A Quasi-Geostrophic Circulation Model of the Northeast Pacific. Part I: A Preliminary Numerical Experiment

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ABSTRACT

A limited-area quasi-geostrophic numerical model with mesoscale resolution is developed to study the circulation in the northeast (NE) Pacific Ocean. The model domain extends from the British Columbia–Alaska coast out to 170°W and down to 45°N, and incorporates the coastline geometry and bottom topography of the region. A long-term integration was conducted using a steady climatological wind stress curl field to drive the circulation. Several statistical properties of the solution are determined and compared with observations.

A cyclonic circulation develops in the model basin with a meandering Alaska Current feeding, at the head of the Gulf of Alaska, into an intense boundary current corresponding to the Alaskan Stream. The head of the Gulf is a region where anticyclonic closed streamline features are occasionally generated with characteristics resembling those of the Sitka Eddy. In the downstream region, the boundary current separates and is subject to lateral meandering due to topographic waves. The occurrence of perturbations with similar characteristics in the Alaskan Stream has recently been verified in satellite IR imagery.

1. Introduction

There have been many observational programs to determine the circulation in the Gulf of Alaska, which lies in the northeast corner of the Pacific Ocean (e.g., Reed 1984; Reed and Schumacher 1984; Reed et al. 1980, 1981). The goal of these studies was to find the mean baroclinic transports, the water property distributions, and the particle velocities. Other research has sought to determine the seasonal variability of the flow and the mesoscale eddy field (Royer 1981; Tabata 1982). In recent years, considerable attention has been devoted to the study of interannual variations in the flow (Schumacher and Reed 1983; Wooster 1983), which are possibly linked with ENSO events in the tropical Pacific (Emery and Hamilton 1985; Mysak 1985). The impetus for much of this work lies in the fact that the northeast (NE) Pacific is a major habitat for many species of commercially harvested fish (e.g., pink and sockeye salmon). Evidence is accruing that interannual variability in oceanic conditions may be an important factor in the year-to-year changes in the stock recruitment and return migration routes of these fish (Hamilton 1985; Hamilton and Mysak 1986; Mysak 1986).

The circulation of the NE Pacific has also been examined in several analytical and numerical modeling studies. Thomson (1972) developed a linear frictional model of the Alaskan Stream and considered the vorticity balances along the westward path of the current. Mysak (1982) applied a barotropic instability model of flow along a trench to explain wave-like fluctuations in the Alaskan Stream off Kodiak Island. Willmott and Mysak (1980) modeled the production of an eddy that is frequently observed off Sitka, Alaska, by multiply-reflected planetary waves driven by interannual wind stress fluctuations. The generation of this same eddy was also considered by Swaters and Mysak (1985), who proposed that it develops through the interaction of a horizontally sheared baroclinic current with the local bottom topography.

A numerical study of the North Pacific circulation was recently conducted by Hsieh (1987), who used a primitive equation model with 1° X 1° resolution to examine the response of the ocean to seasonal variations in the wind stress and surface heat flux. The NE Pacific was also included as a subregion in the North Pacific circulation model of Huang (1978, 1979).
However, the resolution of his model (2.5° × 2.5°) was insufficient to resolve the Alaskan gyre. In fact, the Aleutian Island Chain, a lateral boundary of importance in establishing the gyre, is absent in this model.

A major deficiency of these numerical modeling efforts is that the solutions obtained are dominated by lateral viscosity. A large value of the eddy viscosity is required because of the relative coarseness of the grid resolution. In consequence, the solutions are unrealistic in several respects (e.g., the width of the western boundary current is far too large). The purpose of the present study is to investigate the circulation of the NE Pacific through the use of a limited-area eddy-resolving quasi-geostrophic numerical model. The fine resolution of this model allows the use of a comparatively low value for the eddy viscosity coefficient. As a consequence, spontaneous transient motions arise from instabilities of mean currents. The detailed structure of the solution obtained is generally much more realistic than those of previous models.

In Part I of this study we use here an extension of the Holland (1978) two-layer, rectangular basin ocean model. Specifically, we include an arbitrary number of vertical layers, the coastline geometry and the bottom topography of the NE Pacific. We describe the model and present results from a preliminary numerical experiment using a standard set of parameters and a steady climatological wind forcing. We use the solution to examine and quantify several features of the flow in the Gulf of Alaska and to compare these, when possible, with oceanic observations. In Part II we will present the results of additional experiments that investigate the effects of the frictional parameterization, bottom topography, and transient wind forcing upon the solution.

To obtain the resolution required for an eddy-resolved model, we have had to compromise on two accounts. It was necessary to restrict the model to quasi-geostrophic (QG) dynamics and to consider a limited-area regional model with a domain confined to the subpolar gyre of the NE Pacific. The limitations imposed by quasi-geostrophy are well known: the formal derivation of the equations rests upon the assumptions (among others) of small-amplitude topography and small interfacial displacements. In addition, in the simplified thermodynamics of the system, the density field is prescribed rather than predicted, as it would be for a primitive equation model. Although these assumptions appear to be fairly restrictive, there are, nevertheless, good reasons to be confident that such a model can give a realistic circulation. For the Gulf Stream and Kuroshio Extension regions, Schmitz and Holland (1982, 1986) have shown that a QG model is capable of accurately simulating the spatial distribution of second-order statistical properties of the flow field. In addition, the series of QG limited-area calculations reported by Holland (1986) apparently reproduce the major features of the large-scale circulation in a variety of geographic locations. Thus, it is anticipated that the QG formulation will retain the essential physics of the processes we seek to model.

In recent years there has been a rapid development of regional eddy-resolving ocean circulation models, and several of these have been very successful in obtaining realistic simulations of localized current systems (e.g., Hurlburt and Thompson 1985; Holland 1986). In these models, however, there are artificial boundaries that do not correspond to true lateral boundaries of the ocean. Two approaches are commonly taken to handle these artificial boundaries. One is to allow an open boundary and thus specify an inflow/outflow of fluid into or out of the domain. The other is to close the domain, thus allowing no flow across the boundaries, and to insert special regions of enhanced friction near the artificial boundaries in order to isolate an interior region of interest. In the present study we considered both of these approaches but settled on the latter. Our difficulties in obtaining a realistic flow field with a boundary-driven (inflow/outflow) model are briefly discussed in section 5.

For our model, two artificial boundaries are required; one is to the south near 45°N, and the other to the west near 170°W. The southern boundary is chosen to lie at the approximate latitude of the zero contour of the mean annual wind stress curl, i.e., at the approximate boundary of the subpolar gyre. Near the western boundary, a region of enhanced friction is inserted to dissipate internally generated features of the flow as they impinge upon this boundary (Barnier 1986). This “sponge” layer effectively acts to reduce the influence of the ocean lying west of 170°W on the flow in the model Gulf of Alaska.

It is useful to review briefly the various current systems that comprise the NE Pacific subpolar gyre. The sluggish and diffuse North Pacific and Subarctic Current (also known as the West Wind Drift) bifurcates as it approaches the west coast of North America into the southward flowing California Current and the northward flowing Alaska Current (see Fig. 1). The latter, also a broad, slow-moving current, forms the eastern branch of the large-scale, wind-driven, cyclonic circulation in the Gulf of Alaska. The Alaska Current eventually feeds into the Alaskan Stream, an intense southwest-flowing boundary current adjacent to the Aleutian Island Chain. Some of the water transported by the Alaskan Stream continues westward into the Bering Sea, while a portion may return to the North Pacific Current.

The plan of the paper is as follows: In section 2 the quasi-geostrophic model is described; in section 3 the parameters of the initial experiment are presented; in section 4 results of this experiment are described and analyzed; our conclusions are given in section 5.
2. Quasi-geostrophic numerical model

a. Governing equations

We now consider a model of the ocean in which the vertical variation of density is discretized into a finite number \( N \) of immiscible layers of constant density \( \rho_k \) and of thickness \( H_k \) (Fig. 2). Variations in the depth of the fluid about a mean depth \( D \) are denoted by \( h_B(x, y) \) and are contained exclusively within the lowest layer. The reduced gravity at the interfaces between layers is given by

\[
g'_{k+1/2} = g \left( \rho_{k+1} - \rho_k \right) / \rho_0,
\]

where \( g \) is the gravitational constant and \( \rho_0 \) is a reference density. A whole number subscript denotes a layer while a fractional subscript (e.g., \( k + 1/2 \)) denotes an interface. The quasi-geostrophic equations governing the conservation of vorticity and the interface displacement for a layered ocean on a beta plane have been derived before (e.g., Pedlosky 1979) and are given by

\[
\frac{\partial \nabla^2 \psi_k}{\partial t} = J(f + \nabla^2 \psi_k, \psi_k) + \frac{f_0}{H_k} (w_{k-1} - w_{k+1/2})
\]

\[
+ A_H \nabla^4 \psi_k + \delta_{1,k} \frac{\text{curl}_x \tau}{\rho_0 H_1} - \delta_{N,K} \epsilon \nabla^2 \psi_k
\]

for \( k = 1, \ldots, N \) (2.1)

\[
\frac{\partial (\psi_k - \psi_{k+1})}{\partial t} = J(\psi_k - \psi_{k+1}, \psi_{k+1}) - \frac{g'_{k+1/2}}{f_0} w_{k+1/2}
\]

for \( k = 1, \ldots, N - 1 \). (2.2)

The streamfunction is given by \( \psi_k \), the vertical interfacial velocity by \( \omega_{k+1/2} \), \( \nabla^2 \) is the horizontal Laplacian operator, and \( J(a, b) = a_x b_y - a_y b_x \) is the Jacobian operator. The Coriolis parameter is \( f = f_0 + \beta y \), \( A_H \) is the lateral viscosity coefficient, \( \epsilon \) is the linear bottom friction coefficient, and \( \delta_{i,j} \) is the Kronecker delta. The streamfunction evaluated at the interfaces is a weighted

\[
\begin{align*}
\psi_1 & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \}
average of the streamfunction from adjacent layers: 
\( \psi_{k+1} = \frac{(H_{k+1} \psi_k + H_k \psi_{k+1})}{(H_k + H_{k+1})} \).

