Leakage of Barotropic Slope Currents onto the Continental Shelf

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ABSTRACT

The effect of alongshore variation in continental slope steepness upon the on-shelf penetration of barotropic, slope-trapped currents is investigated using arrested topographic wave dynamics. Results are summarized by a leakage length scale $L_y = fsw^2/r$ with f the Coriolis parameter, s the continental slope steepness, w the slope current width, and r the linear friction coefficient. Leakage is found to be enhanced if the steepness of the continental slope increases suddenly in the direction of current flow. The Newfoundland and Hebridean shelves are suggested as possible locations where this effect may be observed.

1. Introduction

Strong currents trapped against the continental slope and flowing alongslope in the direction of topographic wave propagation (shallow water to the right/left in the Northern/Southern Hemispheres) are a widespread phenomenon. Examples include the Leeuwin Current (western Australia), the offshore Labrador Current, the Alaskan Stream, the Bering Slope Current, the Falkland–Malvinas Current, and the Scottish Slope Current. Several different mechanisms may ultimately generate these flows including adjustment of alongshore oceanic sea-level gradients over the continental slope (Huthnance 1992) and, in some cases, western boundary dynamics. Most slope currents also have a baroclinic component because they either transport warm water poleward or cold water equatorward giving rise to horizontal density gradients across the slope region.

An important question concerning these currents is whether they are capable of significantly influencing the circulation on adjacent continental shelves; that is, do pressure (sea level) disturbances associated with slope currents leak onto the shelf? It is widely held that steep continental slopes have an insulating effect, which prevents penetration of steady, oceanic sea-level gradients onto the shelf. In this paper the problem is reexamined with the focus on slope-trapped currents represented by an inflow along the upper slope on an upstream boundary. Unlike previous analyses, however, specific consideration is given to the possibility that alongslope variation of bottom topography may promote leakage.

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2. The insulating effect of the continental slope

The steady, linearized, depth-averaged equations of motion and continuity are

$$-fv = -g\eta_x - ru/h \qquad (1)$$

$$fu = -g\eta_y - rv/h \qquad (2)$$

$$(hu)_x + (hv)_y = 0, \qquad (3)$$

where subscripts x and y denote partial differentiation. Here η is the sea surface elevation; (u, v) are depth-mean velocities in the x (cross shore) and y (alongshore) directions, respectively; g is the gravitational acceleration; f the Coriolis parameter; h bottom depth; and r a linear friction coefficient.

If the coastline is assumed to be long and straight with no alongshore variation in topography and the dominant cross-shore dynamical balance is geostrophy, then elimination of u and v between (1) and (3) gives a single equation for surface elevation derived by Csanady (1978),

$$\eta_y + (r/fs)\eta_{xx} = 0, \qquad (4)$$

where $s = h_x > 0$ is the bottom slope. At the coast there is no cross-shore transport ($uh = 0$) and $h = 0$, so the coastal boundary condition is $\eta_x = 0$.

Equation (4) has the form of a heat conduction (diffusion) equation in which η and $y_t = -y$ are the analogs of temperature and time, respectively (Csanady 1978). The negative y direction (analog of forward time) is that of topographic wave propagation and defines the downstream direction for which solutions for η may be found. The analog of conductivity is $K = r/fs$, which is inversely proportional to bottom slope, indicating that steep slopes behave like thermal insulators with respect to penetration of the sea-level field in the cross-shore direction. Wright (1986) has argued that there is a sense in which steep slopes may be regarded as poor insulators. Transformation of variables in (4) from x to $h(x) = sx$ shows that diffusion of the elevation field across isobaths is most effective for steep slopes. However, this paper is concerned with lateral (x direction) spreading for which the conventional view of steep slopes as insulators is appropriate.

In Csanady's original analysis there was no shelf break. Later work (Wang 1982; Csanady and Shaw 1983; Chapman et al. 1986) included both a gently sloping shelf and a steeper continental slope. Wang (1982) showed that an alongshore pressure gradient imposed in deep water would experience negligible penetration onto the adjacent shelf. This effect has also been demonstrated in several numerical simulations over realistic bathymetry (e.g., Prandle 1984; Pingree and Le Cann 1989). Wang (1982) considered forcing by upstream cross-shore sea level gradients (inflows) extending across the entire shelf–slope region, and Chapman et al. (1986) considered those confined to the shelf. They found that the shelf pressure field quickly spread offshore across the (poorly insulating) shelf to become trapped over the top of the slope. Chapman et al. (1986) also examined effects of a combined upstream shelf inflow, upstream deep ocean inflow, and an alongshore oceanic pressure gradient over realistic topography representing the Scotian and Mid-Atlantic Bight shelf–slope regions and found that the oceanic parts of the pressure field tended to prevent the shelf component flow from spreading off the shelf. None of these studies, however, has attempted explicitly to isolate the influence of upstream inflow confined to the slope, which is characteristic of so many regions. This is the subject of the next section.

