A Modified Inertial Dissipation Method for Estimating Seabed Stresses at Low Reynolds Numbers, with Application to Wave/Current Boundary Layer Measurements

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(Manuscript received 8 May 1986, in final form 7 November 1986)

ABSTRACT

The inertial dissipation method for estimating seabed friction velocities from near-bed turbulence spectra requires few measurements; it is relatively insensitive to errors in sensor orientation and measurement of mean flows. However, the method is only valid if turbulence spectra are measured at a height above the seabed that is small enough to be within the constant stress layer but large enough to produce an inertial subrange. It is shown that such a height exists only if the friction velocity exceeds a critical value (typically 0.8 cm s^{-1} for a midlatitude ocean). Recent measurements from combined wave and mean flow conditions on the continental shelf do not satisfy this requirement. However, an empirical modification to the inertial dissipation method is suggested to allow estimation of the friction velocity even when a true inertial subrange does not exist. The modified method is applied to the combined wave and mean flow field data; it virtually removes an increase in estimated friction velocity with height, and results in values which are in good agreement with theoretical expectation. If generally applicable, the modified method will significantly extend the range of conditions in which the inertial dissipation method can be used.

1. Introduction

The "inertial dissipation method" has been widely used to estimate bottom stresses in atmospheric boundary layers from measurements of turbulent velocity spectra. The method was originally described by Deacon (1959) and has been reviewed by Champagne et al. (1977).

Recently, Grant et al. (1984) have used the inertial dissipation method to estimate seabed stresses on the Northern Californian continental shelf under conditions in which both steady flows and oscillatory wave flows were present. Huntley and Hazen (1988) describe similar measurements from the continental shelf of Nova Scotia, Canada. In such combined wave and mean flow conditions the inertial dissipation method is particularly important. The presence of large horizontal oscillatory motion close to the seabed makes estimation of seabed stresses by the direct eddy correlation (Reynolds stress) method extremely sensitive to sensor orientation. Generally, the requirement on the accuracy of alignment to the vertical component of flow is much greater than can be achieved in practice. Use of the mean flow profile to estimate the bottom stress is possible, but it requires sensors at several heights above the seabed and measurements accurate enough to measure the velocity shear in the boundary layer. The inertial dissipation method, on the other hand, in principle requires measurements of just one component of turbulent velocity and will yield a stress estimate at each height that such a measurement is made. It is also quite insensitive to sensor orientation.

However, the results of both Grant et al. (1984) and Huntley and Hazen (1988) using the inertial dissipation technique show an unexpected increase in the estimated stress with increased distance from the seabed. The measurements in both cases were made either within the predicted constant stress layer or somewhat above it where the stress is expected to decrease with height.

The purpose of this paper is to show that this increase in estimated bottom stress arises because the fundamental assumptions in the inertial dissipation method are being violated. We show that this is inevitable unless the bottom stress exceeds a critical value. We also suggest a modification of the inertial dissipation method whereby the true bottom stress can be recovered from the inertial dissipation estimates despite this violation of the assumptions. Our modified inertial dissipation method is found to remove the trend of increasing stress with height, and results in stress changes with height which are in agreement with expectations.

If generally valid, this modified inertial dissipation method will significantly extend the range of conditions in which the method can be used.

2. The inertial dissipation method

Three different methods are available for estimating seabed stresses from measurements of flow in the bot-

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tom boundary layer. First is the "mean flow method." Within the lower part of a simple turbulent boundary layer the mean flow u is expected to vary logarithmically with height above the seabed z, according to the equation (e.g., Tennekes, 1973):

$$u = (u_*/\kappa)\ln(z/z_0) \quad (1)$$

where $u_* = \sqrt{\tau/\rho}$, τ is the bottom stress, κ is von Kármán's constant and z_0 is an appropriate bottom roughness length. Thus with measurements of mean flow at a sufficient number of heights above the bed, u_* and z_0 can be estimated from the slope and intercept respectively of a plot of u against $\ln z$. Recent theories of combined wave and mean flow boundary layers assume that Eq. (1) remains valid in the mean flow boundary layer when waves are present, but with both u_* and z_0 enhanced by the presence of a thin wave boundary layer near the seabed (e.g., Smith, 1977; Grant and Madsen, 1979). Field measurements described by Grant et al. (1984) and Wiberg and Smith (1983) appear to confirm this assumption.

