NOTES AND CORRESPONDENCE

The Open Boundary Condition in the United Kingdom Fine-Resolution Antarctic Model

DAVID P. STEVENS

School of Mathematics, University of East Anglia, Norwich, England

5 June 1990 and 21 February 1991

ABSTRACT

The United Kingdom Fine-Resolution Antarctic Model (FRAM project) is a community program to study the Southern Ocean. Central to this is an eddy-resolving three-dimensional primitive-equation ocean general circulation model. The open boundary condition at the northern boundary is described here. The boundary condition is based on that of Stevens (1990).

1. Introduction

FRAM is a United Kingdom community program whose aim is to study the Southern Ocean. The program is based around an eddy-resolving three-dimensional primitive-equation ocean general circulation model. Further details of the model can be found in the article by the FRAM group (1991). To stay within the bounds set by available computer power and memory the project needed to study a limited area (albeit a large one) rather than the World Ocean. There have been a number of requests for information regarding the treatment of open boundaries in this limited-area model. It is the purpose of this short paper to fulfill those requests.

If the model was perfect and there was an exact knowledge of all the fields at open boundaries, then these values could be simply prescribed. This would lead to a perfectly well-posed mathematical problem. Unfortunately models are not perfect and it is unlikely that good enough boundary data will ever be available. Thus, some form of unphysical condition is required at these open ocean boundaries. At points on the boundary where there is inflow, physical quantities will be advected inward. Here it is possible to specify climatological mean values for these fields, as they should give a reasonable estimate. However, difficulties occur in regions where the flow or wave propagation is out of the domain of interest. Prescribed values can cause problems. Fluid advected toward the boundary could have a different density to that on the boundary. This can lead to large variations in density at the boundary, which in turn could produce large unrealistic pressure gradients. There is also a need to make sure any signal traveling toward these open boundaries is transmitted and not reflected back into the region of interest.

The open boundary condition described here is based on that of Stevens (1990). However, that article considered idealized flat-bottomed oceans with steady surface forcing. This paper illustrates the use of the open boundary condition with complex bottom topography and variable surface forcing.

2. The model equations

The open boundary condition was constructed for use in an ocean general circulation model based on that of Cox (1984) in which the equations of motion, temperature, and salinity are solved on a finite-difference grid. The full equations are

$$\frac{\partial u}{\partial t} + \Gamma(u) - fv = -\frac{1}{\rho_0 a \cos\phi}\frac{\partial p}{\partial \lambda} + F^u, \qquad (1)$$

$$\frac{\partial v}{\partial t} + \Gamma(v) + fu = -\frac{1}{\rho_0 a}\frac{\partial p}{\partial \phi} + F^v, \qquad (2)$$

$$\frac{\partial T}{\partial t} + \Gamma(T) = K_h\frac{\partial^2 T}{\partial z^2} + A_h\nabla^2 T, \qquad (3)$$

$$\Gamma(1) = 0, \quad \frac{\partial p}{\partial z} = -\rho g, \quad \rho = \rho(\theta, S, z) \qquad (4)$$

where

$$\Gamma(\mu) = \frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}(u\mu) + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(v\mu\cos\phi) + \frac{\partial}{\partial z}(w\mu),$$

$$F^u = K_m\frac{\partial^2 u}{\partial z^2} + A_m\left(\nabla^2 u + \frac{(1-\tan^2\phi)u}{a^2} - \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial\lambda}\right),$$

Corresponding author address: Dr. D. P. Stevens, University of East Anglia, School of Mathematics, Norwich NR4 7TJ, England.

$$F^v = K_m \frac{\partial^2 v}{\partial z^2} + A_m\left(\nabla^2 v + \frac{(1-\tan^2\phi)v}{a^2} + \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial\lambda}\right),$$

$$\nabla^2(\mu) = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\mu}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{\partial\mu}{\partial\phi}\cos\phi\right).$$

The variables ϕ, λ, z, u, v, w, p, ρ represent latitude, longitude, depth, zonal velocity, meridional velocity, vertical velocity, pressure, and density, respectively. The radius of the earth is a, g is the acceleration due to gravity, ρ_0 is a reference density, and $f = 2\Omega\sin\phi$ is the Coriolis parameter where Ω is the speed of angular rotation of the earth. The variable T represents any tracer including active tracers such as potential temperature θ and salinity S or passive tracers such as tritium. The terms A_m and A_h are the horizontal-mixing coefficients for momentum and tracers; K_m and K_h are the corresponding vertical-mixing coefficients.

