Energetics of the Kuroshio Extension at 35°N, 152°E*

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ABSTRACT

A simplistic interpretation of eddy heat fluxes from a two-year current meter mooring deployment in the Kuroshio Extension leads to the conclusion that the eddy field is decaying at 152°E, contradicting observations from the surface to 300 m that indicate the region to be one of steady or growing eddy energy. Thus, a simplified version of the method used by Hall to construct the velocity field of the current from the moored data has been used to examine the baroclinic and barotropic energy conversions in the cyclonic and anticyclonic portions of the current, for both geographic and “stream” coordinates. Although the error bars are large, in stream coordinates significant conversions of mean to eddy potential energy occur on the anticyclonic side of the current at both 350 and 625 dbar, with smaller average conversions of eddy to mean energy over the cold portion. Barotropic conversions in this coordinate system are small, but qualitatively the calculated Reynolds stresses agree with previous observations showing that $\delta (u'v')/\delta y < 0$ across the current, so that on average they converge mean momentum. For geographical coordinates, integrated energy balances still suggest overall decay of eddy energy, though not as strong as that found in the “simplistic” interpretation. Reynolds stresses are much stronger than for stream coordinates, and are still convergent, resulting in relatively large apparent conversions of eddy to mean kinetic energy in this coordinate system. Comparison with a similar energetic analysis by Rossby in the Gulf Stream at 73°W shows that: 1) the effects of going from geographical to stream coordinates are similar for the two currents; and 2) at locations that are geographically comparable for the two currents, very different energetic regimes prevail. Dynamical differences are also reflected in the vertical velocity structure. It is hypothesized that external factors, such as the nature of the underlying deep flow, may influence the western boundary current systems in the two oceans in an important way.

1. Introduction

Eddy variability in the world oceans acts as a signal of some oceanic processes (e.g., instabilities associated with strong currents) and as a mechanism for others (poleward transport of heat). In addition, the distribution of eddy kinetic energy has been used as a benchmark for validation of numerical ocean models (Holland and Schmitz 1983; Schmitz and Holland 1982, 1986). Since the strongest oceanic eddy variability is associated with western boundary currents and their eastward extensions, it is important to understand the nature of eddy–mean flow interactions in each of these currents, as well as how and why they differ from one another. Schmitz (1988) has already documented marked differences in the abyssal eddy fields associated with the Kuroshio and Gulf Stream, and Schmitz and Holland (1986) have compared them with abyssal eddy fields generated by a numerical model tuned with various wind forcing and frictional strengths.

The time and space scales of eddy variability in the North Pacific have been described by various investigators from a number of viewpoints (Wyrtki 1975; Bernstein and White 1974, 1977; Bennett and White 1986; Talley and White 1987; Nüller and Hall 1988), some with particular attention to the Kuroshio and Kuroshio Extension (Wilson and Dugan 1978; Bernstein and White 1981, 1982; Schmitz 1984; Schmitz et al. 1982, 1987). However, while in-depth analyses and modeling of eddy–mean flow energetics abound for the Florida Current and Gulf Stream (Webster 1961, 1965; Schmitz and Nüller 1969; Brooks and Nüller 1977; Watts and Johns 1982; Rossby 1987; Dewar and Bane 1985, 1989b), most modeling studies in the Kuroshio have tended to focus on explaining the “bimodality” of the current path near Kyushu (White and McCreary 1976; Chao and McCreary 1982; Chao 1984; Masuda 1982; Hughes 1989). Exceptions are the studies by Szabo and Weatherley (1979) for the area south of Japan, and by Nishida and White (1982) for the Kuroshio Extension between 140° and 180°E. Both of these were limited to the upper 300 m of the water column, and the latter examined the kinetic energy balance only.

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Certainly one of the limitations on such studies in the past has been the nature of the existing data base. In-depth analysis of eddy–mean flow interaction requires many realizations of absolute velocity and temperature structure throughout the current, such as from moored data or (as in the Gulf Stream) using a Pegasus profiler. In the 1980s, a vast amount of moored current meter data in the Kuroshio Extension has become available due to two large programs by W. Schmitz (see Levy and Tarbell 1983, 1987 for data reports; Schmitz et al. 1982, 1987 for descriptions of results). One of these moorings at 35°N, 152°E sampled the Kuroshio flow there for about a year, “profiling” the current as it moved back and forth through the mooring site. Hall (1989b) used these data to reconstruct the vertical and cross-stream velocity and temperature structure of the current, estimating transport and making comparisons with an analogous description of the Gulf Stream at 68°W (Hall 1986a). A simplified version of that analysis has here been used to examine the nature of eddy–mean flow energetic interactions in this part of the Kuroshio, for comparison with similar Gulf Stream studies by Rossby (1987) at 73°W, Dewar and Bane (1989b, also near 73°W), and pointwise estimates by Hall (1986b).

In section 2, the heat equation is used to calculate vertical velocities at the mooring site, in order to complete the description of the velocity field. The next section explores and discusses the subtleties involved in defining “mean” and “eddy” flows for various coordinate systems: in particular, one that is geographically fixed and one whose origin is attached to the current axis. Potential and kinetic energy exchanges are calculated for both, and the results are discussed and interpreted in section 4, followed by comparison and contrast with the Gulf Stream analyses noted above. Finally, these local results are considered in a broader geographical and oceanographic context to highlight important differences between the western boundary current systems of the North Pacific and Atlantic oceans.

2. Vertical velocities at site WP05

For a complete description of the methods used to deduce the cross-stream structure of the Kuroshio (henceforth, KS), the reader is referred to Hall (1989b). Basically, cross-stream position can be quantified in terms of temperature at 350 dbar (which is measured), and an alongstream flow direction defined on the basis of the measured current shear. Using this information, time series of velocity and temperature may be transformed into a spatial coordinate system whose axes are attached to the meandering current; cross-stream position relative to the origin is thus deduced from the measured value of $T_{350}$. This coordinate frame will be referred to as “rotating” or “stream” coordinates; in a fixed coordinate system, Schmitz (1984) notes that the average KS flow is 35° south of east, and a fixed rotation of velocities to this direction will be referred to as “geographical” coordinates. Velocity and temperature time series were recorded at 5 depths (350, 625, 1335, 4000, and 6000 dbar), and while the mooring was maintained for two years (689 days), only the last 390 days reflect the presence of the KS Extension. Thus, when considering the energetics of the current, attention is restricted to this latter portion of the records. In the work, Hall (1989b) calculated transport of $87 \pm 21$ Sv ($Sv = 10^6 \text{m}^3 \cdot \text{sec}^{-1}$) for the KS Extension at this location compared with $94 \pm 26$ Sv for the Gulf Stream (henceforth, GS) at 68°W based on analogous datasets and analysis techniques.

In the analogous GS study, to complete the description of the GS velocity field Hall (1986a) calculated time series of vertical velocity at the mooring site. The velocities were found to be highly correlated throughout the water column (estimates existed at 5 depths), and 80% of the variance could be explained by a single empirical orthogonal function (EOF) whose vertical structure was similar to that of the first baroclinic mode, with a midthermocline maximum in the amplitude. The vertical velocity results were useful in interpreting dynamical behavior of the current at the mooring site (Hall 1986b). For comparison with these results, vertical velocity time series at site WP05 were also calculated, from the temperature equation, following Bryden (1980). Since horizontal temperature gradients are evaluated from velocity shear, the vertical resolution (or lack thereof) in the lower 5000 m of the water column presented some problem in calculating vertical velocities there. Time series of $w$ have been calculated at 350 and 625 dbar, and for nominal depths of 980, 2700, and 5000 dbar (halfway points between measurements), with the details of the calculations varying somewhat from level to level, as described in the Appendix.

