The Baroclinic Structure of the Abyssal Circulation*

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ABSTRACT

A simple baroclinic model of the abyssal ocean circulation is formulated in which the reversals of the meridional velocity with depth, and hence the layering of the abyss, is explained by the *longitudinal* variation of upwelling into the main thermocline.

Since the barotropic meridional velocity is connected to the local upwelling velocity by the Sverdrup relation, regions of weak upwelling have meridional velocity fields that are essentially baroclinic. The baroclinic velocities are driven by thermal anomalies that propagate westward by stationary diffusive Rossby waves from regions of relatively strong upwelling in the eastern portion of the basin. These dynamically driven, internally generated vertical velocities produce the layered baroclinic structure in the western interior of the basin. A simple linear model, continuous in the vertical, is developed to illustrate these elements of the conceptual picture.

1. Introduction

Our present understanding of the abyssal oceanic circulation still rests on the elegantly simple model presented by Stommel and Arons (1960a,b, hereafter SA) in a series of papers published over 30 years ago. In their model cold water, occupying the bulk of the oceanic interior, is heated and rises slowly through the base of the thermocline and, participating in the circulation of the upper waters, flows to a few narrow sinking regions where it is cooled and returned to the abyss. The circulation is closed by a network of deep western boundary currents and the Antarctic Circumpolar Current. At the time the theory was presented, the suggestion of interior upwelling into the thermocline was motivated by early thermocline theories that featured the upwelling of cold water to balance the downward diffusion of heat. Our current picture of the thermocline is of a domain more advective than diffusive (e.g., Luyten et al. 1983), but it is at least plausible that in a transition zone between the vigorous thermocline and the sluggish abyss some such balance may arise.

To keep matters as simple as possible, SA imagined the interior upwelling at the base of the thermocline to be horizontally uniform. However, they were aware (SA 1960b) that, theoretically at least, the upwelling should be more intense in the eastern part of the ocean where the thermocline vertical scale is least since the estimate of w_*, the deep vertical velocity of upwelling, is

$$w_* = \frac{\kappa}{h}, \qquad (1.1)$$

where κ is the vertical mixing coefficient of temperature and h is the vertical scale depth of the temperature (in reality the heat flux) at the thermocline's base. Almost all thermocline theories predict h to increase westward from an eastern minimum in each oceanic basin.

The triumphant simplicity of the Stommel–Arons theory, capped by the subsequent observation of the hypothesized deep western boundary currents, makes it an important building block in ocean circulation dynamics. However, as Warren (1981) points out in his excellent review of the subject, "This model was never intended as a realistic description of the deep ocean circulation, and in at least two respects it would be a qualitatively bad one . . . because the model is barotropic, it cannot allow layered deep flow."

The SA model treats the entire abyss as a single homogeneous layer. In the model, the upwelling velocity linearly increases from the (flat) ocean bottom to the base of the thermocline. This yields a depth-independent stretching of planetary vorticity columns. Under the assumption of planetary-scale geostrophy, this uniform stretching must be balanced by a poleward meridional flow independent of depth. The observed layering of the abyss (e.g., the interleaving tongues of Antarctic Intermediate Water, North Atlantic Deep Water, and Antarctic Bottom Water in the Atlantic) is unexplained by the SA model.

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The estimate (1.1) can be rewritten

$$\frac{w_*}{\kappa} = \frac{\partial^2 T/\partial z^2}{\partial T/\partial z} + \frac{1}{\kappa}\frac{\partial \kappa}{\partial z}, \qquad (1.2)$$

so that in principle vertical variations in the upwelling might be due to vertical variations in the turbulent mixing coefficient as well as due to vertical variations in the scale height of the temperature gradient. Although variations in w_* due to vertical variations in κ cannot be excluded, I focus instead on a process related to the large-scale structure of the forcing.

In this paper I suggest that the layering of the interior meridional velocity is due to the longitudinal variation of the upwelling into the thermocline. The physical explanation can be grasped by considering an extreme example. Suppose all the upwelling in an ocean basin is restricted to a region eastward of a longitude ϕ'. From the Sverdrup relation, the vertically averaged meridional transport satisfies

$$\beta \int v dz = f w_*, \qquad (1.3)$$

where the integral is over the depth of the abyssal region. If w_* is locally zero, that is, west of ϕ', then the net transport is zero. However, the temperature disturbance produced by eastern upwelling will tend to propagate westward due to the β effect. In a steady flow this yields an arrested baroclinic Rossby wave in which the westward penetration of the thermal anomaly is balanced by, say, local thermal diffusion. Thus, west of ϕ' the temperature anomaly will yield an internally generated w field and hence a meridional flow. However, (1.3) guarantees that west of ϕ', the average v is zero in the abyss, that is, the meridional velocity must be layered.