The equations of motion are rearranged to form an equation for the barotropic streamfunction ψ

$$\left[\frac{\partial}{\partial\lambda}\left(\frac{1}{H\cos\phi}\frac{\partial^2\psi}{\partial\lambda\partial t}\right) + \frac{\partial}{\partial\phi}\left(\frac{\cos\phi}{H}\frac{\partial^2\psi}{\partial\phi\partial t}\right)\right] - \left[\frac{\partial}{\partial\lambda}\left(\frac{f}{H}\frac{\partial\psi}{\partial\phi}\right) - \frac{\partial}{\partial\phi}\left(\frac{f}{H}\frac{\partial\psi}{\partial\lambda}\right)\right] = -\left[\frac{\partial}{\partial\lambda}\left(\frac{g}{\rho_0 H}\int_{-H}^{0}\int_{z}^{0}\frac{\partial\rho}{\partial\phi}dz'dz\right) - \frac{\partial}{\partial\phi}\left(\frac{g}{\rho_0 H}\int_{-H}^{0}\int_{z}^{0}\frac{\partial\rho}{\partial\lambda}dz'dz\right)\right] + \left[\frac{\partial}{\partial\lambda}\left(\frac{a}{H}\int_{-H}^{0}F^v - \Gamma(v)dz\right) - \frac{\partial}{\partial\phi}\left(\frac{a\cos\phi}{H}\int_{-H}^{0}F^u - \Gamma(u)dz\right)\right] \quad (5)$$

where

$$-\frac{1}{a}\frac{\partial\psi}{\partial\phi} = \int_{-H}^{0} u dz, \quad \frac{1}{a\cos\phi}\frac{\partial\psi}{\partial\lambda} = \int_{-H}^{0} v dz.$$

Details of the equations for the baroclinic velocities and the method of solution have been given by Cox (1984). The equations are solved by finite differences using an Arakawa B grid.

3. Open boundary conditions

The values of variables at open boundary points can be described by the equations of motion if the model boundary is extended. Thus, it seems sensible to use these equations or at least their dominant terms to describe variables at open boundaries. The equations for the barotropic mode, baroclinic mode, and tracers are now discussed separately. Following the notation of Cox (1984), the northernmost row in the finite-difference scheme for streamfunction and tracer points is row $J = \text{JMT}$. The usual model calculations take place for rows $J < \text{JMT}$. For velocity points the northernmost row is $J = \text{JMT} - 1$ and the usual model calculations take place for rows $J < \text{JMT} - 1$.

Providing a condition for the barotropic streamfunction gives the most difficulty. There are very few useful simplifications of the elliptic equation (5) describing the streamfunction that are tractable along an open boundary. In FRAM the open boundary is located in midlatitudes at 24°S. Thus, it is appropriate to use the Sverdrup balance

$$\frac{\partial\psi}{\partial\lambda} = -\frac{a}{2\Omega\cos\phi}\left(\frac{\partial}{\partial\phi}(\tau^\lambda\cos\phi) - \frac{\partial\tau^\phi}{\partial\lambda}\right) \quad (6)$$

to specify the streamfunction at row $J = \text{JMT}$. The flat-bottom balance is used in preference to the topographic balance because stratification reduces the effect of topography (Gill 1982, p. 511). The wind forcing in FRAM is seasonal. Although (6) is a steady equation, the barotropic mode responds quickly to any change in forcing, and thus this balance provides a reasonable estimate. It is assumed that there is no net northward transport into the Atlantic, Indian, and Pacific oceans. A western boundary current of thickness 230 km is imposed to satisfy this condition. The transport varies linearly across this boundary layer from zero at the coast to a maximum value at its outer edge.