The results are shown in Fig. 1, displaying time series of $w$ at all 5 depths, after smoothing with a 5-day running mean. Mean and rms values of $w$, and correlation coefficients between time series at different depths, are presented in Table 1 for the last 390 (372) days of data ($w$ at 5000 dbar exists only for the last 372 days). The time series in Fig. 1 have integral time scales of 10–20 days, suggesting that for 370–390 days of data, there are 20–35 degrees of freedom; on the other hand, horizontal velocity and temperature time series have integral time scales on the order of 30 days, implying ~12 degrees of freedom. Using $N = 20$ yields mean downward vertical velocity that is just barely significantly different from zero at 350 and 625 dbar, with positive mean vertical velocities at 5000 m, while a choice of $N = 12$ implies that the only significantly nonzero mean value in Table 1 is at 5000 m, where it is clearly upward. Note that at 5000 m, calculation of $w$ using a value of $\theta_z$ that is too small would tend to overestimate the size of $w$ (see Appendix) so that the
positive mean value of 0.04 cm s\(^{-1}\) may be too large; however, since a constant value of \(\theta_z\) is used to calculate \(w\) a change in its value would result in proportional changes in both \(w_{\text{rms}}\) and \(\bar{w}\); thus, in any case \(\bar{w}\) is significantly positive. This positive value of \(w\) is particularly noteworthy in view of recent work by Roemmich and McCallister (1989) concluding upward vertical velocities are expected at depth in this region, as abyssal waters originating in the South Pacific flow upward as well as across density surfaces, later to return southward as part of the Pacific thermohaline circulation cell.

The variance in \(w\) at 350 dbar is 2–5 times as great as at any other depth, and vertical velocities are not strongly coherent throughout the water column. This lack of coherence may reflect the fact that going from 350 to 2700 dbar, there is a transition from dominance of advection to dominance of local temperature change in the vertical velocity calculations; this transition is likely due to the difficulty in accurately interpolating \(\partial T/\partial t\) to 980 and 2700 dbar from records at 625 and 1335 dbar, and 1335 and 4000 dbar, respectively. Interestingly, at 5000 m, \(w\) is again dominated by advection, it is more strongly correlated with \(w\) at 350 dbar than at any other depth, and in fact the correlation is negative. Since that portion due to \(\partial T/\partial t\) is positively correlated between 350 and 5000 dbar (\(C = 0.27\)), the negative correlation is entirely due to opposing behavior of the advective portions. The extremely large values of \(w\) at 350 dbar that occur from mid-June to beginning of November 1981 (Fig. 1) are all associated with events during which the mooring was very rapidly pushed over, as evidenced by values of recorded pressure for the original (uncorrected) time series; however, \(w\) as calculated does not reflect residual vertical motion of the instrument at this time, because even during these times advection rather than local temperature change dominates \(w\). There is some visual vertical coherence in the time series during these events. In particular, the strongly negative event at 350 dbar from mid-July to the end of October is mirrored at 5000 dbar as strong upward velocities.

These results are somewhat different from what was found by Hall (1986a) for the GS at 68\(^{\circ}\)W. Although there too mean vertical velocities were not significantly different from zero throughout the water column (including at the bottom), and rms values were similar order of magnitude (0.03–0.1 cm s\(^{-1}\)), vertical velocities were strongly, positively correlated throughout

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>(\bar{w}) (cm s(^{-1}))</th>
<th>(w_{\text{rms}}) (cm s(^{-1}))</th>
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</thead>
<tbody>
<tr>
<td>350</td>
<td>-0.036</td>
<td>0.142</td>
</tr>
<tr>
<td>625</td>
<td>-0.009</td>
<td>0.035</td>
</tr>
<tr>
<td>980</td>
<td>-0.0030</td>
<td>0.029</td>
</tr>
<tr>
<td>2700</td>
<td>-0.061</td>
<td>0.050</td>
</tr>
<tr>
<td>5000</td>
<td>0.041</td>
<td>0.054</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(w_{\text{rms}})</th>
<th>(w_{350})</th>
<th>(w_{625})</th>
<th>(w_{980})</th>
<th>(w_{2700})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_{350})</td>
<td>0.60</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(w_{625})</td>
<td>0.13</td>
<td>0.51</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(w_{980})</td>
<td>0.13</td>
<td>0.23</td>
<td>0.25</td>
<td>-</td>
</tr>
<tr>
<td>(w_{2700})</td>
<td>-0.25</td>
<td>-0.19</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td>(w_{5000})</td>
<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
</tbody>
</table>
water column ($C \geq -0.6$ in nearly all cases), and maximum variance occurred at 875 dbar, dropping to lower values both above and below. An EOF decomposition of the GS vertical velocities thus led to the straightforward result that (as noted above) 82% of the variance was contained in the first mode, which had maximum amplitude at 875 dbar and no zero crossing, so that it resembled the vertical velocity structure associated with a dynamical first baroclinic mode. In the Kuroshio, the thermocline expression is much shallower (Hall 1989b; also Fig. 4) and this effect, in combination with total kinetic energy values at 350 dbar that are 3–4 times greater than those at 625 dbar (Hall 1989a, Table 4), leads to the dominant variance in $w$ at this depth. In order to prevent this overwhelming variance from completely dominating the EOF calculation, time series of $w$ at each level have been normalized by their rms values before computing the covariance matrix, so that it becomes the cross-correlation matrix.

Figures 2a,b show the first and second mode eigenvectors from this calculation, accounting for 42% and 22% of the variance, respectively. The solid curve is the directly computed eigenvector, while the dashed line is the eigenvector after multiplying by its rms at each depth (“reenergizing” the velocities), then normalizing the total eigenvector modulus to 1. Also shown is the percent of energy accounted for by that mode at each level. Thus, the first mode reflects primarily motions in the thermocline (upper 1000 m of the water column), with maximum relative variance at 625 dbar (solid) but maximum actual variance at 350 dbar (dashed). The second mode, on the other hand, reflects more the deep water variance, with a node at 625 dbar, and nearly equal and opposite actual amplitudes at the top (350 dbar) and bottom (5000 dbar). Unfortunately, the normalization has the effect of giving equal emphasis to the (perhaps incorrect) time series of $w$ at 980 and 2700 dbar. EOFs have thus been calculated for the thermocline and deep values alone, and the dominant mode (Fig. 2c) now clearly accounts for the negative correlation coefficients between $w$ at 350 and 5000 dbar. This mode is evidently a combination of the two displayed in Figs. 2(a,b), while the second mode (Fig. 2d) now reveals some bottom-trapped energy in vertical velocity.

The differences between the vertical velocity structure of the KS at 152$^\circ$E and the GS at 68$^\circ$W are striking evidence that two distinctly different dynamical regimes characterize the currents at these locations. Kinematic differences such as the vertical transport distribution were noted by Hall (1989b), and in section 5 a case is made for distinct energetic scenarios as well.

3. Conceptual model and motivations

The heat equation is a useful diagnostic for long term moored velocity and temperature measurements with good vertical resolution. In the previous section, it was used to obtain time series of vertical velocity. Multiplying the heat equation by $\vartheta^T / \theta_z$, and averaging, one obtains the equation for eddy potential energy ($P_E = \frac{1}{2} g a \theta^T / \theta_z$), viz.

$$
\frac{\partial}{\partial t} P_E + \bar{u} \cdot \nabla P_E + u' \cdot \nabla \left( \frac{g a T^T}{2 \theta_z} \right)
= -\bar{u} \cdot \nabla T^T + \frac{g a \vartheta^T}{\theta_z} - g a \vartheta^T t
$$

(3.1)

where

$$
\alpha = -\frac{\partial \rho}{\partial T} = -\left( \frac{\partial \rho}{\partial S} + \frac{\partial \rho}{\partial S} \frac{dS}{dT} \right);
$$

overbars denote time or ensemble means and primes deviations from that mean, subscript $H$ denotes horizontal component, and other notation is standard. The three terms on the lhs represents local growth (or decay) of $P_E$, advection by the mean flow, and self advection by eddies, respectively; terms on the rhs are generally interpreted as exchanges between mean and eddy potential energies (first term), and as exchanges between eddy potential and kinetic energies. In the equation for $P_E$ and $K_E$ (eddy kinetic energy) combined

$$
\frac{\partial}{\partial t} (K_E + P_E) + \bar{u} \cdot \nabla (K_E + P_E)
+ u' \cdot \nabla \left( \frac{1}{2} \left( \rho_0 (u'^2 + v'^2) + \frac{g a T^T}{\theta_z} \right) \right)
= -\nabla \cdot \left( \bar{u} \rho_0 \bar{p}' \right) - \bar{u} \cdot \nabla T^T + \frac{g a \vartheta^T}{\theta_z}
- \rho_0 \bar{u} \cdot \nabla (\bar{u} \cdot \bar{v}' - \bar{u} \cdot \bar{v}) - \rho_0 \bar{u} \cdot \nabla (\bar{u} \cdot \bar{v}') - \rho_0 \bar{u} \cdot \nabla (\bar{u} \cdot \bar{v}')
$$

