 10^{-5} \pm 0.73 \times 10^{-5}\end{aligned}\right\} [\text{cm s}^{-2}]. \quad (11)$$

TABLE 2. Eddy thickness flux in the Local Dynamics Experiment.

Mooring	Depths (m)	Length	$\overline{u'\eta'}$ (cm s^{-2})	$\overline{v'\eta'}$ (cm s^{-2})
(a) 225-day record length				
1	516, 616, 839	225 days	-0.62×10^{-5}	-1.16×10^{-5}
2	514, 611, 706	225	-0.87×10^{-5}	-1.73×10^{-5}
4	531, 634, 734	178	-1.66×10^{-5}	-1.89×10^{-5}
5	469, 567, 665	225	-0.92×10^{-5}	-0.46×10^{-5}
6	625, 728, 830	225	-0.20×10^{-5}	-2.86×10^{-5}
9	492, 587, 681	225	-0.48×10^{-5}	-0.62×10^{-5}
averages $\overline{u'\eta'} = -0.79 \times 10^{-5} \pm 0.53 \times 10^{-5}$				
$\overline{v'\eta'} = -1.45 \times 10^{-5} \pm 0.71 \times 10^{-5}$				
(b) 434-day record length				
1		434 days	0.18×10^{-5}	-0.72×10^{-5}
5		434	0.23×10^{-5}	-0.27×10^{-5}
6		434	-0.40×10^{-5}	-1.78×10^{-5}
9		434	-0.35×10^{-5}	-0.50×10^{-5}
averages $\overline{u'\eta'} = -0.09 \times 10^{-5} \pm 0.56 \times 10^{-5}$				
$\overline{v'\eta'} = -0.82 \times 10^{-5} \pm 0.57 \times 10^{-5}$				

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

TABLE 3. Eddy potential vorticity flux in the Local Dynamics Experiment for 225-day records.

$\overline{u'q'}$ (cm s^{-2})	=	$\overline{u'\zeta'}$	+	$\overline{u'\eta'}$
-0.95×10^{-5}		-0.16×10^{-5}		-0.79×10^{-5}
		estimated errors		
0.55×10^{-5}		0.15×10^{-5}		0.53×10^{-5}
$\overline{v'q'}$	=	$\overline{v'\zeta'}$	+	$\overline{v'\eta'}$
-1.25×10^{-5}		0.20×10^{-5}		-1.45×10^{-5}
		estimated errors		
0.73×10^{-5}		0.15×10^{-5}		0.71×10^{-5}

In terms of magnitude and direction the total flux is to the southwest at 217°T with a range of 191° to 251°. The magnitude is 1.57×10^{-5} cm s^{-2} with a range of 0.66×10^{-5} cm s^{-2} to 2.48×10^{-5} cm s^{-2}.

The direction of the total eddy potential vorticity flux relative to the mean potential vorticity gradient indicates the sign of enstrophy transfer between the time-dependent eddy field and the mean flow. In the LDE region where the mean velocity is typically $U_0 = 2$ cm s^{-1} and the length scale of the mean flow is $L = 200$ km, the advection of mean relative vorticity $(\bar{\mathbf{u}} \cdot \nabla \bar{\zeta})$ is small compared to the advection of planetary vorticity $(\beta \bar{v})$. Therefore, the potential vorticity gradient can be approximated as the sum of the planetary-vorticity gradient β and the effective β_{eff} due to the northward thickening of the isopycnal layer. This can also be expressed in terms of the vertical shear of the mean velocity by using the thermal-wind relation. In this form the mean potential vorticity gradients are

$$\left.\begin{aligned} \frac{\partial \bar{q}}{\partial x} &= f_0^2 \frac{\partial}{\partial z}\left(\frac{\bar{v}_z}{N^2}\right) \\ \frac{\partial \bar{q}}{\partial y} &= \beta - f_0^2 \frac{\partial}{\partial z}\left(\frac{\bar{u}_z}{N^2}\right) \end{aligned}\right\} . \tag{12}$$