The horizontal geostrophic velocity components within a layer are given by

\[
(u_k, v_k) = \left( -\frac{\partial \psi_k}{\partial y}, \frac{\partial \psi_k}{\partial x} \right),
\]

while the interface height perturbations (positive upwards) are given by

\[
h_{k+1/2} = \frac{f_0}{g_{k+1/2}} (\psi_{k+1} - \psi_k).
\]

Bottom topographic variations are incorporated in the model through the bottom boundary condition on the vertical velocity, viz.,

\[
w_{N+1/2} = -J(h_B, \psi_N).
\]

In addition, McWilliams (1977) showed that the constraint

\[\int \int w_{k+1/2} dx dy = 0, \text{ for } k = 1, \ldots, N - 1 \] (2.6)

is necessary to ensure that mass is conserved in each layer of a closed domain.

The upper layer of the model ocean is directly driven by the vertical component of the curl of the wind stress, \( \text{curl}_z \tau \), which is a forcing function that we will specify. The effect of the wind stress curl is to produce an Ekman pumping tendency that is equivalent to a body force acting on the upper layer.

Dissipation is included in the model through the horizontal Laplacian friction and a linear bottom friction acting on the lower layer. Although the more conventional Laplacian friction has been favored here over the biharmonic friction proposed by Holland (1978), it may nevertheless be useful to investigate in future experiments the solutions that this higher-order frictional parameterization yields.

Equations (2.1)–(2.5) may be combined to yield an expression relating changes in the potential vorticity of a fluid parcel to dissipation and the external forcing, viz.,

\[
\frac{D_k Q_k}{Dt} = A_H \nabla^4 \psi_k + \delta_{1,k} \frac{\text{curl}_z \tau}{\rho_0 H_1} - \delta_{N,k} \epsilon \nabla^2 \psi_k
\]

for \( k = 1, \ldots, N \).

The potential vorticity of a layer \( k \), \( Q_k \), is given by

\[
Q_k = \nabla^2 \psi_k + f(y) - \frac{f_0}{H_k} (h_{k-1/2} - h_{k+1/2}) + \delta_{N,k} \frac{f_0 h_B}{H_N},
\]

and

\[
\frac{D_k}{Dt} = \frac{\partial}{\partial t} + u_k \frac{\partial}{\partial x} + v_k \frac{\partial}{\partial y}
\]

is a material derivative for a layer \( k \).

b. Method of solution

Equations (2.1) and (2.2) with boundary condition (2.5) are solved using finite differences over a uniform mesh of grid points representing the Gulf of Alaska. Using (2.2) and (2.5) to eliminate the vertical velocity in (2.1), we obtain a set of \( N \)-coupled equations that can be written in matrix form as

\[
\partial_t (\nabla^2 \Psi - M \Psi) = R,
\]

where \( \Psi \) is a column vector of the layer streamfunction:

\[
\Psi^T = (\psi_1, \psi_2, \ldots, \psi_N)
\]

and \( M \) is the tridiagonal matrix of the coupling coefficients between layers. The nonzero row elements of \( M \) are of the form

\[
-\frac{f_0^2}{g_{k-1/2} H_k}, \quad \frac{f_0^2}{g_{k-1/2} H_k + g_{k+1/2} H_k}, \quad -\frac{f_0^2}{g_{k+1/2} H_k},
\]

where for mathematical convenience we take \( g'_{N+1/2} = g_{N+1/2} = \infty \). The column vector \( R \) contains the nonlinear, dissipative and forcing terms. The elements of \( R \) are given by

\[
J \left( f + \nabla^2 \psi_k + \delta_{N,k} \frac{f_0 h_B}{H_N}, \psi_k \right) + \frac{f_0^2}{g_{k-1/2} H_k}
\]

\[
\times J(\psi_{k-1} - \psi_k, \psi_{k-1/2}) - \frac{f_0^2}{g_{k+1/2} H_k} J(\psi_k - \psi_{k+1}, \psi_{k+1/2})
\]

\[
+ A_H \nabla^4 \psi_k + \delta_{1,k} \frac{\text{curl}_z \tau}{\rho_0 H_1} - \delta_{N,k} \epsilon \nabla^2 \psi_k.
\]

Following Bengtsson and Temperton (1979), the layer equations (2.7) are decoupled into a set of modal equations by diagonalizing the coupling matrix \( M \), which is written as

\[
M = P \Lambda P^{-1},
\]

where \( P \) is a matrix whose columns are the eigenvectors of \( M \). The matrix \( P \) satisfies the relation

\[
P M = \Lambda P,
\]

where \( \Lambda = \text{diag}(\lambda_1^2, \lambda_2^2, \ldots, \lambda_N^2) \) is a diagonal matrix of eigenvalues. The significance of the eigenvalues is that they are the reciprocal of the square of the deformation radii of each mode. Since the determinant of \( M \) is zero, one of the eigenvalues of \( M \) is identically zero. This eigenvalue is associated with the barotropic mode, which, in this rigid-lid model, has an infinite Rossby radius.

Substituting (2.8) into (2.7) and multiplying by \( P^{-1} \) on the left, we obtain the decoupled system

\[
\partial_t (\nabla^2 \bar{\Phi} - A \bar{\Phi}) = P^{-1}R,
\]

where \( \bar{\Phi} = P^{-1} \Psi \) is a column vector consisting of one barotropic mode, \( \phi_1 \), and \( N - 1 \) baroclinic modes, \( \phi_i \), \( i = 2, \ldots, N \).

To numerically integrate the system (2.10) forward in time we evaluate derivatives using centered space and time differences. The Jacobian operator given by
Arakawa (1966) is employed for the nonlinear terms to ensure that the space differencing conserves kinetic energy and enstrophy. The terms in $\mathbf{R}$ are evaluated at time level $n$ with the exception of the dissipation terms, which are evaluated at time level $n - 1$ to ensure numerical stability. A Robert filter (Asselin 1972) with a very weak filter coefficient ($\nu = 0.01$) is applied at each time step to suppress the development of a computational mode in the solution.

Introducing the time differencing into (2.10) and denoting time levels by superscripts, we have

$$\left(\nabla^2 - \Delta\right)\Phi^{n+1} = 2\Delta t\Phi^{n-1}\mathbf{R}^{(n,n-1)} + \left(\nabla^2 - \Delta\right)\Phi^{n-1},$$

(2.11)

where $\Delta t$ is the time step increment. Given the streamfunction fields at time levels $n - 1$ and $n$, we can evaluate the right-hand side of (2.11). To obtain the $\Phi^{n+1}$ fields we then have to solve $N$ Helmholtz equations.

These elliptic equations are solved in a domain $\Omega$, with an irregular (i.e., nonrectangular) boundary $\partial\Omega$, on which Dirichlet boundary conditions are specified. The condition of no normal flow at the side walls of the domain requires that the $\phi_i$ be constant along $\partial\Omega$. The value of the $\phi_i$ on $\partial\Omega$ is determined by the constraint (2.6). Since the interfacial vertical velocity fields are due entirely to the baroclinic modes, this constraint is met by requiring that

$$\int_0^1 \int_0^1 \phi_i dx dy = 0, \quad \text{for} \quad i = 2, \ldots, N. \quad (2.12)$$

Thus, for each baroclinic mode we adopt the method of Holland (1978) and let the solution to each equation of the form

$$\left(\nabla^2 - \lambda_i^2\right)\phi_i = f_i(x, y), \quad \text{for} \quad i = 2, \ldots, N,$$

be $\phi_i = \phi_i' + C_i(t)\varphi_i$, where $\phi_i' = 0$ on $\partial\Omega$ and the $\varphi_i$ satisfy

$$\left(\nabla^2 - \lambda_i^2\right)\varphi_i = 0,$$

$$\varphi_i = 1 \quad \text{on} \quad \partial\Omega, \quad \text{for} \quad i = 2, \ldots, N.$$

The $\varphi_i$ fields are independent of time and are determined only once at the outset of the integration. The constants $C_i(t)$ are determined from (2.12) and are given by

$$C_i(t) = \frac{\int_0^1 \int_0^1 \phi_i dx dy}{\int_0^1 \varphi_i dx dy} \quad \text{for} \quad i = 2, \ldots, N. \quad (2.13)$$

The barotropic mode, $\phi_1$, is taken, without loss of generality, to be zero on $\partial\Omega$.