Current measurements are extremely variable and difficult to obtain accurately. Therefore, it is not profitable to use them to set the vertical variation of velocity. A further disadvantage is that the velocity field could be set such that it is inconsistent with the density field. These two quantities are closely related through the equations of motion, so inconsistencies are undesirable. Fixing the baroclinic velocity field with some form of extrapolation could also lead to the same problem. With this in mind the baroclinic part of the velocity field at the open boundary is calculated from the following linear form of the equations of motion (1) and (2):

$$\frac{\partial u}{\partial t} - fv = -\frac{1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial\lambda} + F^u, \quad (7)$$

$$\frac{\partial v}{\partial t} + fu = -\frac{1}{\rho_0 a}\frac{\partial p}{\partial\phi} + F^v. \quad (8)$$

Neglect of the nonlinear terms is justified by the fact that they are generally small in comparison with other terms in the equations. However, there are certain regions in the ocean where the nonlinear terms are important, so it is sensible to choose open boundaries in positions away from such regions of intense activity, as is the case in FRAM. The neglect of these terms is useful as unknown values of u, v, and w which lie outside the boundary are no longer required. The diffusion terms in (7) and (8) require a single unknown value. Without these terms the solution is unstable, thus, they are retained by extrapolating from the boundary. A convenient method, as it imparts no shear, is to set the value of u and v outside the model domain equal to the value on the boundary. Equations (7) and (8) are then solved by the same procedure as (1) and (2).

Finally the value of tracer quantities on the boundary need to be calculated. If the velocity at a certain level on an open boundary is directed out of the region, then it is likely that the values of tracers on the boundary are determined from inside the domain of interest. Thus, tracers are calculated from a simplification of Eq. (3) for tracers, which allows a tracer quantity to be advected or diffused out of the interior. For simplicity, tracers are only allowed to be advected out of the model region perpendicular to the boundary. The equation used for this is

$$\frac{\partial T}{\partial t} = -\frac{v + c_T}{a}\frac{\partial T}{\partial \phi} + K_h \frac{\partial^2 T}{\partial z^2} + A_h \nabla^2 T. \quad (9)$$

The retention of the vertical diffusion term [both here and in Eqs. (7) and (8)] allows the surface boundary condition at the boundary to be treated in exactly the same way as in the interior. The correction c_T to the advective term is required because the formulation of the tracer equation at the boundary inhibits the propagation of internal waves. This can then lead to a piling up of wave energy at the boundary. An idealized experiment by Stevens (1990) illustrates this point.

The correcting phase speed c_T is calculated from the finite-difference form of the equation

$$\frac{\partial T}{\partial t} = -\frac{c_T}{a}\frac{\partial T}{\partial \phi} \quad (10)$$

at the previous time step for points just inside the boundary. This approach follows Orlanski (1976) in allowing plane waves to pass through the open boundary. If v and the diffusive terms are neglected in (9), an Orlanski-type radiation condition for T is left. However, simple idealized experiments have shown this is not sufficient to correctly model the behavior of T. Using forward and upstream differencing in (10), the correcting phase speed becomes

$$c^n_{Ti,k} = -\frac{a\Delta\phi}{\Delta t}\frac{T^n_{i,\mathrm{JMT}-1,k} - T^{n-1}_{i,\mathrm{JMT}-1,k}}{T^{n-1}_{i,\mathrm{JMT}-1,k} - T^{n-1}_{i,\mathrm{JMT}-2,k}} \quad (11)$$

for $0 \leqslant c^n_{Ti,k} \leqslant a\Delta\phi/\Delta t$. Stability considerations dictate that $c^n_{Ti,k}$ is set to its bounds if it exceeds them. The upper numerical stability bound is reached at less than 0.5% of boundary points during the FRAM integration.

