The Half-Inertial Flow in the Eastern Equatorial Pacific: A Case Study

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ABSTRACT

We address the problem of the oscillatory periods of two observed phenomena in the instability zone in the eastern equatorial Pacific. The first case concerns two high-speed anticyclonic flow with periods of approximately 12 and 15 mean solar days. The second concerns the long waves with an average period of about 25 days. The problem is that, in terms of their respective mean latitudes, these periods are about twice as long as those of the inertial period of one-half pendulum day. As a solution we offer the hypothesis of the half-inertial flow with its period of one pendulum day.

The inertial flow is governed by $KV = -f$, where $K$ is the path curvature, $V$ the speed of the flow, and $f$ the Coriolis parameter. In contrast, the half-inertial flow is governed by $KV = -f/2$ with a speed that is the maximum for a given curvature and latitude and approaches twice that of the geostrophic speed.

In terms of the half-inertial flow, the average long-wave period of 25 days would correspond to a plausible mean latitude of 2.3°N. Efficient at extracting horizontal shear energy, the half inertial flow could also play an important role in the near-surface heat balance.

1. Introduction

In the eastern equatorial Pacific north of the equator and west of 100°W three zonal currents dominate the surface circulation. They are the broad westward South Equatorial Current (SEC) that extends from the Southern Hemisphere across the equator to a latitude generally south of 5°N, the similarly broad westward North Equatorial Current (NEC) generally north of 8–10°N, and between them the relatively narrow eastward North Equatorial Countercurrent (NECC). An equatorial ridge in surface dynamic height in the vicinity of 5°N (Fig. 1) separates the SEC and NECC.

Spanning a width of some 400–500 km the zone of strong lateral shear straddling the equatorial ridge is a zone of flow instability (Philander 1978; Cox 1980). Previous studies have revealed two manifestations of this instability. On the equatorial side a westward progression of long waves (Legeckis 1977). To the north and traveling with the long waves, a train of mesoscale anticyclonic eddies (Miller et al. 1985). The high speed flows tracked by the surface drifters will be shown embedded in such anticyclonic eddies. In studying the flow our primary goals are to document its unusual oscillatory periods, to interpret them in light of the half inertial flow, and to elicit the pattern of the divergence implicit in the accelerating high speed flow. We consider first the characteristics of half inertial flow.

![Fig. 1. Meridional section of mean surface dynamic height relative to 500 db as a function of latitude along 110°W. Open circles at half-degree intervals are averages of nine CTD sections taken at irregular time intervals over three years. Larger error bars indicate standard deviation, smaller bars the standard deviation of the mean. Reproduced from Miller et al. (1985).](image-url)
TABLE 1. Some features of selected flows.

<table>
<thead>
<tr>
<th>Type</th>
<th>$Vg^*/V$</th>
<th>$dV/dt$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geostrophic flow</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Gradient flow</td>
<td>≠0</td>
<td>0</td>
<td>≠0</td>
</tr>
<tr>
<td>Inertial flow</td>
<td>0</td>
<td>0</td>
<td>&lt;0</td>
</tr>
<tr>
<td>Half inertial flow</td>
<td>1/2 cosφ</td>
<td>≠0</td>
<td>&lt;0</td>
</tr>
<tr>
<td>Cross-stream geostrophy</td>
<td>1/cosφ</td>
<td>≠0</td>
<td>0</td>
</tr>
<tr>
<td>Florida current</td>
<td>≠0</td>
<td>≠0</td>
<td>≠0</td>
</tr>
</tbody>
</table>

Also let $N$ be the (scalar) coordinate in the direction of the pressure ascendant $\nabla p$ and $\phi$ be the cross-isobaric angle, positive toward higher and negative toward lower pressure. Finally, on defining $fV^* = \alpha \partial p/\partial N$ the equation of inviscid horizontal motion along the flow is

$$dV/dt = -\alpha \partial p/\partial s = \alpha(\partial p/\partial N) \sin \phi = -fV^* \sin \phi,$$

and across the flow

$$KV^2 + fV = \alpha \partial p/\partial n = \alpha(\partial p/\partial N) \cos \phi = fV^* \cos \phi,$$

or, on using $V_g = V^* \cos \phi$ for convenience

$$KV^2 + fV - fV_g = 0.$$  \hspace{1cm} (2a)

2. The characteristics of half-inertial flow

Let the unit vector $s$ be directed along the local horizontal velocity $V$ such that $V = sV$ where $V$ is the magnitude of $V$. Let $k$ be the vertical unit vector positive upward, with $n = s \times k$ positive to the right of $s$.