However, use of this method requires a number of sensors within the logarithmic layer, and the mean flows must be measured with sufficient accuracy at each level to provide a reliable estimate of the velocity difference between sensors at different heights above the bed. This last requirement is particularly stringent for measurements in wave/mean flow conditions. Here flowmeters must be able to respond accurately to rapidly changing wave flows, and any nonlinearities or zero-drift in the sensors can cause errors in the mean flow estimates. Where measurements are made in relatively low current conditions, these errors can prevent accurate estimation of friction velocity by this method. An additional problem is uncertainty in the heights of sensors above the bed. Both Grant et al. (1984) and Wiberg and Smith (1983) made empirical adjustments, of a few centimeters, to the assumed location of the seabed, a procedure which reduces confidence in their resulting stress estimates.

The second and perhaps most direct method is known as the "eddy correlation" method. The time average of the product of the horizontal, u', and vertical, w', velocity fluctuations, in the form $-\rho\overline{u'w'}$ (where ρ is the water density and the overbar denotes a time average), is known as the Reynolds stress, and measures the turbulent momentum flux, and hence the stress, at the measurement height. Direct measurement of the Reynolds stress has been widely used for estimating stress in steady flow environments (e.g., Bowden and Ferguson, 1980; Grant et al., 1985) but suffers from the disadvantage that it is sensitive to errors in the alignment of the vertical and horizontal axes. The errors introduced by misalignment of the sensor axes depend upon the correlation coefficient between the vertical and horizontal fluctuations (Kaimal, 1969; Hyson et al., 1977). When mean flows dominate, errors of 10% per degree of tilt out of the vertical are typical. However, if oscillatory wave flows are significant, with large horizontal flows in quadrature with smaller vertical flows, the correlation coefficient decreases and sensitivity to alignment errors is enormously increased. An approximate calculation suggests that alignment accuracy of the order of one-tenth of a degree or less may be necessary to provide an accurate estimate of the Reynolds stress. Such alignment accuracy is unattainable in the field. Moreover, the commonly used method of aligning the vertical axis to coincide with zero mean flow during subsequent analysis (e.g., Soulsby, 1980) will be insufficiently accurate in wave flows, as a rule, due to uncertainty in the mean flow measured by the sensors (e.g., Huntley and Hazen, 1988). Other methods for aligning the axes at the data analysis stage are being investigated but none have to date proved viable.

The third possible technique for estimating turbulent stress involves the use of spectra of the turbulent fluctuations and is commonly known as the inertial dissipation method. If the wavenumbers at which turbulent energy is produced are well separated from the (higher) wavenumbers at which turbulent energy is dissipated by viscosity, then the range of wavenumbers between production and dissipation is known as the inertial subrange. In this range the flux of energy from low to high wavenumbers must be equal to the dissipation rate, ϵ, since there are no local sources or sinks for the energy. This leads to the result that the three-dimensional inertial subrange spectrum must be given by (e.g., Tennekes and Lumley, 1972):

$$E(k) = \alpha\epsilon^{2/3}k^{-5/3} \quad (2)$$

where k is the radian wavenumber and α is the three-dimensional Kolmogorov constant, determined experimentally.

In practice, measurements in the boundary layer generally do not provide estimates of the scalar wavenumber spectrum $E(k)$ but of the one-dimensional spectra, which are functions of the wavenumber component in the direction of the mean flow, k_1. These one-dimensional spectra can be denoted $\phi_{ii}(k_1)$, where $i = 1(3)$ corresponds to spectra of longitudinal (transverse) turbulent fluctuations (Hinze, 1975). In the inertial subrange the one-dimensional spectra take a similar form to $E(k)$:

$$\phi_{ii}(k_1) = \alpha_i\epsilon^{2/3}k_1^{-5/3} \quad (3)$$

but the one-dimensional Kolmogorov constant depends upon the value of i considered. The appropriate choices of i and α_i will be discussed later.