The finite difference Helmholtz equations are integrated in an irregular domain using a capacitance matrix technique (Hockney 1970) in conjunction with a direct Poisson solver (Swarztrauber 1984). We have included a brief description of our capacitance matrix algorithm in the Appendix.

Equations (2.1) and (2.2) are of fourth-order and thus require the specification of not only the streamfunction, $\psi_k$, but also the relative vorticity, $\nabla^2 \psi_k$, on $\partial\Omega$. Both the Laplacian friction term and the Arakawa Jacobian require this as boundary data. This allows the relative vorticity generated at a boundary to be diffused into the interior. The boundary data on the relative vorticity depend on whether there is free-slip (zero stress) or no-slip at the boundary. In the free-slip case $\nabla^2 \psi_k = 0$ on $\partial\Omega$, while in the no-slip case the boundary vorticity evolves as part of the solution. For a no-slip wall the boundary vorticity is evaluated according to a first-order accurate method, due originally to Thom (1928) and used by Blandford (1971). At corner points the vorticity is evaluated using a variant of this method, which effectively rounds the corner (see Roache 1973, p. 170, method No. 4).

c. Model domain

The numerical integration is performed in Cartesian coordinates on a uniform mesh with a resolution of 20 km in the two horizontal coordinates. The domain, which extends up to 3200 km in the east–west direction and 1480 km in the north–south direction is shown in Fig. 3 along with contours of the bottom topography. The sidewall boundary along the continental margin was chosen to coincide as nearly as possible with the 1700 m depth contour. A midocean boundary is placed to the south at around 45°N, a latitude corresponding approximately with the southern limit of the subpolar gyre and also with the zero contour of the annual mean wind stress curl. A second midocean boundary forms a western wall at approximately 170°W.

A map projection was used to identify the grid points on the beta plane with a spherical coordinate on the earth. In this way a bottom depth value could be associated with each grid point. The Lambert equal-area projection was employed, but others, such as the Mercator projection, gave nearly identical results. The projection does introduce some distortion, although area is preserved in the transformation. A horizonal line in the domain no longer corresponds with a latitude circle. For example, the southern boundary varies from 44°N at the east and west corners to 46°N at the center. The relations for the map projection were obtained from Snyder (1982).

A weak nine-point filter (Shapiro 1970) was applied to the bottom topography to remove the small-scale bottom features, which are poorly resolved by the grid. The resulting smoothed field was employed in the numerical model. Prominent topographic features, such as the Aleutian Trench and various seamounts, are evident in the bottom contours of Fig. 3.

In the vertical direction a three-layer ($N = 3$) density structure is adopted. The layer thicknesses are: $H_1$
= 200 m, \( H_2 = 600 \) m and \( H_3 = 4200 \) m. The reduced gravities at the interfaces are \( g_{1/3} = 1.0 \times 10^{-2} \) m s\(^{-2}\) and \( g_{2/3} = 0.5 \times 10^{-2} \) m s\(^{-2}\). With these values the deformation radii of the model are 17.2 km for the first baroclinic mode and 9.6 km for the second baroclinic mode. This compares well with average values of 17.6 and 9.8 km for the deformation radii computed from hydrographic data collected in the Gulf of Alaska (Willmott and Mysak 1980).

d. Sponge layer

With the "artificial" western wall inserted near 170\(^\circ\)W, the flow there will be necessarily unrealistic, and consequently, we wish to isolate this region so that it has as little effect as possible on the flow to the east. This is achieved by placing a region of enhanced friction, called a sponge layer, adjacent to the boundary. The method of Barnier (1986) was utilized to implement the sponge layer. This requires that a frictional term, similar to the bottom friction term for the lowest layer, be introduced to the right side of (2.1) for each layer.

The sponge layer was chosen to be 16 grid intervals (320 km) in width, which is wide enough to encompass the current along the western wall. The frictional coefficients gradually increase over 8 grid intervals from their interior values (which are zero for all but the lowest layer) to the maximum value of \( 1 \times 10^{-6} \) s\(^{-1}\), which is an order of magnitude larger than the bottom friction coefficient in the interior. With these coefficients so specified, the sponge layer acts to damp the barotropic mode at the western boundary, independently of the spatial scale of the motion.

The effect of the sponge layer is to remove some of the cyclonic vorticity imparted to the fluid by the wind stress curl. Numerical experiments show that, in the absence of a sponge layer, the flow near the western wall develops intense, vertically coherent eddies. These eddies create the horizontal gradients necessary for the Laplacian friction to dissipate the excess cycloic vorticity of the fluid particles. This must occur (in a steady state) before the particles can reenter the interior flow. The sponge layer is effective in suppressing the formation of such vortices at the western boundary and thus allows the fluid that has reached the western wall to reenter the interior flow smoothly.

3. A preliminary experiment

The model discussed above requires the specification of several parameters involving lateral and bottom friction as well as wind forcing. The choice of parameters for the initial experiment is presented here. In subsequent experiments we will examine effects of variations of some of these parameters.

The wind forcing employed in the initial experiment is a steady climatological (i.e., mean annual) wind stress curl field derived from the Hellerman and Rosenstein (1983) normal monthly wind stress data (Fig. 4). Notice that this curl field has a maximum near 56\(^\circ\)N and that the zero contour has an approximately zonal orientation along 45\(^\circ\)N. The map projection discussed in section 2c was employed to identify a wind stress curl value to each point in the domain.

The coefficient for bottom friction was set to the frequently used value of \( 1 \times 10^{-7} \) s\(^{-1}\) (e.g., Holland 1978), while the value of the lateral viscosity was set to 200 m\(^2\) s\(^{-1}\). To obtain turbulent flow it is necessary to have a low value for this coefficient. A useful criterion for selecting it is to require that the frictional boundary layer be resolved by the grid. The value selected here gives a boundary layer thickness \( d_M = (A_H/\beta)^{1/3} \) of 25 km. Thus, the grid (barely) resolves this boundary layer. A previous experiment with \( A_H = 100 \) m\(^2\) s\(^{-1}\) gave a noisy relative vorticity field near the western boundary. However, with \( A_H = 200 \) m\(^2\) s\(^{-1}\) this noise is greatly reduced.
In our reference experiment the no-slip condition is adopted along the continental boundary, while the western and southern boundaries are free-slip. This seems to be a physically reasonable choice since we do not want to have any boundary-generated vorticity at the midocean boundaries, but we do expect that vorticity should diffuse into the ocean interior from the continental boundary. We have also run a parallel experiment with a free-slip coastal boundary to assess the influence of this boundary condition. Except in the boundary current region, the solution was not greatly different from that with no-slip. We comment below on the effects that the no-slip condition introduces to the boundary current but give no further description of the free-slip calculation.

4. Results

For this first experiment, the model was integrated from an initial rest state until a state of statistical equilibrium was achieved. The model was then integrated for an additional 10 years (with 1 yr = 360 days) of simulated time during which the streamfunction fields were stored on magnetic tape at 4-day intervals. The time history of the basin-integrated kinetic and potential energies per unit area for this latter period shows that these quantities fluctuate around a well-defined mean that is characteristic of the equilibrium state (Fig. 5). The highly baroclinic nature of the model response is indicated here; the potential energies of the interfaces are about 40 times the kinetic energies of the overlying layers. Note, in addition, the very weak kinetic energy associated with the lower layer.

a. Instantaneous fields

Examples of the streamfunction field from each of the three layers are shown in Figs. 6a–c. These snapshots were taken from Day 2900 (8.05 yr) of the 10-year run. Several features present in these instantaneous fields are characteristic of the entire numerical experiment. We will discuss first some features of the upper-layer flow; however, the middle layer has a very similar instantaneous pattern of motion so much of our discussion also applies to this layer. The lowest layer has an entirely different pattern of motion, and we momentarily defer discussion of it.

A cyclonic circulation is established in the upper layer of the domain. This is composed of several regions of distinct flows, some of which have a correspondence to those observed in the NE Pacific. On the eastern side of the domain, a sluggish northward flowing current that corresponds to the Alaska Current is evident.

FIG. 4. Climatological wind stress curl over the NE Pacific Ocean computed from the data of Hellerman and Rosenstein (1983). The contour interval is $0.2 \times 10^{-7}$ N m$^{-3}$.

FIG. 5. Time-series of the basin-integrated kinetic and potential energies per unit area for the 10-year duration of the experiment. The units are $10^3 \times J$ m$^{-2}$. 
The model Alaska Current meanders slowly but incessantly and occasionally breaks off closed streamline eddies. As the head of the Gulf is approached, the model Alaska Current feeds into an intense southwestward flowing boundary current, which corresponds to the Alaskan Stream shown in Fig. 1. Maximum ve-
ocilities in this current reach 60–70 cm s\(^{-1}\). The model Alaskan Stream flows continuously from the head of the Gulf out to the sponge layer and the artificial western boundary of the domain. The horizontal scale of the Alaskan Stream off Kodiak Island was found by Royer (1981) to be about 60–80 km, which is very close to the instantaneous width of the model stream.