The errors in the point measurements of the mean velocities in the LDE are too large to permit a significant finite-difference estimate of the mean potential vorticity gradient. McWilliams (1983) calculated the mean gradients from differencing smoothed vertical profiles of the mean velocities from each instrument at the LDE central mooring. Because the instruments on the central mooring have different record lengths the mean velocity profile can be estimated either by using a uniform record length for all the instruments or by using the longest available record for each instrument. McWilliams's estimates (his Fig. 12) for the north–south gradient of mean potential vorticity at 650 m depth are comparable to β, between 1.5×10^{-13} and 2.5×10^{-13} cm^{-1} s^{-1}, while the east–west gradient is an order of magnitude smaller with the range between -2×10^{-14} and 4.0×10^{-14} cm^{-1} s^{-1}. It is difficult, however, to have much confidence in these estimates given the basic uncertainties in the data values from which the smoothed profiles are generated.

An alternate estimate of the large-scale potential vorticity gradient can be calculated using hydrographic data from two sources—the January–March 1957/58 IGY section along 65°W and a compilation of historical data. McWilliams (1983) used McDowell et al.'s (1982) calculation of isopycnal thickness gradients from the IGY sections as a qualitative check on his estimates. A more complete hydrographic dataset has recently been compiled by Levitus (1982) from the NODC historical data averaged for each degree square in the North Atlantic. Although the Levitus dataset includes the IGY data, there are more than 1000 additional midthermocline observations in the ten-degree square centered around the LDE site. The Levitus dataset has been smoothed using an objective analysis scheme with a half-width filtering over about 1000 km. Sgouros and Keffer (1983) calculated mean potential vorticity defined by McDowell et al. (1982) as

$$\bar{q}_\theta = \frac{f_0}{\rho} \frac{\partial \rho_\theta}{\partial z} \tag{13}$$

from finite-differencing the potential density ρ_θ between constant density surfaces. This q_θ differs from the quasigeostrophic potential vorticity q because it does not include the relative vorticity or the ambient planetary vorticity tendency and it is divided by the thickness of the density layer. The effective β_{eff} due only to the northward thickening of isopycnal layers can be calculated using the definition (13):

$$\beta_{\text{eff}} = \frac{\rho \partial \bar{q}_\theta / \partial y}{\partial \rho_\theta / \partial z} = \frac{f_0 \partial \bar{q}_\theta / \partial y}{\bar{q}_\theta} . \tag{14}$$

Figure 2 shows the latitudinal variation along 70°W longitude of the potential vorticity q_θ in the layer centered at the $\sigma_\theta = 26.75$ density-anomaly surface. This density layer lies between 400 and 700 m depth in the subtropical gyre. It intersects the sea surface to the north of the LDE region, but at 30°N it is deeper than the influence of direct wintertime convective mixing. The IGY section along 65°W shows general agreement with the climatological data, although the local potential vorticity maximum at 32°N in the NODC data is not evident in the 65°W section. However, it is plotted as a local maximum on the contoured 65°W section shown in Fig. 10 of McDowell et al. (1982). Since the zonal derivative of mean potential vorticity is much smaller than the meridional change, the total gradient will be approximated as the sum of the planetary term

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

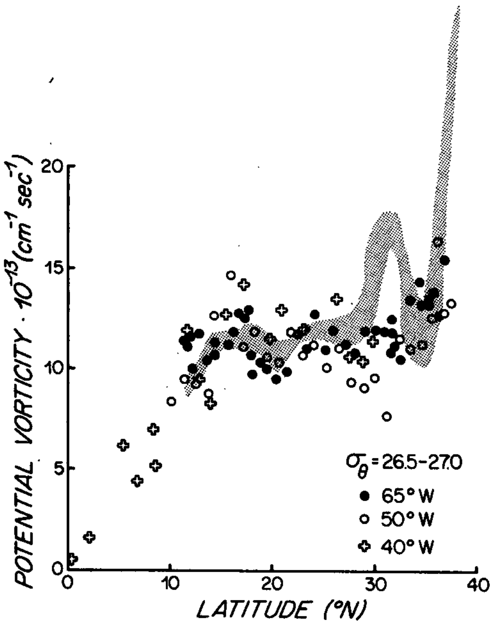

FIG. 2. Large-scale mean potential vorticity q_θ for the density layer σ_θ = 26.5–27.0. The data plotted with symbols are from McDowell et al. (1982), while the shaded values are from the NODC data along 70°W.