In the remainder of this paper, I present a model based on simple linear physics first introduced by Lineykin (1957) and subsequently used by Pedlosky (1969) and Gill (1985) to study the thermocline. In this model the flow is entirely geostrophic, hydrostatic, and incompressible. The heat equation consists of a vertical motion rising against a fixed, average temperature gradient balanced by vertical diffusion of the associated thermal anomaly.

The model is highly simplified and is clearly inadequate at the equator and in western boundary current regions. It is employed here only to show how the verbal argument given above can be realized in a continuous, self-consistent model in which w_* is smooth. It also gives us an opportunity to estimate the degree of layering to be expected. Naturally the weakness of the model is in its linearized thermodynamics. In the abyss, lateral variations of temperature are not very much smaller than vertical variations. Obviously this makes problematic the direct application of the quantitative results that follow. It is, therefore, important to keep in mind that the basic result rests on the most robust feature of the dynamics, namely, the tendency for western propagation of the baroclinic anomalies by the Rossby wave mechanism.

2. The model

a. Formulation

Consider an ocean basin of overall depth D, which occupies a longitudinal extent $\phi_w \leqslant \phi \leqslant \phi_e$. The temperature, T_*, in the abyssal layer $-D \leqslant z \leqslant 0$ is

$$T_* = T_0 + \Delta T_v\left(1 + \frac{z}{D}\right) + (\Delta T_H) T(\phi, \theta, z). \qquad (2.1)$$

The mean temperature gradient $\Delta T_v/D$ is considered to be much larger than that due to the horizontally varying temperature anomaly ΔT_H. This anomaly scale, ΔT_H, is related to the scale W_* of the upwelling imposed at $z = 0$ by

$$\Delta T_H = \frac{W_* 2\Omega R^2}{\alpha g D^2} \qquad (2.2)$$

by application of the thermal wind balance and a scaling for the horizontal velocity of $U = W_* R/D$. Here R is the earth's radius, g is the acceleration of gravity, Ω is the earth's rotation, and α is the coefficient of thermal expansion. The independent role of salinity in affecting the density is ignored. If the pressure anomaly is scaled to hydrostatically balance the temperature anomaly, the nondimensional equations of motion become

$$fv = \frac{1}{\cos\theta}\frac{\partial p}{\partial \phi},$$

$$fu = -\frac{\partial p}{\partial \theta}$$

$$T = \frac{\partial p}{\partial z}$$

$$\frac{1}{\cos\theta}\left[\frac{\partial}{\partial\theta}(v\cos\theta) + \frac{\partial}{\partial\phi}u\right] + \frac{\partial w}{\partial z} = 0,$$

$$wS = \frac{E_T}{2}\frac{\partial^2 T}{\partial z^2}, \qquad (2.3a,b,c,d,e)$$

where $f = \sin\theta$ and

$$S \equiv \frac{\alpha g \Delta T_v D}{4\Omega^2 R^2}$$

$$E_T = \frac{\kappa}{\Omega D^2}. \qquad (2.4a,b)$$

Note that $S = N^2 D^2/4\Omega^2 R^2$, where N is the Brunt–Väisälä frequency.

At the lower boundary,

$$w = T = 0, \quad z = -1 \qquad (2.5)$$

is given, while at the upper boundary, representing the base of the thermocline,

$$w = w_*(\phi, \theta), \quad z = 0$$
$$T = 0. \tag{2.6}$$

It would be easy to generalize (2.6) to include nonzero temperature anomalies at $z = 0$, but they are ignored for simplicity.

The parameter ratio

$$\frac{E_T}{S} = \frac{\kappa 4\Omega R^2}{N^2 D^4} \tag{2.7}$$

is chosen to be O(1). This implies that the Lineykin depth

$$d_L = \left[\frac{\kappa f_*^2}{\beta_* N^2} R\right]^{1/4} \tag{2.8}$$

is of the order of the thickness of the abyssal layer, D. [In (2.8), $f_* = 2\Omega \sin\theta$ and $\beta_* = 2\Omega \cos\theta / R$.]