South of the boundary current we have a region of recirculating flow of moderate intensity with particle velocities of about 10–15 cm s\(^{-1}\). In Fig. 6a we take the -9000 m\(^2\) s\(^{-1}\) contour as separating the recirculating flow from the weaker interior flow to the east. Note that this recirculating flow progressively increases the volume transport of the model Alaskan Stream with distance downstream.

In this regional model we have return flows adjacent to the artificial western and southern boundaries that are not in anyway realistic. Nevertheless, the behavior of the circulation in these regions is of some interest. Along the western wall the "sponge" layer seems effective at stabilizing the flow so that it may rejoin the interior without breaking up into eddies. This occurs in two ways: first, as a northwest flowing recirculation, and second, as a boundary current along the south wall, which eventually feeds the model Alaska Current. Adjacent to the southern boundary there is a region extending about 300–400 km where an adjustment takes place before the wind stress curl begins to drive the circulation to the northeast. This adjustment region is most likely an artifact of the presence of the southern boundary and probably would not appear in a larger model.

The circulation in the bottom layer (Fig. 6c) is in great contrast to that of the two overlying layers. The well-organized circulation pattern found above is absent; instead, the abyssal flow is very weak and consists primarily of numerous eddies of both cyclonic and anticyclonic rotation, mostly concentrated in the northern half of the domain. The scale of these eddies is clearly related to the bottom roughness. In the boundary current region, the eddy-length scale is about the width of the Aleutian trench. In the vicinity of large seamounts, the eddies assume the scale and even to some extent the shape of the seamounts. The reason for the very different circulation pattern of the lower layer lies in the scattering of the barotropic mode of motion by the topography. This will be the subject of further discussion in Part II.

Two sequences of the upper-layer streamfunction are given in Figs. 7a–c and 8a–d to illustrate the transient motions associated with eddy variability in the model. The first sequence, with an 80-day interval between snapshots, shows an episode of anticyclonic ring generation from a meander in the Alaska Current near the head of the Gulf. The meander originally appeared as a small perturbation to the southeast in the Alaska Current. On a time scale of about 250 days it grew in amplitude and moved to the northwest before pinching off into a ring on Day 1120. The ring persisted as an identifiable structure for the next 360 days. Over this time period it propagated to the west at a speed of about 1 km day\(^{-1}\) and, in the process, slowly diminished in size and intensity.

Ring generation, as illustrated above, is largely confined to the Alaska Current and, even there, occurs rather infrequently. The large amplitude meandering of the model Alaska Current often produces regions of cyclonic and anticyclonic curvature in the flow, particularly near the head of the Gulf. Over the course of the 10-year experiment, only a small number (4) of these meanders actually pinched off to form (anticyclonic) rings.

Tabata (1982) noted the repeated occurrence of an anticyclonic, baroclinic eddy located a few hundred kilometers west of Sitka, Alaska (57°N, 138°W). The Sitka Eddy typically has a diameter of 200–300 km and an average 0/1000 db transport of about 5 Sv (1 Sv = 1 \times 10^6 m^3 s\(^{-1}\)). It is a surface-intensified feature with a baroclinic structure extending to depths in excess of 1000 m. Once formed it is thought to drift westward or southwestward at speeds of 1–2 km day\(^{-1}\) and to persist for up to a year.

It is tempting at this point to identify the anticyclonic ring of Figs. 7a–c as a model Sitka Eddy; the comparison holds in some respects. The model eddy is strongly baroclinic, appears in approximately the correct location, and is of the correct rotation and horizontal scale (~200 km initially). In addition, it is a long-lived feature that moves westward in a manner consistent with the few observations. Some discrepancy does exist in the comparison: the model eddy, with a transport of about 2 Sv, is weaker than those typically observed. This may be due to the horizontal friction, which is likely too large and therefore prevents the development of eddies with sufficient intensity at the head of the Gulf. This may also influence the frequency of eddy production in the model Alaska Current.

If the suggestion given above is correct, then it would appear that the formation of the Sitka Eddy is due to a baroclinic instability of the Alaska Current. (As we discuss later, this is the dominant form of instability over interior regions of the domain.) The theory, given by Willmott and Mysak (1980), that the generation of the Sitka Eddy is dependent upon very low frequency oscillations of the wind field may well be unnecessary. Swaters and Mysak (1985) showed that the local bottom topography could have an important role in the generation of the Sitka Eddy. To assess the influence of topography in the generation of eddies in the model Alaska Current, further experimentation will be required.

Figures 8a–d are included to illustrate the characteristic variability of the model Alaskan Stream. This current is typically a parallel flow lying adjacent to the sloping boundary. However, throughout the course of the experiment, the model stream is subject to aperiodic
Fig. 7. (a)–(c) A time-sequence of the upper-layer streamfunction field at 80-day intervals beginning on Day 1100 (CI = 3000 m$^3$ s$^{-1}$).

meandering, which produces localized regions of anticyclonic curvature in the flow. These meanders are perturbations that originate near the head of the Gulf and propagate downstream at a speed of about 2 km day$^{-1}$ causing a lateral displacement of the current axis that may exceed 200 km. The sequence illustrated in Fig. 8 is a particularly large amplitude event of this type. The amplitude of these perturbations decreases
with distance downstream, and rarely do they reach the region off the Aleutian Islands where the boundary is zonal. Since only anticyclonic meanders occur and these never break off to form rings, the model stream remains a continuous, albeit laterally displaced, current.

There is evidence to suggest that the large-amplitude meandering observed in the model boundary current also occurs in the Alaskan Stream. Reed et al. (1980) and Reed and Schumacher (1984) found that the stream was subject to aperiodic, vertically coherent, lateral meandering. Some observations indicated that these perturbations propagated downstream but a phase speed was not given.

Further downstream and offshore from the zonal boundary, the separated model stream undergoes smaller amplitude (40–60 km) lateral motions. In contrast to the episodic variability found farther upstream, the meandering here is quasi-periodic and of high frequency (~0.08 cycles day⁻¹). These perturbations are not traceable back to the head of the Gulf but appear to originate locally. In contrast to these results, the boundary current in the solution with a free-slip coast was virtually unperturbed over the course of a 10-year integration.

**b. Mean fields**

Much of the following analysis depends on a Reynolds's decomposition of the streamfunction fields into mean and eddy components. These are respectively defined as

$$\bar{\psi}_k(x, y) = \frac{1}{T} \int_0^T \psi_k(x, y, t) dt$$  \hspace{1cm} (4.1)

$$\psi_k(x, y, t) = \psi_k(x, y, t) - \bar{\psi}_k,$$  \hspace{1cm} (4.2)

where $T$ is an averaging period of 10 years. Figure 9 shows the mean streamfunction fields in each layer. As is characteristic of this experiment, the mean flows in the upper two layers are generally very close in pattern but are intensified in the top layer. Some difference is found in the circulation pattern of these two layers near the northeastern boundary. The lowest layer shows only a very weak mean flow. It contains a broad eastern interior region with virtually no mean motion.

The mean gyres include a broad weak Alaska Current, which funnels near the head of the Gulf into the model Alaskan Stream. In the model Alaska Current, mean velocities in the upper layer are weak, ranging from 2 to 5 cm s⁻¹ with larger values near the head of the Gulf. In the model Alaskan Stream, mean particle velocities increase downstream from the head of the Gulf. Values range from 35 (9) cm s⁻¹ off Kodiak Island to 55 (15) cm s⁻¹ off the Aleutians Islands for the upper (middle) layer. The current measurements of Reed and Schumacher (1984) provide a point of comparison with our computed values. They obtained 10-month mean speeds of 24 (19) cm s⁻¹ at 300 (500) m depth. The model values at these depths are 22 and 9 cm s⁻¹, which suggests that the mean flow speed in the upper layer of the model is approximately correct while that of the middle layer is somewhat low.

The mean horizontal scale of the western boundary current is about 100 km off Kodiak Island and increases downstream due to recirculation of flow along its southern flank. Separation of the mean boundary current occurs as the boundary becomes zonally oriented. It is notable that the mean boundary current separates from the boundary at the point where the deep isobaths also diverge away from the boundary (Fig. 3).

The depth-integrated transport of the model gyre is about 21.1 Sv of which 11.0 Sv and 9.3 Sv are found in the top two layers, respectively. The model transport between Station P and the coast is about 5.7 Sv and is almost entirely confined to the upper two layers. This compares favourably with the mean 0/1000 db baroclinic transport across Line P of 5.3 Sv (Tabata 1983). Tabata also finds that the use of a deeper reference level does not lead to a significantly different transport, a result that is consistent with the very weak lower-layer transports observed in the model.

