Interaction of Internal Waves with a Topographic Sill in a Two-Layered Fluid

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ABSTRACT

Sills and seamounts may alter or even disrupt internal waves that approach them. The authors study by one-dimensional laboratory experiment: (i) the fission of an internal solitary wave of a two-layered fluid by a triangular obstruction, (ii) the form preservation of the transmitted wave, and (iii) the energy dissipated by the vortex induced by the obstruction. The potential energy stored in the wave signal is taken as a measure to quantify the above notions. Results are presented for upper to lower depth ratio \( r_d = 3 \) and \( r_d = 1/2 \). The results show in general a pronounced dependence on the degree of blocking (\( B = \) depth of obstruction/upper layer depth), but experiments are less conclusive for dependence on particle speeds and slope angles of the triangle. The authors determine for a given degree of blocking how much energy of an approaching wave is fed into the reflected and transmitted wave. For \( B < 0.6 \) the obstruction is virtually ineffective; for \( B \approx 0.8 \) a compact solitary wave hump transforms into a dispersed wave train; and for \( B \approx 1.2 \) wave transmission is practically blocked.

1. Introduction

Internal waves in fluids, that is, waves due to density variations that may exist, play a significant role in oceanography and physical limnology. In lakes they form the dominant processes during summer stratification and greatly affect the transport of nutrients and other particulate substances, not to mention the effect they exert via biophysical coupling mechanisms on the evolution of phyto- and zooplankton. In glacially formed alpine lakes and in complex estuarine regions such as fjords, which may consist of several interconnected basins partly separated from one another by sills, a wave generated in the main basin may or may not be able to enter an arm of the basin system. If such an arm does not have a tributary and the internal waves are blocked by the sill from entering it, then because of the substantial horizontal water displacements that accompany the wave, an important mechanism for water renewal will be missing. All situations between complete blocking and full transmission are conceivable, but we have found no literature in which this question has been adequately addressed; however, limnologists working with Lake Constance state that this might be the case in Lake Überlingen, a long lake arm separated from the main upper Lake Constance by a sill.

The general properties of internal waves are discussed in several textbooks, such as Gill (1982), LeBlond and Mysak (1978), Lighthill (1978), and Phillips (1977). The prevailing stratification is vertical, and during summer, in freshwater lakes, the density structure divides the water body into a light upper layer and a heavier lower layer, suggesting that the real density distribution in lakes may in these situations be approximated by a two-layer configuration with constant density in each layer. Such configurations are often also adequate in the ocean and in fjords. There are a great number of theoretical models covering this situation, linear ones (e.g., see Hutter 1984, 1986, 1993) and nonlinear ones of the KdV type (e.g., Mysak 1984, Grimshaw 1983) or even more complex ones that incorporate variable bottom topography (e.g., Diebels et al. 1994; Bauer et al. 1994). Even though these latter models have demonstrated some potential in elucidating the above question in the Lake Constance situation (Bauer et al. 1994b), all these theoretical models must fail at last because they ignore the local turbulence that may be generated in the vicinity of the sill if the latter functions as a fission mechanism. This is the experimental result of this paper; it was not clear to us before the experiments were performed that the configurations of subsurface obstructions in which the aforementioned nonlinear two-layer models with variable bottom topography are applicable would also be inefficient as fission mechanisms for the internal waves. On the other hand, our experiments also delineate to a certain extent the conditions under which the theoretical models are indeed applicable. The interface of the two layers can be surprisingly close to the sill depth before the theory...
becomes invalid. From these points of view, this work is experimental, and we only speculate about what theoretical implications the results might portend.

Thus, we address the question as to how an internal wave that propagates along the thermocline interacts with variations of the bottom topography such as lake mountains, sills, and other subsurface obstructions. We would like to know which parts of an internal solitary wave that approaches an obstacle are reflected, transmitted, or dissipated by the turbulence that develops on the luv (near) and lee (far) sides of the obstruction, respectively. These vortices or gyres transport, by their enhanced mixing power, tracers (such as gases or nutrients) into the lower layer; and they are also chiefly responsible for the attenuation and complete destruction of the internal wave. This interaction, in turn, also affects the horizontal transport of water, as well as suspended matter across the obstruction. Of interest is to know how this energy fission and annihilation depends upon the relative magnitudes of the total water depth, sill height, thicknesses of the upper (epilimnion) and lower (hypolimnion) layers, and strength of stratification.

