NOTES AND CORRESPONDENCE

Storm-Forced Near-Inertial Waves on a Beta Plane

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ABSTRACT

The influence of the beta effect, that is, the variation of the Coriolis parameter with latitude on the energy flux from moving storms to the oceanic internal wave field is considered. Large-scale, fast-moving storms are emphasized, and the oceanic response is described as forced inertial oscillations on a beta plane. An analytical solution to the energy flux is obtained and discussed for a simple wind stress field.

The beta effect introduces a difference between westward and eastward moving storms. It is most pronounced for storm speeds that move the storms past a fixed point in about an inertial period. In this regime, the energy flux to the internal wave field is, relative to the f-plane case, increased when the storm moves eastward and decreased when it moves westward. However, the influence of the beta effect on the energy flux to the internal wave field is generally small, and the f-plane approximation is expected to give a good description of the energy flux.

1. Introduction

Moving storms are efficient generators of internal waves with near-inertial frequencies (e.g., Geisler 1970; Price 1983; Shay et al. 1989). Preliminary calculations suggest that tropical cyclones might be an important energy source for the oceanic internal wave field (Nilsson 1995). The potential importance of storms in this context motivates development of models for the energy flux from moving storms to internal waves.

Gill (1984) pointed out that the baroclinic response of the ocean to a moving storm can be conveniently divided into two stages. In the first stage, which typically lasts for a few inertial periods, the storm passes by and the ocean is set in motion by the wind. In the second stage, which is more prolonged, the disturbed ocean relaxes toward geostrophic equilibrium by radiating internal waves.

Most storms are large compared to the internal Rossby radius and move fast compared to the internal wave speed. This implies that pressure gradients are of secondary importance during the forced stage of the baroclinic response (e.g., Greatbatch 1983, 1984); the forced response is, essentially, controlled locally by the wind forcing. The energy flux from the moving storm to the internal wave field is determined during the forced phase and therefore can be estimated with a model of forced inertial oscillations (e.g., D'Asaro 1985; Nilsson 1994).

Normally, the f-plane approximation accurately describes the forced stage of the response to a moving storm. In the second stage of the response, the variation of the Coriolis parameter with latitude becomes important after a period varying from a few days to a few weeks (e.g., Gill 1984). Waves that propagate latitudinally will be affected by the beta dispersion, which is induced by the variation of the Coriolis parameter (e.g., Anderson and Gill 1979).

This study considers situations where the storms move sufficiently fast to ignore the effects of pressure gradients and beta dispersion in the forced stage of the response. The central issue is to determine how the variation in the Coriolis parameter over the wind-forced domain affects the energy flux from the storm to the internal wave field.

2. The response of the ocean to moving storms

In the hydrostatic regime, the linear response of a stratified ocean to wind forcing can be described by normal modes (e.g., Gill 1982, §9.10). Each mode has a vertical structure that is determined alone by the stratification. The time evolution of the horizontal mode structure is governed by the shallow-water equations:

$$\frac{\partial \tilde{u}_n}{\partial t} - f\tilde{v}_n = -g\frac{\partial \tilde{h}_n}{\partial x} + \sigma_n X \quad (2.1a)$$

$$\frac{\partial \tilde{v}_n}{\partial t} + f\tilde{u}_n = -g\frac{\partial \tilde{h}_n}{\partial y} + \sigma_n Y \quad (2.1b)$$

$$\frac{\partial \tilde{h}_n}{\partial t} = -H_n\left(\frac{\partial \tilde{u}_n}{\partial x} + \frac{\partial \tilde{v}_n}{\partial y}\right). \quad (2.1c)$$

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Here f is the Coriolis parameter and g the acceleration of gravity. The wind forcing is represented as a body force, which is constant in the surface mixed layer and zero elsewhere; $\mathbf{X} = (X, Y)$ is given by

$$\mathbf{X} \equiv \boldsymbol{\tau}(x, y, t)/H_{\text{mix}}, \tag{2.2}$$

where τ is the wind stress at the sea surface and H_{mix} is the depth of the mixed layer; σ_n is the projection of the wind forcing on the nth mode.