Adjacent to the northeastern boundary, the mean streamfunction field in the middle layer (Fig. 9b) has a weak anticyclonic circulation. Associated with this is a mean southeastward flowing current, which transports a maximum of 0.5 Sv. This feature arises, in a mean sense, from time-averaging the anticyclonic eddies, which appear continuously at the eastern margin of the basin in the middle layer (Fig. 6b). The parallel experiment with free-slip showed the same anticyclonic structure near the eastern boundary.

The model transport between Kodiak Island and 100 km offshore has a value of about 8.0 Sv and again is almost entirely confined to the upper two layers. The observed 0/1500 db mean transport is given by Reed et al. (1980) to be 11.7 Sv so that the model Alaskan Stream transport appears to be somewhat low. In addition, Reed et al. (1980) found that the use of a 3000 db reference level augmented the transport by about 5 Sv.

The deficiency in the model Alaskan Stream transport may be due to a lack of vertical resolution. In models with higher resolution (e.g., the model with eight layers presented by Holland and Schmitz 1985) the interior Sverdrup gyres diminish in scale with depth so that the deep layers contain only tightly confined inertial recirculation cells. We may expect that the incorporation of additional layers in the present model will also lead to an enhanced transport in the western boundary current through recirculation cells in the deeper layers. (The lowest layer, which is directly influenced by topography, would, as in the present experiment, have very little mean transport.) In addition, due to the absence of deep Sverdrup gyres, the circulation in these deep layers would not augment the
model Line P transport. Thus, deep inertial recirculation may account for the difference between the observed transports across Line P and those of the Alaskan Stream.

The mean potential vorticity fields are fundamental to the quasi-geostrophic model and they place an important constraint upon the fluid motion. In the absence of eddies and nonconservative effects (external forcing and dissipation), the flow must follow the mean potential vorticity contours. In the lower layer, the topographic term is the dominant vorticity term of the mean potential vorticity, \( \bar{Q}_2 \). In this case the spatial pattern of the field looks much like the topography of Fig. 3. In the upper layer, \( \bar{Q}_1 \) has a pattern that looks much like the streamfunction of Fig. 9a. The interfacial displacement term is the dominant vorticity term over much of the domain in this layer.

The mean potential vorticity field of the middle layer, \( \bar{Q}_2 \), is contoured in Fig. 10. The two interfacial displacement terms are dominant here. In several locations near the head of the Gulf, the northward gradient of \( \bar{Q}_2 \) changes sign and thus fulfills a necessary condition for the occurrence of baroclinic instability. Holland and Rhines (1980) noticed a similar feature in a region of active baroclinic instability in a two-layer box model. Another feature of Fig. 10 is the region of weak potential vorticity gradients found in the southeastern part of the domain. This is suggestive of the process of vorticity “homogenization” proposed by Rhines and Young (1982) and verified in the quasi-geostrophic model of Holland et al. (1984).

c. Eddy fields

The eddy kinetic energy (EKE) fields are shown in Figs. 11a–c. For each layer the EKE is always largest in the model Alaskan Stream region and much weaker over the interior of the domain. The transition from the quiescent interior to the eddy-intense regions of the Stream is gradual near the head of the Gulf but very sharp further downstream. Within the Alaskan Stream the EKE field has a maximum off the Aleutians Islands that is associated with high-frequency lateral meandering of the current. This maximum is absent in the experiment with a free-slip coastal boundary. The EKE field at the head of the Gulf, away from the western boundary, reflects the eddy energy associated with low frequency meandering of the Alaskan Current.

The ratio of the EKE to the mean kinetic energy, KE, is a measure of the strength of the variability of the flow relative to the mean. In the lowest layer, with its very weak mean flows, the variability dominates everywhere over the mean, with the EKE/KE ratio ranging from 10–50. For the upper layer of the model Alaska Current, this ratio is about 2 and increases to about 3 in the vicinity of the Sitka Eddy region. In the recirculation region farther to the west, the variability is small and the ratio is typically only 0.1–0.2. The western boundary current region is also characterized by comparatively small variability in the flow. Downstream, in the separated current off the Aleutian Islands, the ratio is only about 0.1. Further upstream, where the current is bounded by a sideline, the EKE/KE ratio is about 0.2.

These results are, for the most part, consistent with the observations of Reed and Schumacher (1984). They obtained an EKE/KE ratio of 0.4 in the upper layers of the Alaskan Stream off Kodiak Island, while the model gives a value of 0.2. The magnitude of the observed EKE at 300 m depth was about 80 cm² s⁻², which, as Reed and Schumacher suggest, is quite low in comparison to other western boundary currents. Measurements taken in comparable locations in the Gulf Stream or the Kuroshio Extension indicate both a larger EKE/KE ratio and higher levels of eddy energy. The model gives a EKE of 55 cm² s⁻² at 300 m depth, which indicates that the level of variability, particularly in the middle layer, is somewhat underestimated.

A different view of the eddy variability is obtained from maps of the interfacial eddy potential energy (EPE) as shown in Figs. 12a and 12b. The predominance of eddy fluctuations in the boundary current region is again evident here; however, the contrast with the interior regions is not as pronounced as for the EKE. While the eddy potential and kinetic energies are roughly comparable in the boundary current (an EPE/EKE ratio of 2–3), the EPE is much greater over a large expanse of the interior. This is consistent with our analyses in section 4e, which indicate that baroclinic processes are dominant in accounting for the variability of the model Alaska Current.

Emery (1983) computed the eddy potential energy from temperature variations at 300 m depth over the entire North Pacific. He obtained very large values \[ O(1500 \, \text{cm}^2 \, \text{s}^{-2}) \] in the Gulf of Alaska, which were comparable to those of the Kuroshio Extension. This result was considered implausible and was attributed to having used temperature rather than salinity (the primary determinant of density in the Gulf) to estimate the EPE. The results presented here tend to confirm this assessment.

d. Space–time (Hovmöller) diagrams

Hovmöller diagrams of the upper-layer streamfunction from two sections of the domain (see Fig. 17) are shown in Figs. 13a and 13b. The first is a \( y \) versus \( t \) diagram taken along the N–S line \( x = 360 \, \text{km} \), which cuts diagonally across the Alaska Current. It shows northward phase propagation at a rate of 0.5 km day⁻¹.

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Fig. 8. (a)–(d) A time-sequence of the upper-layer streamfunction field at 40-day intervals beginning on Day 2100 (CI = 3000 m² s⁻¹).
for the northern half of the section ($y > 200$ km). This occurs intermittently; not every year-long record shows northward phase propagation in the Alaska Current. The second section begins near the head of the Gulf at $x = -20$ km and follows along the curving western boundary out to $x = -1040$ km. This diagram allows

FIG. 9. The mean streamfunction fields for (a) the upper layer ($\Gamma = 3000$ m$^2$ s$^{-1}$), (b) the middle layer ($\Gamma = 1000$ m$^2$ s$^{-1}$), and (c) the bottom layer ($\Gamma = 25$ m$^2$ s$^{-1}$).
us to examine phase propagation along the path of the Alaskan Stream in the upper layer. We find that the phase speed increases from about 1 km day$^{-1}$ in the upstream region to about 13 km day$^{-1}$ farther downstream. The change is fairly abrupt and occurs about 400 km along the section. The adjoining boundary is still sloping at this point and is not yet zonally oriented. Notice, in addition, that the fluctuations are of much higher frequency past this point. The lower layer (not shown) shows similar high frequency fluctuations and phase propagation in the downstream region. In the next section we employ cross-spectral analysis to interpret this high-frequency signal in terms of wave modes.

**e. Spectral analysis**

To further examine the temporal variability of the model, spectra and coherences of the streamfunction fluctuations were computed at a number of points over the domain. Selected spectra, plotted in variance-preserving form, are shown in Figs. 14a–c. In comparing these various spectra, a striking feature is the spatially inhomogeneous nature of the variability. Over interior regions in each of the three layers, the spectra are consistently “red”; i.e., low-frequency motions with periods in excess of 300 days contain most of the energy (e.g., Fig. 14a). These spectra are markedly different from those given by Holland (1978), which show a well-defined peak at a 64-day period with very little energy at lower frequencies. The reason once again is the inclusion of topography in our model.

The situation changes considerably in the boundary current region. Higher frequency motions become progressively more energetic with distance downstream. Off Kodiak Island the upper-layer spectra are still largely red, but the “mesoscale” frequency band (50–100 day period) is much more energetic than in the interior. Further downstream, particularly after the stream has separated, the spectra display a high frequency, surface-intensified signal that is due to topographic waves propagating westward along the steep shelf of the Aleutian Trench. This signal is the dominant one very close to the sidewall boundary. Further offshore (say 60 km) the energy is divided between low- and high-frequency bands. The sampling period of four days is too long to properly resolve the high-frequency signal in the Alaskan Stream. To better define this signal, we extended the 10-year experiment for 120 days and sampled at a 1-day interval. Spectra computed from this short run showed a single well-defined peak at 0.07–0.08 cpd, which falls off sharply at both higher and lower frequencies.