(3.2)

(where subscript 3 denotes all 3 components) the $P_E \rightarrow K_E$ exchange term disappears, and the additional terms on the rhs may be interpreted as redistribution of energy by pressure work terms (first term on rhs) and conversions between $K_E$ and $K_M$ (mean kinetic energy) due to eddy momentum fluxes. Downgradient eddy heat fluxes ($- \bar{u} T^T \cdot \nabla \bar{T} > 0$) are a signature of baroclinic instability processes, while downgradient eddy momentum fluxes are associated with barotropic instability; thus these terms will be referred to as baroclinic (BC) and barotropic (BT) conversions, respectively. In the following, $u$ and $x$ will be used to refer to the alongstream flow direction, and $v$ and $y$ to the cross-stream direction. Several terms have been omitted from Eq. (3.2), including vertical momentum transfer due to small-scale shear instabilities, $-\bar{u} \cdot \nabla \bar{w}'$, and a term included by Dewar and Bane (1989b) resulting from spatial variation in the basic, "background" temperature gradient $\theta_e(x)$,
FIG. 2. (a) First- and (b) second-mode eigenvectors for vertical velocity structure using all five depths (see text); (c) first- and (d) second-mode eigenvectors using just three depths. Solid curve is directly computed eigenvector, dashed is "reenergized" eigenvector (see text).

\[
\frac{1}{2} \frac{g}{\partial \partial z} \left[ (\bar{u}'^2 + \bar{u}'T'^2) \alpha \frac{\partial^2 \bar{T}}{\partial x \partial z} + (\bar{v}'^2 + \bar{v}'T'^2) \alpha \frac{\partial^2 \bar{T}}{\partial y \partial z} \right].
\]

Various authors (Brooks and Niiler 1977; Rossby 1987; Dewar and Bane 1989b) have argued that the first of these terms is small in the GS in comparison to remaining terms. Rossby ignored the latter conversion, though Dewar and Bane found that for their analysis it was not negligible; rough estimates of this term for the KS at 152°E, however, suggest that it is typically an order of magnitude smaller than the leading terms.
in (3.2). In any case, both of these conversions would represent highly derived, noisy quantities based on the present dataset, and henceforth they are omitted from the discussion. The pressure work terms cannot be estimated from this dataset, since there is no way to separate the geostrophic and ageostrophic portions of the velocity field. These terms can act as a mechanism for redistribution of eddy energy locally within the current: Brooks and Niler provide a good discussion of their importance to the integrated eddy energy balance.

Now, Nishida and White (1982) have shown that on average at the surface in this part of the Kuroshio Extension, the sum of the last four terms on the rhs in (3.2) (i.e., the BT conversion) tends to be negative north of the axis of mean flow, positive to the south, but approximately zero when averaged across the current. The BC term may be calculated directly from time series of velocity and temperature at 350 and 625 dbar (see Appendix for details), where attention is restricted to the last 390 days. If BC > 0 (<0) and if we assume that this energy supply (deficit) is balanced in equation (3.2) by either temporal or spatial growth of eddy energy, the BC term can be estimated by dividing the rate of energy supply (or sink) by the total energy present. Since $P_E$ dominates $K_E$ by a factor of 3–6 at both depths, this estimate can be made simply as

$$\frac{1}{\Delta t} \sim \frac{-\bar{u} \bar{T} \cdot \nabla \bar{T}}{(1/4 T^2)} \Delta L \sim \frac{-\bar{u} \bar{T} \cdot \nabla \bar{T}}{(1/4 T^2)}.$$

Here, $U_o$ is mean advective downstream velocity while $\Delta t$ and $\Delta L$ represent the e-folding time or downstream distance for growth or decay of energy. Hall (1989a) has cautioned against interpreting such statistics as eddy heat fluxes from a mooring site such as WP05, in the vicinity of a dominant current like the KS, as actual time averages of processes occurring at a point location. Instead, they may be thought of as the biased result of asymmetric horizontal sampling of a flow with strong, coherent cross-stream variability. For example, Fig. 3 shows the alongstream velocity in stream coordinates as a function of $T_{350}$, and a linear least squares fit to the obvious trend. Clearly, the value of $\bar{u} \bar{T}$ will be large and will be dominated by this linear trend, an artifact of the regime being sampled. Nevertheless, it is instructive to carry out the above estimates. Table 2 (top) summarizes the results of these calculations, which imply that there is rapid decay of eddy energy in the KS, corresponding to decay times as short as 3–5 days, or downstream decay over scales of about 100 km; and indeed direct calculation of the total advection of $P_E$ from the (u, T) time series suggests that it is convergent. Yet, Nishida and White (1982) have shown the horizontal distribution of $K_E$ in the Kuroshio Extension region, and various authors (e.g., Talley and White 1987) that of $P_E$: all of these maps bear out that mooring site WP05 at 35°N, 152°E is a region where eddy energy is approximately constant or even growing in the downstream direction (these maps are representative of flow at the surface or at 300 m). In the coordinate frame rotating with the current (Table 2, bottom), the scale estimates are more reasonable, particularly for downstream decay; however, in this frame the BC conversion is indeed dominated by $u \bar{T} \bar{T}_x$, reflecting the overall linear trend of $u$ as a function of $T_{350}$. On the other hand, advection of $P_E$ is divergent in this frame (implying strong $K_E \rightarrow P_E$ conversion, based on the sign of the residual). Indeed, viewing the situation in a rotating coordinate frame only shifts energy between the apparent mean and eddy fields; that is, if the rotation is defined by

$$\hat{u} = u \cos(\phi(t)) + v \sin(\phi(t));$$
$$\hat{v} = v \cos(\phi(t)) - u \sin(\phi(t));$$

($\phi$ being the angle of rotation from east to alongstream, see Hall, 1989b), then

$$\frac{\partial}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y} = \frac{\partial}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} \frac{\partial T}{\partial y}.$$
by sorting the data into the same bins as Hall (1989a) used in the transport calculations. However, many of the bins contain far too few samples (3–10) to make reliable estimates of coherences between velocities and temperatures, even though transport estimates are not compromised, these being a much more robust calculation. Past investigations (Brooks and Nilss 1977; Schmitz and Nilss 1969; Webster 1961) suggest that different energetic regimes may exist in the cyclonic and anticyclonic portions of western boundary currents, so it seemed logical at least to split up the current meter data accordingly.

At this point some discussion is warranted concerning the method used to “synthesize” the current’s structure in the first place. Different methods have been used to parameterize the average cross-stream structure of current like the GS and KS (Hall and Bryden 1985; Hall 1989b; Hendry 1988; Hogg 1986, 1991). That used by Hall and Bryden is relatively simple, recognizing that at thermocline levels the cross-stream temperature gradient $\partial T / \partial y$ is a strong function of temperature, which can be integrated to find cross-stream position $y$ as a function of temperature at the chosen level. This approach has the unfortunate disadvantage of explicitly tying the temperature structure to cross-stream position at that level; additionally, $\partial T / \partial y$ near the cyclonic edge of the KS is quite weak, so that distances between isotherms there are not quite large—perhaps unrealistically so. For example, Fig. 4 shows one realization of the KS from a meridional CTD section at the time the mooring array was in the water, and the current meter data show that the current at the time was flowing southeast. Adjusting distances by a factor of $\cos 45^\circ = 0.72$ to account for the non-normal crossing, the distance from the $6.5^\circ$ to $11^\circ$ C isotherm at 350 dbar is about 73 km, and from $11^\circ$ to $16^\circ$ C about 63 km. These compare with “synthesized” widths of 104 and 43 km, respectively: clearly this inequality will affect results that are integrated over the two parts of the current. Again transport is only moderately affected since it is single signed throughout the current, and most of the contribution comes from the high speed core, where distances are better defined.