It is useful at this point to describe the problem in terms of Fourier vertical modes. With the boundary conditions (2.5) and (2.6), it is natural to use the representation

$$\begin{Bmatrix} w \\ T \end{Bmatrix} = \sum_{n=1}^{\infty} \begin{Bmatrix} W_n \\ T_n \end{Bmatrix} \sin n\pi z, \tag{2.9a}$$

$$\begin{Bmatrix} u \\ v \\ p \end{Bmatrix} = \sum_{n=0}^{\infty} \begin{Bmatrix} U_n \\ V_n \\ P_n \end{Bmatrix} \cos n\pi z, \tag{2.9b}$$

where, of course,

$$\begin{Bmatrix} U_n \\ V_n \\ P_n \end{Bmatrix} = 2\epsilon_n \int_{-1}^{0} dz \begin{Bmatrix} u \\ v \\ p \end{Bmatrix} \cos n\pi z,$$

$$\begin{Bmatrix} W_n \\ T_n \end{Bmatrix} = 2 \int_{-1}^{0} dz \begin{Bmatrix} w \\ T \end{Bmatrix} \sin n\pi z, \tag{2.10a,b}$$

and

$$\epsilon_n = \begin{cases} 1, & n > 0 \\ 1/2, & n = 0. \end{cases}$$

Multiplying (2.3a,b,d) by $\cos n\pi z$ and (2.3c,e) by $\sin n\pi z$ and integrating over the depth yields

$$fV_n = \frac{1}{\cos\theta} \frac{\partial P_n}{\partial \phi},$$

$$fU_n = -\frac{\partial P_n}{\partial \theta},$$

$$-n\pi P_n = \epsilon_n T_n,$$

$$\frac{1}{\cos\theta}\left[\frac{\partial}{\partial\theta}(V_n \cos\theta) + \frac{\partial U_n}{\partial\phi}\right] + 2\epsilon_n w_* + n\pi\epsilon_n W_n = 0,$$

$$W_n = -n^2\pi^2 \frac{E_T}{2S} T_n. \tag{2.11a,b,c,d,e}$$

b. The barotropic mode (n = 0)

The model solution for $n = 0$ yields the barotropic solution for the horizontal velocity, that is, for $n = 0$

$$T_0 = 0,$$

$$V_0 = \frac{f}{\beta} w_*,$$

$$U_0 = \frac{1}{f} \int_{\phi}^{\phi_e} \frac{\cos\theta}{\beta} d\phi' \frac{\partial}{\partial\theta}(f^2 w_*) \tag{2.12a,b,c}$$

where I have used the boundary condition of zero zonal flow on $\phi = \phi_e$. The solution for the barotropic mode is precisely the same as the SA solution. In particular, note that the barotropic meridional velocity is given entirely in terms of the *local* value of w_*. As long as $w_* > 0$, the barotropic velocity is always poleward.

c. The baroclinic modes (n > 0)

The baroclinic fields have $T_n \neq 0$. The continuity equation yields, in conjunction with the geostrophy and hydrostatic balance,

$$W_n = -\frac{2}{n\pi} w_* - \frac{\beta}{n^2\pi^2 f^2 \cos\theta} \frac{\partial T_n}{\partial\phi}, \tag{2.13}$$

which with the thermal equation (2.11e) yields an equation for T_n, namely,

$$\frac{\partial T_n}{\partial\phi} - a_n(\theta) T_n = -\frac{2n\pi f^2}{\beta} \cos\theta w_*, \tag{2.14}$$

where

$$a_n = \frac{n^4\pi^4 \cos\theta}{2\beta} \frac{E_T}{S} f^2. \tag{2.15}$$

Recall that in these units $\beta = \cos\theta$ and $f = \sin\theta$. The solution of (2.14) in which θ enters only parametrically and which satisfies $U_n = 0$ on $\phi = \phi_e$ is

$$T_n = \int_{\phi}^{\phi_e} e^{a_n(\phi-\phi')} \frac{2n\pi f^2}{\beta} \cos\theta w_* d\phi' + C_n e^{-a_n(\phi_e-\phi)}, \tag{2.16}$$

where C_n is an arbitrary constant. Before further discussion of the baroclinic solution, it is useful to point out that (2.14) is a balance between the westward propagation effect due to β (the first derivative term), the vertical diffusion (the term proportional to a_n), and the thermal forcing produced by w_*. It is clear from this equation as well as its solution, (2.16), that the baroclinic fields are *not* locally related to w_* but depend on the distribution of w_* east of the point under consideration.