BUOY 4416

Fig. 2. Plots of trajectory for drifter 4416: (a) from deployment (23 March 1983; 4.50°N, 85.28°W) to last message (26 October 1984; 11.33°N, 165.93°W). The windowshade drogue was lost on 18 January 1984 at 10.01°N, 121.59°. (b) Daily locations for period between 28 July 1983 and 5 December 1983, and (c) for period between 0600 UTC 1 September 1983 and 0600 UTC 20 October 1983, the interval selected for analysis. The motion is anticyclonic in the sense indicated by the arrowheads. Numbers 1–4 refer to the corresponding loops in the lower part of Figs. 6 and 7, and in Table 2. For reference to Pazos and Acero (1985) which reports on this drifter, the Julian Day count corresponding the first period is 1670–1800, and to the second 1705.25–1754.25.
In this paper $d/dt$ is the time change following only the horizontal component of a parcel in three-dimensional motion, $K$ the (scalar) path curvature or the reciprocal of the radius of path curvature, $f$ the Coriolis parameter, and $V^g$ the geostrophic speed.

The quadratic equation (2) has the solution

$$ V = \frac{1}{2K}[-f \pm (f^2 + 4KfV^g)^{1/2}] \quad (3) $$

or

$$ \omega = \frac{-f}{2} \pm \frac{1}{2}(f^2 + 4KfV^g)^{1/2} \quad (3a) $$

since the circle of radius of $1/K$ is the instantaneous radius of path curvature so that $\omega = KV$ is the instantaneous angular speed.

The solution (3) is extensively discussed in meteorological textbooks (e.g., Halitner and Martin 1957). Within the constraints of positive $V$ and positive square root, (3) show a large range of dynamically possible speed $V$ and period $2\pi/\omega$.

One special flow is obtained when $V^g$ is identically zero. This gives the well-known inertial flow

$$ K_1V_1 = \omega_1 = -f \quad (4) $$

with a constant speed and a period $P_1$ of

$$ P_1 = 2\pi/\omega_1 = 2\pi/f. \quad (5) $$

Another special but much more general flow is obtained when only the sum inside the square root term is zero. This gives the anticyclonic flow

$$ KV = \omega_2 = -f/2 \quad (6) $$

with

$$ V = 2V^g \quad (7) $$

and period $P_2$

$$ P_2 = 2\pi/\omega_2 = 4\pi/f \quad (8) $$

or one pendulum day. This we call the half inertial flow.

While the form of (4) and (6) differs only by a factor of $\frac{1}{2}$ the latter represent a dynamical regime of much greater generality. One way to exhibit this is shown in Table 1.

The minus sign before the square root term in (3) is appropriate only in occasional anomalous atmospheric flows. Accordingly the speed of the anticyclonic half inertial flow is the highest possible for a given latitude and curvature. This association of high speed with anticyclonic curvature tends also to occur in the Florida Current (Chew and Bushnell 1987), and in the winter atmospheric subtropical jet stream (Krishnamurti 1961). Moreover the relation $dV/dt = 2dV^g/dt$ from (7) appeared in Newton (1959) in an analysis of atmospheric jets. Hence the half inertial flow is likely to share much of the dynamics of the Florida Current and other high speed flows.

3. The drifter motion

Attached to a 2 m × 10 m windowshade drogue by a 10-m tether, both drifter buoy 4416 and 6862 were deployed as part of the Equatorial Pacific Ocean Climate Study program of the National Oceanic and Atmospheric Administration. Data processing is consisted primarily of editing the raw positions of these satellite-tracked drifters to remove spurious points, smoothing by passing through a three-point triangular running average and interpolating to uniform six-hour intervals. As shown in Figs. 2a and 2b drifter 4416 traveled westward for some time and distance before entrained into the high speed anticyclonic flow. In contrast, Fig. 3a and 3b show drifter 6862 was in the anticyclonic flow from the time it was launched. Both drifters, however, were later detrained, drifted eastward for a time in the NECC and eventually northward into the westbounding NEC. Fig. 2c and 3c show the observation intervals selected for analysis. This is presented in three subsections: the oscillatory period, the relation to anticyclonic eddy, and relation to long waves.