Greatbatch (1983, 1984) demonstrated that during the forced response to a moving storm, the pressure gradients can be neglected provided that

$$c_n^2/f^2L^2 \ll 1, \quad c_n^2/U^2 \ll 1. \tag{2.3a,b}$$

Here L is the length scale, U is the speed of the storm, and c_n is the wave speed:

$$c_n^2 \equiv gH_n. \tag{2.4}$$

When relations (2.3) are satisfied, the baroclinic response is dominated by internal waves with near-inertial frequencies. We henceforth ignore the slight frequency shift (above the inertial frequency) of the waves. The response is described as pure inertial oscillations and the horizontal velocity field is thus governed by

$$\frac{\partial \Pi_n}{\partial t} + (if)\Pi_n = F\sigma_n, \tag{2.5}$$

where the following complex notation has been used:

$$\Pi_n \equiv \tilde{u}_n + i\tilde{v}_n, \quad F \equiv (X + iY). \tag{2.6a,b}$$

Note that the dependence on the mode number only enters via σ_n. The vertical structure of the baroclinic velocity field is simple in this case; the velocity field is constant in the mixed layer and below it is also constant but flowing in the opposite direction (e.g., Gill 1984).

For a moving storm, the rate of change of the wind vector at a fixed point is generally dominated by the movement of the wind field. Therefore, the main features of the oceanic response can be modeled with a steady wind stress pattern that moves at a uniform velocity. For convenience, we introduce a coordinate system with the negative x axis aligned with the storm's velocity. In this coordinate system the forcing is of the form $F = F[(x + Ut), y]$. The solution to Eq. (2.5), with the ocean in a state of rest as initial condition, can be obtained as

$$\Pi_n = \sigma_n \exp(-ift) \int_{-\infty}^{t} \exp(ift')F[(x + Ut'), y]dt'. \tag{2.7}$$

We use the beta-plane approximation and the Coriolis parameter is given by

$$f = f_0 + \beta[\cos(\alpha)y + \sin(\alpha)x] \tag{2.8a}$$

$$f_0 \equiv 2\Omega \sin(\theta), \quad \beta \equiv 2\Omega \cos(\theta)/R. \tag{2.8b,c}$$

Here θ is the latitude, Ω and R are the rotation rate and the radius of the earth, and α is the angle between the velocity of the storm and the westward direction.

3. Energy flux to inertial motion

Beneath a moving storm the wind stress performs continuous work on the ocean. The main fraction of the energy input is distributed to inertial oscillations in the wake of the storm. These currents carry energy away from the wind-forced region. The energy flux associated with the inertial motion, which is the emphasis of this study, can be calculated from the velocity field in the wake of the storm. We focus on a fixed cross-track section and place the origin of the coordinate system there ($y = 0$ corresponds to the center of the track). Using Eq. (2.7), the velocity field after the storm has moved past is easily calculated:

$$\Pi_n = \exp(-if_*t)(2\pi)^{1/2}\hat{F}(f_*/U, y)\sigma_n U^{-1} \tag{3.1}$$

$$f_* \equiv f_0 + \beta \cos(\alpha)y. \tag{3.2}$$

Here we have introduced

$$\hat{F}(f_*/U, y) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} \exp[ix(f_*/U)]F(x, y)dx, \tag{3.3}$$

the Fourier transform of $F(x, y)$ with respect to x, evaluated at the wavenumber $k = f_*/U$. The Doppler frequency associated with this wavenumber (i.e., Uk) is equal to the local inertial frequency f_*.

We can calculate the kinetic energy in the wake of the storm from Eq. (3.1); the contribution to the kinetic energy per unit area, from the nth mode, is given by (e.g., Gill 1984)

$$E_k(y, n) = \pi U^{-2}|\hat{F}(f_*/U, y)|^2 \rho_0 H_{\text{mix}}\sigma_n. \tag{3.4}$$

Relative to the storm, the kinetic energy in the wake recedes at the speed U. Accordingly, the associated energy flux is obtained as

$$P(n) = U \int_{-\infty}^{\infty} E_k(y, n)dy. \tag{3.5}$$

This formula gives an estimate of the energy flux to near-inertial waves in the wake of the storm. When the pressure gradients can be neglected for all baroclinic modes, the sum over σ_n is readily evaluated (e.g., Nilsson 1995); the result is close to unity. Thus, an estimate of the net energy flux to near-inertial waves is obtained by replacing σ_n with unity in Eq. (3.5).

We note briefly that the beta-plane distribution of kinetic energy in the wake may be obtained on an f plane by modifying the forcing $F(x, y)$. An equivalent stress field that produces the same energy distribution and energy flux, say $G(x, y)$, is found from Eq. (3.3):

$$G(x, y) = \exp[i(\beta/U)\cos(\alpha)xy]F(x, y). \tag{3.6}$$

The new stress field has everywhere the same amplitude as the original one, but the stress vector is rotated to compensate for the varying turning rate of the inertial oscillations on the beta plane.

