Spectra of Deep Vertical Temperature Profiles

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ABSTRACT

Repeated temperature profiles were obtained at a slowly sinking and rising free float in a depth range of 1300 to 3600 m at 10°24'N, 50°22'W during the period 15–17 October 1967. Temperature deviations from a linear gradient had spectra with approximately a -2 power dependence on wavenumber over a range of 5 m to a few hundred meters; a $-5/3$ power dependency is not excluded. No significant coherence was obtained between successive profiles even though some were scanned a few hours later at the same location. Spectra of the ratio of temperature deviation from a linear gradient to the average temperature gradient over different depth intervals at this as well as two other distant locations are similar in slope and in magnitude. The standard deviations of temperature from a linear profile were found to be comparable with the deviations measured at free hovering floats at three locations.

1. Introduction

The vertical temperature profiles obtained by means of free falling and rising floats were analyzed. Temperatures obtained in this way are of interest because they are uncontaminated by the motions in the horizontal and vertical directions present at cabled sensors. Furthermore, many of the observations were obtained in a series of drops made within a mile or so of each other, making it possible to compare successive profiles at the same location. Although most of the profiles were made at vertical velocities near 1 kt, some were made at slower speeds with the slowest being made at a velocity near 6 cm sec⁻¹. Any pronounced effect of the velocity of measurement should be revealed. The analyses led to some surmises on the nature of temperature variations over vertical wavelengths of a few hundred meters and less.

2. Measurements

Most of the measurements to be described were made at the location 10°24'N, 50°22'W but a few profiles obtained far from this location also are included for comparisons. Some details of the procedures as well as descriptions of the profiles for the first location have been described (Pochapsky, 1969). Present attention is directed toward the results of spectrum analyses made over sections of profiles which are relatively straight; thus, they do not include regions of large curvature such as the knee below the thermocline or the sharp gradients in the AABW flow. Sections were picked to be near 1 km long whenever reasonable. Analyses proceeded with the attitude that typical overall shapes of temperature profiles are constant features and that

temporal perturbations are probably most successfully extracted if attention is paid to wavelengths of a few hundred meters or less in waters below the thermocline.

The temperature sensor was a single crystal thermistor of 5-sec time constant as installed, with the data being transmitted to a listening ship by acoustic means. Temperature determinations were made every 20 sec and recording was done to a least count of 0.0008°C at the 10N location but only to 0.004°C at the other two locations. Shifts in calibration with time were small enough not to affect the spectra determined. Listening conditions usually allowed measurements to better than ± 1 least count but double this error could occur in heavy seas. Overall transmission noise is expected to be such that the noise contribution to the temperature spectrum is approximately white. Experience has tended to confirm this. Depths were determined from smoothed pressure gauge readings.

Individual profiles and associated data are listed in Table 1. Most of these were obtained at the 10N, 50W location and specific trajectories there are denoted by lower case letters. Thus, drop d was started from the surface at 1100 GMT 15 October 1967 and return profile e was obtained immediately after the float touched bottom, 4900 m, at 1400 GMT. Drop f was initiated at 0520 GMT 16 October and g is the return profile started 2½ hr later. For profile h the float was secured 15 m above a thin-walled plastic sack containing water with a displacement equal to nine times that of the float. It sank below the 1300 m level at 1700 on 16 October and reached 3500 m at 0300 on 17 October, giving an average sinking speed near 6 cm sec⁻¹ instead of the 45 cm sec⁻¹ speed of the other profiles. The inertial period at this location is 66.3 hr.

TABLE 1. Summary of temperature characteristics.