For comparison, then, a parameterization scheme like that used by Hogg (1986) to correct temperature mooring motion in the GS was applied to the KS data. Briefly, this method assumes that thermocline temperature can be described by a family of curves

$$T = A \tanh[B(p_r - p)] + C. \quad (3.3)$$

Parameters $A$, $B$, and $C$ are fit from a regression on observations (in this case, the daily $(T, p)$ values), and then $p_r$, which is a function of cross-stream position, can be determined for each day by solving (3.3) using the observed values of $(T, p)$. Finally, differentiating (3.3) with respect to $y$ and using thermal wind to evaluate the lhs, viz.,

$$f \frac{\partial u}{\partial p} = g \alpha \frac{\partial T}{\partial y} = g \alpha \frac{\partial}{\partial y} \left[ A \tanh[B(p_r - p)] \right] (p_r(y))$$

it is possible to determine the function

$$p_r(y) = R(p_r) \quad (3.4)$$

where the rhs is known from the data, and (3.4) is then integrated to obtain $p_r(y)$. Corrected temperature at 350 dbar using this method is very nearly a function of $p_r$, and hence $y$, alone just as in the original method; but the cross-stream positions associated with isotherms at 350 dbar are somewhat different from those deduced before. In particular, the cycloonic side is more compressed and the anticyclonic side just slightly stretched, so that their widths as described in the previous paragraph are respectively 69 and 55 km, in much closer agreement with the CTD data. The value of $11^\circ$ C of $T_{350}$ has been chosen as the division for the two sides of the current: in Fig. 3, $u$ is clearly increasing (cycloonic) up to this point, though exactly where it attains its maximum is somewhat ambiguous (see also Fig. 3 of Hall, 1989b). This choice also has the advantage of dividing the current into relatively equal portions, and Fig. 4 shows that this isotherm is centered in the strong gradient region at 350 dbar. In subsequent calculations, $\Delta y$ values for the cycloonic and anticyclonic portions of the current are taken as 69 and 55 km.

There are several other points worth noting about (3.1) before going on to the results of the calculations. First of all, the terms involving horizontal advection and exchanges can all be calculated directly from the data, and (for the moment ignoring $\partial P_r / \partial t$) the product $wT \theta_y \sim wT \theta_\theta$ would be obtained as a residual: this calculation is essentially the same one as that used.
To complete the picture of energetic exchanges in the current, one needs to consider also the BT exchange terms of (3.2). Only those terms involving cross-stream structure are accessible with the present analysis method, and scale analysis suggests that $v^{12}u_y \ll u^2v u_y$. Thus, only this one term has been investigated, though previous authors (Nishida and White 1982; Brooks and Niiler 1977) have shown that, e.g., $u^2u_x$ may contribute importantly as well. Schmitz et al. (1987) described the average structure of $u^2v$ along 152°E from the line of ten moorings there and found that it was negative north of the average KS location and positive south of there. In their more detailed energetic analysis, Nishida and White (1982) found a very complex distribution of $u^2v$, showing many sign reversals with both latitude and longitude. Only in taking zonal averages of this term does the above meridional structure emerge ($u^2v < 0$ north of the current axis, >0 to the south). However, they also found an important momentum flux attributable to the asymmetric structure of quasi-stationary meanders in the mean current flow, which flux is negative south of the mean axis and negligible to the north. In a study of Kuroshio surface kinetic energy south of Japan, Szabo and Weatherley (1979) found the opposite cross-stream structure for $u^2v$ (i.e., $u^2v > 0$ to the north), with the additional feature that the zero line tends to lie to the cyclonic side of the axis rather than be aligned with it.

The present dataset is clearly inadequate for addressing these intricacies of structure, and instead the momentum fluxes have been evaluated on the two sides of the current. Even within these large bins, the standard error of $u^2v$ is much greater than the values themselves, and the correlation coefficients $C_{uv}$ are typically 0.1–0.2. Comparison with time series of $(u, v)$ from other current meter records shows that these standard errors are usually large, even when $C$ is $O(0.3–0.4)$. Instead, another error estimate may be made based on the notion that sampled values of $u$ and $v$ contain no error, so that $\overline{uv}$ (time average of the total = mean + eddy momentum flux) contains no error; but individually the estimates of the mean and eddy components have a range of uncertainty due to the standard error of the estimated mean. That is,

$$u = \overline{u} + u', \quad v = \overline{v} + v', \quad \overline{uv} = \overline{\overline{u}v} = \overline{\overline{u}v} + \overline{u'v'}$$

where $\overline{u}$ and $\overline{v}$ are the true values of the mean and variable parts, but

$$\overline{u}_{est} = \overline{u} \pm S_E(\overline{u}), \quad u'_{est} = u' \pm S_E(u')$$

where "est" stands for "estimate." Thus,

$$\overline{u'v'_{est}} = \overline{u'v'} \pm S_E(\overline{u} + \overline{v})$$

and the estimated Reynolds stresses are in error by the product of the standard error of the mean velocities. These estimates are given in Table 4 and are used in determining the error ranges presented in Table 3 for the BT conversion (see section 4).
4. Results

Figures 5 through 7 visually summarize the results of the heat balance calculations, which are also tabulated in Table 3. Shown in the figures and table are the values of all the terms in (3.1), as well as the BT conversion, and their associated uncertainty ranges. The identifying labels used in the figures are defined precisely in the table; for example, “Mean” is the value of $\overline{u}_J \times \nabla_3 P_E$. The first five terms in the figures and tables are the five terms of (3.1), and all are evaluated as if they were on the lhs of the equation, while $\text{BT} = -u'v'u_J$ is from the rhs of (3.2). For consistency in nomenclature, since the terms BC and BT are meant to refer to conversion of mean to eddy energy when positive, “HEHF” (horizontal eddy heat flux) = $-\text{BC}$ is used to label this term. Thus, in summary, BC > 0 implies $P_M \rightarrow P_E$, BT > 0 implies $K_M \rightarrow K_E$, and VEHF > 0 implies $P_E \rightarrow K_E$.

Figures 5 and 6 show the cross-stream structure at both depths for stream and geographic coordinates, respectively, while Fig. 7 integrates results from the cyclonic and anticyclonic sides of the current. This average does not produce the same results as are obtained by averaging (3.1) over the whole time series, per the discussion in section 3. Also, note that in the figures the vertical scale at 625 dbar is twice as large as for 350 dbar. In Table 3, values that are significantly nonzero are marked with an asterisk.

Consider first the results of Fig. 5 and Table 3a, for the rotated (or stream) coordinates. “Loc.”, “Mean,” and “Eddy” represent temporal and advective changes to $P_E$, and their sum might be thought of as the total change of $P_E$ following a fluid parcel $D(P_E)/Dt$, while the remaining terms are the various energy exchanges. Figure 5 emphasizes the similarity at 350 and 625 dbar of the pattern of energy exchanges, and the contrast between the cyclonic and anticyclonic sides at both depths. On the cyclonic side, weak upgradient horizontal eddy heat flux ($P_E \rightarrow P_M$) is balanced mostly by negative vertical eddy heat flux ($K_E \rightarrow P_E$) at 350 dbar, and at 625 dbar by $D(P_E)/Dt$. Eddy advection is divergent at both depths while mean advection is negligible (and not significantly nonzero); however, the temporal change $\delta P_E/\delta t$ is of the same order as the other dominant terms in the balance, suggesting that insufficient sampling has been achieved to determine the long-term average balance of (3.1). Results are markedly different on the anticyclonic side of the current, where the average isotherm depth has dropped about 250 m from the cyclonic side (Fig. 4). BC is now strongly downgradient at both depths, with magnitudes approximately three times the upgradient values for $T_{350} < 11^\circ$C, indicating strong conversion of...
mean to eddy potential energy. Interestingly, vertical eddy heat flux is about the same size at both levels, so that at 625 dbar, all of this energy derived from $P_M$ is then converted to $K_E$ (VEHF $> 0$), and $D(P_E)/Dt$ is close to zero; but at 350 dbar, VEHF accounts for only about 40% of the balance, while the remaining portion feeds a net advective divergence, or downstream growth, of $P_E$. Furthermore, the value of $\partial P_E/\partial t$ at both depths is considerably less important in the balance of terms (it is completely negligible at 350 dbar), implying that this portion of the current has in fact been adequately sampled. In stream coordinates, the BT exchange is very small and significantly nonzero only at 350 dbar. These will be discussed further below.