![Fig. 3. Plots of the trajectory for drifter 6862: (a) from deployment (24 November 1987; 5.17°N, 121.55°W) to last message (3 January 1988; 17.42°N, 160.23°W). The windowshade drogue was lost on 28 January 1988 at 5.64°N, 137.31°W. (b) Daily locations from deployment to 31 January 1988, and (c) from 1800 UTC 2 December 1987 (Julian Day 336.75) to 1800 UTC 4 January 1988 (Julian Day 369.75), the interval selected for analysis.](image)
a. Oscillatory period

In Figs. 4 and 5 we show plots of the eastward, northward and total horizontal speed with vertical lines marking the observation intervals corresponding to Fig. 2c and 3c. In both cases we see large changes in speed. For drifter 4416 the speed ranged from a low of about 75 to a high of over 200 cm s\(^{-1}\); for drifter 6862, from some 50 to 175 cm s\(^{-1}\). And in both cases we see three features: a mean westward advection, a meridional speed nearly symmetric about zero, and most evident of all an oscillation in the speeds of period of about 12 days in Fig. 4, and about 15 days in Fig. 5.

At these low latitudes the Coriolis parameter \(\beta\) is almost linear. Hence to find the average angular speed over a cycle according to (6) or the average period according to (8) we simply use the arithmetical mean of the latitudes. When all the six-hourly locations are taken into account we find the average latitude, which we call the latitude center, is 4.6°N for the flow in Fig. 2c and 3.7°N for that in Fig. 3c. The corresponding one-pendulum-day periods are respectively 12.4 and 15.4 mean solar days, in good agreement with the observed periods. While clearly the corresponding inertial periods from (5) are too small by about one-half.

b. Relation to anticyclonic eddy

Here we confine attention to the dataset represented in Fig. 2c in part because it is longer and in part because there is concurrent observation of long waves. The conclusion will be seen to be equally applicable to the other set. First of all the 4.6°N latitude center about which the flow oscillate meridionally corresponds well, fortuitously it must be stated, to the position of the mean ridge in Fig. 1, an indication that the flow was embedded in a westward travelling anticyclonic eddy.
This can be made more explicit by subtracting out the advection component from the observed trajectory to reveal the anticyclonic eddy component that would be observed if simultaneous observation over the whole region were made. Moreover, the inferred streamline field will facilitate matching up with the long waves to the south since they too are based more or less on synoptic observation. From Fig. 2c we first form four separate loops and compute their respective mean zonal \( \langle \nu \rangle \) and meridional speed \( \langle u \rangle \) components, as shown in Table 2. Then assuming the anticyclonic eddy to advect at a constant speed without significant change in shape we multiplied these mean speeds by the time elapsed since the start of each loop converting them to zonal and meridional distances which are finally subtracted from corresponding observed drifter locations. These steps transform the trajectories into flow streamlines for the instant corresponding to each start as shown in Fig. 6 and 7 above the trajectories from which they are derived. The shape of these streamlines and the colocation of their latitude centers with the equatorial ridge are good evidence that the half inertial flow was embedded in a generally westward moving anticyclonic eddy.

The latitude center of the flow in Fig. 3c, tracked in December, is farther south at 3.7°N suggesting that the colocated equatorial ridge was also farther south. This is in line with the knowledge that the ridge is farthest north during the northern summer (Hayes et al. 1983), and thus further suggesting that the half inertial flows are confined to the equatorial ridge. The corresponding streamline patterns for the December flow are very similar to those in Fig. 6 and 7 and are not shown.

c. Relation to long waves

Long waves from satellite sea surface temperature (SST) charts for the season beginning in August 1983 are reported in Legeckis and Pichel (1984). They labeled the long waves and gave their average positions at weekly intervals as well as their mean phase speeds, wavelengths and periods. By matching the times and locations of the anticyclonic eddy as represented by the inferred streamlines in Fig. 6 and 7 with those of the long waves we find the eddy and the wave E–F traveling together westward. This is seen in Fig. 8 where we have superposed on each SST chart a heavy black dot representing the approximate center of the nearest concurrent inferred streamline from Fig. 6 or 7. We note that each SST chart is an average over one week, that there is no chart for 6–13 September because of missing SST data, and that there is some ambiguity in the earlier locations of the wave crest F. Nonetheless there is good overall agreement in westward progression. A more subtle indication of coupling is the fit between the half inertial flow and the long wave. For as the wave E–F becomes longer in Fig. 8c there is a corresponding increase in the dimensions of the last inferred streamline in Fig. 7.