Inspection of the spectra from the free-slip experiment showed that the high-frequency signal, though still present, was reduced in amplitude by a factor of 5 to 10 compared to the reference experiment. This indicates the importance of the production of anticyclonic vorticity by the sidewall boundary condition for the generation of topographic waves in the Alaskan Stream.

The modal transformation discussed in section 2b was applied to the layer streamfunction fluctuations, and spectra of the resulting modal fluctuations were computed to determine the partition of energy among the three vertical modes. The results showed that the first baroclinic mode was dominant over the entire domain. For interior regions, the ratio of the spectral values of the first baroclinic mode spectra to those of the barotropic and second baroclinic mode spectra was about 10:1 and 5:1 respectively. In the Alaskan Stream the most energetic fluctuations are again in the first mode, but the relative importance of the other modes is augmented by a factor of 2 to 3. It is conceivable that the importance of the second baroclinic mode in this calculation is limited by the horizontal resolution.

Coherences were calculated between the streamfunction fluctuations at points over the domain that
are separated either vertically or horizontally. The vertical coherences discussed below are between fluctuations of the upper and middle layers. In all cases very similar results are obtained from coherence calculations that involve the lower layer with those above. Figs. 15a and 15b show the coherence and phase between fluctuations in the top and middle layers at two representative locations, one in the interior and the other in

Fig. 11. The eddy kinetic energy fields for (a) the upper layer (CI = 10 cm$^2$ s$^{-2}$), (b) the middle layer (CI = 0.5 cm$^2$ s$^{-2}$), and (c) the bottom layer (CI = 0.05 cm$^2$ s$^{-2}$).
the boundary current (corresponding to Figs. 14a and 14b). At the interior point, we see that the fluctuations are coherent at the 99% confidence level over almost the entire range of frequencies. There is a characteristic drop in the coherence at about 0.02 cpd. Below this frequency the lower layer fluctuations have a significant phase lead over those of the upper layer, while at higher frequencies there is essentially no phase difference. The phase lead with depth suggests that the low-frequency fluctuations are the result of a baroclinic instability process. This may be substantiated by examining the signs of the rates of conversion of mean kinetic and mean potential energies to eddy kinetic and eddy potential energies (see Holland 1978 for details and formulae). These two types of energy conversion are the signature of barotropic and baroclinic instability processes respectively. Away from the boundary current region the dominant energy conversion is from mean potential to eddy potential energy. Within the boundary current the conversion rate of mean kinetic to eddy kinetic energy is comparable to that of mean potential to eddy potential energy.

Figure 15b indicates that the fluctuations of the upper and middle layers in the model Alaskan Stream are coherent over the entire resolved spectrum. The phase shows that the lower-layer fluctuations slightly lead those of the upper layer for all but the lowest frequencies. The current measurements obtained by Reed and Schumacher (1984) also indicated a high degree of coherence at points separated vertically by several hundred metres in the axis of the Alaskan Stream.

Additional coherence and phase plots are shown in Figs. 16a and 16b for points in the western boundary current separated by 40 km in the cross-stream direction and 60 km in the downstream direction. The cross-stream diagram shows that fluctuations are laterally coherent over a wide frequency range with the offshore fluctuations leading those inshore. The model of barotropic instability that Mysak (1982) applied to the Alaskan Stream predicted a similar phase relation in
the cross-stream direction. However, taken together, Figs. 15b and 16a suggest that both baroclinic and barotropic instability processes are important in the region. Over the length of the Alaskan Stream, the energy conversion due to both of these processes is important.

The coherence and phase for two points separated by 60 km in the model Alaskan Stream off the Aleutian Islands are given in Fig. 16b. As in the previous figures, the fluctuations are coherent across the entire band of frequencies; however, in this case the phase lag of the downstream point increases in a nearly linear fashion with increasing frequency. Other phase diagrams computed for points separated by 120 km (not shown) also indicate a linear increase in phase lag with frequency but at double the rate of Fig. 16b. The simplest interpretation of this phenomena is that the fluctuations are due to topographic waves propagating in the downstream direction of the Alaskan Stream. It is possible to estimate the wavelength of the fluctuations from Fig. 16b. At 0.08 cycles day$^{-1}$ the downstream phase lag is about 135°. For two points separated by 60 km, this implies a wavelength of 160 km, which corresponds quite well with a direct estimate of the wavelength of the fluctuations from the eddy fields. The separation between successive highs in the eddies in the down-
stream region of the Alaskan Stream is about 8 grid intervals, or 160 km. Given a frequency of \( f = 0.08 \) cpd and a wavelength of \( L = 160 \) km, the phase speed, \( c = f \times L \), is 12.8 km day\(^{-1}\). This is in excellent agreement with the 13 km day\(^{-1}\) phase speed estimate of section 4d. It is notable that the maximally unstable wave obtained in Mysak's (1982) highly idealized barotropic model of the Alaskan Stream also had similar properties, with a wavelength of 138 km and a phase speed of 10.4 km day\(^{-1}\).

Direct measurements from the downstream region of the Alaskan Stream are not yet available for comparison with the results indicated here. However, recently analyzed satellite IR images (Royer, pers. comm., 1987) show that the Alaskan Stream in this region is subject to quasi-periodic, cusp-shaped, lateral perturbations with a wavelength of about 150 km. Based upon the model results described above, we conjecture that these disturbances in the Stream are due to topographic waves.
\[ \frac{\partial \psi_k}{\partial t} = \text{PADV} + \text{MADV} + \text{EADV} + \text{STR}_{k-\nu} \\
- \text{STR}_{k+\nu} + \text{DISS} + \text{BFRIC} + \text{CURL} + \text{TOPO}. \]

(4.3)

The terms in this equation have the following definitions:

\[ \text{PADV} = J(f, \overline{\psi}_k), \]

\[ \text{MADV} = J(\nabla^2 \overline{\psi}_k, \overline{\psi}_k), \]

\[ \text{EADV} = J(\nabla^2 \psi_k, \psi_k), \]

\[ \text{STR}_{k-\nu} = \frac{f_0^2}{g'_{k-\nu} H_k} [J(\psi_{k-1} - \psi_k, \psi_{k-\nu}) \]

\[ + J(\psi'_{k-1} - \psi_k, \psi'_{k-\nu})], \]

\[ \text{STR}_{k+\nu} = \frac{f_0^2}{g'_{k+\nu} H_k} [J(\psi_k - \psi_{k+1}, \psi_{k+\nu}) \]

\[ + J(\psi'_k - \psi_{k+1}, \psi'_{k+\nu})], \]

\[ \text{DISS} = A_H \nabla^2 \overline{\psi}_k, \]

\[ \text{BFRIC} = \delta_{N,k} \epsilon \nabla^2 \overline{\psi}_k, \]

\[ \text{CURL} = \frac{\text{curl}_T \tau}{\rho_0 H_1}, \]

\[ \text{TOPO} = \delta_{N,k} J \left( \frac{f_0 h_B}{H_N}, \overline{\psi}_k \right). \]

PADV is the mean advection of planetary vorticity tendency, MADV the mean advection of mean relative vorticity tendency, and EADV the eddy advection of eddy relative vorticity tendency. The two stretching terms STR_{k-\nu} and STR_{k+\nu} include contributions of mean and eddy vortex stretching tendency for the interface above and below the layer, respectively. DISS represents the lateral diffusion of mean vorticity tendency, while BFRIC gives the mean contribution from the bottom friction. The last two terms, CURL and TOPO, give the contribution to the vorticity balances from the external forcing and the topographic vortex stretching tendency. In a statistical steady state, the term on the left side of (4.3) should vanish provided that the time-averaging is done over a sufficiently long period.

The terms in (4.3) were integrated over five subregions, which were considered to be dynamically distinct. These are shown in Fig. 17; notice that the sponge layer and a strip along the southern boundary are omitted. There are two distinct western boundary current subregions, a southwestern recirculation subregion, and two interior subregions adjacent to the eastern boundary. The boundary current region was partitioned into two halves to check whether different dynamical balances hold in the separated Alaskan Stream compared to the wall-bounded Stream. The interior

FIG. 15. Coherence squared and phase between streamfunction fluctuations in the upper and middle layers at locations in (a) the Alaska Current (Point X, Fig. 17), and (b) the Alaskan Stream (Point Y, Fig. 17). Positive phase implies that the middle-layer fluctuations are leading. The dashed horizontal line indicates the 99% significance level for coherence.

f. Vorticity budgets

As a further diagnostic of the model response we have integrated the various terms in the vorticity equation over subregions of the model domain. This allows us to establish the dominant vorticity balances and to examine the importance of mean and eddy vorticity terms in these subregions. The analysis we conducted follows closely that of Harrison and Holland (1981). Separating the streamfunction into mean and eddy components according to (4.1), then substituting this into (2.1) and time-averaging, we obtain the following mean vorticity equation for layer \( k \):
in Fig. 17 were chosen on the basis of the different flow regimes suggested by the mean streamfunction. However, as we discovered, the western boundary current has a complex structure, with some terms changing sign over the width of the current. Thus the boundary layer current can contain several sublayers with different dynamical balances. With this caveat in mind we present vorticity budgets for both the depth-integrated flow and the upper and middle layers of the model.