4. An analytical solution for a simple stress pattern

Analytical solutions are useful and instructive, since they offer a comprehensive view over the parameter space. Such information can be valuable for numerical investigations of less tractable cases. An analytic solution to the energy flux can be obtained for the following axisymmetric wind stress field:

$$\boldsymbol{\tau}(r) = \tau_{\max}(-\hat{r}\sin\varphi + \hat{\theta}\cos\varphi)(r/L) \times \exp[0.5(1 - (r/L)^2)], \quad (4.1)$$

where r is the radius, $\hat{r}$ and $\hat{\theta}$ are unit vectors in the radial and the azimuthal directions, φ is a constant inflow angle, and L is the radius of maximum wind. This field captures the main features of a hurricane wind field.

In terms of $F(x, y)$, we have

$$F(x, y) = (\tau_{\max}/H_{\text{mix}})(ix/L - y/L) \times \exp[i\varphi + 0.5(1 - (r/L)^2)]. \quad (4.2)$$

For this field, we can integrate Eq. (3.3) and thus determine the kinetic energy per unit area:

$$E_k(y, n) = \frac{\rho_0 \tau_{\max}^2}{f_0^2 H_{\text{mix}}} \Gamma(y_*, K, \epsilon)\sigma_n, \quad (4.3)$$

where we have introduced

$$\Gamma(y_*, K, \epsilon) \equiv \pi K^2[K + (1 + K\epsilon)y_*]^2 \times \exp[1 - y_*^2 - K^2(1 + \epsilon y_*)^2] \quad (4.4)$$

and

$$K \equiv f_0 L/U, \quad \epsilon \equiv \cos(\alpha)\beta L/f_0, \quad y_* \equiv y/L. \quad (4.5\text{a–c})$$

Finally, we calculate the energy flux, Eq. (3.5) and obtain

$$P(n) = \frac{\rho_0 \tau_{\max}^2 L^2}{f_0 H_{\text{mix}}} \Phi(K, \epsilon)\sigma_n. \quad (4.6)$$

The nondimensional function Φ is defined by

$$\Phi(K, \epsilon) \equiv \pi^{1.5}[0.5(1 + \lambda)\Lambda + (1 - \lambda)\Lambda^3] \exp(1 - \Lambda^2), \quad (4.7)$$

where

$$\Lambda \equiv K(1 + \epsilon^2 K^2)^{-1/2}, \quad \lambda \equiv \epsilon K(1 + \epsilon^2 K^2)^{-1}. \quad (4.8\text{a,b})$$

Evidently, the f-plane formula for the energy flux is obtained by putting $\epsilon = 0$. By replacing Λ with K, and λ with ϵK in Eq. (4.7), we find the two first terms in a power series expansion in ϵ.

5. Discussion of the energy flux

On an f plane, the dependence of the energy flux on the speed and the form of the wind field can be elucidated by a simple mechanical analogy, namely a harmonic oscillator (with eigenfrequency f_0) subjected to transient forcing. A fixed cross section in the ocean is exposed to the wind stress a finite duration of time and the characteristic forcing frequency can be estimated as U/L, where L is the typical length scale of the storm and U is the storm speed. The match between the forcing frequency and the inertial frequency determines the strength of the response and, consequently, the energy flux to inertial motion. Depending on the value of U/f_0L, we may identify three distinct regimes.

(i) $U/f_0L \ll 1$: it takes the storm many inertial periods to move past a fixed point. The forcing frequency is much lower than f_0 and the energy flux to inertial motion is insignificant.

(ii) $U/f_0L \approx 1$: the storm moves past a fixed point in about an inertial period. A resonant coupling occurs between the forcing and the inertial oscillations, which results in a strong energy flux.

(iii) $U/f_0L \gg 1$: the time a fixed point is exposed to the wind is short compared to the inertial period. The response is only weakly affected by the Coriolis acceleration during the storm passage. In this regime, the energy flux is mainly controlled by the time a fixed point is exposed to the wind.

Even when pressure gradients are retained and wave propagation cannot be neglected, these three regimes still delineate the main features of the energy flux from the storm (Nilsson 1995); the contribution to the restoring force from the pressure gradients merely increases the eigenfrequency above f_0.