For the 105 days wave E–F was tracked, Legeckis and Pichel found an average wavelength of 1100 km, and an average westward phase speed of \(-54\) cm s\(^{-1}\) for an average period of 23.6 days. The phase speed is seen to be somewhat less than the \(-59\) cm s\(^{-1}\) in Table 2, and 23.6 days is also seen to be somewhat less than twice the period that is so evident in Fig. 4. Owing to the Doppler effect the period in Fig. 4 is longer than the period of the flow around the eddy. But assuming the half-inertial-flow dynamics to hold otherwise, and considering the differing numbers of days in the above averages as well their uncertainties we can state the following. As the coupled anticyclonic cell and the long wave traveled westward the embedded half inertial flow at the higher latitude completed two periods of one
### Table 2. Various data for drifter buoy 4416 and 6862.

<table>
<thead>
<tr>
<th>Loop</th>
<th>Average center</th>
<th>Days</th>
<th>( N_o )</th>
<th>( \langle u \rangle ) (cm s(^{-1}))</th>
<th>( \langle v \rangle ) (cm s(^{-1}))</th>
<th>Speed (cm s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drifter 4416</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1705.25</td>
<td>50</td>
<td>–62.4</td>
<td>(12.3)(^b)</td>
<td>(10.5)</td>
<td>121.6</td>
</tr>
<tr>
<td>2</td>
<td>1717.75</td>
<td>50</td>
<td>–49.8</td>
<td>(12.2)</td>
<td>9.8</td>
<td>115.4</td>
</tr>
<tr>
<td>2</td>
<td>1730.25</td>
<td>47</td>
<td>–62.8</td>
<td>(13.7)</td>
<td>–10.5</td>
<td>131.1</td>
</tr>
<tr>
<td>4</td>
<td>1742.00</td>
<td>50</td>
<td>–61.1</td>
<td>(16.1)</td>
<td>–2.7</td>
<td>156.2</td>
</tr>
<tr>
<td>All</td>
<td>4.6°N</td>
<td>197</td>
<td>–59.0</td>
<td>(16.1)</td>
<td>–1.5</td>
<td>131.1</td>
</tr>
<tr>
<td>Drifter 6862</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>All</td>
<td>3.7°N</td>
<td>336.75</td>
<td>133</td>
<td>–46.6</td>
<td>(6.5)</td>
<td>100.8</td>
</tr>
</tbody>
</table>

\( N_o \): number of 6-hourly observations.
\( \langle u \rangle, \langle v \rangle \): mean zonal and meridional speed, positive eastward and northward respectively.
\(^b\) Standard deviation of the mean in parenthesis.

A pendulum day while the long wave at lower latitude completed one of one pendulum day.

Satellite sea surface temperature maps for December 1987 show indications of long waves but their analysis is still pending (R. Legeckis, personal communication). In any case Miller et al. (1985) have found westward

![Fig. 6. Plots of original loop 1 and 2 with daily locations (lower) and their respective inferred streamlines for 1 and 13 September 1983 obtained by removing the effect of mean drifter velocity (upper).](image)
traveling anticyclonic eddies even when long waves were not detected. We turn now to explore some aspects of the dynamics of the half inertial flow.

4. Acceleration and divergence

The changing speed along the flow shown in Fig. 4 and 5 implies a downstream acceleration. With reference to Fig. 2c and 3c there is generally downstream acceleration in the southward and deceleration in northward leg of the flow. In either case there is cross-isobaric flow. This cross-isobaric angle is given by the ratio of the downstream to Coriolis acceleration,

\[
\frac{dV}{dt}/fV = \left( fV_r \sin \phi \right)/2fV_r \cos \phi = (\tan \phi)/2, \quad (9)
\]

or on solving for the angle of the cross-isobaric flow

\[
\phi = \tan^{-1} \left[2(\frac{dV}{dt})/fV\right]. \quad (9a)
\]

This is twice the angle in an accelerating flow in cross-stream geostrophic balance for the same acceleration ratio. To illustrate the magnitude of this angle consider the flow in Fig. 2c at 4.6°N. Here we have approximately \(dV/dt = 2 \times 10^{-4}\) cm s\(^{-2}\) and \(fV = 1.3 \times 10^{-3}\) cm s\(^{-2}\) for an angle of 18°. Thus, as in other inviscid flows generally, the half inertial flow is toward higher pressure when decelerating and toward lower pressure when accelerating. This is schematically illustrated in Fig. 9 for Loop 3 of Fig. 7.