3. Slope current inflow

a. Uniform alongshore topography

An infinitely long, straight shelf–slope region is considered as shown in Fig. 1. The x axis points offshore and the y axis is parallel to the coast and located at the shelf break, which is at $x = 0$. In what follows, subscripts 1 and 2 denote variables defined on the shelf and slope, respectively. The shelf is located in the region ($-L \leqslant x \leqslant 0$) and has constant bottom slope $h_x = s_1$ and zero depth at the coast. The continental slope is in the region ($x > 0$) and has constant bottom slope $h_x = s_2$ ($>s_1$). The upstream boundary of the shelf–slope region is at $y = 0$ and negative y direction is that of topographic wave propagation.

The slope current is treated as a steady, barotropic inflow across an upstream boundary and flowing in the sense of topographic wave propagation. By implication, no attempt is made to explain the physical origin of the flow. Neglect of baroclinic effects is justified on the grounds that barotropic forcing (oceanic sea-level gradients) naturally generates flow over steep slopes and thus may be regarded as the primary forcing

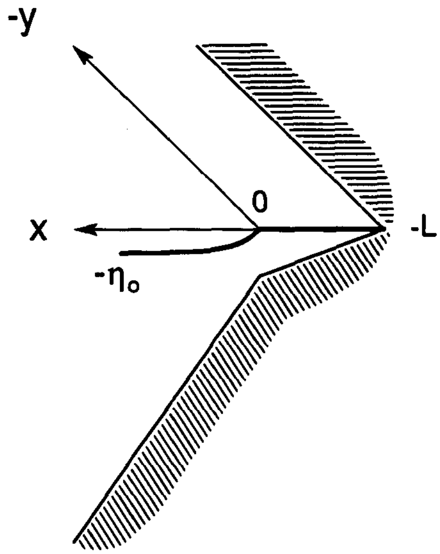

FIG. 1. A semi-infinite, straight shelf–slope region.

agent with baroclinic effects playing a modifying role. As shown in Fig. 1, the sea level on the continental shelf at the upstream boundary is taken to be flat (unperturbed), decreasing exponentially to a level $-\eta_0$ in deep water over an e-folding width, $w = 1/d$; that is, $\eta_1(x, 0) = 0$ for $-L \leqslant x \leqslant 0$ and $\eta_2(x, 0) = \eta_0(e^{-dx} - 1)$ for $x > 0$.

The problem is to determine how the shelf sea-level field is disturbed downstream in response to the slope current inflow. Previous authors have tackled such problems numerically. However, it is possible to find an analytical solution in this case using Laplace transform methods applicable to heat conduction problems in composite solids (Carslaw and Jaeger 1959). Equation (4) governs the solution in the shelf and slope domains. Solutions obtained in each region are matched subject to the conditions of continuity of surface elevation and of cross-shore surface slope at the shelf edge. In the shelf region the coastal boundary condition $\eta_x = 0$ applies. At large distances offshore in the slope region $\eta_2(x, y_t) \rightarrow -\eta_0$ as $x \rightarrow \infty$.

The shelf part of the solution is

$$\eta_1(x, y_t) = \left(\frac{\eta_0}{1+\gamma}\right)\left[\sum_{n=0}^{\infty}(-\gamma_r)^n(F(2nL - x) + F(2[n+1]L + x))\right], \quad (5)$$

where

$$F(x) = \operatorname{erfc}\left(\frac{x}{2(K_1 y_t)^{1/2}}\right) - \exp(d\gamma x + K_1 d^2\gamma^2 y_t) \times \operatorname{erfc}\left(\frac{x}{2(K_1 y_t)^{1/2}} + d\gamma(K_1 y_t)^{1/2}\right), \quad (6)$$

with $\gamma = (K_2/K_1)^{1/2}$ and $\gamma_r = (1-\gamma)/(1+\gamma)$.