A 10-year period (e.g., as employed by Harrison and Holland) was used for the time-averaging in (4.3). In the following tables, the effect of a small residual time-derivative term will be noticed. (In the absence of a residual, the terms in each of the columns of Tables 1 and 2 would sum to zero.) For all the cases discussed, this residual is much smaller than the dominant vorticity terms.

The most readily interpreted budgets are the depth-integrated ones in which the vortex stretching terms exactly cancel. The resulting budgets are summarized in Table 1, where the various terms such as PADV now represent a summation over the three layers. In both interior subregions the Sverdrup balance between planetary advection and the wind stress curl is obtained, although the advective terms also make a modest contribution in subregion INT2 (17% of PADV). In the RCIRC subregion the dominant balance is again between planetary advection and the wind curl. The MADV term also makes a significant contribution (46% of PADV), most of which derives from large negative values near the BC2 subregion.

In the upstream boundary current subregion, BC1, the main balance is between southward advection of planetary vorticity and the diffusion of mean vorticity. This Munk-type frictional boundary-layer balance for the model Alaskan Stream does not entirely apply, however, because the inertial terms are significant. The mean and eddy advection terms are opposite sign, and both have a magnitude of about half of PADV.

Inertial effects are also likely to be important in setting the lateral scale of the boundary current. The frictional boundary layer theory predicts an O(25 km) lateral scale, which is much smaller than the O(100 km) scale obtained. The topography, in particular the width of the Aleutian Trench, may also be important in establishing this length scale.

In subregion BC2, the model Alaskan Stream has separated from the boundary and dissipation is much less important. There is no clear balance between two terms here; however, we note that the advective terms (especially EADV) have much greater relative importance and that PADV is much smaller because the mean flow is nearly zonal in this subregion.

The upper- and middle-layer vorticity budgets given in Tables 2 and 3 indicate that simple two-term balances are obtained only in one or two of the subregions for the middle layer. In the upper layer of subregion INT1, the CURL term is balanced by PADV and

region to the east was also partitioned into two for similar reasons.

As Harrison and Holland (1981) have discussed, the choice of subregions is sometimes a difficult matter. One problem that can arise is that, over a subregion, a term may vary in value from a large positive to a large negative, so that cancellation results from integration and a small net value is obtained. This may then leave the erroneous impression that the term is unimportant over the subregion. The subregions given

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Fig. 16. Coherence squared and phase between upper-layer streamfunction fluctuations at locations in the Alaskan Stream. In (a) the points are separated by 56 km in the cross-stream direction and are located off Kodiak Island (Point Y, Fig. 17). Positive phase implies that the offshore point is leading. In (b) the points are separated by 60 km in the downstream direction and are located off the Aleutian Islands (Point Z, Fig. 17). Positive phase implies that the downstream point is leading.
STR_{1+1/8}. This implies that the wind curl drives the fluid northward in the top layer and deforms the upper interface. This then induces northward advection in the middle layer. The balance in subregion INT1 for the middle layer is essentially the classical vorticity balance between vortex stretching and planetary advection (see Table 3). Harrison and Holland (1981) obtained a similar result for the interior of the lower layer of a two-layer model.

The balance in the RCIRC subregion in the middle layer resembles that of subregion INT1. Vortex stretching and planetary advection are again the important terms of this layer. In the upper layer there is an additional contribution from MADV to balance the CURL term.

The balances for the upper and middle layers in subregion INT2 are much more complex. Dissipation is important in both layers—but of opposite sign in each. This led to a virtual cancellation in the depth-integrated budget and the illusory impression that dissipation was unimportant in INT2. A closer examination of the dissipation term revealed that its importance in INT2 derives from large values adjacent to the eastern wall. We also note that the small value for the STR_{1+1/8} term is due to a partial cancellation of the mean and eddy components. Thus, these budgets suggest that all the vorticity terms contribute significantly near the head of the gulf.

For the upper-layer BC1 subregion, DISS is balanced by PADV and STR_{1+1/8}. The middle layer displays the classical balance between vortex stretching and planetary advection. The budgets for the BC2 upper- and middle-layer subregions are as difficult to sort out as the depth-integrated budgets in which no simple balances are apparent.

We will not discuss the vorticity balances in the lower layer because the residual time derivative term for this layer is comparable to the largest terms. It appears that an averaging period of ten years is insufficient to obtain reliable statistics in a deep layer with topography. However, we note that all the vorticity terms in this layer are very small in comparison with the important terms of the overlying layers. In addition, it is apparent that the topographic vortex-stretching tendency is an important component of the vorticity balances in this layer.

Thomson (1972) developed a depth-integrated, frictional model for the Alaskan Stream in which the dif-

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**Table 1. Depth-integrated vorticity budgets. The units of the spatially integrated vorticity tendency terms are m^3 s^{-2}.**

<table>
<thead>
<tr>
<th>Vorticity terms</th>
<th>INT1</th>
<th>INT2</th>
<th>RCIRC</th>
<th>BC1</th>
<th>BC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PADV</td>
<td>-71.4</td>
<td>-43.8</td>
<td>57.5</td>
<td>40.6</td>
<td>9.2</td>
</tr>
<tr>
<td>MADV</td>
<td>-1.1</td>
<td>-5.0</td>
<td>-31.1</td>
<td>18.4</td>
<td>-4.9</td>
</tr>
<tr>
<td>EADV</td>
<td>-0.4</td>
<td>-2.6</td>
<td>4.7</td>
<td>-20.3</td>
<td>-9.8</td>
</tr>
<tr>
<td>DISS</td>
<td>-1.0</td>
<td>-1.6</td>
<td>5.6</td>
<td>-49.0</td>
<td>-3.9</td>
</tr>
<tr>
<td>CURL</td>
<td>73.9</td>
<td>49.8</td>
<td>82.5</td>
<td>11.1</td>
<td>10.1</td>
</tr>
<tr>
<td>BFRC</td>
<td>0.0</td>
<td>0.2</td>
<td>-0.1</td>
<td>0.2</td>
<td>0.0</td>
</tr>
<tr>
<td>TOPO</td>
<td>0.0</td>
<td>2.8</td>
<td>-4.3</td>
<td>0.4</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

---

**Table 2. Upper layer vorticity budgets. The units of the spatially integrated vorticity tendency terms are m^3 s^{-2}.**

<table>
<thead>
<tr>
<th>Vorticity terms</th>
<th>INT1</th>
<th>INT2</th>
<th>RCIRC</th>
<th>BC1</th>
<th>BC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>PADV</td>
<td>-39.5</td>
<td>-25.6</td>
<td>-28.1</td>
<td>22.9</td>
<td>5.0</td>
</tr>
<tr>
<td>MADV</td>
<td>-0.8</td>
<td>-4.4</td>
<td>-23.7</td>
<td>10.8</td>
<td>-3.9</td>
</tr>
<tr>
<td>EADV</td>
<td>-0.3</td>
<td>-2.9</td>
<td>3.5</td>
<td>-18.7</td>
<td>-9.2</td>
</tr>
<tr>
<td>DISS</td>
<td>-30.0</td>
<td>-4.2</td>
<td>-34.0</td>
<td>22.4</td>
<td>7.2</td>
</tr>
<tr>
<td>STR_{1+1/8}</td>
<td>-5.9</td>
<td>-15.8</td>
<td>3.1</td>
<td>-48.4</td>
<td>-10.1</td>
</tr>
<tr>
<td>CURL</td>
<td>73.9</td>
<td>49.8</td>
<td>82.5</td>
<td>11.1</td>
<td>10.1</td>
</tr>
</tbody>
</table>
fusion of anticyclonic vorticity from the coast balances the gain of anticyclonic (planetary) vorticity from the southward movement of water parcels. He suggested that, with the increasingly zonal orientation of the boundary, planetary advection would, at some point, be incapable of balancing dissipation and that the frictional boundary layer flow would break down. He anticipated that the stream would then separate from the boundary and that other terms in the vorticity equation (e.g., the inertial terms) would become more important.

There are some similarities between this model and our vorticity budgets. The primary balance in the wall-bounded portion of the depth-integrated model Alaskan Stream is between planetary advection and dissipation, as in Thomson’s model. However, in contrast to his model, the advective terms are not negligible over the width of the boundary current. We also find that, in a mean sense, the current separates from the boundary once it is zonally oriented and that the contribution of the dissipation term to the vorticity budget is much reduced there. The inclusion of a no-slip boundary condition at the sidewall is essential to obtain separation of the boundary current in our model.

5. Conclusions

The results presented above show that the limited-area OG model driven by steady winds reproduces many of the observed features of the Alaskan gyre. Specifically, the near-surface signature of the major current systems and their associated variability are generally well reproduced. The paucity of oceanic measurements from the NE Pacific region prevents an extensive intercomparison with data necessary to validate the model. It is apparent, however, that the model is deficient in certain respects. The volume transport of the model Alaskan Stream is probably too low, which may be due to a lack of vertical resolution. In addition, there is some indication that variability in the model boundary current may be somewhat smaller than what is found from current meter data.