Note that C_n in (2.16) is *not* determined by the condition that U_n (or $\partial T_n/\partial\theta$) vanish on the eastern boundary. Instead, an additional condition must be used to determine C_n.

d. The integral condition

Suppose that our ocean basin is girdled by a boundary Γ through which, at several locations, mass enters to represent the entry of cold water formed in polar regions. These sources must deliver an amount of water equal to the volume of water leaving at the upper level of the abyss due to upwelling into the thermocline. Suppose also there are J such sources, each yielding locally a velocity normal to the boundary (see Fig. 1) $u^{(j)}\cdot\mathbf{n}$. The *inward* transport of each source is

$$S^{(j)} = \iint dzdls^{(j)}(\phi, \theta, z), \quad j = 1, 2, \cdots J, \tag{2.17}$$

where

$$s^{(j)} \equiv -u^{(j)}\cdot\mathbf{n}, \tag{2.18}$$

and in particular, each $s^{(j)}$ can be represented in the Fourier cosine series:

$$s^{(j)} = \sum_{n=0}^{\infty} s_n^{(j)} \cos n\pi z. \tag{2.19}$$

When the continuity equation (2.11d) is integrated over the basin containing the flow,

$$2\epsilon_n \iint w_* \cos\theta d\phi d\theta + n\pi\epsilon_n \iint W_n \cos\theta d\phi d\theta - \sum_{j=1}^{J} \oint s_n^{(j)} dl = 0. \tag{2.20}$$

In (2.20) I have used the fact, easily verified, that only the *interior* vertical velocity and not the vertical velocity in western boundary currents is significant in the overall mass balance. A similar integral condition was derived in Pedlosky (1969); see also Tziperman (1986).

The balance obtained from the $n = 0$ term yields the integral balance of SA, which equates the overall upwelling to the sources of cold water formed in narrow sinking zones, that is, for $n = 0$

$$\iint w_* \cos\theta d\phi d\theta = \sum_{j=1}^{J} \oint s_0^{(j)} dl \equiv \sum_{j=1}^{J} s^{(j)}. \tag{2.21}$$

This allows us to relate the net upwelling strength to estimates of the deep water formation rates (or vice versa), as is done in SA (1960b). Otherwise, it places no constraint on the solution.

The integral condition for $n > 0$ becomes, using (2.11e)

$$2\iint w_* \cos\theta d\theta d\phi - \sum_{j=1}^{J} \oint s_n^{(j)} dl = \frac{n^3\pi^3 E_T}{2S} \iint T_n \cos\theta d\phi d\theta. \tag{2.22}$$

If (2.16) is used in (2.22), this yields an algebraic condition for each C_n in terms of integrals over w_* and each $s_n^{(j)}$. Thus, C_n is determined *in principle*. Unfortunately, as is evident from (2.22), the solution for each C_n will depend upon each Fourier coefficient of each source term, $s_n^{(j)}$, and these are entirely unknown.

To avoid the dependence of the solution on the unknown (and at this point, arbitrary) vertical structure of the source terms, the integral condition for the baroclinic modes is replaced by the condition that the temperature *anomaly* vanish on the eastern boundary. This requires

$$C_n = 0 \tag{2.23}$$

and thus completely determines T_n. It then follows that (2.22) can be satisfied by considering (2.22) as a set of algebraic equations for the vertical structure of the sources, that is, for the modal distributions $s_n^{(j)}$. This seems no more artificial than specifying $s_n^{(j)}$ to determine C_n and is considerably simpler.

e. The solution

For $C_n = 0$, the solution for U_n, V_n, T_n, and W_n can be written for $n > 0$,

$$\left.\begin{aligned}
V_n &= 2\frac{f}{\beta} w_* - 2\frac{f}{\beta} a_n \int_{\phi}^{\phi_e} e^{a_n(\phi-\phi')} w_* d\phi' \\
T_n &= \int_{\phi}^{\phi_e} 2n\pi \frac{f^2}{\beta} \cos\theta w_* e^{a_n(\phi-\phi')} d\phi' \\
W_n &= -2\frac{a_n}{n\pi} \int_{\phi}^{\phi_e} e^{a_n(\phi-\phi')} w_* d\phi' \\
U_n &= \frac{1}{f}\frac{\partial}{\partial\theta} \int_{\phi}^{\phi_e} 2\frac{f^2}{\beta} \cos\theta w_* e^{a_n(\phi-\phi')} w_* d\phi'
\end{aligned}\right\}, \tag{2.24a,b,c,d}$$

which, with the barotropic mode (2.12), completes the solution.

