On ENSO Coastal Currents and Sea Levels

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ABSTRACT

Sea-level measurements along the western coast of the Americas have shown that there is a strong signal at ENSO frequencies (approximately \(2\pi/2\) yr\(^{-1}\) to \(2\pi/5\) yr\(^{-1}\)) that propagates poleward at about 40 to 90 cm s\(^{-1}\). This ENSO sea level signal must be associated with ENSO coastal currents, but because adequate interannual current time series are unavailable, the structure and strength of these currents are not known. Coastal ENSO currents must be fundamentally affected by bottom topography and bottom friction, but previous theory has not taken these effects into account. A near-boundary numerical model with realistic bottom friction, stratification, and shelf and slope topography was therefore constructed to study ENSO coastal flow.

(i) At ENSO frequencies, previous results for models with no bottom topography and no bottom friction suggest that sea level should not propagate poleward. With realistic bottom friction and bottom topography coastal sea level propagates poleward at speeds similar to those observed. A simple mechanism based on the effect of bottom friction on offshore propagating geostrophic alongshore flow explains why coastal poleward alongshore propagation occurs. Calculations also show that the depths of the 20°C and 15°C isotherms also propagate poleward at approximately the observed speeds.

(ii) Sea levels change little across the shelf and slope at lower latitudes and slowly decrease in amplitude alongshore due to bottom friction. The small sea level change across the shelf and slope implies that the long sea level records available are useful for analyzing the nearby deep ocean variability.

(iii) Lower-order deep-sea vertical modes incident at the equator are rapidly scattered mainly by bottom friction into other (higher) vertical modes. Scattering has two main effects. Those vertical modes equatorward of their critical latitudes propagate offshore as Rossby waves interfere with each other and produce a complicated deep-sea velocity field, especially at low latitudes where most of the vertical modes are propagating offshore. Those vertical modes poleward of their critical latitudes only exist as coastally trapped motion and give rise to a trapped ENSO jet over the continental slope. This jet has an amplitude peak of order 20 cm s\(^{-1}\) and is trapped within about 300 m of the bottom. The jet core is approximately 180° out of phase with near-surface currents over the continental shelf and slope. Therefore, during (say) the El Niño part of the ENSO cycle when the sea level is high and the near-surface flow over the continental shelf and slope tends to be poleward, the flow in the jet core tends to be equatorward. Present observations are inadequate to prove or disprove the existence of this ENSO continental slope jet. Due to bottom friction, the alongshore velocity decreases shoreward of the shelf break and is negligible at the coast.

(iv) Critical latitudes for vertical modes change when the coastline angle changes and so motion near the boundary is affected by coastline angle. When the coastline is less meridional, coastal sea level and the 20°C and 15°C isotherm depths propagate poleward more rapidly (although still at approximately the observed speeds). The ENSO jet has its maximum amplitude nearer the equator.

(v) In the biologically important top 100 m or so of the ocean alongshore particle displacements seaward of the shelf can be ~1000 km. Interannual near-surface alongshore currents over the continental shelf and slope lead coastal sea level by several months.

1. Introduction

Several years ago Enfield and Allen (1980) and Chelton and Davis (1982) showed that sea level fluctuations at ENSO frequencies (periodicity 2–5 years) propagate poleward at about 90 cm s\(^{-1}\) (Enfield and Allen) or 40 cm s\(^{-1}\) (Chelton and Davis) along the western coast of the Americas (see Fig. 1). Kessler (1990) showed that the coastal 15°C and 20°C isotherms also propagate poleward at a similar speed (32 cm s\(^{-1}\)). Such propagation is strange because at El Niño–Southern Oscillation (ENSO) frequencies and for a solution dominated by vertical modes 1 and 2, the solution along most of the coast is in the form of westward group velocity Rossby waves that do not propagate along the coast (Clarke 1983). This no-propagation result can also be anticipated without detailed theory by recognizing that at small enough frequencies, the current perpendicular to an eastern ocean boundary is

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in geostrophic balance. This balance implies that there can be no gradient of pressure along a vertical coastal wall, for, if there were, there would be a flow into the wall. Therefore coastal pressure, and hence coastal sea level, are of constant amplitude and phase along the boundary; that is, they do not propagate.

We emphasize that the sea level and isotherm propagation observations by Enfield and Allen (1980), Chelton and Davis (1982), and Kessler (1990) were either for filtered signals containing only ENSO frequencies or for signals dominated by ENSO frequencies. Propagation has been seen in numerical models of eastern boundary flow [e.g., the 1½-layer model of Pares-Sierra and O’Brien (1989)], but this propagation, at about 2.5 m s⁻¹, is much faster than that for the low-frequency observations discussed above and is distinct from it. Their model boundary signals have significant high-frequency content and the poleward propagation is associated with high-frequency coastal Kelvin waves that travel at 2.5 m s⁻¹.

The no-propagation ENSO frequency result is based on a constant depth frictionless model with a vertical coastal wall. Clarke (1983) found that dissipation due to vertical diffusion of mass could give rise to poleward propagation at ENSO frequencies, but that this dissipation was too weak to explain the observed sea level propagation speeds. More important factors not taken into account by the constant depth model are bottom friction and the shelf and slope bottom topography. When these effects are included in a model, the coastal boundary condition changes fundamentally. Does one obtain poleward sea level propagation when a more realistic coastal boundary condition is used? And what of the coastal ENSO currents? Typical frictional decay time scales over the shelf and slope are of the order of a few days to a few weeks, so the very low-frequency ENSO currents over the shelf and slope must be strongly distorted by bottom friction. Previous theory and modeling (e.g., Clarke 1983; Pares-Sierra and O’Brien 1989) have not taken bottom friction and the order one changes in bottom topography into account. The structure and magnitude of these ENSO currents are important biologically; Chelton et al. (1982) showed that these flows are the primary cause of the interannual variability of the zooplankton biomass off California.

Motivated by the need to understand the coastal ENSO frequency dynamics, we constructed a high-resolution low-frequency numerical model of the coastal region. In section 2 we describe this model in detail and then discuss sea level and alongshore current model results in sections 3 and 4. We find that the strange sea level propagation is due to bottom friction and that the velocity field bears little resemblance to the vertical
wall inviscid results. There is a trapped alongshore ENSO jet over the continental slope and a complicated velocity field over the constant depth deep-sea region. Coastline angle can be expected to affect our results because for flow of a given amplitude, less planetary vorticity change will be “seen” by a parcel of water if it is forced to oscillate along a less meridional coast. In section 5 we discuss results for a nonmeridional coast. Section 6 deals with some difficulties. In particular, we discuss buoyancy effects within the sloping turbulent bottom boundary layer. Under certain conditions such effects can make the bottom much more slippery (e.g., see MacCready and Rhines 1991, 1993; Trowbridge and Lentz 1991); this is of importance since our results depend strongly on turbulent bottom friction. Section 7 contains some concluding remarks.