To find at least a partial answer to this question experiments were performed in a laboratory channel 10 m long, 0.33 m wide. In a two-layer-fluid system consisting of a saltwater and freshwater layer with free surface soliton-like internal waves at the salt-freshwater interface were generated at one end of the channel. These waves traveled along the channel; in the middle they encounter a triangular symmetric sill and are (possibly) reflected, transmitted, and (partly) dissipated by the turbulence that may be generated at the sill. Measuring the energies of the incoming, reflected, and transmitted waves determines the amount of energy dissipated at the obstruction. We present here results of how these quantities depend on the relative depth of the two layers and the degree of blocking by the triangular obstruction. The work summarizes and extends results obtained by Schuster (1992) and primarily by Wessels (1993).

In Fig. 1 details of the vicinity of a flat-topped triangular obstruction for two experiments are described. In the left column of photographs the degree of blocking is \( B = 0.7 \), and in the right column \( B = 1 \). In the first photograph (top) the wave (approaching from the left) has not yet reached the area of view. As one moves downward through the snapshots, one sees how the wave overcomes the triangle. The disturbances on the lee side are larger than on the luv side, but in the experiment on the right a pronounced vortex appears on the lee side, while in the experiment on the left no gyre is formed but some local diffusion (mixing) takes place. Less, as well as more, dramatic situations have also been encountered in our experiments.

2. Experimental setup and measuring technique

The wave channel consists of four Plexiglass modules 2.5 m long and 10 mm thick (see Fig. 2 and its figure caption for details) with a quadratic cross section of side length 0.33 m, solidly connected and sealed. The horizontal level of the channel over its 10-m total length is accurate to within ±0.4 mm and the width varies by ±1.0 mm at most. On the left, an additional module with the wave generator is added. The channel is filled with deaerated fresh water, which is underlain by deaerated saltwater. Six gauges record the displacement of the interface between fresh and saltwater by using electrical conductivity variations that are induced by the passage of the interface wave. The gauges consist of two parallel thin wires subjected to an alternating voltage. The voltage difference between the two wires depends on their gap width, the salinity (electrical resistivity) of the lower-layer fluid, and the depth of submergence of the two wires into this saline water. Thus, variations in the vertical position of the interface will cause variations in the current flowing between the gap of the two wires. With proper calibration, the voltage difference at the two wires is a direct measure of the interface elevation. These gauge signals are analog signals, and the voltage differences they cause are small. Thus, they need be amplified and digitized by an analog—digital (AD) converter, transferred to a PC, and then processed for further use as time series of interface elevation at the position of the gauge. The experimental method, including a detailed error analysis, is documented in Schuster (1992); interested readers can contact the second author.

A wave generator is installed at the left end of the channel; it consists of two pistons, separated by a horizontal plate at the level of the interface, moving in opposite directions. By an appropriate gear it is assured that the displaced volumes in the upper and lower layers are exactly balanced. This guarantees that—except for experimental error—only the baroclinic mode is excited, that is, the free surface disturbance is virtually nil, while a large internal wave arises. The construction allows experimentation with thickness ratios \( r_H \) of the layers 1:3, 1:2, 1:1, 2:1, and 3:1. There have been earlier attempts to construct wave generators for baroclinic waves in the laboratory; see Davis and Acrivos (1967), Thorpe (1968), Walker (1973), Lewis et al. (1974), Koop and Redekop (1981), Koop and Butler (1981), Segur and Hammack (1982), Kao et al. (1985), Helfrich and Melville (1986), Wallace and Wilkinson (1988), and Renouard et al. (1987). However, in none of these is the barotropic signal filtered out as successfully as with our wave generator.