3. An example

Consider the upwelling distribution

$$w_* = KW_* e^{-s(\phi_e-\phi)}. \tag{3.1}$$

The upwelling is a maximum on the eastern boundary and decays westward at a rate dependent on s. If

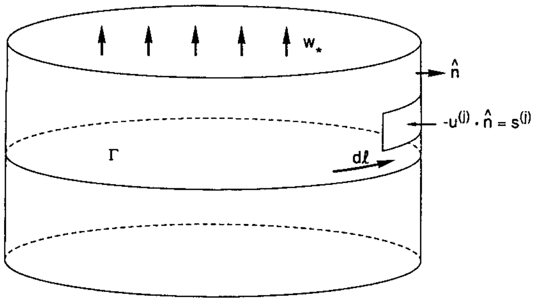

FIG. 1. A schematic of the ocean basin in the abyss. An upwelling, w_*, into the mean thermocline is balanced by boundary sources $s^{(j)}$ representing the entry of deep water formed in sinking zones adjacent to the basin interior. The contour girdling the basin is Γ, and its outward normal is **n**.

$s = 0$, the simple SA case of uniform upwelling is again obtained. The constant K is chosen such that

$$\int\int_{\phi_w}^{\phi_e} w_* \cos\theta d\theta d\phi = W_* \int\int_{\phi_w}^{\phi_e} \cos\theta d\theta d\phi;$$

namely,

$$K(s) = (\phi_e - \phi_w)s/(1 - e^{-s(\phi_e-\phi_w)}). \qquad (3.2a,b)$$

Hence, by varying s, only the distribution of upwelling is changed, and not its strength. Using (3.1) in (2.12) and (2.24), we obtain (after a partial resum of the series for w to improve convergence)

$$w = W_* K\Bigg[e^{-s(\phi_e-\phi)}(1 + z) + \sum_{n=1}^{\infty} \frac{2\sin n\pi z}{n\pi(s - a_n)}\{se^{-s(\phi_e-\phi)} - a_n e^{-a_n(\phi_e-\phi)}\}\Bigg] \qquad (3.3a)$$

$$v = \frac{f}{\beta} W_* K\Bigg[e^{-s(\phi_e-\phi)} + \sum_{n=1}^{\infty} \frac{2\cos n\pi z}{(s - a_n)}\{se^{-s(\phi_e-\phi)} - a_n e^{-a_n(\phi_e-\phi)}\}\Bigg], \qquad (3.3b)$$

while for the temperature anomaly

$$T = W_* K \sum_{n=1}^{\infty} \frac{2n\pi f^2}{(s - a_n)} \times \{e^{-a_n(\phi_e-\phi)} - e^{-s(\phi_e-\phi)}\} \sin n\pi z, \qquad (3.3c)$$

where a_n is given by (2.15). Note that a_n is a function of latitude.

For very small stratification, a_n becomes enormous, and the reader may quickly verify that the solution reduces to the homogeneous solution of SA. Figure 2a shows v and w as a function of depth for the case where

$$\alpha \equiv \frac{\pi^4 E_T}{2S} = \frac{\pi^4 \kappa 2\Omega R^2}{N^2 D^4} \qquad (3.4)$$

is 10^3. The meridional velocity is depth independent, while w is a linear function of z. The sums in (3.3a,b) are entirely negligible. The solutions in this limit are local; that is, they depend only on the local value of w_*.

For smaller values of α, the meridional velocity becomes vertically sheared. Figure 2b shows the velocity profiles at $\phi/\phi_e = 0.5$ for the case where $\theta = 30°$N and $\alpha = 10$. Although v is a function of depth, the velocity is everywhere poleward.

It is, naturally enough, difficult to give a reliable estimate for α since it depends on κ. However, using a value of N of 3 rads h^{-1}, $D = 4$ km, $\Omega = 7.3 \times 10^{-5}$ s^{-1}, and $R = 6 \times 10^3$ km, I find that

$$\alpha = 0.24\ \kappa$$

if κ is measured in cgs units. A larger buoyancy frequency will reduce α further. For a value of κ of 1/2 cm^2 s^{-1}, this leads to an α of 0.12.

Figure 2c shows v and w as a function of depth at $\phi/\phi_e = 0.25$ for the case $s = 0.001$ (i.e., w_* essentially independent of longitude) for $\alpha = 0.1$. There is now, at $\theta = 30°$N, a slight reversal at depth of the meridional velocity. At most depths, it is strongly poleward. Figure 2d shows the same case except that now $s = 5$ so that the upwelling is strongly localized near the eastern boundary. The meridional velocity is now highly layered, and it takes only the slightest imagination to identify the lobes in v with the alternating intrusions of Antarctic Bottom Water, North Atlantic Deep Water, and Antarctic Intermediate Water as we rise through the water column. However, given the simplicity of the model, it is perhaps unwise to insist on detailed comparison with observations. The important point is, as emphasized in the Introduction, that the meridional velocity is much larger than the vertically averaged value, given by V_0 and shown in Fig. 2d. The meridional velocity in the western basin, where w_* is small, is driven by the internally generated vertical velocity forced by the temperature anomaly propagated westward from the eastern forcing region and hence, has a small barotropic component compared with the baroclinic transport. Figure 3 shows the temperature anomaly as a function of depth, which is responsible for the vertical velocity shown in Fig. 2d.