For geographic coordinates (Fig. 6, Table 3b), there are two important differences that emerge in the calculations. First, error bars become much larger, particularly on the anticyclonic side, so that fewer of the estimates are significantly different from zero. Second, while the sign of virtually all terms remains the same, their relative sizes change dramatically, painting a much different picture of the $P_E$ balance (3.1) for the current. [Note that $\partial P_E/\partial t$ and the vertical eddy heat flux are necessarily almost identical for the two systems: their small differences are due to the fact that in stream coordinates samples are selected according to several “Kuroshio” criteria (see Hall 1989a) while in geographic coordinates they are not.] At both depths, an increase in mean convergence of $P_E$ in the cyclonic zone is offset by much stronger upgradient horizontal eddy heat flux, while at 350 dbar, the reverse occurs in the anticyclonic zone, though the changes are only half as large as on the cyclonic side. BT also increases at both depths, especially on the cyclonic side where it is now significantly nonzero at 625 dbar.

To summarize, then, in the cyclonic part of the current the balance is largely between relatively weak horizontal and vertical eddy heat fluxes such that eddy energy is being fed back into $P_M$ in stream coordinates; but in geographic coordinates, the dominant balance is between strong convergence of $P_E$ feeding relatively strong upgradient BC conversions, so that in either case there is a net loss of eddy to mean energy. On the anticyclonic side, there is a signature of active baroclinic instability processes for both coordinate systems, with energy derived from the mean flow (through BC $< 0$) feeding the eddy kinetic energy of the flow (VEHF $> 0$). At 350 dbar in rotated coordinates excess energy also feeds a net divergence of $P_E$ following fluid parcels, but in all other cases, $D(P_E)/Dt$ constitutes a somewhat weaker part of the balance.

Consider now how the relative size changes on the two sides of the current, going from rotated to geographic coordinates, affect the $P_E$ balance for the current as a whole. The terms of Tables 3a,b have been integrated over the whole current width, taking $\Delta y = (69 \text{ km}, 55 \text{ km})$ for the (cyclonic, anticyclonic) side, in accordance with the discussion in section 3. At 350 dbar, where all five terms of (3.1) change sign from the cyclonic to anticyclonic side of the current in both
coordinate systems, none of the integrated values is significantly nonzero. The balance of terms in the stream coordinates again suggests active baroclinic instability of the current ($BC < 0$), balanced by a growth of $P_E$ following fluid parcels, with little exchange between $K_E$ and $P_E$ occurring. That conclusion is consistent with the distribution of eddy energy in this region discussed in section 1, where it was pointed out that 152°E is a location of stable or growing eddy energy. However, for geographic coordinates the energy flow is reversed and is about three times as strong, with upgradient eddy heat fluxes ($P_E \rightarrow P_M$) fed by a mean convergence of $P_E$. At 625 dbar, there is little qualitative or quantitative difference between the stream and geographic coordinates, and $\partial P_E / \partial t$ is the dominant term in both cases, so that the overall averages are questionable in any case. At this depth in geographic coordinates, conversions to both $P_M$ and $K_E$ (via BC > 0 and VEHF > 0) are draining $P_E$ from the flow; in rotated coordinates, $P_M$ is feeding $P_E$ (BC < 0) but overall there is still a net loss to $K_E$.

The conclusions that would be obtained by calculating the terms of (3.1) over the whole current in just one bin—i.e., averaging over all samples such that $6^\circ C < T_{350} < 16^\circ C$—are very different indeed. These values are shown in Table 3a,b beneath the corresponding integrated values (again asterisked when significant). No BT conversion is given as this estimate requires cross-stream resolution. At 350 dbar, most of the terms are an order of magnitude larger for the one-bin values, and in all cases strong upgradient horizontal eddy heat flux is indicated (as was anticipated in Table 2), accompanied by equally strong negative VEHF, together implying an energy flow $K_E \rightarrow P_E \rightarrow P_M$. Net changes in $P_E$ following fluid parcels are divergent in the rotated case, convergent in the geographic coordinates, but overall somewhat weaker than the remaining term. Evidently it is at least as important to separate the cyclonic and anticyclonic zones as it is to rotate into stream coordinates.

Reynolds stresses and barotropic exchanges. The Reynolds stresses, horizontal shears and barotropic conversion estimates are presented in Table 4 for both coordinate systems and show perhaps the most striking differences between rotated and geographic coordinates of any of the terms thus far. Clearly, a simple calculation of $u'v'$ from time series at the mooring site includes contributions from both translation and meandering of the stream since much of the alongstream flow may actually be contained in $v$. Separation of samples into bins partially eliminates the translational effect, but rotation into stream coordinates is required to compensate for the changing current direction. Therefore, in geographic coordinates $u'v'$ represents net southeastward transport of northeastward momentum (or vice versa); in stream coordinates, the cross-stream velocity $\theta$ is a measure of motion relative to the coherent velocity/temperature structure of the
Table 3. Estimates of all terms in the equation for $P_{E}$ (3.1); also listed is the barotropic energy exchange. Units are $10^{14}$ ergs cm$^{-2}$ s$^{-1}$. Uncertainty ranges are in parentheses beneath the estimates, and significantly nonzero values are asterisked. Values are given for the cyclonic and anticyclonic sides of the current, their integrated (average) value, and the “one-bin” case described in the text.

<table>
<thead>
<tr>
<th></th>
<th>Local $(\partial P_{E}/\partial t)$</th>
<th>Mean advection $(\bar{u} \cdot \nabla P_{E})$</th>
<th>Eddy advection $(u' \cdot \nabla P_{E})$</th>
<th>HEHF $(\bar{w}'T' \cdot \nabla T')$</th>
<th>VEHF $(\bar{v} \cdot w'T')$</th>
<th>BT $(-u'w' \delta u/\partial y)$</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cyclonic</td>
<td>-6.90</td>
<td>-0.98</td>
<td>8.79*</td>
<td>4.76*</td>
<td>-5.66*</td>
<td>-1.48*</td>
</tr>
<tr>
<td>Anticyclonic</td>
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<td>-5.66*</td>
<td>-13.86*</td>
<td>5.95</td>
<td>-1.90*</td>
</tr>
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<td>-9.28, -2.05</td>
<td>-24.24, -5.78</td>
<td>-0.96, 13.43</td>
<td>-4.57, -0.37</td>
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<tr>
<td>“One-bin”</td>
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<td>45.3*</td>
<td>12.11, 5.97</td>
<td>8.40, 0.71</td>
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<td>-0.95, 7.37</td>
<td>9.94</td>
<td>0.19, 20.9</td>
<td>121.0, -78.5</td>
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</tr>
<tr>
<td>625 dbar</td>
<td>-5.19*</td>
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<td>1.72*</td>
<td>-0.12</td>
<td>-0.28</td>
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<td>Cyclonic</td>
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<td>1.68, 4.35</td>
<td>-6.36, 2.36</td>
<td>1.76, 4.35</td>
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<tr>
<td>Anticyclonic</td>
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<td>1.56</td>
<td>3.01*</td>
<td>4.16*</td>
<td>1.56, 4.35</td>
<td>1.76, 4.35</td>
</tr>
<tr>
<td>Integrated</td>
<td>-4.06, -0.44</td>
<td>-1.17</td>
<td>-0.89</td>
<td>1.48, 8.74</td>
<td>-0.08, 0.13</td>
<td>0.04</td>
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<tr>
<td>“One-bin”</td>
<td>-3.90, -0.86</td>
<td>2.29, 5.33</td>
<td>2.08, 0.34</td>
<td>0.34, 4.03</td>
<td>-0.38, 0.03</td>
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<td>-2.81*</td>
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<td>-13.55*</td>
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<td></td>
<td>-5.30, -0.32</td>
<td>-8.63, 10.56</td>
<td>4.63, 11.14</td>
<td>-19.74, -8.42</td>
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<tr>
<td>350 dbar</td>
<td>-6.68</td>
<td>-20.47*</td>
<td>10.23</td>
<td>22.20*</td>
<td>-5.29*</td>
<td>-19.41*</td>
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<tr>
<td>Cyclonic</td>
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<td>-36.83, -8.23</td>
<td>-15.70, 36.17</td>
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<td>-4.34</td>
<td>5.82</td>
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<td>“One-bin”</td>
<td>-7.82, 1.06</td>
<td>-10.08</td>
<td>3.39</td>
<td>10.43</td>
<td>-0.36</td>
<td>-9.37*</td>
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<td>-3.07</td>
<td>-24.34, 2.31</td>
<td>-11.50, 18.28</td>
<td>-0.88, 23.37</td>
<td>-4.68, 3.95</td>
<td>-15.77, -3.39</td>
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<tr>
<td></td>
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<td>-144.2, -3.8</td>
<td>145.73</td>
<td>-100.7*</td>
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<tr>
<td>625 dbar</td>
<td>-5.53*</td>
<td>-4.40</td>
<td>4.75</td>
<td>4.58*</td>
<td>0.42</td>
<td>-4.23*</td>
</tr>
<tr>
<td>Cyclonic</td>
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<td>-9.61, -0.72</td>
<td>-0.89, 10.39</td>
<td>1.97, 8.64</td>
<td>-1.14, 2.04</td>
<td>-6.56, -2.03</td>
</tr>
<tr>
<td>Anticyclonic</td>
<td>-2.17*</td>
<td>-0.05</td>
<td>1.77</td>
<td>-4.17</td>
<td>4.61*</td>
<td>0.11</td>
</tr>
<tr>
<td>Integrated</td>
<td>-3.94*</td>
<td>-2.47</td>
<td>3.43</td>
<td>0.70</td>
<td>2.89*</td>
<td>-2.40</td>
</tr>
<tr>
<td>“One-bin”</td>
<td>-5.92, -1.96</td>
<td>-5.98, 0.50</td>
<td>(0.02, 6.77)</td>
<td>-2.11, 3.64</td>
<td>(0.52, 4.18)</td>
<td>-3.74, -1.29</td>
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<tr>
<td></td>
<td>-2.85*</td>
<td>-8.92</td>
<td>6.95*</td>
<td>18.57*</td>
<td>-13.75*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5.36, -0.34</td>
<td>-22.41, 3.94</td>
<td>(2.30, 11.60)</td>
<td>(5.46, 33.41)</td>
<td>-20.02, -8.55</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Reynolds stresses, horizontal velocity shears, and barotropic energy conversions for the two sides of the current, at 350 and 625 dbar. Uncertainty ranges for energy conversions are in parentheses below estimates.