Figure 1 shows the energy flux, Eq. (4.7), as a function of U/f_0L for a westward ($\epsilon > 0$) and an eastward ($\epsilon < 0$) traveling storm on a beta plane. The wind stress field is defined by Eq. (4.1). For comparison, the f-plane ($\epsilon = 0$) case is displayed. A rather large ϵ value is used to clearly illustrate the beta effect.

To appreciate how the beta effect influences the energy flux, it is important to recognize that the storm produces an asymmetric oceanic response. The reason is that the wind stress vector, relative to a fixed point, turns clockwise to the right and anticlockwise to the left of the storm center. Accordingly, the coupling between the inertial oscillations, which are turning clockwise, and the wind forcing is stronger on the right-hand side of the storm track (e.g., Price 1981, 1983). Thus, the energy flux is mainly affected by how the beta effect shifts the inertial frequency to the right of the storm center. For a westward moving storm the inertial frequency is increased (relative to f_0, the midtrack fre-

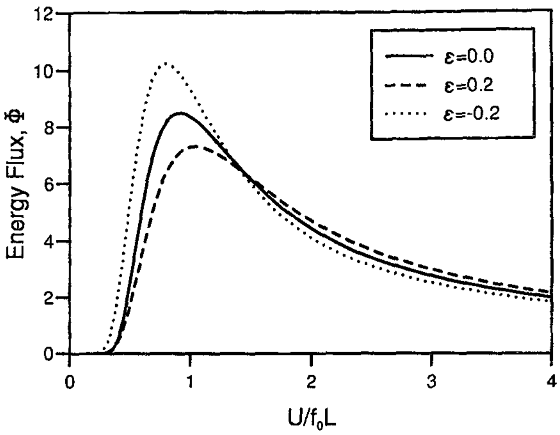

FIG. 1. The (nondimensional) energy flux from a moving storm to inertial oscillations on a beta plane, Eq. (4.7), shown as a function of U/f_0L: f_0 is the Coriolis parameter at the storm center, U is the speed, and L is the length scale of the storm. Here ϵ measures the importance of the beta effect and is positive for westward storm velocities and negative for eastward velocities; $\epsilon = 0$ corresponds to the f-plane case. Resonance occurs when U/f_0L is about unity.

quency) to the right and decreased to left of the storm center, and vice versa for an eastward moving storm ($\epsilon < 0$).

The harmonic oscillator analogy explains why the energy flux peaks at a larger U/f_0L value for the westward moving storm; the effective eigenfrequency is increased above f_0 and the forcing frequency U/L is therefore larger when resonance occurs. At resonance, the energy flux is proportional to the time a fixed point is exposed to the wind. The eastward traveling storm moves slower when the resonance occurs and therefore the resonant peak is stronger in that case (see Fig. 1).

Figures 2a–c show the kinetic energy distribution in the wake of the storm, Eq. (4.4), for three different values of U/f_0L. Each illustration contains the cases $\epsilon = \{-0.2, 0, 0.2\}$. Note that the rightward bias of the velocity field is most pronounced in the resonant regime. These illustrations give an approximation of the kinetic energy distribution in the mixed layer directly after the storm has passed. As time evolves, however, the energy is dispersed laterally and vertically (e.g., Price 1983; Gill 1984).

It is clarifying to discuss the response on the two sides of the storm center separately:

(i) To the left of the storm center, the wind stress vector and the inertial oscillations are turning in opposite directions. Thus, a decrease in the inertial frequency diminishes the difference in turning rate between oscillations and forcing, which leads to an enhanced coupling. On a beta plane, the amplitude of the oscillations is therefore larger when the storm moves to the west. This occurs for all values of U/f_0L; see Fig. 2.

(ii) To the right of the storm center, the wind stress vector and the inertial oscillations are turning in the same direction. Here, the beta effect acts differently depending on the value of U/f_0L. When U/f_0L is large,