Profile ¹	Depth range (m)	Sample ² interval (m)	Data points	θ' Adiabatic linear trend ($^{\circ}\text{C m}^{-1}$) $\times 10^3$	δT Standard deviation from linear trend ($^{\circ}\text{C}$)	$\delta\theta'$ Standard deviation of gradient ³ ($^{\circ}\text{C m}^{-1}$) $\times 10^3$	$\frac{\delta\theta'}{\theta'}$	δT_u Standard deviation, unwhitened ($^{\circ}\text{C}$)	δT_f Standard deviation after high-pass filter ($^{\circ}\text{C}$)	$\frac{\delta T_f}{\theta'}$ (m)
h	1300–2100	5.25 I	141	1.7	0.037	1.2	0.71	0.009	0.011	6.5
	1300–2100	5.25 II	140	1.7	0.037	1.26	0.74	0.011	0.012	7.0
	1300–2100	10.5 I	70	1.7	0.037	0.94	0.55	0.009	0.011	6.5
	1300–2100	10.5 II	70	1.7	0.038	0.83	0.49	0.010	0.012	7.0
	2260–3485	8.3 I	148	0.67	0.023	0.40	0.60	0.0043	0.0049	7.3
	2260–3485	8.3 II	147	0.67	0.023	0.46	0.69	0.0054	0.0046	6.9
	2260–2700	1.5	273	0.81	0.010	1.53	1.89	0.0029	0.0028	3.5
	2730–3485	4.5	191	0.56	0.0053	0.49	0.88	0.0035	0.0027	4.8
	3240–3500	1.1	224	0.65	0.0035	1.27	1.95	0.0020	0.0020	3.1
d	1300–2100	11.5	63	1.67	0.051	1.18	0.71	0.013	0.012	7.2
g	1300–2100	13.6	52	1.69	0.036	0.69	0.41	0.009	0.010	5.9
d	2100–3600	11.2	130	0.69	0.013	0.35	0.51	0.005	0.0035	5.1
e	2100–3600	13.3	114	0.69	0.010	0.34	0.49	0.005	0.0040	5.8
f	2100–3600	13.3	116	0.69	0.019	0.30	0.43	0.005	0.0050	7.3
g	2100–3600	14.3	103	0.69	0.020	0.27	0.39	0.005	0.0040	5.8
C	900–1950	11.0	94	2.02	0.072	1.4	0.69	0.015	0.018	6.9
C	2060–3820	11.0	159	0.64	0.066	0.63	0.98	0.008	0.007	9.8
B	976–2111	7.34	155	2.3	0.255	2.5	1.09	0.035	0.022	10.9
B	976–1859	8.33	106	2.79	0.198	2.4	0.86	0.028	0.023	8.6
B	2010–3588	9.30	170	0.79	0.044	0.81	1.03	0.011	0.009	10.3

¹ Lower case profiles at 10°24'N, 50°22'W; C at 12°N, 27°W; B at 27°56'N, 55°22'W.² Points of series II alternate with those of series I.³ Temperature gradient as measured by the difference in temperature between successive readings divided by the sampling distance.

3. Analysis procedure

Spectrum analyses were made following the procedures outlined by Blackman and Tukey (1958). Since the lag is 10%, the 90% confidence limits for random data are within factors of 0.54 and 1.6 of the true averages. Actual transmitted data were used in all analyses unless otherwise noted even though the sampling distance changed approximately 10% over a given series because transmissions were made at fixed intervals of time. Sample analyses showed that any associated error is appreciably less than incurred as a result of the filtering which accompanies interpolation to obtain a constant sampling distance. Spectra were determined for a variety of depth intervals and data point separation distances as well as for alternating or staggered sequences of points in some of the profiles. The effects of such variations were explored particularly for the slow drop, h. The object was to try to make evident any unexpected source of "noise" in the data such as may be attributable to least count error, aliasing, etc. The number of data points and the depth intervals used in each analysis are listed in Table 1. Other information included in that table will be described later.

For any sequence of temperature readings over a given depth interval the average temperature gradient was removed by the method of least squares. That

gradient is listed in Table 1; it can be used along with the total depth range in order to determine the overall temperature change for any profile. The standard deviations of the actual temperatures from that gradient are also listed. Spectra for such differenced temperatures were determined but are not reported here. At high wavenumbers they had spectrum densities which were practically the same as obtained from more accurate procedures but at the lowest few wavenumbers, excluding zero, they grew rapidly to become 10–100 times larger than was true for spectra obtained by using pre-whitening or filtering procedures.