$\beta = 2 \times 10^{-13}$ cm^{-1} s^{-1} and the isopycnal thickening $\beta_{\rm eff}$. At the LDE latitude the NODC data estimate for the contribution due to the northward isopycnal thickening in the midthermocline is $\beta_{\rm eff} \approx 5 \times 10^{-13}$ cm^{-1} s^{-1}, while for the 65°W data $\beta_{\rm eff} \approx 3 \times 10^{-13}$ cm^{-1} s^{-1}. Thus the combined mean potential-vorticity gradient is approximately (5–7) $\times 10^{-13}$ cm^{-1} s^{-1} in the north–south direction. The mean potential-vorticity gradient is shown in Fig. 3 along with the eddy potential vorticity flux.

5. Discussion

The observational data from the Local Dynamics Experiment show that the eddy potential vorticity flux at 31°N, 69.5°W is directed to the southwest owing mostly to the thickness flux. Although the LDE is the only dataset from which the exact form of the total eddy potential vorticity flux (Eq. 4) can be computed, a partial comparison can be made with another dataset from the recirculation region by using the two-layer approximation. In the layer-model representation (Holland and Rhines, 1980) the interface height perturbation is analogous to minus the temperature fluctuation divided by the mean vertical temperature gradient T_z^s. The thickness flux with this approximation is proportional to the heat flux at the interface:

$$\overline{\mathbf{u}'\eta'} = \frac{f_0}{H_1} \frac{\overline{\mathbf{u}'t'}}{T_z^s}. \qquad (15)$$

In the LDE only the meridional component of the heat flux is significant, with the maximum values in the thermocline. The spatially averaged meridional heat flux at ~700 m for a uniform 225-day record at the nine moorings is -1.89°C cm s^{-1} which gives a thickness flux of $\overline{v'\eta'} = -0.91 \times 10^{-5}$ cm s^{-2} using an upper-layer thickness of 700 m and a mean vertical temperature gradient, $T_z^s = 2.22 \times 10^{-4}$ °C cm^{-1}, calculated from the average CTD profile. Thus, the two-layer approximation gives a value for the upper-layer thickness flux which is comparable to the exact LDE calculation from Equation (4).

The two-layer approximation also gives values from Array II which are consistent with the LDE results. Hogg (1983) estimated the meridional lower-layer thickness flux at 600 and 1000 m from the POLYMODE Array II data using a two-layer model approximation to the continuous form. Although the Array II dataset is longer (500–700 days) than the LDE set, the total relative vorticity flux cannot be calculated because of the array configuration. In the Array II data there is a large spatial variation in the heat fluxes, but