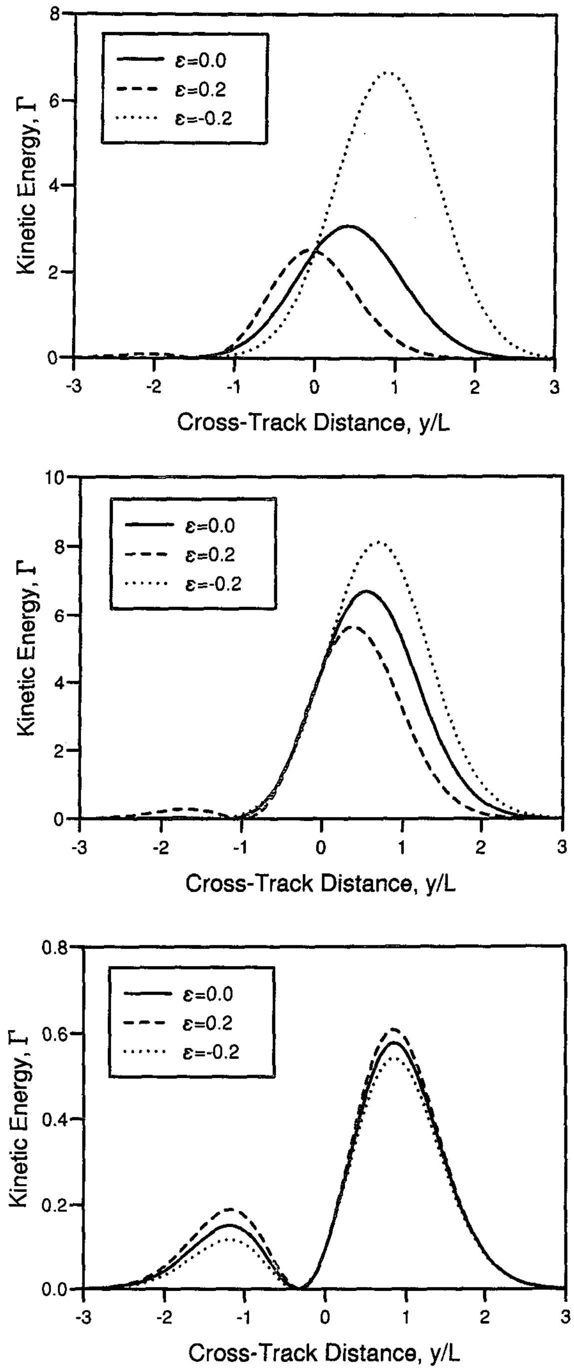

FIG. 2. Spatial distribution of kinetic energy in the wake of the storm, Eq. (4.4). Three regimes are displayed: (a) $U/f_0L = 0.5$, (b) $U/f_0L = 0.8$, and (c) $U/f_0L = 3.0$. Each illustration shows the energy distribution for the beta plane case, with a westward ($\epsilon > 0$) and an eastward moving storm ($\epsilon < 0$), and the f-plane case ($\epsilon = 0$).

the stress vector turns rapidly compared to the oscillations, and coupling is consequently promoted by an increase in the inertial frequency. In this regime, the storm is better tuned with the inertial oscillations when it moves to the west. When U/f_0L is small, the stress vector turns slowly compared to the inertial oscillations. Now, the situation is reversed and the eastward moving storm, for which the inertial frequency is reduced to the right of the storm center, is better tuned with the oscillations. The transition to this regime occurs for this particular wind field when $U/f_0L \approx 1.5$.

6. Concluding remarks

A simple axisymmetric wind stress distribution has been used to study how the beta effect influences the energy flux to inertial oscillations. We believe, however, that the important physics is captured by this idealized stress field. A more complicated stress pattern, for example a nonaxisymmetric storm, may have more than one characteristic length scale. Thus, resonance can occur at different storm speeds for the different, characteristic length scales. The energy flux may now have several resonance peaks, instead of a single as in Fig. 1. However, the preceding discussion still predicts, qualitatively, how the beta effect will alter the position and strength of the resonance peaks.

Tropical cyclones are storms for which the present study may be of relevance. Consider a tropical cyclone at 10° latitude with a typical length scale L (1×10^5 m) and moving at the speed U (5 m s^{-1}). With a wave speed representative for the midocean ($c_n = 3/n$ m s^{-1}), we obtain

$$c_n^2/f_0^2L^2 \approx 1.4/n^2, \quad c_n^2/U^2 \approx 0.4/n^2, \quad \beta L/f_0 \approx 0.1. \quad (6.1a–c)$$

This example suggests that the pressure gradients are generally more important than the beta effect for the first few baroclinic modes. The energy flux to the higher modes is, to some extent, influenced by the beta effect. Thus, the f-plane formula for the energy flux presented by Nilsson (1995), which includes pressure gradients, is normally expected to work well. However, when c_n/U is close to unity and the storm moves at low latitudes, the beta dispersion of the generated waves may not be negligible during the passage of the storm. The f-plane formula for the energy flux probably fails in this case. Further studies of the energy flux in this regime would be interesting.

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