The finite difference form of the equation for tracers is

$$\frac{T^{n+1}_{i,\mathrm{JMT},k} - T^n_{i,\mathrm{JMT},k}}{\Delta t} = -\frac{c^n_{Ti,k} + v^n_{i,\mathrm{JMT}-1,k}}{a}\frac{T^n_{i,\mathrm{JMT},k} - T^n_{i,\mathrm{JMT}-1,k}}{\Delta\phi} + F^{Tn}_{i,\mathrm{JMT},k} \quad (12)$$

where $F^{Tn}_{i,\mathrm{JMT},k}$ is the usual diffusion term. The advective term is written in a nonconservative form using upstream differencing, which enables it to be retained without having to use values that would otherwise be unknown. Forward differences are used instead of leapfrogging for time differencing. No problems occur with the mixing of modes. The value of a tracer at the unknown point just outside the model domain is set equal to that on the boundary for the calculation of the diffusion term. In the above, if $v < 0$ then it is set equal to zero. If neither c_T or v are greater than zero (outward) then the following procedure is used.

If the velocity on the boundary is into the domain of interest then tracers can be set to a prescribed value. However this can lead to undesirable discontinuities in the vicinity of the boundary when the direction of the velocity at the boundary changes from outward to inward. To eliminate this problem, tracers are smoothly forced toward prescribed values when the velocity is into the region of interest by using the following equation

$$\frac{\partial T}{\partial t} = \frac{1}{\alpha}(T_b - T). \quad (13)$$

In (13) T_b is the prescribed boundary value of T and α is a time scale for the relaxation of T to T_b. The values of T_b are taken from the Levitus (1982) atlas. The time scale α is 360 days. This is chosen to correspond to the relaxation time scale used in the initial spinup of the interior thermohaline fields [see the FRAM Group (1991) for details].

It is of interest to quantify how often Eq. (9) is used in preference to Eq. (13), and under what circumstances. The average of four time steps from FRAM are examined. The results are presented in Table 1. Equation (9) is used at 74.5% of grid points and Eq. (13) at the remaining 25.5% of locations. It is clear that the correction term c_T plays a major role in determining the solution at the boundary.

4. Conclusion

The formulation of the open boundary condition in the United Kingdom FRAM model has been described. To demonstrate its effectiveness some examples of FRAM output are presented. Figure 1 shows the velocity normal to the open boundary in the Indian Ocean, between Africa and Australia, at 2556 and 2922

TABLE 1. The percentage of grid points at which the four possible inflow and outflow criteria hold.

Criteria	Percent of grid points
$c_T > 0$ and $v \leqslant 0$	17.5
$c_T \leqslant 0$ and $v > 0$	22.5
$c_T > 0$ and $v > 0$	34.5
$c_T \leqslant 0$ and $v \leqslant 0$	25.5