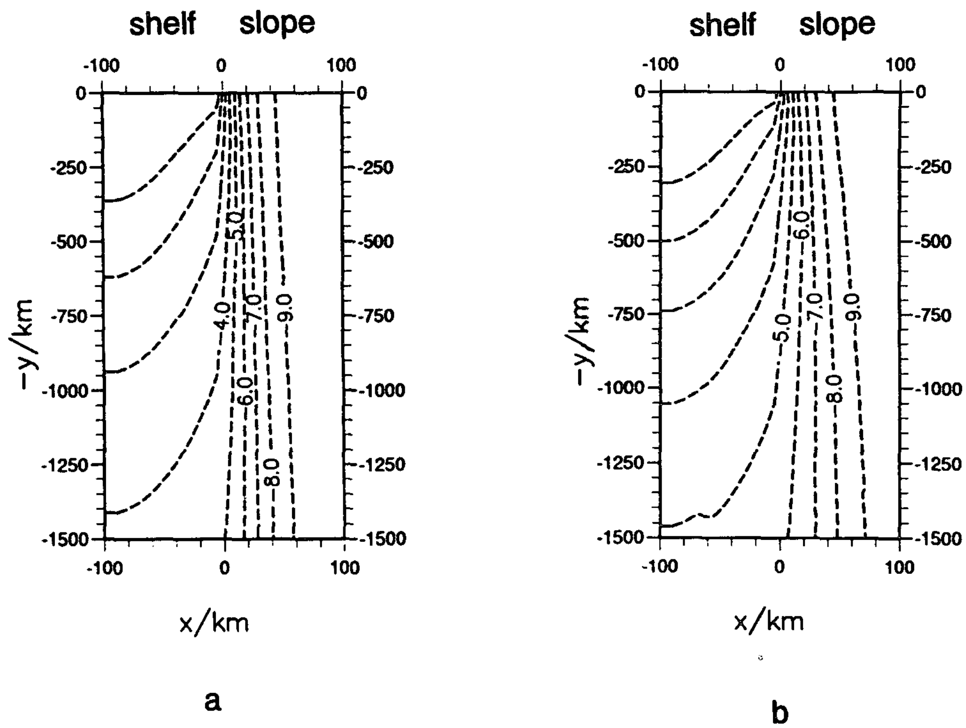

FIG. 2. Analytical solutions of (4) for a shelf–slope region. Surface elevation at $y = 0$ is prescribed as zero on the shelf, decreasing to $\eta = -\eta_0 = -0.1$ m on the slope over an e-folding width, $w = 20$ km. Elevations (cm) below $\eta = 0$ are shown: (a) Shelf gradient $s_1 = 1 \times 10^{-3}$, slope gradient $s_2 = 5.0 \times 10^{-2}$; (b) $s_1 = 1 \times 10^{-3}$, $s_2 = 2.5 \times 10^{-2}$.

The following values were assigned to the principal parameters: $g = 9.81$ m s^{-2}, $f = 10^{-4}$ s^{-1}, $r = 10^{-3}$ m s^{-1}, $\eta_0 = 0.1$ m, $L = 100$ km, and $w = 20$ km. The value of r has been chosen to be characteristic of the shelf region and hence will probably overestimate friction over the slope part of the region. Figure 2a shows the elevation field (cm) over the shelf–slope region for which $s_1 = 1$ m km^{-1} and $s_2 = 50$ m km^{-1} corresponding to a fairly steep continental slope. The basic features of the solution are that the elevation gradient on the upstream boundary relaxes onto the shelf in the downstream direction, inducing both alongshore and cross-shore sea-level gradients on the shelf. There is little corresponding spread of the elevation field in the off-slope direction. Figure 2b shows a similar solution but for a gentler continental slope ($s_2 = 25$ m km^{-1}), the gradient of the continental shelf remaining unchanged. Clearly in the latter case there is much more penetration of the slope elevation field onto the shelf, consistent with the view that steep slopes are the most effective insulators of the shelf with respect to the elevation field.