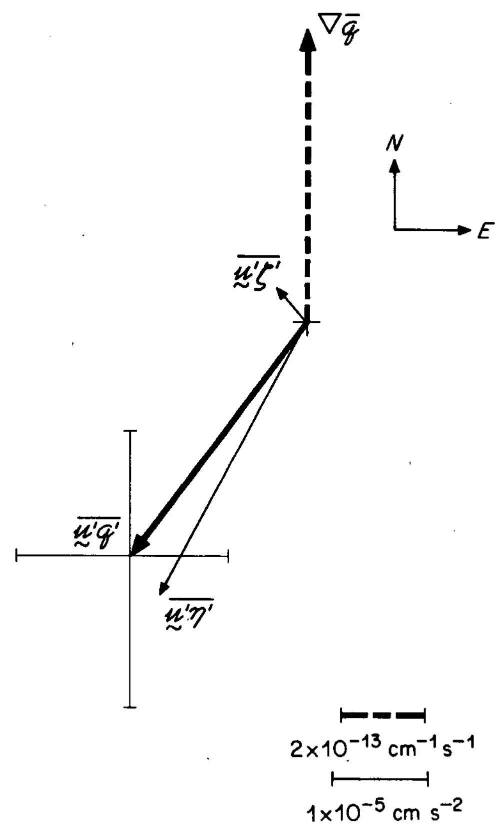

FIG. 3. The direction of the eddy potential vorticity flux $\overline{\mathbf{u}'q'}$ and its components—the relative vorticity flux $\overline{\mathbf{u}'\zeta'}$ and the thickness flux $\overline{\mathbf{u}'\eta}$—are plotted in relation to the mean potential vorticity gradient $\nabla\bar{q}$. The 90-percent confidence limits are also shown for the total eddy flux.

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

TABLE 4. Heat fluxes and thickness fluxes in the thermocline for Array II measurements along 55°W.

Mooring	Lat/long	Record length (days)	Depth (m)	Correlation coefficients ρ_{ut}	ρ_{vt}	$\overline{u't'}$ (°C cm s^{-1})	$\overline{v't'}$ (°C cm s^{-1})	$\overline{u'\eta'} \times 10^{-6}$ cm s^{-2}	$\overline{v'\eta'} \times 10^{-6}$ cm s^{-2}
7	31.6°N, 55°W	380	600	0.34*	0.05	0.94	0.13	7.16*	0.99
		762	1000	0.18	−0.18	0.35	−0.33	2.66	−2.51
5	34.9°, 55.1°	724	600	−0.05	0.01	−1.06	0.19	−9.51	1.70
		724	1000	−0.18	−0.19	−1.05	−0.95	−6.57	−5.94
3	35.6°, 55.1°	776	600	0.07	−0.09	1.67	−2.13	15.01	−19.15
		476	1000	0.05	−0.19	0.27	−1.02	1.98	−7.48
1	35.9°, 55.1°	557	600	−0.03	−0.16	−0.74	−3.29	−7.67	−34.09
		663	1000	−0.002	−0.21*	−0.02	−1.72	−0.10	−8.42*
2	35.9°, 54.8°	527	600	−0.06	−0.13	−0.98	−2.03	−11.21	−23.22
		744	1000	−0.19*	−0.26*	−2.00	−2.28	−1.07*	−1.47*
4	36.0°, 53.8°	697	600	−0.01	−0.20*	−0.26	−4.70	−2.41	−43.56*
		480	1000	−0.22	−0.20	−1.98	−1.10	−10.49	−5.83
6	35.9°, 59.0°	284	600	−0.10	0.40*	−3.51	18.70	−31.92	+170.08*†
		648	1000	−0.06	−0.13	−0.43	−1.32	−2.38	−7.30
8	37.5°, 55.0°	779	600	−0.28*	−0.27*	−10.39	−9.63	−86.40*	−80.08*
		516	1000	−0.40*	−0.30*	−4.70	−3.63	−37.93*	−29.30*

* 90-percent level of correlation.
† Mooring 6 record length is only 9 months long.

at 600 and 1000 m depths all of the thickness fluxes in the moorings (with the exception of mooring 6 at 600 m) north of 35°N are southward (Table 4). However, as pointed out by Hogg (1983), the 600-m record from mooring 6 is only nine months long and it contains the signal due to two Gulf Stream rings. The magnitude of the correlation coefficients is less than 0.4, so there are large uncertainties even in the two-year records. The amplitudes of the meridional components vary by an order of magnitude between a maximum of -8×10^{-5} cm s^{-2} at 600 m at 37.5°N to a minimum of 0.99×10^{-6} cm s^{-2} at 31.6°N. Although the two-layer-approximation estimates are basically consistent with our result, the LDE data and the associated errors can be much better evaluated.