<table>
<thead>
<tr>
<th>$T_{350}$</th>
<th>Cyclonic</th>
<th>Anti-cyclonic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta y$ (km)</td>
<td>6°-11°C</td>
<td>11°-16°C</td>
</tr>
<tr>
<td>69</td>
<td>55</td>
<td></td>
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</tbody>
</table>

(a) BT conversions calculated for the rotating (or stream) coordinate system

350 dbar

$\overline{u'v'}$ (cm s$^{-2}$) | -16.86 ± 15.67 | 56.27 ± 35.62 |
$-\overline{\Delta u'dy}$ (cm s$^{-1}$) | 58.86 ± 6.81 | -18.11 ± 8.53 |
$-\overline{\partial u'v'dy}$ (s$^{-1}$) | 8.53 × 10$^{-6}$ | -3.29 × 10$^{-4}$ |
$-\rho \overline{\mu v'ddy}$ (ergs cm$^{-3}$ s$^{-1}$) | -1.48 × 10$^{-4}$ | -1.90 × 10$^{-4}$ |
(ers $^{-3}$ s$^{-1}$) | (-3.18, -0.12) × 10$^{-4}$ | (-4.57, -0.37) × 10$^{-4}$ |

625 dbar

$\overline{u'v'}$ (cm s$^{-2}$) | -7.03 ± 7.67 | -2.06 ± 9.02 |
$-\overline{\Delta u'dy}$ (cm s$^{-1}$) | 27.06 ± 4.26 | -1.40 ± 4.70 |
$-\overline{\partial u'v'dy}$ (s$^{-1}$) | 3.92 × 10$^{-6}$ | -2.55 × 10$^{-7}$ |
$-\rho \overline{\mu v'ddy}$ (ergs cm$^{-3}$ s$^{-1}$) | -0.28 × 10$^{-4}$ | 5.38 × 10$^{-7}$ |
(ers $^{-3}$ s$^{-1}$) | (-0.69, 0.03) × 10$^{-4}$ | (-7.93, 12.62) × 10$^{-6}$ |

(b) BT conversions calculated for the geographic coordinate system (x-axis rotated 35° south of east)

350 dbar

$\overline{u'v'}$ (cm s$^{-2}$) | -170.79 ± 71.40 | -142.96 ± 100.12 |
$-\overline{\Delta u'dy}$ (cm s$^{-1}$) | 76.34 ± 8.70 | -12.10 ± 9.36 |
$-\overline{\partial u'v'dy}$ (s$^{-1}$) | 11.06 × 10$^{-6}$ | -2.20 × 10$^{-8}$ |
$-\rho \overline{\mu v'ddy}$ (ergs cm$^{-3}$ s$^{-1}$) | -1.941 × 10$^{-4}$ | 3.23 × 10$^{-7}$ |
(ers $^{-3}$ s$^{-1}$) | (-30.65, -10.01) × 10$^{-4}$ | (0.22, 9.74) × 10$^{-4}$ |

625 dbar

$\overline{u'v'}$ (cm s$^{-2}$) | -66.81 ± 25.32 | -41.23 ± 30.08 |
$-\overline{\Delta u'dy}$ (cm s$^{-1}$) | 42.52 ± 5.33 | 1.45 ± 5.12 |
$-\overline{\partial u'v'dy}$ (s$^{-1}$) | 6.16 × 10$^{-6}$ | 2.64 × 10$^{-7}$ |
$-\rho \overline{\mu v'ddy}$ (ergs cm$^{-3}$ s$^{-1}$) | -4.23 × 10$^{-4}$ | -0.11 × 10$^{-4}$ |
(ers $^{-3}$ s$^{-1}$) | (-6.56, -2.30) × 10$^{-4}$ | (-0.87, 0.49) × 10$^{-4}$ |

5. Discussion

Complete energetic analyses of the KS and GS are difficult because they require statistical information on velocity and temperature not only at a single point, but as a well-resolved function of cross-stream distance and depth; additionally, though many authors have argued that components of the equation depending on downstream development of the currents are negligible, that is not proven to be the case, and ideally they would be included as well. Finally, pressure work terms are by nature ageostrophic and are hence very difficult to assess accurately from observations. The issue is particularly complicated in regions where the currents are no longer confined spatially to the western boundary but are free to meander; Hall (1986c) and Rossby (1987) have discussed the conceptual difference between the energetics of the current-containing region versus the internal dynamics and energetics of the current's own coherent velocity and temperature structure.
By comparing the results for a geographic and a stream coordinate system in the GS at 73°W, Rossby (1987) clearly illustrates these differences. The present analysis of the KS offers a similar—and far less detailed—comparison for this current: one important difference lies in the derivation of the geographic coordinate system, which in this case is spatially locked to the current axis and translates with it, even though it does not capture the changing orientation of the current (as does the stream coordinate system). Also, the present analysis is for a region well downstream of the current’s coastal separation, while 73°W is only 200 km northeast of Cape Hatteras. Nevertheless, the two studies are similar in concept, and it is worthwhile comparing results for the KS and GS, in geographic and stream coordinates.

Other energetic analyses of the GS exist, especially south of Hatteras (e.g., Dewar and Bane 1985) and in the Florida Straits (Webster 1961; Schmitz and Niiler 1969; Brooks and Niiler 1977); Hall (1986b) assessed pointwise barotropic and baroclinic conversions in the GS at 68°W, also from a single current meter mooring. Recently, Dewar and Bane (1989b) have combined all these results (as well as new estimates based on current meter data near Hatteras) to suggest a picture of downstream development of the GS eddy field from Florida to 68°W, and they conclude that the Cape Hatteras region is unique in its eddy–mean flow interactions with respect to regions both upstream and downstream, possibly because it is the location where the GS begins to move into deep water. This conclusion should be borne in mind as the KS results at 152°E are compared with Rossby’s (1987) work, because the KS location is well downstream from coastal separation.