2. Theory

To resolve the shelf and slope bottom topography as well as the vertical variation of the stratification adequately, many grid points are needed. For this reason, and also because we are mainly interested in the flow near the boundary, we limit our attention to a boundary-layer strip extending northward from the equator rather than solve the problem for the whole Pacific Ocean. We make use of a low-frequency boundary-layer approximation to simplify the problem and carefully choose the “open” boundary conditions. We will discuss the field equation and boundary conditions in sections 2a and 2b and some numerical and parameter details in section 2c.

a. Field equation and bottom boundary condition

The equations of motion, linearized about a state of rest and horizontally uniform stratification, are

\[
\begin{align*}
\frac{\partial^2}{\partial t^2} + f^2 & \nabla u = -\nabla p/\rho_0 + f k \times \nabla p/\rho_0, \quad (2.1) \\
p_t & = -g \rho' \quad (2.2) \\
\nabla \cdot u + w & = 0 \quad (2.3) \\
\rho'_t + w \rho'_z & = 0. \quad (2.4)
\end{align*}
\]

In (2.1)–(2.4), \( z \) represents distance vertically upward from the ocean surface \( z = 0 \), \( t \) the time, \( u \) the horizontal velocity, \( w \) the upward vertical velocity, \( f \) the Coriolis parameter, \( k \) the unit upward vector, \( \nabla \) the horizontal gradient operator, \( \rho_0 \) the constant mean water density, \( X \) the horizontal stress in the water due to turbulence, \( p \) the perturbation pressure associated with the motion, \( g \) the acceleration due to gravity, \( \rho' \) the density perturbation due to the motion, and \( \rho' \) the density when there is no motion. We assume that the stress is confined to thin boundary layers near the surface and bottom so that in the ocean interior \( X_z \) is negligible. Taking \( \rho''' = 0 \) of (2.1) gives

\[
\nabla^2 p + \beta p_x + \frac{f^2}{N^2} \left( \frac{p_{xx}}{N^2} \right)_z = 0, \quad (2.11)
\]

and the bottom boundary condition to

\[
p_{xx} + f^{-2} h_x (p_{xx} + f p_y) + f^{-2} (r p_x)_x \\
- r h_x p_{xx} = 0 \quad \text{on } z = -h(x). \quad (2.12)
\]

For the low frequencies \( \omega \) of interest \( \omega^2 \ll f^2 \), so the \( \frac{\partial^2}{\partial t^2} \) term can be omitted from (2.5). Substituting this modified version of (2.5) into (2.3) and eliminating \( w \) and \( \rho' \) between (2.2)–(2.4) leads to the vorticity equation

\[
\nabla^2 p + \beta p_x + \frac{f^2}{N^2} \frac{p_{xx}}{N^2} = 0, \quad (2.6)
\]

where \( \beta = d f/dy \), \( N \) is the buoyancy frequency, \( y \) is distance north from the equator, and \( x \) is distance eastward from the origin at the intersection of eastern boundary and the equator.

For simplicity, we will usually consider a model in which the coastline is north–south. Isobaths are everywhere parallel to the coast so the water depth \( h \) is a function of \( x \) alone. Following the bottom boundary-layer analysis of Clarke and Brink (1985), the bottom boundary condition for a bottom boundary layer that is thin compared to the total water depth is

\[
w = -u h_x + w_{Ek} \quad \text{on } z = -h(x) \quad (2.7)
\]

where \( u \) and \( w_{Ek} \) refer, respectively, to the onshore horizontal velocity and the Ekman pumping velocity. The latter is given by

\[
w_{Ek} = k \cdot \nabla \times (\tau_B/\rho_0 f), \quad (2.8)
\]

where \( \tau_B \) is the bottom stress. In (2.8) \( \tau_B \) is evaluated along the sloping bottom \( z = -h(x) \); strictly speaking, \( w_{Ek} \) is perpendicular to the sloping bottom rather than vertical but the slope is small enough (~10⁻¹ to 10⁻²) that these directions are nearly the same.

In standard fashion we write

\[
\tau_B = \rho_0 f u_B = r f^{-1} k \times \nabla p, \quad (2.9)
\]

where \( u_B \) is the geostrophic velocity just above the bottom boundary layer and \( r(x) \) is the commonly used bottom friction coefficient. Substitution of (2.9) into (2.8) gives

\[
w_{Ek} = \nabla \cdot \left[ f^{-2} r \nabla p/\rho_0 \right]_{z=-h(x)}, \quad (2.10)
\]

Both the field equation (2.6) and bottom boundary condition [(2.7) combined with (2.10)] can be simplified by noting that at low frequencies variations alongshore are slow in comparison with variations perpendicular to the coast (see appendix A). To within an excellent approximation, the field equation reduces to

\[
\frac{p_{xx}}{N^2} + \beta p_x + \frac{f^2}{N^2} \left( \frac{p_{xx}}{N^2} \right)_z = 0, \quad (2.11)
\]
b. Other boundary conditions

The coastal and surface boundary conditions are the same as those used in other studies. Following Mitchum and Clarke (1986), the coastal boundary condition will be taken to be

\[ p_{ct} + \frac{r_p x}{h} + f p_y = \frac{f \tau^x}{h} \quad \text{at} \quad x = b, \quad (2.13) \]

where \( \tau^x \) is the alongshore component of the wind stress. Physically, (2.13) states that the depth integrated flow perpendicular to the model coast is zero at location \( x = b \) where the surface and bottom turbulent mixed layers begin to separate. In all cases we let \( x = b \) correspond to the 25-m isobath. At the free surface \( w \) vanishes and from (2.2) and (2.4) this means that

\[ p_{z}/N^2 = 0 \quad \text{on} \quad z = 0. \quad (2.14) \]

The offshore boundary condition at \( x = -L \) is more complicated. We choose \( x = -L \) to lie in the constant depth deep sea region well beyond the shelf and slope topography. Bottom friction has a negligible influence on the flow in the deep-sea region and for convenience we allow \( r(x) \) to gradually approach zero near \( x = -L \) so that \( r \) vanishes at and near \( x = -L \). Then in this region we may represent the solution as a sum of inviscid vertical modes satisfying (2.11), (2.14), and

\[ w = -\frac{p_z}{N^2} = 0 \quad \text{on} \quad z = -H. \quad (2.15) \]

For convenient study we take the motion to be harmonic in time. There is no loss of generality in doing this because a linear solution for general low-frequency time dependence can be found by appropriately summing the response at several different frequencies. When the motion is harmonic in time the solution in the constant depth deep sea is of the form

\[ p = \sum_{j=0}^{\infty} A_j(y) \exp[\lambda_j(x + L)]R_j(z) \exp(i \omega t), \quad (2.16) \]

where \( R_j(z) \) satisfies the standard Sturm–Liouville vertical mode problem

\[ (R_j / N^2)z + R_j / c_j^2 = 0 \quad (2.17a) \]

and

\[ R_j / N^2 = 0 \quad \text{on} \quad z = 0, -H. \quad (2.17b) \]