Nondimensionalization of (4) or inspection of the solution (5)–(6) shows that the characteristic alongshore distance over which spreading of the elevation field onto the shelf occurs is

$$L_y = w^2/K_2 = fs_2w^2/r, \tag{7}$$

where w is the slope current width and s_2 is the continental slope steepness. This implies that narrow inflow, gentle bottom slope, and high friction all promote leakage (small L_y). For the examples shown in Fig. 2, $L_y = 2 \times 10^3$ km and 1×10^3 km for slopes of 50 m km^{-1} and 25 m km^{-1}, respectively.

b. Alongshore variation in topography

When alongshore variation in bathymetry is allowed, we require the generalized form of (4), obtained by eliminating u and v between (1)–(3) and retaining the cross-shore friction term in (1). Surface elevation is then governed by the elliptic equation (Schwing 1992)

$$\alpha(\eta_{xx} + \eta_{yy}) + (\alpha_x - \beta_y)\eta_x + (\beta_x + \alpha_y)\eta_y = 0, \tag{8}$$

where $\alpha = R^2\epsilon/(1 + \epsilon^2)$, $\beta = R^2/(1 + \epsilon^2)$, $R = (gh)^{1/2}/f$, and $\epsilon = r/fh$.

Equation (8) was solved numerically, using the method described by Lindzen and Kuo (1969), by expressing it in terms of centered finite differences on a grid with mesh size $dx = 5$ km (cross-shore) and $dy = 10$ km (alongshore). Depth is taken to be zero at the coast, hence the coastal boundary condition remains as before. At the ocean boundary, surface elevation is clamped ($\eta = -\eta_0$), which is the numerical