In an inviscid fluid a decelerating flow works against the horizontal pressure gradient force and conversely. Hence we may expect the half inertial flow to undergo significant exchange between kinetic and potential energies. To explore this and related baroclinic processes in the flow let the circulation in the surface layer in this region be approximated by the so-called 1\(\frac{1}{2}\)-layer model; in particular, by a thin active layer of thickness \(H\) overlying an infinitely thick, inert layer. The change in kinetic energy following the flow is then given by

\[
\frac{d}{dt}V^2/2 = -g'V \partial H/\partial s \\
= -g'(dH/dt - \partial H/\partial t), \quad (10)
\]

where \(g'\) is the reduced gravity and \(dH/dt = \partial H/\partial t + V\partial H/\partial s\). The divergence \(D\) in this model is \(D = -(1/H)dH/dt\). And additionally when the local change is set equal to the change arising from the advection (with velocity \(C\)) of the anticyclonic eddy in which the flow is embedded we have

\[
D = (1/g'H)(d/dt)V^2/2 + C \cdot \nabla H/H. \quad (11)
\]

The divergence is thus a sum of two components. If
the flow in the anticyclonic eddy were gradient the kinetic energy component would be zero, and $D$ would be given entirely by the advection component, which for a westward travelling anticyclonic eddy would be convergence in its western and divergence in its eastern part. In the present case there is generally a pattern of decreasing speed and hence decreasing kinetic energy in the northward flows and increasing speed and kinetic energy in the southward flows. As kinetic energy is converted to potential energy along the decelerating flow the thickness $H$ becomes larger as the column stretches to reach greater depths in the process of convergence. Similarly, as potential energy converts to kinetic energy along the accelerating flow the thickness $H$ decreases as the column shrinks to smaller depths in a divergence. In this regime a parcel would be at its most shallow depths when moving fastest and conversely. In all, in one pendulum day we see one cycle of accelerating-ascending and decelerating-descending motion.

The close structural relationship between the half inertial flow and the anticyclonic eddy in which the flow is embedded is also evident in the general reinforcement of the two divergence components in (11). This reinforcement could lead to divergence magnitudes approaching those in the Florida Current (Chew

FIG. 8. Reproduction of three "Weekly average sea surface temperature charts" for the region north of $5^\circ S$ from Legeckis and Pickel (1984) with wave E-F indicated. (a) For week from 30 August to 6 September 1983 with the (approximate) center of the (inferred) streamline for 1 September 1983 from Fig. 6 superposed. (b) For week 13–20 September 1983 with the center of the streamline for 13 September 1983 superposed. (c) For week 4–10 October 1983 with the center of the streamline for 8 October 1983 from Fig. 7 superposed. No temperature chart is available for location comparison with streamline for 26 September 1983. Note the earlier ambiguity of wave crest F, the averaging period of the charts, and the later increase in the length of wave E-F.
5. Discussion and conclusion

The geostrophic method is often used to estimate the transport of equatorial currents. With a speed at nearly twice that of the geostrophic speed, during the occurrence of the half inertial flow the use of the geostrophic method will be highly inaccurate.

The shapes of the inferred streamlines in Fig. 6 and 7 all have major axes oriented along northwest to southeast quadrants. This pattern is remarkably like the idealized perturbation streamlines discussed in Pedlosky (1979, p. 436, Fig. 7.3.1). And like them the inferred streamlines "lean" against the horizontal shear of the SEC and the NECC. This is the most efficient configuration for extracting energy from the horizontal shear of the basic currents, and is likely to be the primary mechanism responsible for the horizontal eddy fluxes reported in Hansen and Paul (1984) for this region.

We have seen in the eastern tropical Pacific the half inertial flow is closely coupled to the equatorial ridge in surface dynamic heights and the westward travelling long waves. Both an equatorial ridge (Katz 1981) and long waves (Weisberg 1984) are found in the tropical Atlantic. Hence we expect that region of the Atlantic also to have a regime of half inertial flow.

Numerical modeling of the region with various winds (e.g., Hayes et al. 1989) has been successful, to a degree, in reproducing the long-wave phenomenon; but there appears to be no direct connection between the periodicity of the half inertial flow or of the long wave with those of the winds. Together, the half inertial flows and the long waves may well be a coupled oscillatory system in a setting that allows it to seek its own free period, the period of one pendulum day. Given the high speed and large acceleration of the half inertial flow how it couples dynamically with the long waves may well hold the key to the dynamics of this instability region. We recommend further study of the half inertial flow in all aspects.

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