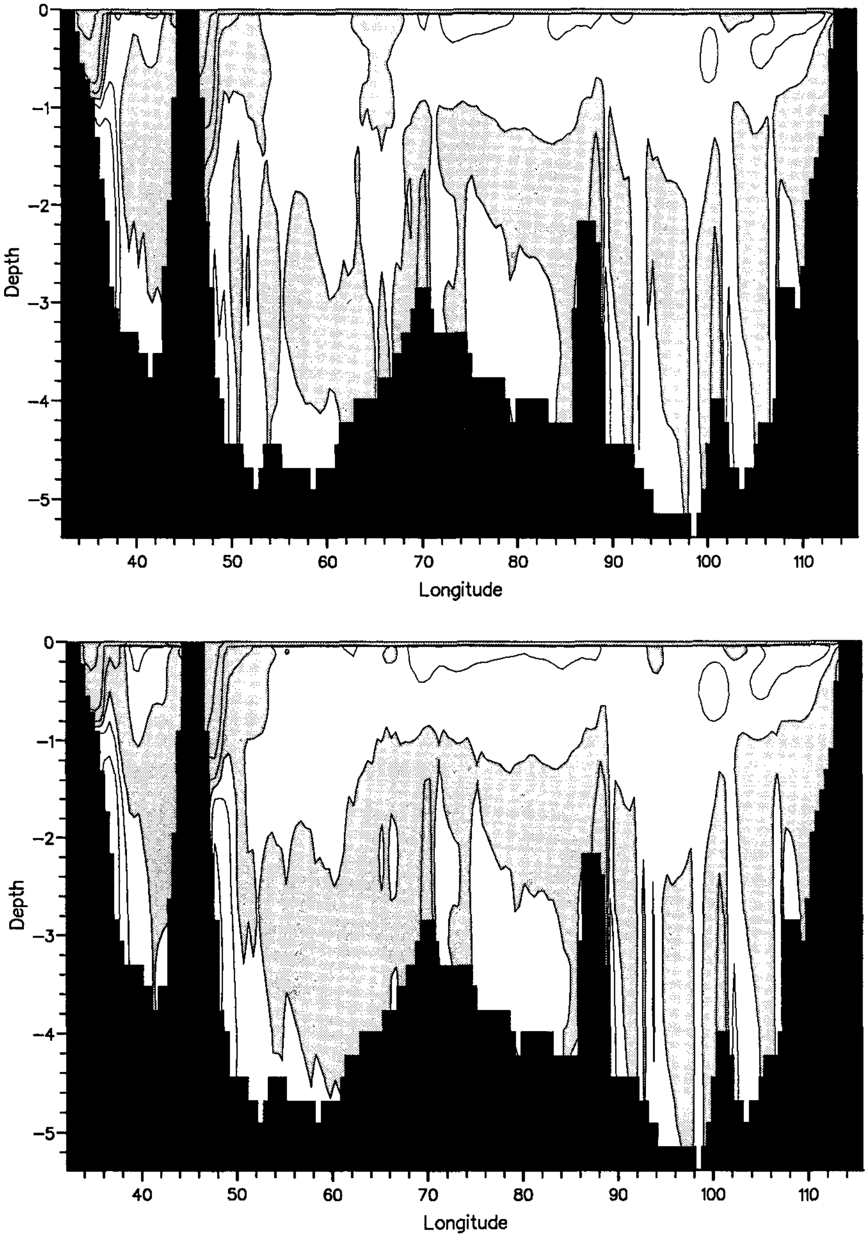

FIG. 1. The velocity normal to the open boundary in the Indian Ocean at (a) 2556 days and (b) 2922 days. Southward flow is shaded. Contours are at 0, ±1, ±3, ±10 cm s^{-1}.

days. The two times are exactly a year apart, so the surface forcing is identical. There are a number of features that occur at both times. At the surface there is a southward-flowing Ekman layer. Strong currents occur at the western boundary and over the large topographic features. However, there are a number of differences in the regions of inflow and outflow. This illustrates that the direction of the current at the boundary can readily change as the model integration proceeds.

Figure 2 depicts the streamfunction in the region of the Pacific Ocean open boundary after 2750 days. There is a broad northward flow in a gyre that reaches entirely across the ocean. A southward-flowing western boundary current, to the east of Australia, ensures that the net northward transport into the Pacific Ocean is zero. To the south of New Zealand, the Antarctic Circumpolar Current takes the form of an intense narrow jet with strong meanders. Farther east the current is broader, except where it passes over the Pacific–Ant-