As indicated earlier, several additional experiments are currently in progress that will yield insights into the influence of bottom topography and transient forcing on the solution. Since seasonal fluctuations in wind stress are very substantial over the Gulf, a natural extension of the present work is to determine the response of the gyre to a seasonal cycle superimposed upon the mean wind field.

Our model illustrates the possibility of obtaining a reasonable simulation by isolating a limited area of the ocean and driving the region with only the local wind stress curl. As mentioned in the introduction we have also conducted some experiments with our model using open boundaries. For these experiments the wind forcing was turned off and a broad inflow port was placed along the southern boundary. The outflow was at a narrow port crossing the Aleutian Trench at the western boundary. The results obtained from this configuration were found to be dependent on the specified boundary conditions. For example, with a barotropic inflow specified, the interior flow was locked to the topography. On the other hand, with a surface-intensified baroclinic inflow current, the upper-layer response consisted of a flow following the planetary vorticity (beta) contours along the southern boundary out to the western wall. There was very little penetration of the upper- or middle-layer inflows into the interior. These experiments indicated that it was important to include the local wind stress curl field to drive the flow to the north across the planetary vorticity gradient.

At this point we elected to specify the wind stress curl forcing and to omit the specification of the poorly known inflow/outflow boundary conditions. While this approach proved adequate, it may be more promising in the long term to use the method that Holland (1987) applied to the California Current region. For the NE Pacific this method would involve embedding a fine-resolution model of the Gulf region within a coarser-resolution one of the North Pacific. Both models are driven by the wind stress curl, and the coarse-resolution model provides the open boundary conditions required by the fine-resolution one.

Acknowledgments. We have benefited from discussions with Drs. A. F. Bennett, W. R. Holland, and G. Holloway on this work, and we are grateful for financial support from the Canadian Natural Sciences and Engineering Research Council (NSERC) and the U.S. Office of Naval Research. During the latter stages of this work L.A.M. was supported by an Atmospheric Environment Service/NSERC Industrial Research Chair in Climatology.

APPENDIX

Capacitance Matrix Algorithm

The solution to a finite difference Helmholtz or Poisson equation can be obtained through iterative methods such as relaxation, or through direct methods such as the FACR(1) algorithm (Swarztrauber 1984). The direct methods are usually preferred, since they
are much more efficient than iterative methods and do not require that a convergence criterion be specified. However, they are not as general as iterative methods; in particular, they are most frequently constructed to solve the Helmholtz equation in rectangular domains. The capacitance matrix method (Hockney 1970) is a technique for extending the usefulness of direct solvers to nonrectangular domains. The major computational burden of the method is that it requires that the direct solver be called twice in a program to obtain the solution. In addition, a large capacitance matrix must be stored. Nevertheless, this is usually much more efficient than resorting to an iterative solver. A simple algorithm is presented here for the application of the technique to the Helmholtz equation with Dirichlet boundary conditions. We first present a formulation of the method in a continuous domain, followed by a brief discussion of its implementation in the finite difference context.

We wish to obtain a field, \( \phi(x, y) \), which satisfies

\[
(\nabla^2 - \lambda^2)\phi = \theta(x, y) \quad (A1)
\]

in a domain \( \Omega \) with the boundary condition \( \phi = \phi_B \) on \( \partial\Omega \), the contour bounding \( \Omega \) (Fig. 18). Although \( \phi_B \) is constant along \( \partial\Omega \) for a closed domain QG model, this is not in general required to apply the technique given below.

Let \( \Omega \) be embedded in a rectangular domain \( \Omega_1 \), with boundary \( \partial\Omega_1 \). Some portion of the contours \( \partial\Omega \) and \( \partial\Omega_1 \) may coincide, as is the case in Fig. 18, but this is not necessary. The portion of the \( \partial\Omega \) contour which does not coincide with \( \partial\Omega_1 \) is denoted by \( \partial\Omega' \); the area between \( \partial\Omega \) and \( \partial\Omega_1 \) is denoted by \( \Omega' \). Also let \( \mathbf{s} \) be a coordinate along the contour \( \partial\Omega' \).

We obtain first a field, \( \phi_1 \), that satisfies

\[
(\nabla^2 - \lambda^2)\phi_1 = \theta_1 \quad (A2)
\]

in the regular domain \( \Omega_1 \). The boundary condition is \( \phi_1 = \phi_B \) on \( \partial\Omega_1 \); \( \theta_1 = \theta \) in \( \Omega \) but is arbitrary in \( \Omega' \) and on \( \partial\Omega' \) and may be taken as zero there. Formally we may write the solution to (A2) as

\[
\phi_1 = \int_{\partial\Omega} G(x, y; x', y')\theta_1(x', y')dx'dy' + \int_{\partial\Omega_1} \phi_1 \frac{\partial G}{\partial n} dl'. \quad (A3)
\]

The Green’s function, \( G \), satisfies

\[
(\nabla^2 - \lambda^2)G = \delta(x-x')\delta(y-y') \quad (A4)
\]

in the domain \( \Omega_1 \) where \( \delta \) is the Dirac delta function and \( G = 0 \) on \( \partial\Omega_1 \).

The function \( \phi_1 \) will not in general have \( \phi_1 = \phi_B \) on \( \partial\Omega' \). The essence of the capacitance matrix method is to modify \( \theta_1 \), the right hand side of (A2), on \( \partial\Omega' \) so that the solution to the Helmholtz operator in \( \Omega_1 \) has the value \( \phi_B \) on \( \partial\Omega' \) and hence is our desired solution in \( \Omega \). To make this more explicit, let the function modifying \( \theta_1 \) be denoted by \( \Theta(\mathbf{s}) \) where \( \Theta \) is nonzero only on \( \partial\Omega' \). Now consider a function \( \mu(x, y) \) that satisfies

\[
(\nabla^2 - \lambda^2)\mu = \theta_1 + \Theta \quad (A5)
\]

in \( \Omega_1 \) with \( \mu = \phi_B \) on \( \partial\Omega_1 \). If \( \Theta(\mathbf{s}) \) is chosen such that the solution to (A5) has \( \mu = \phi_B \) on \( \partial\Omega_1 \), then \( \phi(x, y) = \mu(x, y) \) in the domain \( \Omega \), and the solution to (A1) is found.

To determine \( \Theta(\mathbf{s}) \) we use (A3) and (A4) to write the solution to (A5) as

\[
\mu(x, y) = \phi_1 + \int_{\partial\Omega'} \Theta(\mathbf{s}')G(x, y; \mathbf{s}')d\mathbf{s}'. \quad (A6)
\]

We now require that \( \mu = \phi_B \) on \( \partial\Omega' \) so that (A6) reduces to

\[
\phi_B = \phi_1(\mathbf{s}) + \int_{\partial\Omega'} \Theta(\mathbf{s}')G(x, y; \mathbf{s}')d\mathbf{s}', \quad (A7)
\]

which is an integral equation that determines \( \Theta(\mathbf{s}) \).

It is straightforward to apply this technique to a finite difference form of (A1) on a uniform mesh. The curve \( \partial\Omega' \) passes through a set of grid points referred to as the irregular boundary points. The numerical algorithm first requires that a direct solver be applied to obtain the \( \phi_1 \) field of (A2), in the rectangle, given the forcing field \( \theta_1 \). Next the modifying function \( \Theta \) is obtained from the values of \( \phi_1 \) along \( \partial\Omega' \) by using a discretized version of (A7). The direct solver is employed a second time to solve (A5) and the required solution is obtained.
For a finite difference mesh, (A7) can be written in matrix form as

$$\phi_B = \phi_I + \Delta s^2 G \cdot \Theta,$$

(A8)

where $\Delta s$ is the grid interval, and $\Theta$, $\phi_I$, and $\phi_B$ are now column vectors of length $M$, where $M$ is the number of irregular boundary points. The elements of $\Theta$ give the correction to the forcing at the irregular boundary points, those of the vector $\phi_I$ give the value of the $\phi_I$ field at these points, and those of $\phi_B$ give the boundary values, also at the same points.

The finite difference Green’s function, $G$, is an $M \times M$ matrix. It is determined in the following manner. A delta function source of strength $\Delta s^{-2}$ is placed at one irregular grid point and the solution to (A4) is obtained in the rectangle to yield the response at (all) the irregular boundary points. The determined response forms one row of $G$; successive rows are obtained in the same way by moving the location of the unit source along the irregular boundary, until the matrix is filled. Glaser (1970) showed numerically that the finite difference Green’s function for the Helmholz equation is a convergent approximation as $\Delta s \to 0$.

By rearranging (A8), the vector $\Theta$ is found from

$$\Theta = \Delta s^{-2} C (\phi_B - \phi_I),$$

(A9)

where $C = G^{-1}$ is the capacitance matrix. Provided that the geometry and the Helmholz constant $\lambda^2$ do not change, the capacitance matrix has to be determined only once, at the outset of the integration. For a 4G model with three layers it is necessary to construct and store these three matrices, one for each mode.

REFERENCES