In order to use Eq. (3) to estimate bottom stress we need an expression linking the dissipation rate ϵ to the bottom stress. Two assumptions are made in finding such an expression. First, a local balance is assumed between the production and dissipation of turbulent energy, an assumption which is approximately valid (with an error probably much less than 30%; Wyngaard and Coté, 1971) in the lower part of the boundary layer. The production of turbulent energy is given by $(\tau/\rho)\partial u/\partial z$. In the logarithmic part of the boundary layer, $\partial u/\partial z = u_*/\kappa z$ [Eq. (1)], and in the lower part of the logarithmic layer the local stress is equal to the bottom stress, $\tau = \rho u_*^2$. Thus if we make the second assumption, that measurements are made within the constant stress part of the logarithmic layer, we can write $\epsilon = u_*^3/\kappa z$. Substituting this expression into Eq. (3) and rearranging gives

$$u_* = (\phi_{ii}(k)k^{5/3}/\alpha_i)^{1/2}(\kappa z)^{1/3} \tag{4}$$

where k is now used to denote the along-flow wavenumber component k_1. Thus this method of estimating bottom stress involves calculating a one-dimensional wavenumber spectrum of turbulent velocity, finding the range of wavenumbers over which the energy density falls off as $k^{-5/3}$ (usually limited by sensor dimensions as well as limits to the inertial subrange; see Soulsby, 1980) and using the level of the spectrum in this range in Eq. (4) to estimate u_*.

The spectrum of any of the three orthogonal components of turbulent velocities can be used in Eq. (4) provided that the appropriate value of Kolmogorov constant is chosen. In steady flows the longitudinal (along-flow) turbulence spectra have frequently been used (e.g., Champagne et al., 1977; Williams and Paulson, 1977), but in combined wave and mean flow conditions, it is better to use vertical flow spectra since these will include much less contamination from any wave motion in the inertial subrange (Grant et al., 1984).

The major attraction of this method of estimating bottom stress is its relative insensitivity to errors in axis alignment. Since the expected ratio of longitudinal to vertical spectral amplitude in the inertial subrange is 4:3, the stress error even from a gross misalignment of axes should not exceed 33%. In fact an empirical test of sensitivity to alignment error by Huntley and Hazen (1988) showed only 0.8% error in stress per degree of misalignment.

A potential complication in the use of the method arises from the fact that measurements of turbulence are generally in the form of time series and therefore provide spectra as functions of frequency rather than wavenumber. To convert to wavenumber spectra, we need to invoke the Taylor concept of "frozen turbulence," in which

$$\phi_{ii}(k) = \phi_{ii}(f)/(2\pi/\bar{u}) \tag{5}$$

where $\bar{u}$ is the mean velocity and f the frequency in Hz. In order for this frozen turbulence concept to be valid we require the time scale of an eddy to be much larger than the time for that eddy to be advected past the measurement point by the mean flow. Tennekes and Lumley (1972) estimate the time scale of an eddy with wavenumber k to be $2\pi/(k^3\phi(k))^{1/2}$, where $k\phi^{1/2}$ is an estimate of the velocity in the eddy based on the spectrum $\phi(k)$. The time scale for such an eddy to pass a point is $2\pi/k\bar{u}$. Hence for frozen turbulence we require $k\phi/\bar{u}^2 \ll 1$. In practice this criterion is generally easily met. For the data discussed later the parameter on the left of the inequality is typically 10^{-3} or less.

It might also be anticipated that the Taylor hypothesis would need to be substantially revised in an environment with significant oscillatory wave flows. However, Lumley and Terray (1983) show that, for isotropic turbulence and horizontal wave velocities much larger than vertical velocities, the friction velocity corrected for the influence of wave advection is given approximately by

$$u_* = (1 - 0.16(u_{\rm rms}/\bar{u})^2)^{1/2}\hat{u}_* \tag{6}$$

where $u_{\rm rms}$ is the root-mean-square horizontal wave velocity and $\hat{u}_*$ is the value of friction velocity found from vertical spectra using the Taylor hypothesis (Lumley and Terray (1983) Eq. (A16) to order $(u_{\rm rms}/\bar{u})^2$). Corrections using Eq. (6) have been made to the friction velocity estimates discussed later, but the largest reduction in friction velocity is only about 5% so this effect is not of major importance compared to other sources of measurement error.