The function \( \lambda_j(y) \) is given by

\[ \lambda_j = \frac{i \beta}{2 \omega} + \left[ f^2 / c_j^2 - \beta^2 / (2 \omega)^2 \right]^{1/2} \]

for \( f^2 / c_j^2 > \beta^2 / (2 \omega)^2 \quad (2.18a) \]

and

\[ \lambda_j = \frac{i \beta}{2 \omega} - \left[ \beta^2 / (2 \omega)^2 - f^2 / c_j^2 \right]^{1/2} \]

for \( \beta^2 (2 \omega)^2 > f^2 / c_j^2 \). \hspace{1cm} (2.18b)

Note that \( \lambda_j \) has been chosen so that each mode either decays westward from the boundary or has group velocity westward away from the boundary. In the low-frequency, near-equator limit \( (\beta / 2 \omega)^2 \gg (f / c_j)^2 \), \( \lambda_j = i f^2 \omega / \beta c_j^2 \) corresponding to long, westward propagating Rossby waves while in the large latitude, higher-frequency limit \( (\beta / 2 \omega)^2 \ll f^2 / c_j^2 \), \( \lambda_j = f / c_j \) corresponding to motion trapped near the boundary. The critical latitude at which motion becomes trapped can be found from

\[ \frac{|f|}{c_j} = \frac{\beta}{2 \omega}. \quad (2.19) \]

A physical discussion of why motion should be trapped at high frequencies and latitudes and propagate offshore as Rossby waves at low frequencies and latitudes has been given by Clarke and Shi (1991).

In formulating an open boundary condition at \( x = -L \), we must be careful to make sure that no eastward group velocity waves enter \( x > -L \) because of numerical error. Since velocity is proportional to the gradient of pressure and short eastward group velocity Rossby waves have enormous wavenumbers compared with the wave numbers of their long westward group velocity counterparts, even tiny pressure errors can give large errors in velocity. We begin the boundary condition formulation by first noting that, from the orthogonality of the eigenfunctions \( R_j(z) \),

\[ A_j(y) \exp(i \omega t) = (p, R_j)/(R_j, R_j), \quad (2.20) \]

where

\[ (q_1, q_2) = \int_{-H}^{0} q_1 q_2 dz. \quad (2.21) \]

Differentiation of (2.16) twice with respect to \( x \) and substitution of the result (2.20) gives the boundary condition

\[ p_{xx} = \sum_{j=0}^{\infty} \lambda_j^2 R_j(p, R_j)/(R_j, R_j) \quad \text{at} \quad x = -L. \quad (2.22) \]

We found that this boundary condition has less numerical error than an analogous one involving \( p_{ct} \). In practice the sum in (2.22) is finite since there are only 25 gridpoints in the vertical and therefore only 25 possible vertical modes. Further numerical details concerning (2.22) are given in appendix B.

The problem to be solved is thus (2.11) subject to (2.12), (2.13), (2.14), (2.15), and (2.22). Since \( \partial \theta / \partial t = i \omega \), the problem reduces to one in \( x, y \), and \( z \). We solve it by integrating northward along the coast using backward differences for \( \partial / \partial y \) (cf. McCreary and Chao 1985). Thus at every \( y = \text{const} \) cross section we solve
an elliptic problem in $x$ and $z$ [see (2.11)] that is "forced" by coastal and bottom pressure available from the previous $y$ grid point. Such an integration assumes that $p$ is known at some initial latitude. We begin the integration at the equator utilizing the $y = 0$ solution of the problem; namely,

$$p = p_0(z) \exp(i \omega t) \quad (2.23)$$

with $p_{0z}$ vanishing on $z = 0$ and $z = -h(-L)$. In what follows we will take $p_0(z)$ to be some linear combination of vertical modes. To summarize in physical terms, the coastal flow is driven by an alongshore pressure gradient associated with a signal that originates at the equator.

c. Some numerical and parameter details

We adopted $1^\circ$ $y$ steps, solving the $(x, z)$ problem at each $y = \text{const}$ section using 151 points in the $x$ direction for a total distance of 200 km and 25 points in the vertical over water depth varying from 25 m at the coast to 3000 m in the constant depth deep sea. To maximize the physical resolution in the vertical using the 25 grid points at each $x$ location, the spacing between the points was stretched in a WKB sense by changing the vertical coordinate to

$$\theta = -\int_0^z N dz / \int_{-h}^0 N dz. \quad (2.24)$$

The above choice puts more points where $N$ varies strongly and transforms the problem into a rectangular grid with $\theta = 0$ at the surface and $\theta = -1$ at the variable bottom. The choice is based on the WKB result (e.g., see Clarke and Van Gorder 1986)

$$R_j(z) = (N/\bar{N})^{1/2} \cos(j \pi \theta), \quad (2.25)$$

where $R_j(z)$ is the $j$th vertical eigenfunction and $\bar{N}$ is the vertical average of $N$ from the deep-sea bottom to the surface. We solved the finite difference version of the $(x, \theta)$ problem at each $y = \text{const}$ section by solving the band matrix problem corresponding to the finite-difference formulation.

We chose $N(z)$ to be that shown in Fig. 2. This profile is an average of $N$ along the northeastern Pacific boundary from the equator to $59^\circ$N. Similarly, we took the topography $z = -h(x)$ to be typical of the shelf and slope topography along the North Pacific eastern boundary. The friction coefficient $r(x)$ will be varied, but in the standard case it will take the reasonable values shown in Fig. 3 (see Clarke and Brink 1985). The increase in $r(x)$ on the shelf is primarily due to the influence of surface gravity waves on bottom stress (Grant and Madsen 1979). A representative ENSO frequency is $2\pi / 3$ years and this is the frequency we will take for the standard case. We allowed $f$ and $\beta$ to vary alongshore appropriately according to latitude.

To complete the specifics for the model in the standard case, we must choose the linear combination of modes making up $p_0(z)$ in (2.23). The analysis of Busalacchi and Cane (1985) suggests that the interannual sea level near the equator and the eastern Pacific boundary is dominated by the first two vertical modes. Using a simple forced wave model similar to that in Clarke and Liu (1993), we also found this result. In
addition (see Fig. 4) we found that mode 2 and mode 1 were in phase and that on average mode 2 was about 1.5 times mode 1. Why should these modes be in phase with one another? The interannual sea level at the eastern Pacific–Indian Ocean boundary is dominated by the mode 1 and mode 2 equatorial Kelvin wave signals forced largely in the central equatorial Pacific Ocean. At interannual frequencies the Kelvin wave signals differ negligibly in phase by the time they reach the boundary.

Based on the preceding analysis we will take, as our standard case, that the initial $y = 0$ solution [see (2.23)] is given by

$$p = [0.4 R_1(z) + 0.6 R_2(z)] \rho_0 g \eta_0 \exp(i \omega t),$$

(2.26)

with $\rho_0 = 1026 \text{ kg m}^{-3}$, $g = 9.8 \text{ m s}^{-2}$, and $\eta_0$, the interannual sea level at the equator, having the observationally reasonable amplitude of 5.5 cm.