Figure 4 shows a zonal cross section of v at 30°N for the case $s = 5$ and $\alpha = 0.1$. Note that the layering extends over the whole basin, although the meridional velocity in the eastern part of the basin is dominated by the strong poleward flow produced by the local upwelling. At the same depth, that is, for $z \geqslant -0.2$, the abyssal velocity is actually southward in the western part of the basin.

Since $a_n = \alpha f^2 n^4$, lower latitudes are parametrically equivalent to greater stratification. This simply corresponds to the increase of the Rossby deformation radius with decreasing latitude. Figure 5 shows the meridional and vertical velocities as a function of depth at 11.5°N

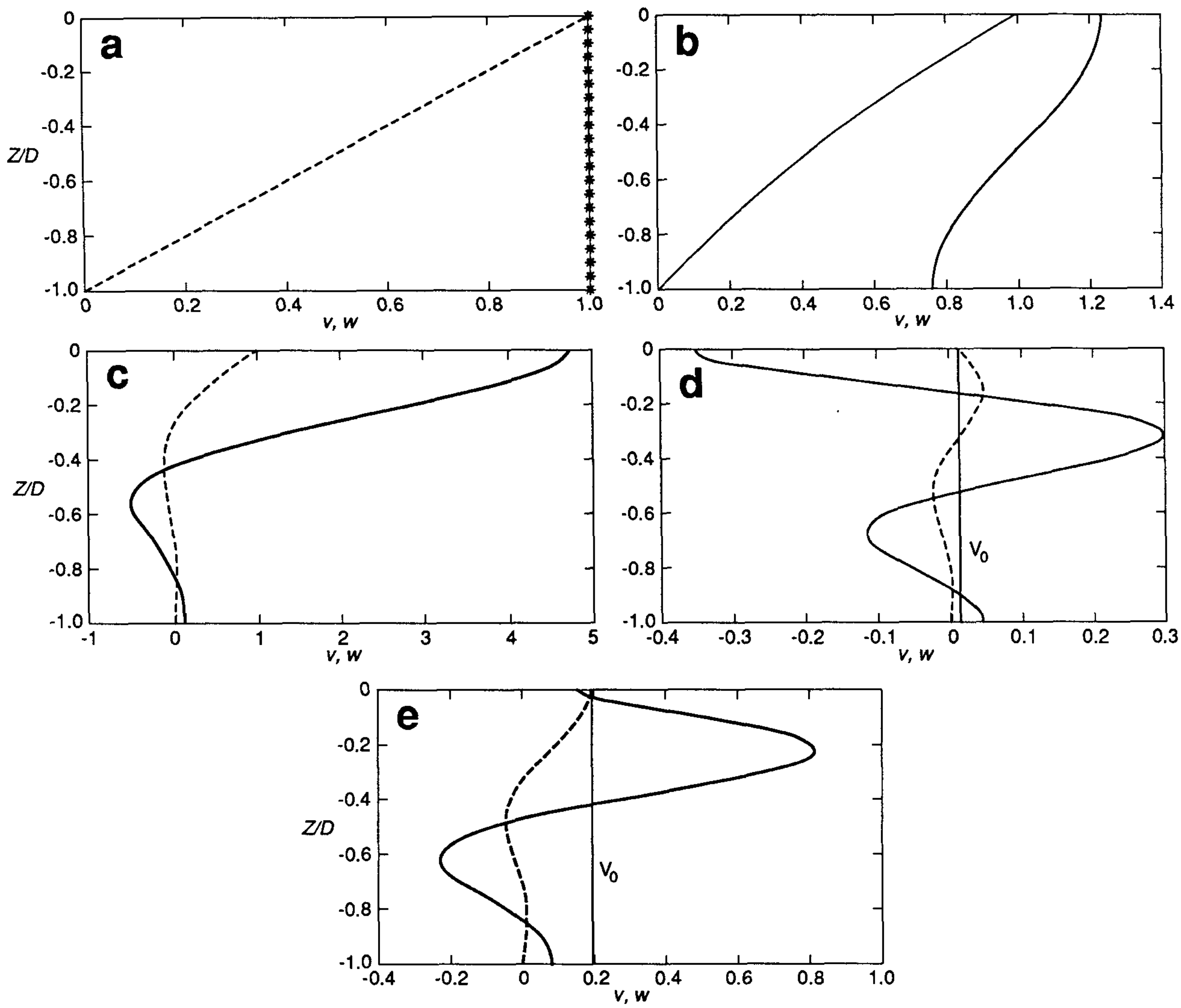

FIG. 2. The meridional and vertical velocities as a function of depth in the abyss. In each case shown, the sums in (3.3a,b,c) retain 20 modes in the vertical. In each figure, w is scaled by KW_* and v is scaled by $KW_* f/\beta$. (a) Homogeneous model ($\alpha = 10^3$) at $\theta = 30°$N with uniform upwelling ($s = 10^{-3}$). The vertical velocity is shown by the dashed line, while the (constant) meridional velocity is indicated by asterisks. (b) As in (a) except $\alpha = 10$. The profiles as shown at $\phi/\phi_e = 0.25$. The meridional velocity is shown by the solid line. (c) As in (a), with $\alpha = 0.1$. (d) As in (c) except $s = 5$ (strong localization of w_* near $\phi = \phi_e$). (e) As in (d) except $s = 2$.