Thus, the inertial dissipation method appears to be well suited to estimation of bottom stress, particularly in combined wave and current conditions. However, we show in the next section that the assumptions leading to Eq. (4) are frequently violated in oceanic boundary layers. In particular, the measurements of Grant et al. (1984) and Huntley and Hazen (1988) generally do not comply with these assumptions, so the values of friction velocity deduced using Eq. (4) are in error.

3. Criteria for the validity of the inertial dissipation method

The existence of a true inertial subrange depends upon full separation in wavenumber space between the low wavenumber production and the high wavenumber dissipation of turbulence. By considering scales of turbulent production and dissipation, Tennekes and Lumley (1972) suggest that, in a steady flow boundary layer, this separation will only occur if the turbulent Reynolds number is greater than some critical Reynolds number, Re_c:

$$\mathrm{Re} = u_*\kappa z/\nu > \mathrm{Re}_c \tag{7}$$

where ν is the kinematic viscosity of water. Tennekes and Lumley (1972) estimate that Re_c might be around 4000, though the actual value is sensitive to assumptions about the degree of separation required between turbulence production and dissipation. Gross and Nowell (1985b) show field measurements of one-dimensional longitudinal turbulence spectra in tidal flows that support the hypothesis of a critical Reynolds number. Based on their Figs. 9 and 10, the observed critical Reynolds number lies in the range 2500–3500.

Gross and Nowell (1985b) also describe spectra for steady canal flow; we shall show that these spectra are not inconsistent with $\mathrm{Re}_c = 2500$, though the measurements do not extend close enough to the boundary to provide definite evidence for the existence of a critical Reynolds number in this steady flow, higher friction case. Zimmerman (1967) compares his results from upper atmosphere turbulence with previous measurements from a round turbulent air jet and from open channel flow, and concludes that the critical Reynolds number, as defined in Eq. (7), is about 3000. The critical Reynolds number is further discussed in section 4.

Assuming that Eq. (7) is generally valid, we can rewrite it to give a critical height above which measurements must be made to ensure an inertial subrange:

$$z_{cr} = \mathrm{Re}_c \nu/(\kappa u_*). \tag{8}$$

In deriving Eq. (4), we also required that the measurements be made within the constant stress part of the logarithmic layer. In fact, we will now show that, for low values of friction velocity, there is no height at which measurements are high enough above the bed to satisfy the Reynolds number criterion [Eq. (7)] while also being close enough to the bed to be within the constant stress layer, and hence no height at which Eq. (4) is valid.

The thickness of a steady, unstratified, not-depth-limited boundary layer, defined in terms of the mean velocity profile, is generally taken to lie within the range (0.25 to 0.40)u_*/f, where f is the Coriolis parameter (Blakadar and Tennekes, 1968). The thickness of the logarithmic layer is expected to be about 10%–15% of this height (Hinze, 1975; Businger and Arya, 1974; Tennekes, 1973). The constant stress layer is considerably thinner than the logarithmic layer. The stress defect equations given by Tennekes (1973) lead to a constant (to within 10%) stress layer thickness equal to about one-half of the logarithmic layer thickness. Thus, the thickness of the constant stress layer is given approximately by

$$z_\tau = (0.013 \text{ to } 0.030)u_*/f \tag{9}$$

Comparing Eqs. (8) and (9), we find that there will only be a height within the constant stress layer with a true inertial subrange if

$$z_\tau > z_{cr}$$

and this leads to the inequality

$$u_*^2/(f\nu) > \mathrm{Re}_c/[(0.013 \text{ to } 0.030)\kappa].$$

The dimensionless parameter on the left-hand side of this inequality can be written $(u_*/f)/(\nu/u_*)$ and is therefore a ratio between the geophysical boundary layer thickness scale and the viscous length scale, in other words a boundary layer Reynolds number. It can also be thought of as a viscous Rossby number. For typical values, for a midlatitude ocean, of f (10^{-4} s^{-1}) and ν (0.015 cm^2 s^{-1}), and with $\mathrm{Re}_c = 3.0 \pm 0.5 \times 10^3$ the inequality suggests that