3. Coastal ENSO sea level and isotherms

The best documented observations of the coastal ENSO signal are those discussed earlier for coastal sea level and coastal isotherms. In this section we will examine these fields.

a. Sea level propagation

Figure 5 shows results for coastal sea level propagation for the standard case for the three ENSO frequencies $2\pi/2$ years, $2\pi/3$ years, and $2\pi/6$ years. The propagation is noisy close to the equator, but for all these ENSO frequencies it soon settles down to values similar to those observed.

The friction coefficient $r$ may vary along the coast and across the shelf, so we tested how strongly the phase propagation depends on it. The results are summarized by Fig. 6. Propagation speeds are close to those observed for all cases where $r$ takes reasonable values based on Clarke and Brink (1985). For a lower value over the continental slope ($r = 10^{-4} \text{ m s}^{-1}$) the coastal sea level propagation is too large.

Why does poleward coastal sea level propagation occur? Although the pressure field is distorted by bottom topography and friction, one would expect (and calculations confirm this) that the pressure field is quasigeostrophic and, at least qualitatively like the long, westward propagating Rossby waves, it propagates offshore. Consider now the geostrophic offshore propagating pressure and velocity fields shown schematically in Fig. 7a. The northward geostrophic alongshore velocity near the coast weakens in the turbulent frictional bottom boundary layer and consequently so does the Coriolis force but the westward geostrophic pressure gradient does not. This pressure gradient drives an offshore boundary-layer transport which, by continuity (see appendix C), is balanced by an interior onshore geostrophic flow. The latter results in alongshore pressure distribution with high pressure to the south and low pressure to the north. But one-quarter of a period later (Fig. 7b), the westward propagation has resulted in a high pressure at the coast. Therefore, to a coastal
The coastal boundary condition (2.13) with $\tau_r = 0$ and $\omega \ll r/h$ implies that

$$il = p_j/p = -rp_x/h\beta p,$$

which, using (3.1), leads to the phase speed estimate

$$\omega/l = -(1 + a_1/a_2)\beta hc^3/[f(1 + c_s^2a_1/c^2a_2)].$$

For the values $c_s = 2.0 m s^{-1}$, $c_2 = 1.1 m s^{-1}$, $\beta = 2.3 \times 10^{-11} m^{-1} s^{-1}$, $a_1/a_2 = 2/3$ (the value at the equator), $h = 25 m$, and $r = 5 \times 10^{-4} m s^{-1}$ (the coastal value in the standard case), we have poleward sea level propagation at speeds of 7.6 cm s$^{-1}$, 3.9 cm s$^{-1}$, and 2.6 cm s$^{-1}$ at 20°N, 20°N, and 30°N, respectively. While the direction of propagation is correct, these propagation speed values are much lower than those either observed or calculated. The discrepancy arises because we have not taken into account the decrease in velocity near the coast due to bottom friction (see section 4a). Since $v = p_x/\rho c$, from (3.2) such a velocity decrease will lead to decreased $l$ and hence an appropriate increase in the alongshore phase propagation speed $\omega/l$.

Note that we would expect this effect on the velocity to increase as we integrate poleward and the deep-sea solution is forced to adjust to the influence of friction. This frictional effect apparently decreases $l$ (and increases $\omega/l$) faster poleward than the decrease in $\omega/l$ predicted by (3.3) because the phase velocity increases poleward (see Fig. 5).

**b. Variation of sea level amplitude**

Figure 8 shows that the coastal sea level decays alongshore. This decay, due to turbulent bottom friction, is substantial and is very different than that for a frictionless vertical wall coastal boundary. In the latter case sea level amplitude for a given vertical mode
increases slightly alongshore from the equator (see Fig. 9).

Near the equator, Fig. 8 shows that, despite the presence of friction and large amplitude bottom topography, sea level amplitude does not change much perpendicular to the coast. Why do friction and the bottom topography not distort the pressure field? The sea level field in the deep ocean region is dominated by the lower-order Rossby wave modes. Calculations (e.g., see Fig. 13 and Fig. 12a at 30°N) show that the surface alongshore velocities $v_{surf}$ of these modes are reduced by bottom friction over the continental shelf and slope. Therefore,

$$v_{surf} = g \eta_s / f \lesssim \eta_D / \rho_0 f,$$  \hspace{1cm} (3.4)

where $\eta_D$ is the deep-sea long Rossby wave sea-level field. But the lower-order modes in the latter have an east–west reciprocal wavenumber equal to $Bc^2 / \omega f^2$, which, at ENSO frequencies and lower latitudes for the lower-order vertical modes, is much greater than the width of the shelf and slope. By (3.4) the cross-shelf scale is at least as large as the Rossby wave scale and so sea level changes negligibly across the shelf and slope for the lower latitudes. This means that the large coastal sea level dataset is a useful antenna for the adjacent deep-sea ENSO variability.

c. Coastal isotherm propagation

Using a long time series of bathythermograph observations from a near-coast shipping route between the U.S. West Coast and the Panama Canal, Kessler (1990) found that at interannual frequencies 15°C and 20°C isotherm depths propagated poleward at about 32 cm s⁻¹. To test whether our model predicts propagation at a similar speed, we assume that upward isotherm displacement corresponds to upward isopycnal displacement $d$. The isotherm depth, $D$, is then given by the average isotherm depth, $\bar{D}$, minus $d$. Since $\bar{D}$ is a constant, we can estimate any propagation characteristics of $D - d$ at depth $\bar{D}$ by calculating those characteristics for $d$ at depth $\bar{D}$ or, since $w = iw \bar{D}$, by calculating those characteristics for $w$ at depth $\bar{D}$. Along almost all of the coast of North America between the equator and 30°N (where Kessler estimated his phase speed), the mean depth of the 20°C isotherm is about 50 m [see Fig. 4 of Kessler (1990)], while that of the 15°C isotherm is about 100 m (see Robinson 1976). Therefore, we calculated the phase speed of $w$ for the standard case at the depths 50 m, 70 m, and 100 m. We note that $w$ is available from the pressure field as

$$w = -p_{20^\circ} / \rho_0 N^2,$$  \hspace{1cm} (3.5)

which follows from (2.2) and (2.4).

The results of these calculations are shown in Fig. 10. We comment below.

(i) The phase speed at each depth is really a function of $x$ and $y$ but in Fig. 10 we have averaged it in $x$ over the shelf and slope region. This was done both for a clear presentation and also to take into account the fact that Kessler’s analysis used data that varied in distance from the coast but nevertheless kept close to it. To indicate how the phase speed varies from this average at each latitude, Fig. 11 is a plot of phase speed across the shelf and slope at 30°N.