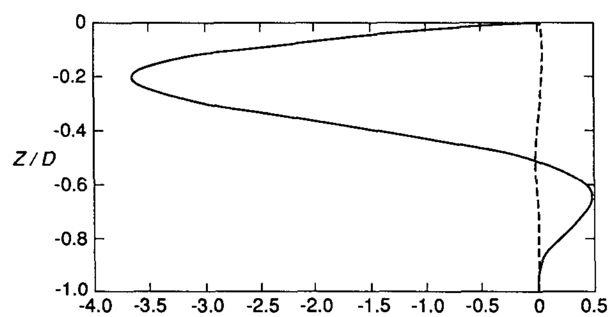

FIG. 3. The temperature anomaly (solid line) and associated vertical velocity. The temperature anomaly is scaled by $KW_* f^2\epsilon/s$, where ϵ is the Rossby number.

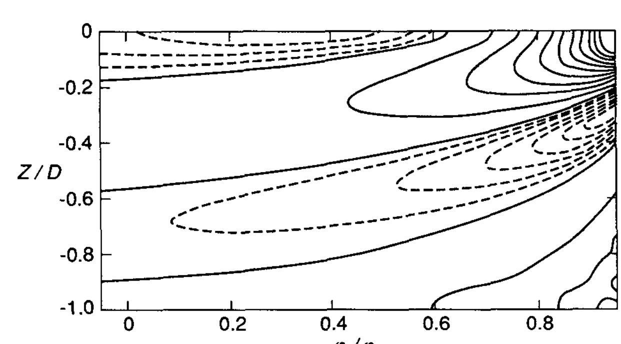

FIG. 4. A cross section showing contours of meridional velocity at $f = 0.5$, $s = 5$, and $\alpha = 0.1$. The dashed contours indicate $v < 0$ (contour interval = 0.1); the solid lines indicate $v > 0$ (contour interval = 0.5).

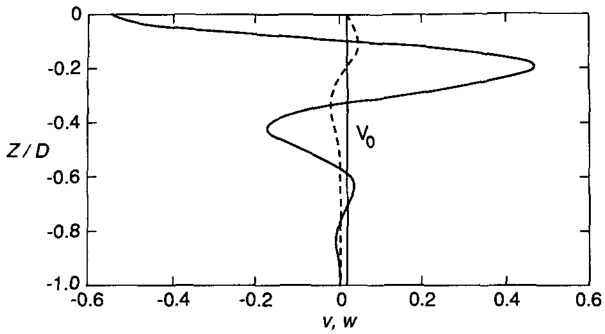

FIG. 5. The meridional and vertical velocity profiles at $\sin\theta = 0.2$ for the case shown in Fig. 2d.

(where $\sin\theta = 0.2$). The same major lobes are present; they are moved somewhat higher in the water column. This corresponds to the dynamical scaling explicit in (2.15) that smaller f corresponds to greater S (i.e., N^2) and hence greater baroclinicity in the velocity field.

Figure 6 shows the cross section of meridional velocity at the same parameter values as in Fig. 4, except that s has been reduced from 5 to 2. Figure 2e shows the profiles of v and w at $\phi/\phi_e = 0.25$. The same qualitative features are evident, which is reassuring since a precise estimate of the eastward intensification of the upwelling into the thermocline is not available. The insensitivity of the layering to the value of s is, I believe, a robust result related to the simple physical argument presented in the Introduction.