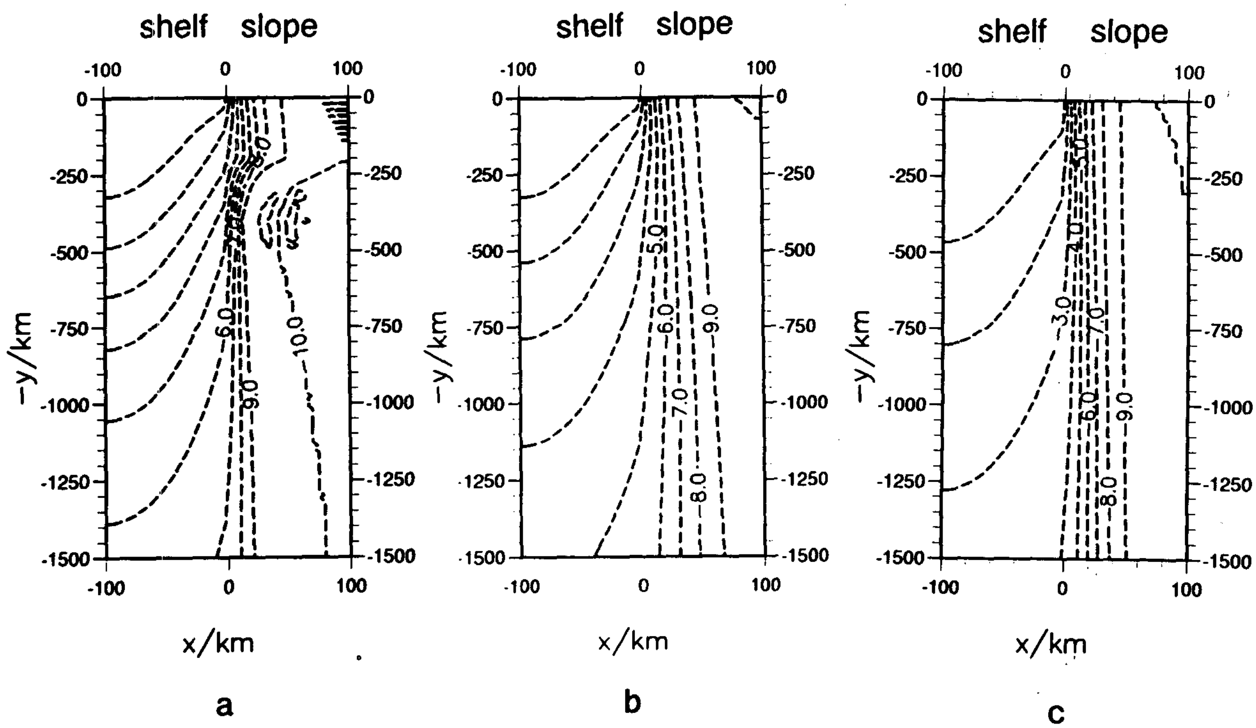

FIG. 3. Numerical solutions of (8). Surface elevation at $y = 0$ is prescribed as zero on the shelf, decreasing to $\eta = -\eta_0 = -0.1$ m on the slope over an e-folding width, $w = 20$ km. Elevations (cm) below $\eta = 0$ are shown. (a) Slope steepness increasing alongshore. Shelf gradients $s_1 = 1 \times 10^{-3}$, slope gradient constant $s_2 = 2.5 \times 10^{-2}$ for ($0 > y > -200$ km), increasing linearly between (-200 km $> y > -300$ km), then constant with $s_2 = 1 \times 10^{-1}$ ($y <= -300$ km). (b) Shelf gradient $s_1 = 1 \times 10^{-3}$, slope gradient $s_2 = 2.5 \times 10^{-2}$. (c) $s_1 = 1 \times 10^{-3}$, $s_2 = 1 \times 10^{-1}$.

implementation of the far-field condition used above. The upstream boundary condition is as before, and at the downstream boundary the condition is that there be no elevation gradient in the alongshore direction, ($\eta_y = 0$). The model was tested by ensuring that it was able to reproduce the analytical solutions shown in Fig. 2.

Figure 3a shows the elevation field resulting from a slope current inflow of width $w = 20$ km onto a continental slope for which $s_2 = 25$ m km^{-1} from the upstream boundary to 200 km downstream. Slope steepness then increases linearly from 25 m km^{-1} to 100 m km^{-1} between 200 and 300 km downstream. Beyond 300 km, the slope remains constant with $s_2 = 100$ m km^{-1}. Figures 3b and 3c show equivalent numerical solutions for $s_2 = 25$ m km^{-1} and $s_2 = 100$ m km^{-1} when there is no alongslope change in steepness (Figs. 2b and 3b provide a direct comparison between the analytical and numerical solutions). In all cases, the continental shelf has constant steepness $s_1 = 1$ m km^{-1}. Thus, while the conventional view is that steep slopes should reduce on-shelf leakage of the elevation field, the results above clearly demonstrate that alongshore steepening of the slope actually increases leakage.

4. Discussion

The paradoxical increase in slope current penetration with increasing alongshore slope steepness can be explained by consideration of the leakage length scale (7). To first order, elevation contours over the continental slope are almost parallel to isobaths because the steep slope makes topographic steering the dominant process. When the slope steepens rapidly (over a distance much less than fs_0w^2/r where s_0 is the initial slope), the width of the elevation field narrows as isobaths bunch together. Characteristically, an increase in the bottom gradient by a factor k decreases the width of the slope current to w/k and, because L_y is linear in s but quadratic in w, reduces the leakage length scale to L_y/k. For the simulation shown in Fig. 3 the increase in bottom slope by a factor of 4 reduces the leakage length scale from $L_y = 1 \times 10^3$ km to 250 km. Physically, therefore, steepening of the slope causes the slope current to become narrower by topographic steering, which in turn sharpens the offslope elevation gradient, increasing the potential for leakage, by relaxation of the elevation field onto the shelf. Similarly, alongshore reduction of bottom slope would widen the flow and reduce the capacity for leakage.

There are at least two locations where there is some evidence that the dynamics described above may apply.

Hukuda et al. (1989) performed a numerical simulation of flow on the Newfoundland shelf–slope and found that at least part of the westward flow across the shallow Grand Banks could be accounted for by leakage onto the continental shelf of the sea-level field associated with the offshore Labrador Current. This onbank spread of the flow, which is also apparent in the simulation of Greenberg and Petrie (1988), appears to be associated with a steepening of the slope in the vicinity of Carson Canyon.

There is evidence from current meter records that the Scottish continental slope current spreads over the shelf edge to the north of Ireland, particularly in autumn and winter (Booth and Ellett 1983; Ellett et al. 1986). A broad eastward incursion of Atlantic water onto the Malin–Hebrides shelf north of Ireland as well as eastward flow along the north coast of Ireland are also documented (Ellett 1979). The onshelf incursions appear to occur where the slope current flows from a region of fairly gentle slope west of Ireland to a location at about 55°N where the upper slope becomes extremely steep (about 70 m km^{-1}).

Other processes may be implicated in slope current leakage such as enhanced shelf friction, inertial effects, and variations in shelf bathymetry resulting in onshelf steering of flow. In this paper a particularly simple explanation of the phenomenon has been provided, based solely upon alongshore steepening of the continental slope with no variation in shelf bathymetry. Slope current leakage is important as a possible coupling mechanism between coastal ocean circulation and the oceanic processes, that ultimately drive slope currents.

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