$$u_* > 0.8 \pm 0.2 \text{ cm s}^{-1}$$

if Eq. (4) is to be valid at any height. If we relax this condition to include measurements within the logarithmic layer but above the constant stress layer [Eq. (4) will not be strictly valid here but values of friction velocity can, in principle, be corrected for the reduction in stress as will be shown later (see also Grant and Williams, 1985)], then the right-hand side of the inequality becomes (0.55 ± 0.15) cm s^{-1}. It should be noted that these u_* conditions are necessary but not sufficient conditions for the correct application of Eq. (4). Even where u_* exceeds these minimum conditions, Eq. (4) is valid only for measurements above z_{cr} [Eq. (8)] and below the top of the constant stress or the logarithmic layer. It should also be noted that for oscillatory or depth-limited flows, determination of the thickness of the constant stress layer will be more complex than shown here (e.g., Soulsby, 1983). Nevertheless, a minimum value of u_* will still exist if the inertial dissipation method is to be valid, and its magnitude will typically be of the same order as that estimated here for steady, unlimited-depth flows.

Thus, for much of the time on continental shelves, and particularly during calmer periods when near bed measurements are easiest to make, the use of Eq. (4) is not justified. Even where the friction velocity exceeds these minima, measurements have generally been made below the critical height, z_{cr}, and the use of Eq. (4) is again unjustified.

4. A proposed extention of the inertial dissipation method to low Reynolds number conditions

The longitudinal turbulence measurements of Gross and Nowell (1985b) in a tidal flow boundary layer suggest that spectra of turbulent velocity measured below the critical height [Eq. (8)] do not increase with decreasing height as expected for an inertial subrange in a constant stress layer [Eq. (4)], but remain at the magnitude appropriate to the critical height. Soulsby (private communication) and Andreas and Paulson (1979) have also made measurements below the critical height in tidal and atmospheric boundary layers, respectively, which show anomalies which may be consistent with this observation. As we shall show in section 5, the ocean wave/current boundary layer spectra also appear to support these observations of Gross and Nowell (1985b). The universality of this behavior below the critical Reynolds number is not clear, particularly in laboratory studies of turbulence. It is generally agreed that a $k^{-5/3}$ region in turbulence spectra persists to Reynolds numbers much smaller than critical values for a true inertial subrange (e.g., Champagne, 1978; Coantic et al., 1981), but the wind tunnel measure-

TABLE 1. Inertial dissipation of friction velocity from Gross and Nowell (1985b, Table 2) denoted $\hat{u}_*$, and values recalculated using the modified inertial dissipation, Eq. (11), denoted u_*.

	Accelerating		Decelerating		Canal flow	
Height Z (cm)	$\hat{u}_*$ (cm s^{-1})	u_*	$\hat{u}_*$ (cm s^{-1})	u_*	$\hat{u}_*$ (cm s^{-1})	u_*
19	1.35	1.96	1.57	2.19	2.88	3.47
32	1.74	2.07	1.74	2.08	3.17	3.29
42	1.85	2.03	1.96	2.12		3.17
55	1.91	1.94		2.07		3.27
70		2.14		2.31		—
90		2.18		2.40		3.36
110		2.07		2.44		3.52
158		2.16		2.35		3.65
210		2.07		2.35		3.55
270		1.89		2.29		3.49
360		1.93		2.05		3.68
Critical heights (based on average u_* above z_{cr})	55 cm		50 cm		32.5 cm	

ments of Ligrani and Moffat (1986), for example, lead to spectra of longitudinal turbulence which continue to scale as kz to Reynolds numbers below those observed by Gross and Nowell (1985b) (and also below the Reynolds numbers observed in the wave/current boundary layers discussed in section 5). Reasons for this apparent disagreement between laboratory and geophysical boundary layers are not clear and merit further investigation. Nevertheless, the observed limiting behavior does appear to be a feature of a wide range of geophysical boundary layers.

Based on the observations of Gross and Nowell (1985b), we tentatively propose the following scheme for correcting inertial dissipation estimates of u_* from measurements from below the critical height, z_{cr}, but within the constant stress layer.