The overall flow pattern at the selected depths $z = -0.3$ and $z = -0.7$ is shown in Fig. 7. As in the classical SA theory, no fluid crosses the equator in the interior. The burden is placed on the unresolved western boundary current system to transfer mass across the equator. In the figures presented here w_* is chosen to be independent of latitude. Certainly a different choice for the latitudinal structure of the upwelling will affect the structure of the zonal velocity field and the horizontal pattern of baroclinic currents shown here.

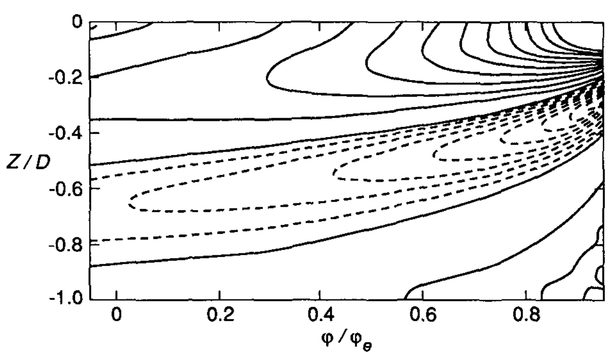

FIG. 6. As in Fig. 4 but with $s = 2$.

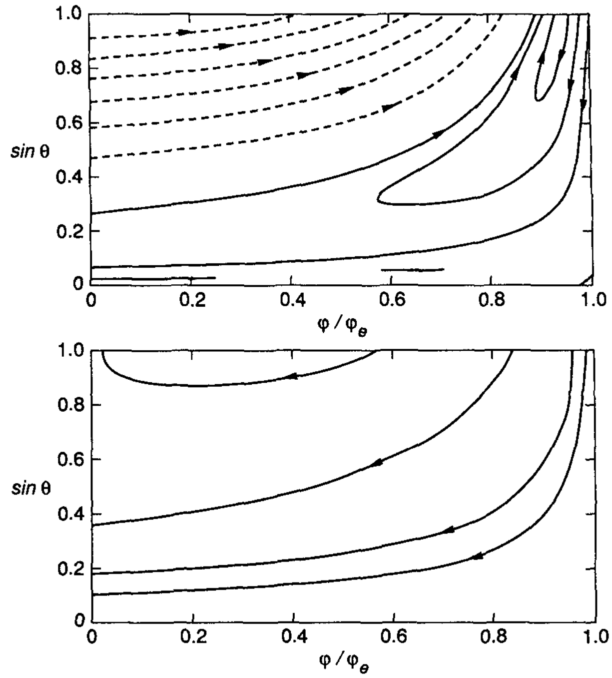

FIG. 7. The pattern of the abyssal pressure field at (a) $z/D = -0.3$ and (b) $z/D = -0.7$ for $s = 5$, $\alpha = 1$.

4. Discussion

The simple model presented here suggests that the layering of the abyssal circulation is connected to the longitudinal inhomogeneity in the upwelling w_* of cold water into the main thermocline. The basic idea is composed of two elements: first, that the barotropic, abyssal meridional velocity is tied to the local value of upwelling by the Sverdrup relation and hence, is small where w_* is small and second, that the planetary vorticity gradient yields a propagation mechanism by which the thermal field produced where the upwelling is large will propagate westward into regions where w_* is small, thus producing a field of vertical velocity not locally related to w_*. This field of thermally induced vertical velocity yields a baroclinic meridional velocity with values much greater than the mean.

These ingredients are so simple that the results can be expected to transcend the limitations of the rudimentary linear model used to illustrate the consequences of the basic ideas. Naturally, horizontal advection of the thermal field will distort the patterns of the linear model. However, as long as the abyssal fluid velocities are smaller (or at least not larger than) the baroclinic Rossby wave speed, we can expect the principal qualitative results to stand.

The use of a constant buoyancy frequency in the theory is also not of fundamental importance. An $N^2(z)$ that is not constant would lead to a quantitatively different set of expansion modes with numerically different eigenvalues replacing the a_n of sections 2 and 3. However, the general properties of the baroclinic modes are reasonably unchanged.

Relatively little is known directly about the velocity structure of the abyssal flow apart from uncertain inferences from property fields. There are hints, however, of deep layering in the velocity field. Armi and Stommel (1983) made careful repeated measurements in the beta-triangle area (27°N, 32°30′W). They commented on the reversal of geostrophic shears observed in the abyss (see especially their Fig. 7.4). There is then some evidence directly from the baroclinic mass field for the layered structure suggested by the ideas of this study.

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