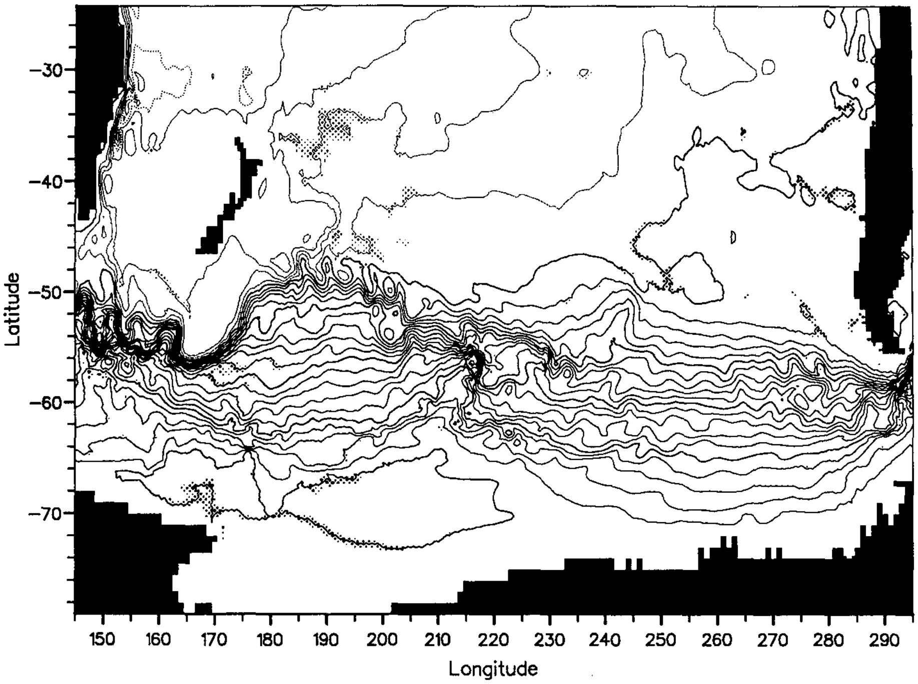

FIG. 2. The streamfunction after 2750 days in the region of the Pacific Ocean open boundary. The contour interval is 10 Sv, negative values are dashed, bold contours occur every 50 Sv.

arctic ridge. The current becomes narrow and intense again as it passes through the Drake Passage. Figure 3 shows the potential density at 120 m and 2922 days in the vicinity of the Indian Ocean open boundary. The boundary has regions of both inflow and outflow, which can be seen in Fig. 1b. Without resorting to a larger model as a control, it is difficult to state conclusively that the open boundary condition has performed successfully. However, it is clear from the aforementioned two figures that there is no irregular or unphysical be-

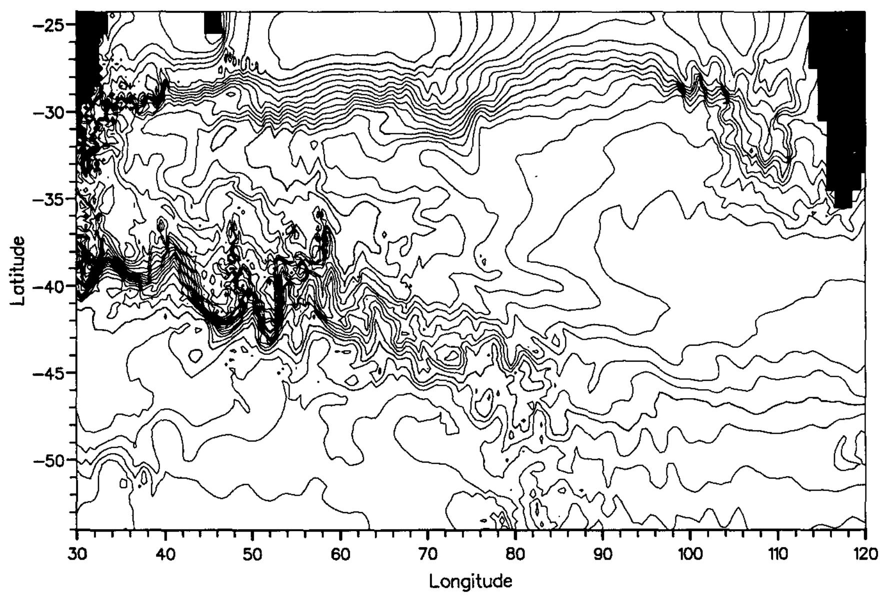

FIG. 3. The potential density at 120 m after 2922 days in the vicinity of the Indian Ocean open boundary. The contour interval is $0.1\sigma_\theta$, every tenth contour is bold.