The LDE eddy potential vorticity flux estimates can also be qualitatively compared with the results from numerical model experiments, although the LDE result is only a single point measurement. In the numerical model flows (Holland and Rhines, 1980; Harrison and Holland, 1981) the relative vorticity flux dominates the western boundary current and jet-separation regions, but the thickness flux is larger in the gyre interior. The magnitude and direction of the eddy potential vorticity flux with respect to the mean potential vorticity gradient indicates the effect of the eddy field on the mean potential vorticity balance. The thickness fluxes in the near field of the model western boundary current are directed opposite to the mean potential vorticity gradient. This downgradient flux is consistent with eddy generation by baroclinic instability.

During the early 1970s, technological developments made possible long-term measurements of current and temperature fields. Since that time one important goal of oceanographic research has been to determine the effect of the low-frequency variability on the time-mean circulation. The approach taken in this paper is the calculation of the eddy potential vorticity flux. In analyses of numerical models, the flux is area-integrated because the divergence of this quantity is dynamically important in forcing the mean potential vorticity balance. The oceanic data is sufficient only for calculating the flux in a small region of the ocean. During the Local Dynamics Experiment in the Gulf Stream recirculation region the flux was dominated by the thickness flux term which was directed opposite to the large-scale mean potential vorticity gradient determined from hydrographic measurements. The eddy potential vorticity flux was nonzero and its sign was in accordance with the predictions of instability theory.

Acknowledgments. This research has been conducted under NSF grants OCE 77-19403 and OCE 82-00154. Special thanks are due to Mary Ann Lucas for typing the manuscript.

REFERENCES

Bryden, H. L., 1982: Sources of eddy energy in the Gulf Stream recirculation region. *J. Mar. Res.*, **40**, 1047–1068.

Guttman, I., S. S. Wilks and J. S. Hunter, 1971: *Introductory Engineering Statistics.* Wiley and Sons, 549 pp.

Harrison, D. E., and W. R. Holland, 1981: Regional eddy vorticity transport and the equilibrium vorticity budgets of a numerical model ocean circulation. *J. Phys. Oceanogr.*, **11**, 190–208.

Hogg, N. G., 1983: A note on the deep circulation of the western North Atlantic: Its nature and causes. *Deep-Sea Res.*, **30**, 945–961.

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC

Holland, W. R., and P. B. Rhines, 1980: An example of eddy-induced ocean circulation. *J. Phys. Oceanogr.*, **10**, 1010–1031.
Levitus, S., 1982: *Climatological Atlas of the World Ocean.* NOAA Prof. Paper 13, U.S. Govt. Printing Office, 173 pp.
McDowell, S., P. Rhines and T. Keffer, 1982: North Atlantic potential vorticity and its relation to the general circulation. *J. Phys. Oceanogr.*, **12**, 1417–1436.
McWilliams, J. C., 1983: On the mean dynamical balances of the Gulf Stream recirculation zone. *J. Mar. Res.*, **41**, 427–460.
——, W. B. Owens and B.-L. Hua, 1986: An objective analysis of the POLYMODE Local Dynamics Experiment. Part I: General formalism and statistical model selection. *J. Phys. Oceanogr.*, **16**, 483–504.
Mills, C. A., S. A. Tarbell and R. E. Payne, 1981: A compilation of moored instrument data and associated hydrographic observations, Vol. 28, (POLYMODE Local Dynamics Experiment, 1978–79). WHOI Tech Rep. WHOI-81-73, 47 pp.
Owens, W. B., J. R. Luyten and H. L. Bryden, 1982: Moored velocity measurements on the edge of the Gulf Stream recirculation. *J. Mar. Res.*, **40**(Suppl.), 509–524.
Pedlosky, J., 1979: *Geophysical Fluid Dynamics.* Springer-Verlag, 624 pp.
Rhines, P. B., and W. R. Holland, 1979: A theoretical discussion of eddy-driven mean flows. *Dyn. Atmos. Oceans,* **3**, 289–325.
Sgouros, T. A., and T. Keffer, 1983: *The CAMS Interactive Atlas Package:* A computerized program for the archiving and presentation of oceanographic data. WHOI Tech. Rep. WHOI-83-39, 51 pp.

Unauthenticated | Downloaded 10/22/21 01:36 AM UTC