Let $\hat{u}_*$ be the value of friction velocity estimated from Eq. (4) at a height $z < z_{cr}$, and let u_* be the true value. Then the observations of Gross and Nowell (1985b) suggest that

$$u_* = \hat{u}_*(z_{cr}/z)^{1/3} \tag{10}$$

and if we substitute for z_{cr} from Eq. (8) we get

$$u_* = [\hat{u}_*^3 \, \mathrm{Re}_c \nu/(\kappa z)]^{1/4} \quad \text{for} \quad z < z_{cr}. \tag{11}$$

Although the corrections made to $\hat{u}_*$ using Eq. (11) will be seen to be nearly as large as a factor of two for some of the data to be discussed below, the one-quarter power ensures that the correction is not strongly dependent on the value assumed for the critical Reynolds number: changing the number by ±1000 about a mean value of 3000 results in changes of less than ±10% in u_*.

Table 1 shows the results of applying Eq. (11) to the inertial dissipation estimates of friction velocity measured by Gross and Nowell (1985b, Table 2). The critical Reynolds number has been taken to be 3000. Not surprisingly, since Eq. (11) and the chosen value of Re_c were based on the results from the accelerating and decelerating tidal flow, application of Eq. (11) increases the low values of friction velocity near the boundary, bringing them into good agreement with the estimates higher in the water column. Of interest is the fact that applying Eq. (11) to the steady canal flow, also with $\mathrm{Re}_c = 3000$, similarly brings the values from the lowest two current meters closer to the average of the values from higher in the water column. Thus the average u_* for $z > 32$ cm is 3.46 cm s^{-1}, compared with 3.47 and 3.29 cm s^{-1} for the modified values at 19 and 32 cm, respectively. Gross and Nowell (1985b) comment that the low values at the lowest meter might be due to a spatial resolution problem owing to the proximity of the bottom. However, their discussion of the spatial resolution of the sensors in Gross and Nowell (1985a) suggests that its effect on the inertial subrange spectrum at 19 cm should be much smaller than can account for the reductions in $\hat{u}_*$ shown in Table 1. Clearly,

TABLE 2. Friction velocity estimates from Grant et al. (1984) with corrections.

Height (cm)		U_*^{GWG} (cm s^{-1})	U_*^A (cm s^{-1})	U_*^B (cm s^{-1})	U_*^C (cm s^{-1})	$U_*^C/U_*^{profile}$	Z_{cr} (cm)	Z_{log} (cm)
53	(a)	0.41	0.49	0.68	0.71	1.26	137	235
	(b)	0.39	0.47	0.66	0.69	1.22	141	228
103		0.44	0.53	0.61	0.67	1.18	147	220
203		0.48	0.58	0.58**	0.69	1.22	141	228

U_*^{GWG}—quoted by Grant et al. (1984)
U_*^A—without ½ applied to spectra, and with $\alpha = 0.69$
U_*^B—after correction using Eq. (11)
U_*^C—after correction for decrease of stress with height
** No correction with Eq. (11) is made because height is above Z_{cr}
(a) Value at 53 cm from Grant et al. (1984)
(b) Value at 53 cm from mean of range from Grant and Williams Table 3d, (1985a).

modification using Eq. (11) provides an alternative explanation for these low values.

5. Application to wave/current field data

Grant et al. (1984) report values of friction velocity for the CODE-1 site on the Northern Californian continental shelf which were estimated using both the mean velocity profile and the inertial dissipation method. They find that the estimates made by the inertial dissipation method are consistently smaller than the single estimate found from the mean flow profile. In addition, the friction velocities estimated by the inertial dissipation method at three heights above the seabed increase with height.

Since their friction velocities are considerably below the critical value that we have proposed for the validity of the inertial dissipation method, we have revised their estimates using Eq. (11), making the assumption that the equation is equally valid for the transverse turbulence spectra used in these wave-current studies as for the longitudinal spectra measured by Gross and Nowell (1985b). For this revision we have used $\nu = 1.3 \times 10^{-2}$ cm^2 s^{-1}, appropriate for the approximately 10°C bottom water at the CODE site. Three other changes have also been made to the estimates reported by Grant et al. (1984):

1) Gust (1985) points out that the correct spectrum to use in Eq. (4) is one which integrates, over positive frequencies, to the variance of the turbulent motion, not to the total turbulent kinetic energy. The wavenumber spectra of Grant et al. (1984) should therefore be increased by a factor of two.