Important errors in the analysis caused by leakage from the high spectrum density at the lowest wavenumbers into wavenumbers of concern were avoided by following two procedures. One was to analyze data after having been sent through a high-pass filter. The filter passed 67% of the variance at the lowest wavenumber of each analysis and only 10% and 0.7% at one-half and one-fourth that wavenumber, respectively. No inverse filter was used subsequently and the spectra include such rejection. The second procedure used differenced data to obtain whitened spectra which represent the temperature gradient fluctuations. Temperature spectra were obtained upon unwhitening. Plots of the latter were visually similar to those obtained for data passed through the filter.

FIG. 1. Spectra for the ratio of temperature deviation from a linear temperature gradient to the mean adiabatic gradient. Symbols specify the profile letter listed in Table 1 and the depth range in kilometers. The sampling distance in meters is included for profile h.

4. Results

a. Temperature deviations

Only plots of the spectra based on filtered data are presented here (Fig. 1). Densities are given for the ratio of temperature deviation divided by the mean adiabatic temperature gradient. The results for various circumstances during the slow drop (h) are illustrated in the left group of points. To the right are points representing other profiles at this location as well as profiles obtained at $12^{\circ}\text{N}, 27^{\circ}\text{W}$ and $28^{\circ}\text{N}, 55^{\circ}\text{W}$ as listed in Table 1. Descents at the latter locations took place at speeds of 17 and 12 cm sec^{-1} , respectively. The measurements at the two locations were made with a least count of 0.004C or at a sensitivity of one-fifth that used at the first location; the results must be contaminated by some extraneous noise. The spectra based on unwhitened differenced data did not exhibit the relative lowering shown at the lowest wavenumber.

The standard deviations from the linear trend obtained in the filtered and unwhitened slope spectrum analyses are listed in Table 1. Both sets are in fair

agreement and are smaller by a factor of $\sim \frac{1}{3}$ than the standard deviations of the unprocessed data from that trend. The agreement between the deviations obtained from the filtered and unwhitened spectra probably results because both fortuitously omit about the same contribution to the variance at wavelengths longer than the lowest estimate. The latter as shown in Fig. 1 ranges from 200 to 300 m.

Ratios of the standard deviation of temperature from a linear gradient to the mean temperature gradient are almost all in the range of 5–15 m (Table 1). Most of this contribution is from the longest wavelengths and so might be thought associated with the presence of low vertical mode numbers. If such a pattern persisted some coherence is expected between profiles d–h which were all completed within a time of two-thirds the local inertial period. No coherence significantly higher than that for a random Gaussian field was found between any of these profiles. Such comparisons were made from data as recorded as well as from temperatures for exact 10 m intervals as obtained by interpolation for each profile. Phase angle

correlations did not exhibit the randomness to be expected and tended to linger near fixed in- or out-of-phase conditions at wavelengths 50 m or more in length. No relationship between such angles and the time of the profile was evident.

Return profiles were traversed approximately 3 hr after their descents; thus, these tracks should not have been displaced horizontally relative to the descending trajectory by a distance exceeding 1 km. Nevertheless, such horizontal displacements could be responsible in large measure for the lack of coherence if temperature fluctuations have properties similar to velocity fluctuations. Previous measurements with these floats have shown small horizontal coherences of the horizontal velocity at this length scale (Pochapsky, 1972).

b. Gradient deviations

Although the spectra of temperature gradients can be deduced from the spectra of the temperatures, it was thought preferable to obtain them directly. The results are shown in Fig. 2. The density presented is the ratio of the difference of temperature in the sampling distance divided by the product of that distance and the mean temperature gradient. One set of curves and points summarizes all results except those for the slow drop profile h. The other set represents different sections

FIG. 2. Spectra for the ratio of temperature difference divided by sample interval times mean gradient. Symbols specify profile letter, depth range in kilometers, and the sampling distance in meters. Curves in upper set are averages for the various profiles at the 10N location.

of profile h analyzed over a number of sample lengths. The small sampling interval extends spectra into large wavenumbers where the errors of slope measurement may dominate. The error noise spectrum expected when data are sampled every 1.5 m at plus or minus the least count in a gradient of $0.7 \times 10^{-3} \text{C m}^{-1}$ is illustrated.