What is immediately obvious from any of these analyses is the elaborate structure of the BC and BT conversion terms as functions of position in the current: therefore, pointwise estimates such as Hall’s (1986b) must be interpreted with caution if trying to assess the integral energetic balances of the currents. It is also noteworthy that in both currents, energetic conversions in geographic coordinates may be considerably larger than their counterparts in stream coordinates, as found by Rossby and the present analysis. Rossby explains the reduction of the cross-stream contributions as due to the “stiffness” of the current in stream coordinates, and of the downstream contributions as due to the uniformity of current structure downstream. Inferred growth or decay rates based on energetic conversions in geographic coordinates turn out to be just a few days for both the KS and GS analyses.

In comparing the more detailed structure of the energetics, consider first the BC conversion terms. Using his direct velocity and temperature measurements, Rossby evaluated only the cross-stream contribution to this quantity, in the present notation, \( \nu T' \dot{\theta}_Y \). Although he warns that the downstream component could be large, Dewar and Bane’s (1989b) direct estimates in fact support the dominance of the cross-stream term. In the KS, where both components have been estimated, the net is generally dominated by \( \dot{v} T' \dot{\theta}_Y \), since the large-scale trend is eliminated by separation of the current into its cyclonic and anticyclonic sides. [In the Florida Current, on the other hand, Brooks and Niiler (1977) found that the alongstream component dominated.] In the GS, Rossby shows a ridge of maximum \( P_M \rightarrow P_E \) BC conversion, which more or less follows the sloping main thermocline and is approximately aligned with a similar ridge in temperature variance (though not exactly \( P_E \)). In geographic coordinates, the ridge decays in amplitude with depth, and the BC conversion is positive almost throughout the entire current; in stream coordinates, the ridge values are much smaller and less surface intensified, and the overall distribution shows substantial areas of negative conversions as well. Hall’s (1986b) and Dewar and Bane’s (1989b) results suggest the presence of such a ridge as well, with BC values that are lower at 400–600 m depth than at 875 m depth, in the anticyclonic part of the current. In the KS, the spatial coverage is inadequate for detecting any ridge, although the geographic results do suggest increasing importance of BC at 625 dbar relative to values at 350 dbar on the anticyclonic side (Table 3). An important obvious difference for the geographic coordinate results are the strong negative values of BC at 350 dbar on the cyclonic side of the current: it is precisely in this part of the GS that Rossby finds maximum positive values. In stream coordinates, the upper level dominates the positive BC conversion in the anticyclonic zone of the KS, in contrast to Rossby’s “ridge,” showing dominant values of BC at a depth of 800–1000 m. While the evidence suggests baroclinic instability is active in both currents, in that area average values of BC tend to be positive, the exchanges in the GS are approximately an order of magnitude greater than those in the KS (O(10^{-3}) ergs cm^{-3} s^{-1}) versus O(10^{-2} ergs cm^{-3} s^{-1}), for stream coordinates.

A comparison of the barotropic conversions must be prefaced with several cautionary notes. First of all, the error bars on the calculations for the KS are very large so that, as we have seen, most of the estimates are barely significant. On the other hand, previous work (Nishida and White 1982; Schmitz et al. 1987) supports the general patterns of \( u' \dot{v}' \) deduced here. Second, only the cross-stream contribution \( -\dot{u}' \dot{v}' \), \( \dot{u}_Y \), has been computed, both here and by Rossby (1987). Dewar and Bane (1989b) find this to be the dominant contribution to BT near 73°W, but Nishida and White’s (1982) analysis of near-surface \( K_E \) in the Kuroshio Extension suggests other terms may be important at 152°E. Thus, there is no a priori reason to suppose that the distributions of \( -\dot{u}' \dot{v}' \) at these two locations should resemble one another. In fact, they are quite different, once again emphasizing the complexity of eddy–mean flow interactions for western boundary currents.
As in the case of the baroclinic conversions, the magnitudes of BT in the KS are generally an order of magnitude smaller than their GS counterparts (except at 350 dbar, cyclonic side, for geographic coordinates). In both currents and in both coordinate systems, the conversions are rather strongly surface-intensified, with values at (350, 575) dbar 3–10 times larger than those at (625, 875) dbar in the (KS, GS) [Hall (1989b) shows that these levels are comparable in terms of velocities and transports for the two currents.] Unlike BC, however, area averaged BT conversions are of opposite sign in the two currents, being positive for the GS but negative for the KS. That is, while it has been possible to characterize this region of the KS as one of a fully developed baroclinic instability process, with eddies feeding energy back into the mean flow barotropically, the GS near Cape Hatteras (and, apparently, at 68°W) is both baroclinically and barotropically unstable. That these differences exist is not altogether surprising: 152°E is downstream of the primary maximum in KS mean and eddy kinetic energy, upstream of a secondary maximum (Nishida and White 1982), while 73°W and even 68°W are still upstream of the GS maximum at 63°W. Satellite imagery of this area clearly shows that the eddy field of the GS continues to develop considerably downstream (Cornillon 1986).

It is instructive to consider the patterns of BT throughout the currents, since they are essentially the result of the Reynolds stress distributions $\overline{u'v'}$ when only the cross-stream contribution to BT is estimated. As briefly mentioned in section 4, the net barotropic conversion in the current will be from $K_e \rightarrow K_M$ when the Reynolds stresses converge momentum into the downstream flow, and from $K_M \rightarrow K_e$ when the flux is divergent. Note that $\overline{u'v'}$ need not change sign across the current: in the latter case, e.g., $\overline{u'v'}$ could be negative across the entire current, but as long as $\partial(\overline{u'v'})/\partial y > 0$ (for an eastward current) eddy momentum is divergent. In fact, that was precisely the case found by Hall (1986b) at 68°W, so that north of the current axis BT was less than 0 and south of the axis it was positive, and had a larger magnitude. In the KS, $\overline{u'v'}$ is also less than zero throughout the current for geographic coordinates (for comparison with Hall), but here it is stronger north of the axis, leading to stronger values of BT < 0 there that overcome positive BT values to the south. In stream coordinates the Reynolds stresses remain convergent, becoming positive at 350 dbar on the anticyclonic side, so that the resulting net values of BT remain negative. Rossby’s analysis of the GS is markedly different from both of the above, as well as from energetic studies in the Florida Current, which tend to show that though local BT conversions are significant, they largely cancel when integrated across the current. In both geographic and stream coordinates, Rossby (1987) shows BT > 0 everywhere in the GS (except for small areas, at depth and at the offshore edge, of weakly negative values for the geographic frame). This distribution implies not only that $\overline{u'v'}$ is into the current on both sides, but that the sign change is precisely aligned with the stream axis at all depths, since $\overline{u_v}$ also changes sign there. This result seems rather remarkable in view of the variety of $\overline{u'v'}$ distributions observed both upstream and downstream of Cape Hatteras, particularly since it holds in both coordinate systems.

6. Conclusions

This investigation of eddy–mean flow interactions in the Kuroshio Extension, and comparison of results with the Gulf Stream, demonstrates important differences between the two currents, and raises some interesting questions concerning more subtle interconnections with external influences on the current—for example, the nature of the deep flow. Hall (1989b) pointed out kinematic differences between these two currents, primarily in terms of their respective transport distributions: while total transports were comparable, the KS was found to be more barotropic, while the GS had a substantially deeper thermocline expression. The present work extends the comparison to identify dynamical and energetic differences as well.

Calculations of vertical velocity and its vertical structure at mooring site WP05 (152°E, KS) and at the GUSTO mooring site (68°W, GS), suggest that the deep flow may be an important factor in the dynamics of these currents. In the GS, vertical velocities are strongly, positively correlated throughout the water column, and it is possible to view the vertical motion as a response to a shift of the current structure up or down the topographic slope (Hall 1986b). In the KS, there is significant upward mean vertical velocity near the bottom, and variations in $w$ there are negatively correlated with those at 350 dbar ($C = -0.25$). Both of these currents appear to extend to the bottom on average, and it is possible that deep vertical velocities reflect dynamical interactions with whatever deep flow underlies the current. Because of the strong coherent structure of the overlying currents, these deep vertical velocities may then play a role in the internal current dynamics. The structure of $w$ in the thermocline is also markedly different for the two currents. Although in a kinematic sense it was shown that levels of (350, 625) dbar in the KS were comparable to levels of (575, 875) dbar in the GS, dynamically that appears not to be the case: $w_{rms}$ in the GS exhibits a midthermocline maximum at 875 dbar, but in the KS, $w_{rms}$ is largest (by a factor of 3) at 350 dbar. Hall (1989b) also observed that while cross-stream velocity variance $\overline{u'^2}$ is barotropic in the GS, in the KS it is barotropic only below 350 dbar, and its value at 350 dbar exceeds that below, also by a factor of three.