Figure 2 gives an overview of the experimental setup and the figure caption provides some further detailed information. Here we simply state that there are six gauges \( F_1 - P_a \) distributed along the 10-m-long channel, a wave generator on the left and a bottom obstruction
Fig. 1. Interaction of internal waves with a flat-topped triangular obstruction. The total height of both layers is 15 cm and the layer ratio is $r_H = 3$. The triangle has a base length of 41 cm and height of 7 cm (left) and 10 cm (right), respectively. The saltwater is fluoresced; the back wall of the channel was coated with a red plastic folio. Time difference between the snap shots is 1 s. Waves propagate from left to right. For $B = 0.7$ (left column) the transmitted wave seems to undershoot the lower layer along the back ramp and thus generates the transmitted wave on the right end of the photographs. For $B = 1.0$ (right column) a typical overshooting takes place and an internal hydraulic jump seems to develop, forming a large eddy that in turn generates the wave farther to the right [reproduced from Schuster (1992) with permission].
Fig. 2. Sketch of the experimental arrangement. (a) Top view of the wave channel consisting of four modules and the wave generator on the left. On the right are two water tanks for deaerated fresh and saltwater, respectively. They feed the two distribution pipes, which are themselves connected (not shown) with 24 entrance valves in the bottom of the channel. Through these valves the channel is first filled with fresh water, which then is slowly underlain with saltwater. (b) Side view of the wave channel. On the left are two pistons, separated by a thin plate at the interface level, which synchronically move in opposite directions and displace the same volume. Six electrical resistivity gauges P1–P6, 1.45 m apart from one another, record the interface elevation. At the position of gauge P3 the obstruction is positioned. It causes an approaching wave (amplitude $a_{app}$) to split into a reflected ($a_{refl}$) and a transmitted wave ($a_{trans}$). (c) Detail of the geometry of the triangular obstruction, which is slightly flat topped [reproduced in parts from Schuster (1992) with permission].
below gauge $P_3$. We also mention that the thin plate between the two pistons of the wave generator was an essential structural detail, as it avoided almost completely the formation of vortices that would otherwise have been introduced by shearing of the oppositely moving pistons. In panel (c) of Fig. 2 the triangular obstruction is flat-topped (10 mm) as sketched and two-dimensional as indicated in the side view, that is, it does not change shape across the channel width. We have in some isolated experiments also used other geometries of the obstruction (i.e., ramp–plateau combinations) but shall specify these only when experiments with these are discussed.

We adopt the following notation: $H_1$, $H_2$, $h$, $s$, and $a$ denote the depths of the upper and lower layer respectively, the height and the side length of the triangle, as well as the initial amplitude of the solitary wave generated by the wave machine. The densities $\rho_1$ and $\rho_2$ of the two layers give rise to the density anomaly $\sigma = (\rho_2 - \rho_1)/\rho_1$, which in all experiments has been chosen as $\sigma = (2.25 \pm 0.02) \times 10^2$. Thus, the phase speed of the linear baroclinic wave is

$$c_0 = \sqrt{g \sigma H'}, \quad H' = \frac{H_1 H_2}{H_1 + H_2},$$

of which an order of magnitude in the experiments is $0.08 - 0.09$ m $s^{-1}$. An additional quantity of some relevance will be the maximum piston velocity $v_{\text{max}}$ of the wave generator. Thus, the following dimensionless quantities can be constructed:

- Froude number, $Fr = v_{\text{max}}/c_0$, a measure for the horizontal particle velocities
- layer ratio $r_H = H_1/H_2$ (with values $r_H = 1/3$, 2, and 3 in this paper)
- degree of blocking $B = h/H_2$, a measure for how much of the layers is obstructed by the triangle; $B < 1$ means that the tip of the triangle does not reach the undeformed interface, $B > 1$ means that it reaches into the top layer
- base angle of the triangle $\alpha = \arcsin(h/s)$
- scaled (with the lower layer depth) amplitude of the solitary wave, $a^* = a/H_2$.

For $r_H > 1$ the soliton generated by the wave machine is formed as a hump; alternatively, $r_H < 1$ leads to a soliton trough, which suggests introduction of an effective degree of blocking according to

$$B_{\text{eff}} = \frac{h}{(1 + a^*)H_2}, \quad r_H \neq 1.$$  

(Of course, troughs and humps can be generated for opposite layer ratios, but they will break apart as time proceeds since they are not stable.)

The generation of the solitons was somewhat tricky and required that for $r_H > 1$ ($r_H < 1$), the lower piston be adequately pushed forward (pulled backward), while the top piston is correspondingly pulled back (pushed forward). Figure 3 displays the “best” scaled soliton signals produced experimentally in the unobstructed channel in the case when $r_H = 3$, a distance of 2.45 and 16.45 m from the wave generator, and compares these with the SECH-profile of the K–dV theory in the same situation (Korteweg and de Vries 1895). There are some trailing oscillations that can be recognized in the measured curves, but the general soliton-type character is fairly well reproduced by the mea-
sured interface displacement. In all experiments, the signals approaching the obstruction have about this quality.