2) Gust (1985), Huntley (1985) and Grant and Williams (1985a) agree that Grant et al. (1984) used a low value of Kolmogorov constant by choosing $\alpha = 0.5$. We have recalculated their friction velocities using $\alpha = 0.69$. This value is 4/3 the average longitudinal Kolmogorov constant estimated by Champagne et al. (1977) and Williams and Paulson (1977), where the 4/3 factor is the theoretically expected ratio of the spectra of vertical and along-flow turbulent velocities (e.g., Tennekes and Lumley, 1972). Bowden and Ferguson (1980) present field measurements of this spectral ratio from tidal flows. Their observed ratio becomes essentially constant for $kz \geqslant 2\pi$, with an average value of 1.44 but with error bounds that readily encompass the theoretical value of 1.33. Since Grant et al. (1984) (and Huntley and Hazen (1988) discussed below) used wavenumber ranges with $kz \geqslant 2\pi$, use of 4/3 for the ratio seems justified, though its validity has not been demonstrated at low Reynolds numbers.

3) Grant and Williams (1985a) point out that their sensors extended above the constant stress layer, and the friction velocity is therefore expected to show a decrease with height. Based on their analysis, we have corrected their measured values to the values expected within the constant stress layer. For this correction we have used Eqs. (3), (4) and (5) from Grant and Williams (1985a; after Tennekes, 1973) and have iteratively chosen δ such that the final corrected u_* values are consistent with $\delta = \kappa u_*/f$.

The results of applying these various corrections are summarized in Table 2. The original estimates of friction velocity from the inertial dissipation method are given in column 2. We have added a revised value for 53 cm, based on the average (of two values) of the range of dissipation estimates for this height given in Table 3d of Grant and Williams (1985a). The original estimate of friction velocity given by Grant et al. (1984) for this height is larger than that based on the largest estimate in Table 3d, so we have included in Table 2 both their original estimate, labeled (*a*), and the revised estimate, labeled (*b*). At the other levels the inertial dissipation estimates given in Table 3d are consistent with the friction velocity estimates given by Grant et al. (1984). As noted by these authors, the values in column 2 of Table 2 are consistently below the value of 0.556 cm s^{-1} found from the mean velocity profile, up to 30% lower at 53 cm and 14% lower at 203 cm. They also show a systematic trend, with increasing friction velocity as the height above the bed increases. This increase is up to 23% from 53 to 203 cm.

Column 3 of Table 2 gives the values of friction velocity corrected for the factor of two and the revised Kolmogorov constant (points 1 and 2 above). Column 4 of Table 2 shows values corrected using Eq. (11), with the critical Reynolds number taken as 3000. No correction has been applied to the value at 203 cm since the critical height, based on the final estimate of friction velocity, is below this height. Application of equation 11 has clearly reversed the trend of the values, the estimates now decreasing with height about 15% (using estimate *a* at 53 cm) or 12% (using *b*) from 53 to 203 cm.

Finally, in column 5 of Table 2, the estimates corrected for the expected decrease in local friction velocity with height are shown. Encouragingly this final correction substantially reduces the spread of friction velocity estimates to 6% using estimate *a* at 53 cm and to 3% using estimate *b*. The systematic trend in the values of u_* with height is also no longer evident, providing additional support for the corrections, particularly the large correction based on Eq. (11).

Unfortunately, the corrected values are now larger than the estimate from the mean velocity profile by between 18% and 26% (column 6, Table 2). Some potential problems with the profile method have been pointed out earlier, but it is difficult to assess their importance for the CODE-1 data. In any event the disagreement is no worse than that found by Grant et al. (1984) for the uncorrected values. [Note that the corrected dissipation estimates would be in much better agreement with the profile estimate (between 3% and 11% below the profile estimate) if the factor of 2 applied

TABLE 3. Measured values of friction velocity (Huntley and Hazen, 1988).

	Height (z (cm))	$\hat{u}_*$ (cm s^{-1})	u_* (cm s^{-1})	Estimated critical height (cm)	Estimated height of constant stress layer (cm)
Cow Bay	52	0.42	0.64	176	96
	52	0.38	0.59	191	89
	22	0.28	0.58	194	87
		51%	11%		
Sable Island Bank	44	0.58	0.83	136	124
	21	0.48	0.89	126	134
		19%	7%		

to the spectra (point 1 above) were incorrect. However, on the basis of information given by Grant et al. (1984) and Grant and Williams (1985b), it appears that the factor of two must be included.]