(ii) Kessler’s 32 cm s⁻¹ poleward phase speed for the 15°C and 20°C isotherm depths was based on an analysis between the equator and 30°N. Figure 10

![Fig. 9. For the inviscid vertical wall case, the ratio of the coastal pressure amplitude to the coastal pressure amplitude at the equator plotted as a function of the variable $\xi = 2\omega |y|/(c \cos \phi)$. In the expression for $\xi$, $\phi$ is the angle that the coastline makes with due north. Note that the amplitude increases poleward. Past the critical latitude (corresponding to $\xi = 1$) the amplitude increases like $|y|$. The function $A(\xi)/A(0)$ can be found in closed form using the modulus of Eq. (2.15) of Clarke (1992).](image)
shows that over this range the model average poleward speed at depths of 100 m and 50 m is smaller but still of about the right size. Over most of this latitude range the coastline is tilted at about a 60° angle from due north and this has the effect of increasing the phase speed. For a 60° coastline angle, $\omega = 2\pi/3$ years, and other standard parameters the average phase speed between 5°N and 30°N is about 28 cm s$^{-1}$ at a depth of 100 m (the 15°C isotherm depth) and about 36 cm s$^{-1}$ at 50 m (the 20°C isotherm depth). These speeds are close to the 32 cm s$^{-1}$ observed speed. Further discussion of the nonmeridional coastal case will be given in section 5.

4. ENSO coastal currents

ENSO coastal currents are much more strongly distorted by bottom friction and bottom topography than sea level is. In this section we will examine coastal ENSO flows.

a. ENSO coastal currents for standard parameters

Figure 12 shows amplitude and phase of the alongshore velocity for the standard case at several coastal sections. We describe the main features of the velocity field below.

(i) Except near the equator, the largest velocities occur in a jet trapped to the bottom along the continental slope. The core of this ENSO jet is at a depth about 1200–1400 m and is approximately 180° out of phase with near-surface currents over the continental shelf and slope.

(ii) From about 15°N to 40°N the velocity amplitude varies in a seemingly confused manner over short scales, especially in the deep constant depth water seaward of the continental shelf and slope bottom topography. At higher latitudes the velocity field is much less complex so that by 60°N the constant depth deep sea velocity field is quite smooth.

(iii) Due to bottom friction, over the continental shelf the velocity field decreases toward the coast where it is nearly zero.

(iv) The velocity field differs markedly from that expected near an inviscid vertical wall (compare Fig. 13 with the corresponding 30°N result shown in Fig. 12a).

b. Physics

We will use the vertical mode theory of appendix D to understand qualitatively the numerical results. In that appendix we show that if $|Nh_x/f| \gg 1$, then the motion can be described by a sum of vertical modes, even over the continental shelf and slope. Calculations show that $|Nh_x/f| \gg 1$ near the equator and elsewhere is O(1). Thus, while we do not expect to obtain precise estimates using vertical modes, we will use them to understand the numerical results.

Near the equator only the two lower-order vertical modes given initially at the equator exist in the deepsea region and the velocity field is smooth. As the integration proceeds northward, each initial lower-order vertical mode is scattered into higher-order vertical
Fig. 12. (a) Alongshore velocity amplitude (in cm s$^{-1}$) and (b) alongshore velocity phase (in deg) at 10° latitude intervals from 10°N to 60°N for the standard case. The velocity amplitude contour interval is 2 cm s$^{-1}$ and the velocity phase contour
Fig. 12. (Continued) interval is 45°. The phase is relative to the phase of the pressure field at the equator. A positive phase means that the alongshore velocity leads the equatorial pressure field.
The alongshore velocity field converges more slowly because $\lambda_j \sim c_j^{-2} \sim j^2$ near the equator and $\lambda_j \sim c_j^{-1} \sim j$ at higher latitudes.

c. Sea level and near-surface ENSO flow

As mentioned earlier, ENSO flows are the primary cause of the interannual variability of the zooplankton biomass off California. We analyzed our results to see if we could find simple relationships between the simply measured coastal sea level and the biologically important top 100-m alongshore flow. The near-surface alongshore flow over the continental shelf and slope leads sea level by several months but, because of offshore propagation, lags it farther offshore. Total alongshore particle displacements are $2|\bar{v}|/\omega$, so seaward of the shelf these can be $\sim 1000$ km. The ratio $|\bar{v}|/\eta_c$, where $|\bar{v}|$ is the average alongshore velocity amplitude from the surface to 140 m over the shelf and $\eta_c$ is the coastal sea level, is about 0.2 s$^{-1}$ for standard parameters. When $|\bar{v}|$ represents the average top 140-m alongshore velocity amplitude over the continental slope, the ratio is much larger, becoming $\sim 1$ s$^{-1}$ in midlatitudes.

d. Observations

To date there has only been one detailed analysis of interannual flow variability near the western coast of

![Diagram](image-url)
the Americas. Chelton et al. (1982) examined the large-scale interannual variability of the California Current using an empirical orthogonal function analysis of the dynamic height derived from the California Cooperative Oceanic Fisheries Investigation (CalCOFI) data. Their empirical orthogonal function analysis, however, did not allow for wave propagation, and seaward propagation of Rossby waves is fundamental to the physics. Therefore, their estimation of alongshore transport, involving gradients of steric sea level, is likely to be inaccurate.

The CalCOFI data are limited to depths shallower than 500 m, so they offer no opportunity to see the slope-trapped ENSO jet predicted by our model. Deep continental slope measurements are surprisingly rare. Hydrographic observations taken from cross-isobath sections made during the Northern California Coastal Circulation Study (NCCCS) do show a current trapped over the continental slope at about 1400 m depth (see Fig. 16). Bray and Greengrove (1993), who have analyzed the NCCCS dataset, suggest that there is considerable nonseasonal variability at depth. However, their time series is far too short to determine whether there is an ENSO signal in slope currents at depth. In addition to a possible ENSO signal, the slope-trapped signal in Fig. 16 may have contributions from mean, seasonal, intraseasonal, and eddy flows.

Direct current measurements by Hickey (1979, see her Fig. 10) over the Washington continental slope show no evidence of a deep slope-trapped jet, but perhaps this is not surprising. First, at this high latitude the ENSO jet is weak and the measurements were at shallow enough locations that current amplitudes would only be expected to be about 4 cm s\(^{-1}\) or less. This estimate is based on the meridional and 60 deg angle coastline results with our standard parameters. It does not take into account the Gulf of California, which, based on coastal sea level data, seems to attenuate further the ENSO signal [see Fig. 9 of Baumgartner and Christensen (1985)]. Second, Hickey’s data were for alongshore currents averaged over 5 weeks. Therefore, as discussed above, mean, seasonal, intraseasonal and eddy currents may have contributed to the flow and made it difficult to discern the ENSO flow. But even if the flow measured were due solely to the ENSO jet, the measurement would still be inconclusive because it could have been made near a zero part of the alongshore velocity oscillation and thus not be more than the few centimeters per second observed. In summary, no measurements exist at present to test adequately the prediction of a deep ENSO jet trapped over the continental slope.

5. Nonmeridional coastline

At low frequencies currents are nearly parallel to the coastline and isobaths. As noted by Clarke and Shi (1991), this implies that particles undergoing a certain amplitude harmonic oscillation near a nonmeridional coastline will experience less planetary vorticity change when the coastline is less meridional. Since Rossby wave propagation depends on the planetary vorticity change particles go through during an oscillation, the more nonmeridional the coastline, the more the motion is likely to be trapped nearer the coast. Thus changes in coastline angle considerably influence the character of the flow. For a coastline making an angle \(\phi\) with due north, the critical latitude for vertical mode \(j\) and for motion of frequency \(\omega\) is determined from [see Clarke and Shi (1991) or a similar formula in Grimshaw and Allen (1988)]

\[
\frac{|f|}{c_l} = \beta \cos \phi / 2 \omega.
\]  

(5.1)

Consistent with the tendency for increased trapping along a nonmeridional coastline, (5.1) indicates that critical latitude decreases when the coastline is less meridional.