Within the confidence limits the spectra of temperature gradients are approximately white and the correlation function is thus peaked sharply at the origin. Successive temperature differences are uncorrelated and coherence distances for such differences must be less than ~ 5 m. White gradient spectra are consistent with the -2 slopes obtained for the temperatures. The scatter of readings remains such that a $-5/3$ temperature slope is not excluded.

The ratio of temperature difference to sampling distance represents temperature gradients fairly well for wavenumbers up to near the limiting Nyquist high-wavenumber cutoff. It clearly underestimates gradients existing above that wavenumber and this is fortunate because it reduces the severe aliasing that would be present if true gradients were determined. Such aliasing is expected to contribute errors outside the confidence limits only possibly in the immediate vicinity of the Nyquist wavenumber.

The gradient spectra have densities which vary under the different experimental circumstances listed in Table 1 over a range of 5–25 as illustrated in the upper section of Fig. 2. For any particular wavenumber the ratio of the standard deviation of the temperature slope to the adiabatic mean gradient then changes only over a factor of $\sqrt{5}$, or 2.2, in the differing conditions. Because aliasing is expected to have only a small contribution, the spectrum levels should remain near the same values when the upper limiting wavenumber of the analysis is increased. The standard deviation of that slope then increases as the sampling interval decreases and at the 10N location it was found to vary as

$$\frac{\text{standard deviation of slope}}{\text{adiabatic mean gradient}} = \frac{1.8}{\sqrt{L}},$$

when the sample interval L is in the range 5–15 m. This form is consistent with a constant spectrum density and a Nyquist wavenumber proportional to $1/L$. The constant is determined from values listed in Table 1. Values of the standard deviation exceeded this expectation for the values of L near 1 m, possibly because of the contribution from transmission noise. There the constant in the equations is 2.6. The constant is near 2.8 at the other two locations where measuring noise contributed. It is evident that changes in the slope which exceed the average local temperature gradient are not unusual occurrences in deep water over sampling distances of ~ 5 m. On a smaller scale more drastic changes may take place as shown by the

precise and high-resolution measurements of Gregg and Cox (1971) obtained at a slowly sinking float at depths near 300 m.

c. Variations at a hovering float

When a neutrally buoyant float hovers in deep water it tends to stay at a fixed level while water moves past it vertically at low frequencies with amplitudes measured in tens of meters (Pochapsky, 1972). At frequencies near the stability frequency of the water, floats do move up and down to follow the water more closely but amplitudes there amount to only a few meters. A temperature sensor on the float measures low-frequency temperature oscillations which have a magnitude determined by the product of the amplitude of the relative water motion and the average vertical temperature gradient. Superimposed on this are increasingly more rapid and smaller temperature fluctuations. Horizontal temperature gradients as well as vertical ones probably contribute to the temperature variations at a hovering float. In the profiles described above the temperature changes are such as would be produced by displacements of 5–10 m and so should have magnitudes comparable to those for the slow periods at hovering floats. Measurements at such floats have yielded the following information: At the 10N station a float hovering at a depth of 4400 m showed a standard deviation of temperature of 0.0055°C in an adiabatic gradient of $0.48 \times 10^{-3} \text{ } ^\circ\text{C m}^{-1}$, while one at the 28N station at a depth of 2300 m revealed 0.015°C in a gradient of $0.9 \times 10^{-3} \text{ } ^\circ\text{C m}^{-1}$. At the 12N station the measurements showed 0.005°C at 3800 m in a gradient of $0.35 \times 10^{-3} \text{ } ^\circ\text{C m}^{-1}$. Measurements every 20 sec or so were accumulated over a period of at least a few days. It is concluded that temperature fluctuations along a profile will have a contribution from active water movements and need not be primarily “frozen in.”

5. Discussion

The temperature variations relative to a linear gradient may originate as a result of non-mixing motions as in a chaotic internal wave field or by turbulent mixing which is inherently disordered, or by a combination of both. Spectra with a -2 wavenumber dependence such as observed could be produced by a random “frozen in” vertical step structure of the temperature field such as discussed by Phillips (1971). An examination of the temperature profile obtained in the slow drop h shows several sharp drops which are reminiscent of steps but the general impression is not that of the definite step structure such as is found in some near surface waters.