Huntley and Hazen (1988) describe measurements from combined wave and mean flow boundary layers on the continental shelf off Nova Scotia. The measurements were made at two sites, at Cow Bay in 25 m water depth where the ratio of root-mean-square wave amplitude to mean flow was around 0.7, and over Sable Island Bank in 45 m depth where the ratio was around 0.2. Table 3 shows their estimates using the inertial dissipation method, $\hat{u}_*$, and their corrected estimates after application of Eq. (11), u_*. The two values for 52 cm height at Cow Bay result from having two independent vertical velocity sensors at that height. As with the CODE-1 data, the uncorrected estimates show an increase in friction velocity with height, but the trend is essentially removed by correcting the values using Eq. (11). Huntley and Hazen (1988) also show that the corrected values are in good agreement with predictions based on the theory of Grant and Madsen (1979), if the significant wave orbital velocity (the average of the highest one-third of measured wave amplitudes in a time series) is used in the prediction scheme.

6. Summary and conclusions

The inertial dissipation method for estimating bottom stress, through spectra of turbulent fluctuations, is very useful, particularly in conditions of coexisting wave and mean flows where alternative methods are very sensitive to measurement errors. However, we have shown that the conditions for the validity of the inertial dissipation method are very restrictive. The twin requirements of being within the constant stress layer and yet high enough above the seabed to allow full separation between the production and dissipation wavenumbers [the Reynolds number criterion of equation (7)] can only be met if the friction velocity exceeds a critical value, typically for the oceanic boundary layer about 0.8 ± 0.2 cm s^{-1}. Even where the friction velocity exceeds this value, the Reynolds number criterion severely restricts the range of heights above the bed for which the inertial dissipation method is valid.

Using the observations of Gross and Nowell (1985b), we have suggested a modification of the inertial dissipation method which extends its use to Reynolds numbers much lower than the critical Reynolds number. The proposed correction, Eq. (11), is simple to apply and is relatively insensitive to the value assumed for the critical Reynolds number.

When applied to the field data of Grant et al. (1984) and Huntley and Hazen (1988), the modified method removes an apparent increase in friction velocity with height above the bed. The resulting modified friction velocity estimates of Huntley and Hazen (1988) are independent of height, as expected for measurements from within the constant stress layer, while the modified estimates from the data of Grant et al. (1984) extend above the constant stress layer and show a decrease with height which is well accounted for by predictions based on the stress defect equations of Tennekes (1973).

The corrections to the original inertial dissipation estimates of friction velocity are almost a factor of two for some of these field data, and are applied to data with Reynolds numbers as small as 13% of the critical Reynolds number. Nevertheless, it is encouraging that the results are consistent with the expected behavior of the stress as the height above the bed changes. Huntley and Hazen (1988) also show that their corrected estimates are consistent with predictions based on the theory of Grant and Madsen (1979).

A remaining problem is the lack of agreement in the data of Grant et al. (1984) between the modified inertial dissipation estimates and the profile estimate of friction velocity. The lack of agreement, ranging from 18% to 26%, is perhaps marginally smaller than that found with the uncorrected inertial dissipation estimates, and is probably within the broad measurement error bounds expected. However, it is not clear why the difference occurs.

The modification of the inertial dissipation method proposed here is, as yet, purely empirical and based on a very small number of observations. The results of applying it to field data are very encouraging but are far from conclusive owing to the small amount of data available. Nevertheless, we have shown that some form of correction to the inertial dissipation method is essential for measurements made in low friction velocity conditions or made below a critical height above the seabed. If the modification to the inertial dissipation method proposed here proves generally applicable, it will significantly increase the range of conditions over which the method can be applied.

Acknowledgments. This work was carried out while the author was on sabbatical leave at the University of Southampton, England. The hospitality and assistance of the Department of Oceanography at Southampton are gratefully acknowledged. Discussions with Drs. Terry Chriss and Richard Soulsby were also helpful.

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