Calculations for standard parameters but with a coastline tilted at an angle 60 deg west of north show that the change in the coastline angle does influence the pressure, velocity, and density fields but not dramatically. As mentioned earlier, the 15°C and 20°C isotherm depths propagate poleward slightly faster. Comparison of Fig. 17 with Fig. 5 shows that the sea level propagation speed is also slightly greater in the nonmeridional case. Figures 14 and 18 indicate that the ENSO jet in the meridional and tilted coastline cases has a similar structure but with the maximum
ENSO jet amplitude nearer to the equator in the tilted coastline case. This is to be expected. The trapped ENSO jet owes its existence to scattered vertical modes that exceed their critical latitudes and by (5.1) these are closer to the equator in the tilted coastline case.

6. Some difficulties

Turbulent friction in the bottom boundary layer is of fundamental importance to the results discussed in the previous sections. But in obtaining those results, we did not take into account buoyancy effects that are important in the dynamics of a sloping bottom boundary layer. In this critique we will discuss this first and then another possible weakness in our theory, namely, the neglect of nonlinear effects.

a. Buoyancy effects in the bottom boundary layer

Consider, for example, an equatorward geostrophic interior flow on an eastern ocean boundary and above the bottom boundary layer. The standard argument is that, due to bottom friction, the alongshore flow decreases in the bottom boundary layer due to turbulent...
friction, the Coriolis force consequently weakens, and, because the shoreward pressure gradient force does not, there is an onshore transport in the bottom boundary layer. But such an onshore transport will carry heavier water upward over the sloping bottom, causing, by hydrostatic balance, a seaward horizontal pressure gradient force acting in opposition to the horizontal pressure gradient force associated with the equatorward alongshore geostrophic flow. Eventually the buoyancy effect stops the onshore bottom boundary-layer transport and the turbulent bottom friction vanishes. For a poleward geostrophic interior flow the bottom boundary-layer transport is seaward, light water is forced under heavier water, the bottom boundary layer thickens as mixing occurs and, as in the equatorward alongshore flow case, the buoyancy effect is eventually strong enough to stop the seaward bottom boundary-layer transport and cause the turbulent bottom friction to vanish. This buoyancy effect, first noticed by Weatherly and Niiler (1974), has also been discussed more recently in terms of its effect on bottom stress (e.g., MacCready and Rhines 1991, 1993; Trowbridge and Lentz 1991).

If this mechanism were of importance in our case, where in the development of our equations do we err? The results (2.7) and (2.8) are not in error, for they essentially depend on the reasonable approximation that interior vertical scales are large compared to the bottom boundary-layer thickness. The crucial assumption is made in the next step (2.9) where we write the bottom stress in terms of the interior alongshore geostrophic flow; as we have seen, because of the neutralizing horizontal pressure gradient force developed in the bottom boundary layer, bottom stress can be zero on a sloping bottom even when there is a nonzero interior flow. Therefore, with commonly used values of $r$, the balance (2.9) would be incorrect when the boundary-layer buoyancy mechanism is operating.

Does this buoyancy mechanism operate in our case? An assumption of the mechanism is that there is no convergence or divergence in the boundary-layer flow when, in fact, in our case, there is. Such convergence or divergence causes an exchange of mass between the boundary layer and the interior. This must seriously modify the buoyancy mechanism which works best when the density of bottom boundary layer water is not modified by exchange or mixing with the interior. We also note that the observed slow propagation of interannual sea level and isotherm depth along the coast indicates that in the real ocean an alongshore pressure gradient exists. At interannual frequencies such an alongshore pressure gradient must be balanced by bottom friction for any other balance, with either linear or nonlinear terms, results in unrealistically large interannual alongshore flows. While buoyancy effects in the boundary layer may have an influence on the interannual flow, they do not shut down bottom friction.
b. Neglect of nonlinearity

Another possible weakness in the theory is the neglect of the nonlinear terms, which could be important at the very low ENSO frequencies under consideration. An estimate of the error in omitting the nonlinear terms is

\[
\frac{u}{\omega} \frac{\partial}{\partial x} + \frac{\partial}{\partial t}
\]

or \( u/(\omega)(x \text{ scale}) \). In a small region where the flow is maximum at the core of the continental slope jet, this error rises to about \( 1/3 \), but over almost all the field the error is less than 10\%. Therefore, to a first approximation, we expect nonlinear effects do not affect our results.

7. Concluding remarks

There are two main results in this paper. First, the curious poleward propagation of coastal interannual sea level and isotherm depth along the coast of North America seems to be associated with bottom friction via the physical mechanism discussed earlier in the text. For any reasonable bottom friction, the theory predicts propagation in the same direction (poleward) and magnitude (\( \sim 1/2 \text{ m s}^{-1} \)) as that observed. Second, the theory predicts that there should be an ENSO jet trapped over the continental slope at middepth. Present measurements are inadequate to resolve whether such a jet exists in the ocean. However, since the theory is able to predict observed coastal ENSO sea level and isotherm depth propagation in the correct direction and at about the right speed, we suspect that the trapped ENSO jet does indeed exist.

In our analysis we only considered coastal ENSO motion of equatorial origin. We recognize that poleward of about 30\(^\circ\)N, the coastal ocean experiences alongshore wind forcing that is correlated with ENSO events (e.g., see Enfield and Allen 1980; Chelton and Davis 1982; Huyer and Smith 1985; Emery and Hamilton 1985; Rienecker and Mooers 1986; and Pares-Sierra and O’Brien 1989). This forcing will generate ENSO currents and sea-level fluctuations that will add another ENSO signal to coastal interannual flow north of about 30\(^\circ\)N. We intend to discuss this in future work.