In the analysis of eddy–mean flow interactions in the KS, evaluating terms for two sides of the current helps to clarify the results by eliminating the effect of
a large-scale horizontal trend. Again, the interactions appear to be dominated by processes at 350 dbar, where in stream coordinates there is evidence for baroclinic instability feeding net growth of $P_E$ following fluid parcels; energy conversions also suggest a net flow of some energy at $O(10^{-4} \text{ ergs cm}^{-3} \text{ s}^{-1})$ through the system, viz., $P_M \rightarrow P_E \rightarrow K_E \rightarrow K_M$. In geographic coordinates, on the other hand, there is apparent strong upgradient eddy heat flux in the cyclonic part of the current balanced primarily by mean convergence of eddy potential energy, both of which nearly vanish in stream coordinates, suggesting the latter is a more "natural" reference frame for the current energetics.

The results of the energetic analysis in the KS have been compared with a similar analysis by Rosby (1987) for the GS at 73ºW. As a function of both depth and coordinate system, all the energetic conversions are uniformly nearly an order of magnitude weaker in the KS than in the GS, despite the fact that mean and eddy kinetic energies are roughly the same for the two currents (Hall 1989b). While the GS at Cape Hatteras may admittedly be unique in its eddy--mean flow exchanges, these same general comments also apply to comparisons with a few pointwise values of baroclinic and barotropic conversions near 68ºW by Hall (1986b). These locations—152ºE in the KS and 68ºW in the GS—have been established as geographically comparable for the two currents in terms of downstream distance from coastal separation (Schmitz and Holland 1986), but they clearly represent different energetic regimes. Indeed, the composite of energy analyses that have been performed at different locations in both the KS and GS demonstrate that patterns of eddy--mean flow interaction exhibit complex spatial distributions.

The variety of patterns is likely to depend in some manner on factors external to the boundary currents themselves. In the GS system, the maximum in mean kinetic energy is attained well downstream of coastal separation at Cape Hatteras, probably somewhere near 63ºW ($K_E$ may yet increase farther downstream); in the KS, on the other hand, the primary maximum is attained just after coastal separation at 143ºE, and a weaker, secondary maximum occurs near 158ºE (Nishida and White 1982, their Fig. 2). Since the nature of the eddy--mean flow interactions is likely to depend strongly on the background mean flow, it might be anticipated that energetics would be different for the two currents at comparable alongstream locations. Certainly it would be informative to investigate KS energetics near coastal separation: another important difference between the KS and GS is the nature and existence of any deep western boundary currents that must cross under them. Cape Hatteras appears to be such a crossover point in the North Atlantic, but the existence of an analogous deep current has yet to be verified in the North Pacific. Thompson and Schmitz (1989) have suggested that the existence of the deep western boundary current in the North Atlantic affects the separation of the GS from the coast. The present analysis shows that the thermocline and deep flows are evidently dynamically coupled through the vertical velocity structure, but the explicit effect of the deep flow on the overlying currents has yet to be determined.

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## Appendix

### 1. Vertical velocity calculations

The original data records of $(u, v, T)$ at site WP05 existed at nominal depths of 200, 500, 1200, 4000 and 6000 m. Vertical motion of the mooring was considerable, so Hall (1989a) corrected both velocities and temperatures at the two uppermost instruments, where pressure was also recorded, to fixed pressures of 350 and 625 dbar, respectively.

Now, vertical velocities may be calculated diagnosed from the heat equation as

$$w = -\left(\frac{\partial T}{\partial t} + \frac{\rho f}{g \alpha} \left( v \frac{\partial u}{\partial z} - u \frac{\partial v}{\partial z} \right) \right) / \frac{\partial \bar{z}}{} \quad (A.1)$$

where

$$\alpha = -\frac{d \rho}{d T} = -\left( \frac{d \rho}{d S} \frac{d S}{d T} + \frac{d \rho}{d T} \frac{d S}{d T} \right)$$

and horizontal temperature gradients are estimated from measured velocity shear. The velocity correction scheme applied by Hall (1989a) used either exponential or linear extrapolation/interpolation for each velocity component, depending on whether measured values of $u$, e.g., had the same sign at both depths (exponential) or opposite signs (linear). Two values of $(u, P)$ in the vertical define either an exponential or linear velocity profile, and consistent with that profile, the values of shear $\partial u / \partial z$, $\partial v / \partial z$ in (A.1) were determined at 350 and 625 dbar. (Note that for a linear profile, shear will be the same for both depths.)

Also, the temperature corrections involved defining an analytic form for the basic structure of $\theta$ between 200 and 900 dbar, based on CTD data, which also depended on the measured value of $T_{350}$. Thus, on a given day, the instantaneous value of $T_{350}$ was used to compute a profile $\theta(z)$, so that $\partial \theta / \partial z$ in (A.1) could be evaluated for both thermocline depths. Local time change of temperature at each depth was a simple centered finite difference. Equation (A.1) may also be used to calculate time series of $w$ at depths midway between two instruments, and this was done in the lower 5 km of the water column. Originally, no corrections were made to records at the 1200 m instrument, but because this instrument probably moved vertically as much as
the upper two, the motion could significantly affect values of \( \frac{\partial T}{\partial t} \), so that \( w \) would reflect mooring motion rather than water velocities. Assuming a fixed difference of pressure (~710 dbar) between the nominal 500 and 1200 m instruments, measured \((P, T)\) at the former and \( T \) at the latter may be used to define an exponential temperature profile between 625 and 1335 dbar. Then, \( w \) halfway between the two (nominally 980 dbar) can be calculated from

\[
w = \left[ \frac{\partial T}{\partial t} \right]_{980} + \frac{\rho_0 f}{g \alpha} \left( \frac{v_{625} u_{1335} - v_{1335} u_{625}}{700 \text{ m}} \right) \left( \frac{\partial \theta}{\partial z} \right)^{-1} \left|_{980} \right. \quad (A.2)
\]

In (A.2), \( \alpha = 2.6 \times 10^{-4} \text{ gm cm}^{-3} \text{ oC} \) (from CTD data).

Finally, time series at 2700 and 5000 m were generated from the 1335, 4000, and 6000 m records by the simple formulation

\[
w = -\left[ \frac{\partial}{\partial t} \left( \frac{T_i + T_{i+1}}{2} \right) \right] + \frac{\rho_0 f}{g \alpha} \left( \frac{v_{0} u_{i+1} - u_{0} v_{i+1}}{\Delta z} \right) \left( \frac{\partial \theta}{\partial z} \right)^{-1} \quad (A.3)
\]

where \( \alpha = (1.79, 1.4) \times 10^{-4} \text{ gm cm}^{-3} \text{ °C}^{-1} \) and \( \frac{\partial \theta}{\partial z} = (4.23 \times 10^{-4}, 7 \times 10^{-3}) \text{ °C m}^{-1} \) at (2700, 5000) m, respectively, as determined from CTD data.

2. Heat balance calculations

The heat balance calculations are a simple extension of the calculations made in (A.1), since it is necessary only to multiply that equation by \( T' \), and take the time average, to obtain Eq. (3.1) except for the factor \( g \alpha / \theta' \). Energy exchanges are expected to be important only at the upper two levels, so attention was restricted to these in evaluating (3.1). To make the calculations, first time series of \((T_x, T_y, \theta_x)\) at 350 and 625 dbar were generated consistent with the formulation described above. These were used in conjunction with the time series \((u, v, T, w)\) to estimate terms in (3.1). When making the calculations in the two temperature bins, \(6.5^\circ C < T_{350} < 11^\circ C\) and \(11^\circ C < T_{350} < 16^\circ C\), first the time series were subsampled, and observations within each bin were grouped together. These observations were used to define mean values of all quantities, which were then subtracted to find the “eddy” portion. These eddy portions were used in the usual way to calculate eddy heat fluxes and other correlations required to estimate advective terms (e.g., \( T' T'' x \) to calculate \( \bar{u} \bar{b} (T'' x^2) / \partial x \)). The rms values of variables for each side of the current were also calculated to use in estimating a standard error for the mean values.

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