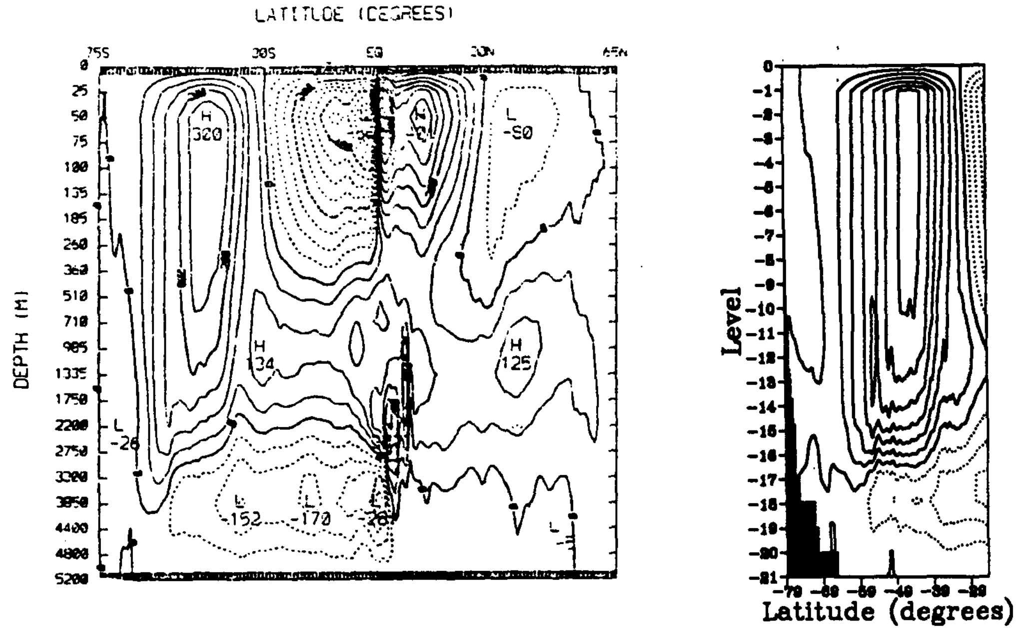

FIG. 4. The meridional overturning streamfunctions from (a) the global model of Semtner and Chervin (1988) and (b) FRAM after 2191 days. The contour interval in both cases is 5 Sv, with negative contours dashed. Labeled values in (b) are units of 0.1 Sv.

havior in the region of the open boundary. Experience has shown that when problems occur at open boundaries the resulting solutions can be spectacular.

Although running a larger control experiment is not feasible,[1] it is possible to make comparisons with larger models of other researchers. Figure 4 illustrates the meridional overturning streamfunction of (a) Semtner and Chervin (1988) and (b) FRAM after 2191 days (interpolated onto the same vertical grid as the former for comparison purposes). Even if the open boundary condition were perfect, one would not expect exact agreement between the two models, since the model resolution, mixing coefficients, and configurations are different. However, they agree well. All the main cells of the global model that fall within the FRAM area are well represented. In fact, they agree well right up to the boundary. This would certainly not be the case with a buffer-zone-type treatment of the boundary. The large cell between 30° and 60°S is of comparable magnitude, 30 Sv (Sv ≡ 10^6 m^3 s^{-1}) in the global model and 35 Sv in FRAM. It would appear that this important part of the large-scale circulation is largely unaffected by the presence of the open boundary. Indeed, the results presented here suggest that the boundary condition is performing well and can continue to be used with confidence.

Acknowledgments. This work would not have been possible without the help of many members of the FRAM group, in particular Peter Killworth and John Johnson. I would also like to thank an anonymous referee who, while making life difficult, considerably improved the paper. The support of NERC Grant GST/02/408 is gratefully acknowledged.

[1] It took over a year of work on a Cray XMP to produce the results shown here.

REFERENCES

Cox, M. D., 1984: A Primitive Equation, 3-Dimensional Model of the Ocean. GFDL Ocean Group Tech. Rep. No. 1, GFDL, Princeton, NJ, 144 pp.

Gill, A. E., 1982: *Atmosphere–Ocean Dynamics.* Academic Press, 662 pp.

Levitus, S., 1982: Climatological atlas of the world ocean. NOAA Prof. Pap., 13, U.S. Dept. of Commerce, 173 pp.

Orlanski, I., 1976: A simple boundary condition for unbounded hyperbolic flows. *J. Comput. Phys.,* **21,** 251–269.

Semtner, A. J., and R. M. Chervin, 1988: A simulation of the global ocean circulation with resolved eddies. *J. Geophys. Res.,* **93,** 15 502–15 522.

Stevens, D. P., 1990: On open boundary conditions for three-dimensional primitive equation ocean circulation models. *Geophys. Astrophys. Fluid Dyn.,* **51,** 103–133.

The FRAM Group, 1991: Initial results from a fine resolution model of the Southern Ocean. *Eos, Trans. Amer. Geophys. Union,* **72,** 169, 174–175.