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APPENDIX A

Simplification of the Field Equation and Bottom Boundary Condition

Using (2.10), (2.2), (2.4), and (2.5) (with \( \partial^2 \partial t^2 \) negligible), we can write the bottom boundary condition (2.7) in terms of the pressure:

\[
\frac{p_{x1}}{N^2} + f^2 \bar{h}_x (p_{x1} + \bar{f} \bar{p}_y) + r \frac{\partial}{\partial y} [f^{-2} p_y] \\
+ f^{-2} [r \bar{p}_x - \bar{r} \bar{p}_{xx}] = 0
\]

on \( z = -h(x) \). (A1)

At the low ENSO frequencies the terms \( f \bar{h}_x \bar{p}_y \) and \( \bar{r} \bar{p}_{xx} \) are of the same order so

\[
\frac{\partial}{\partial y} \left[ f^{-2} \frac{\partial \bar{p}}{\partial y} \right] = 0
\]

where \( d \) is the vertical scale of the low-frequency flow near the boundary. A comparison of \( 2\beta f^{-2} \bar{p}_y \) with \( \bar{h}_x f^{-1} \bar{p}_y \) and \( f^{-2} \bar{r} \bar{p}_{yy} \) shows that the term

\[
\frac{\partial}{\partial y} \left[ f^{-2} \frac{\partial \bar{p}}{\partial y} \right]
\]

is negligible in (A1) provided that \( 2\beta f^{-2} (h_x)^{-1} \) and \( (f/\bar{d})^2 \) are both small. For the representative values \( r = 2 \times 10^{-4} \text{ m s}^{-1} \), \( \beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \), \( h_x = 2 \times 10^{-2} \), and \( d = 500 \text{ m} \), both terms are less than 10\% poleward of 1\(^\circ\) of the equator. We expect \( p \) to have large spatial scales near the equator and that therefore a negligible error is made if

\[
\bar{r} \frac{\partial}{\partial y} \left[ f^{-2} \bar{p}_y \right]
\]

is neglected everywhere in (A1). An a posteriori check of the error in omitting this term confirmed that negligible error results if it is dropped. Thus (2.12) in the main text is valid.

The field equation (2.6) of the main text can also be simplified using the above scaling. A comparison of \( 2\beta \bar{p}_{yy}/f \) to \( \bar{p}_x \) and \( \bar{p}_{yy} \) to \( \bar{p}_{xx} \) shows that \( 2\beta \bar{p}_{yy}/f \) and \( \bar{p}_{yy} \) are negligible in (2.6) provided that

\[
\left( \frac{2\omega}{f} \right) \left( \frac{r}{\bar{d}} \right) \text{ and } \left( \frac{r}{\bar{d}} \right)^2
\]

are both small. Poleward of about 1/2\(^\circ\)N or S this is the case, so by the large-scale pressure argument advanced earlier, we expect that \( 2\beta \bar{p}_{yy}/f \) and \( \bar{p}_{yy} \) can be neglected in (2.6); that is, (2.11) is valid. Note that this argument assumes that the \( x \) to \( y \) scale ratio determined at the bottom boundary remains valid away from that boundary. It is not clear that this assumption is correct but it is of no consequence since numerical a posteriori
checks show that negligible error results if $2\beta p_{\theta\theta}/f$ and $p_{\theta\theta}$ are dropped in (2.6).

**APPENDIX B**

**Numerical Details for the Open Boundary Condition (2.22)**

As mentioned in the main text, the boundary condition at $x = -L$ must be formulated very accurately. To make the open boundary condition accurate enough we obtained finite difference analogs of the $\lambda_j$ and the dot product $(,)$ in (2.22). At $x = -L$, where the ocean is of constant depth and frictionless, the motion can be separated into vertical modes. In $(x, \theta)$ coordinates, where $\theta$ is the WKB coordinate defined in (2.24), the analogous equation to (2.16) is

$$p(x, y, \theta) = \sum_{j=0}^{\infty} A_j(y) G_j(x) Q_j(\theta), \quad (B1)$$

where we have dropped the common factor $\exp(i\omega t)$ from both sides of (B1) and the $G_j(x)$ and $Q_j(\theta)$ satisfy

$$N(Q_{j\theta}/N)_\theta + Q_j(S_H/c_j)^2 = 0, \quad (B2)$$

$$Q_{j\theta} = 0 \quad \text{on} \quad \theta = 0, -1, \quad (B3)$$

and

$$G_{jxx} - \frac{i\beta}{\omega} G_{jx} - \frac{f^2}{c_j^2} G_j = 0, \quad (B4)$$

with

$$S_H = \int_{-H}^{0} Ndz. \quad (B5)$$

To find the finite difference analog of $\lambda_j$, first write (B4) in centered difference form for a uniform grid $\Delta x$. Then, analogous to the continuous case, look for solutions proportional to $\exp(\lambda_j n \Delta x)$, where, in this context, $n$ is an integer. Then it follows that $\lambda_j$ is a solution of

$$(1 - i\beta \Delta x/2\omega) \exp(2\lambda_j \Delta x) - [2 + (f\Delta x/c_j)^2] \exp(\lambda_j \Delta x) + 1 + i\beta \Delta x/(2\omega) = 0. \quad (B6)$$

For small $\lambda_j \Delta x$ we obtain the continuous result

$$\lambda_j^2 - i\beta \lambda_j/\omega - (f/c_j)^2 = 0, \quad (B7)$$

which leads to (2.18) of the text. Although we used (B6) to estimate the $\lambda_j$, we chose $\Delta x$ small enough that the continuous result (B7) was valid for all the vertical modes contributing significantly to the response.

In attempting to apply the open boundary condition (2.22) in finite form we found that the discrete form of the integral dot product was not accurate enough. To see the approach we adopted, consider the finite form of (B1), namely,

$$p(\theta_m) = \sum_{j=0}^{M} A_j Q_j(\theta_m), \quad m = 0, 1, \cdots, M, \quad (B8)$$

where $M + 1$ is the total number of modes and points in the $\theta$ coordinate, and $G_j$ at $x = -L$ is unity. Instead of using the finite form of the integral dot product to obtain the $A_j$ from (B8), it is more appropriate to invert the matrix with elements $q_{mj} = Q_j(\theta_m)$. If the inverse of this matrix is $B$ then

$$A_j = \sum_{k=0}^{M} B_{jk} p(\theta_k), \quad j = 0, 1, \cdots, M. \quad (B9)$$

Substitution of (B9) into $\partial^2/\partial x^2$ of (B1) gives, after changing the order of summation,

$$p_{xx}(\theta_m) = \sum_{k=0}^{M} \left( \sum_{j=0}^{M} G_{jxx} B_{jk} Q_j(\theta_m) \right) p(\theta_k). \quad (B10)$$

In finite difference form, using the solution $G_j = \exp(\lambda_j n \Delta x)$, (B10) becomes

$$[p_1(\theta_m) - 2p_0(\theta_m) + p_{-1}(\theta_m)]/(\Delta x)^2$$

$$= \sum_{k=0}^{M} E_{mk} p_1(\theta_k), \quad (B11)$$

where

$$E_{mk} = \sum_{j=0}^{M} [\exp(\lambda_j \Delta x) - 2$$

$$+ \exp(-\lambda_j \Delta x)] B_{jk} Q_j(\theta_m), \quad (B12)$$

and the subscripts 0, 1, and −1 on $p$ refer to grid points at $x = -L$, $x = -L + \Delta x$, and a computational point at $x = -L - \Delta x$. The latter is eliminated from the problem using the field equation stencil.