If vertical motions were coherent over vertical distances of a few hundred meters the concentration of temperature fluctuation density at low wavenumbers could be explained in terms of internal waves. Such motions in the existing stratification, however, should

take place at a period longer than the sinking and rising time of a trajectory and might be expected to be significantly coherent between both profiles unless the horizontal coherence is poor over distances of, say, a kilometer. Past work with floats has shown the latter condition to be true for horizontal velocities at frequencies above inertial. It is then likely that significant coherence in both the horizontal and vertical directions occurs at scales of at most a few hundred meters but coordinated movement on such a vertical scale could produce much of the temperature variance observed. Because vertical velocities have been found to have spectra which are approximately white, the spectra of temperature gradients would also have that character, as observed.

Any explanation must recognize the presence of a limited coherence which is more characteristic of turbulence than of a simple internal wave field. Although the spectra of Fig. 1 suggest a possible $-5/3$ power dependence which is characteristic of inertial turbulence, some enhancement by aliasing at the high wavenumber end and the filtering at low frequencies may let a -2 power dependence seem more likely. In the latter case the temperature difference from one sampling level to the next proceeds independently and the net temperature change increases with distance so as to favor long wavelengths in the temperature spectrum similar to the displacements of path in a random walk. Similarly, the temperature variance will be concentrated at the longer wavelengths in an inertially turbulent field. Profiles will differ from water column to water column and an overall coherence in both directions of a profile trajectory will probably not exist. These experiments thus cannot distinguish between the contributions made by internal waves and turbulence.

In a somewhat different approach to the spectra of near surface waters of the Pacific in the depth range 0 to 1500 m, Roden (1971) finds that a $-8/3$ law fits his spectra in the range 0.003–0.03 cycle m^{-1} . The larger slope could be the result of using a different filtering procedure. His spectra show standard deviations typically of 0.1°C which are contributed primarily in the depth range 200–500 m. The zero crossings show temperature perturbations which have thicknesses of 13–16 m, distances which are about double the distances found here for the ratio of the standard deviation of the temperature to the average slope. No gradients were reported but points in his spectra at a wavelength of 10 m would lie near our normalized points if the temperature change was approximately 2°C over the 200–500 m interval of large temperature fluctuation illustrated in his records. There is a suggestion of consistency between the type of results obtained when using a cabled STD sensor in shallow water and ours with free floats in deep water, except for the differing spectrum slopes. Procedures, however,

are sufficiently dissimilar to dissuade one from pursuing comparisons in more detail.

6. Conclusions

The present measurements obtained at slowly sinking floats show that the temperature differences in successive discrete steps of constant length in the vertical direction are incoherent over steps at least as small as 5 m. As a consequence, in profiles which exclude regions of high curvature, the temperature spectra fall as the inverse square of the wavenumber. This was shown for wavelengths shorter than a few hundred meters. The reliability of the determinations is not sufficient to exclude the presence of the slight coherence associated with a Kolmogoroff $-5/3$ power dependence. When the temperature deviation is divided by the average adiabatic temperature gradient for a series of data the spectra of such normalized quantities change little from location to location. No significant coherence is observed between profiles at the same location even when measurements are made during the descent and subsequent ascent of a given float. A lack of important horizontal and presumably of vertical temperature coherence occurs over distances larger than a few hundred meters. Since the variance of the temperature from a linear gradient compares with that obtained at hovering, neutrally buoyant floats, the local motions caused by either internal waves or turbulence

can form an important part of that variance. The float sinking at a speed of 6 cm sec^{-1} gave results similar to those obtained at a speed near 1 kt and this fact suggests that cabled sensors will give similar results provided that cable motions are not excessive. These experiments do not enable a separation of mixing and non-mixing motions. The ultimate significance of the separation of turbulent and internal wave motions may be to determine the nature of mixing, and for this it may prove to be more appropriate to investigate the diffusion process directly.

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