**APPENDIX C**

**The Approximate Mass Balance along the Bottom**

Both scaling and numerical calculations show that at ENSO frequencies the dominant balance at the bottom boundary is [cf. (2.12)]

$$f h_x p_y + (r p_x)_x - r h_x p_{xx} = 0 \quad \text{on} \quad z = -h(x). \quad (C1)$$

This equation may also be written

$$f h_x p_y + \frac{d}{dx} [r p_x]_{x=-h(x)} = 0 \quad \text{on} \quad z = -h(x) \quad (C2)$$

and integrated along the bottom from the "coast" at $x = b$ to some general point $x$ to give

$$\int_{b}^{x} f p_x h_x dx + [r p_x]_{b}^{x} = 0 \quad \text{on} \quad z = -h(x). \quad (C3)$$

Since there is no wind forcing and the frequency is so low, the coastal boundary condition (2.13) reduces to
\[ r_p \cdot x = -hfp_y \text{ at } x = b. \quad (C4) \]

Consequently, (C3) can be written

\[ r_p(x) = - \int_{h(b)}^{h(x)} \int f \ p_y \ dh \ - \ h(b)fp_y \text{ on } z = -h. \quad (C5) \]

or, since \( p \) is nearly independent of \( z \) at \( x = b \),

\[ r_p(x) = - \int_{b}^{h} f \ p_y \ dh \text{ on } z = -h. \quad (C6) \]

Because \( u \) and \( v \) are essentially geostrophic at ENSO frequencies, we may also write (C6) as

\[ \tau_{y//\rho_0f}^{*} = \int_{0}^{b} udh \text{ on } z = -h. \quad (C7) \]

Physically, (C7) states that the bottom boundary-layer transport perpendicular to the coast (\( \tau_{y//\rho_0f}^{*} \)) at depth \( h \) is equal to the interior transport onto the bottom over the depth \( h \).

**APPENDIX D**

**Coastal ENSO Theory When the Shelf and Slope Topography Is Dynamically Steep**

When the continental shelf and slope topography is steep enough it can be regarded as a vertical (frictional) wall and the motion can consequently be separated into vertical modes. In this appendix we will show that the shelf and slope topography is dynamically like a vertical wall when \( |Nh_x/f| \gg 1 \). Note that this condition does not imply that the bottom topography must be geometrically steep since usually \( |N/f| \gg 1 \). We will also show that at ENSO frequencies vertical modes are mainly scattered by bottom friction rather than bottom topography.

**a. Separation into vertical modes when \( |Nh_x/f| \gg 1 \)**

It is convenient to follow Huthnance (1978) and represent the continental shelf and slope \( z = -h(x) \) by \( x = b(z) \) and use the coordinates \((\xi, \zeta)\) defined by

\[ \xi = x - b(z) \]
\[ \zeta = z. \quad (D1) \]

In these coordinates the continental shelf and slope topography is defined by \( \xi = 0 \) and the equations and boundary conditions for the flow, (2.11)–(2.15), become

\[ \rho \frac{\partial v}{\partial \xi} + \beta p_t + \frac{f^2}{N^2} \frac{\partial p_t}{\partial \xi} + \left[ \frac{f^2}{N^2} \frac{\partial p_t}{\partial \xi} \right] \frac{\partial^2}{\partial \zeta^2} \rho = 0, \quad (D2) \]

\[ p_{\xi\xi} + \frac{\partial^2 p_{\xi}}{\partial \xi^2} - \frac{b'f^2}{N^2} = 0 \quad \text{on} \quad \xi = 0, -H < \zeta < -h_0 \quad (D3) \]

\[ p_{\xi\xi} + \frac{\partial^2 p_{\xi}}{\partial \xi^2} = \frac{f^2}{N^2} \quad \text{on} \quad \xi = 0, -h_0 < \zeta < 0 \quad (D4) \]

\[ \frac{\rho}{N^2} = 0 \quad \text{on} \quad \xi = 0, -H, \quad (D5) \]

where \( h_0 \) is the depth of the nearshore region where the coastal boundary condition is applied, \( H \) is the depth of the constant depth deep sea region, and \( b' \) and \( b'' \) denote \( \partial b/\partial \xi \) and \( \partial^2 b/\partial \xi^2 \), respectively. Note that in obtaining (D5) we have ignored the small friction effects in the constant depth deep sea.

It follows from (D2) and (D5) that if the terms in brackets in (D2) are negligible, then the solution can be written in the form

\[ p = \sum_{m=0}^{\infty} a_m(y, t) \exp(\lambda_m \xi) \rho \left( \frac{\xi}{\zeta} \right), \quad (D6) \]

where the \( R_m \) and \( \lambda_m \) are defined in (2.16)–(2.18). For each vertical mode \( (\partial / \partial \xi) \approx (N/c) \) and \( (\partial / \partial \xi) \lambda \approx (f/c) \). Under this scaling a comparison of terms V and VI with term III and term IV with term I shows that terms IV, V, and VII are negligible provided that \( b'f/N \ll 1; \) that is, \( |Nh_x/f| \gg 1 \). Since also term VI \ll term V, all terms in brackets in (D2) are indeed negligible and (D6) is a valid solution under the condition that \( |Nh_x/f| \) is large enough.

The functions \( a_m(y) \) in (D6) can be found by substitution of (D6) into the \( \xi = 0 \) boundary conditions (D3) and (D4) and using the orthogonality relations for the eigensolutions \( \rho(\xi) \). The resulting equations for the \( a \) describe how the amplitude of each vertical mode evolves under the influence of wind forcing and scattering by bottom topography and bottom friction.

**b. Scattering of the vertical modes by bottom friction alone**

As noted in appendix C, at ENSO frequencies the dominant balance in the bottom boundary condition (2.12) is

\[ fN_x p_x + (rp_x)_x - nh_x p_{xx} = 0 \quad \text{on} \quad z = -h(x). \quad (D7) \]
At ENSO frequencies we also have $\omega \ll r/h_0$ so the coastal condition (2.13) reduces to

$$rp_x/h_0 + fp_y = f\tau^y/h_0.$$  \hspace{1cm} (D8)

We show below that under (D7) and (D8) vertical modes are scattered by bottom friction alone. In $(\xi, \zeta)$ coordinates, (D7) and (D8) can be written

$$p_y - \frac{dr}{d\zeta}p_{\theta y}/f - rp_{\theta y}/f = 0$$  \hspace{1cm} (D9)

on $\zeta = 0, -H < \zeta < -h_0$ and

$$p_x + rp_{\theta y}/h_0 = \tau^y/h_0$$  \hspace{1cm} (D10)

Substitution of (D6) into (D9) and (D10) and use of the orthogonality relation for the eigenfunctions $R_\zeta(\zeta)$ gives

$$a_{ij} + \sum_{m=0}^{\infty} K_{jm}a_m = v_j\tau^y,$$  \hspace{1cm} (D11)

where

$$v_j = \int_{-h_0}^{0} R_j d\zeta/h_0(R_j, R_j)$$  \hspace{1cm} (D12)

and

$$K_{jm} = \frac{\lambda_m}{(R_j, R_j)f} \left[ \frac{r_0h_0}{r_0h_0} \int_{-h_0}^{0} R_m R_j d\zeta \right. \right.$$

$$\left. - \int_{-H}^{H} R_j d\zeta (rR_m) \right],$$  \hspace{1cm} (D13)

where $r_0$ is the value of $r$ at the coast. Equations (D11)–(D13) show that in the limit $|N\zeta_x/f| \gg 1$, at low enough frequencies vertical modes are scattered solely by bottom friction.

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