Since the performance of the experiments and their analysis was rather elaborate, only the most significant of the above parameters could be varied. Only one density anomaly $\sigma = (2.25 \pm 0.02) \times 10^{-2}$ was used, Froude numbers were restricted to $Fr = 0.44$ and $Fr = 0.88$, respectively, and layer ratios were mostly chosen to be $r_H = 3$ and $r_H = 1/3$, respectively. Table 1 summarizes these cases.

3. Preliminary wave data analysis

a. The basic phenomenon

In the first experiments our interest was in finding the conditions under which the expected fission mechanism would be generated. Furthermore, we wanted to see how the wave developed while it was passing the obstruction. To this end two ramp–plateau combinations were used by Schuster (1992) with ramps 20 cm long and a plateau of 1-cm and 50-cm length, respectively. Furthermore, the total water depth was 20 cm and $r_H = 2$, $B = 0.85$. For these conditions the measurements at the six gauges did demonstrate what one expected: namely, the fission of the approaching soliton at the triangular obstruction into a reflected and transmitted signal and the secondary reflection of the reflected and transmitted signal at the location of the generator and the channel end. Figure 4 summarizes the findings from this experiment. Time series for gauges $P_1$ to $P_6$ are plotted one below the other with distances proportional to the gauge separation. One can clearly recognize the amplification of the soliton-type interface displacement at the first interaction ($P_1$). The first reflected wave is recorded at $P_2$ and then at $P_3$; this wave is reflected at the channel end (wave generator), and subsequently a second interaction occurs at the obstruction ($P_3$). The first transmitted wave travels toward the channel end; its signal is attenuated along its path but is clearly seen at $P_4$, $P_5$, and $P_6$; it is reflected at the channel end, travels backward in the channel ($P_5$ and $P_6$), and encounters the rear end of the obstruction, where the first interaction at the rear end arises with a transmission toward the wave generator and a reflection to the channel end, etc. Solid lines signify approaching waves, dashed lines show the trace of the reflected waves, and dashed-dotted lines show the transmitted waves.

In later analyses of time series we will only be concerned with the approaching, the first reflected, and the first transmitted signal; see also Fig. 8. We could present a large number of such figures—one for each experiment—but not much would be learned from them. Important is the result that for degrees of blocking that are too high, say $B \geq 1.2$, virtually no signal is transmitted and all are reflected. Conversely, when $B$ is too small, say $B \approx 0.6$, the entire wave is transmitted and virtually no signal is reflected.

A second, interesting demonstration is the transformation of an approaching soliton when it encounters a ramp–plateau combination. For a water depth of 20 cm, $r_H = 2$, and $B = 0.85$, the evolving soliton transformation over the ramp–plateau combination with the indicated geometry is shown in Fig. 5, which shows the time series of the interface displacement at 17 positions equally distributed over the obstruction. The measurements were made by repeating the same experiment 17 times and recording with gauge $P_4$ at the respective positions (reproducibility was tested beforehand and found valid). The plotted curves have been scaled such that the approaching soliton at $P_4$ has a maximum amplitude that corresponds exactly to the distance between two neighboring reference lines, numbered in Fig. 5 from 1 to 18. Thus, within the ascending ramp the soliton is steepening at first. At the crest between the ramp and plateau, the effects of separation into reflected and transmitted waves damps the amplitude (positions 4, 5). On the plateau the front steepens and the amplitude grows, whereas the rear flank of the hump spreads considerably, generating a trough that forms the beginning of the trailing oscillation. This trough disappears later over the plateau and on the rear ramp where an entirely new signal forms. This signal has a very steep frontal flank, a conspicuously higher amplitude, and a long tail. We have studied this transformation for a ramp–plateau combination with an arbitrary long ramp (such as a shelf model) Maurer et al. (1994). In that work the nonlinear wave theory of Diebels et al. (1994) is applied to such shelf situations, and it is demonstrated that theory and experiments yield fair agreement as long as (i) waves do not break and (ii) no turbulence is generated on the shelf front (cf. Maurer et al. 1995; Bauer et al. 1994a,b; Diebels et al. 1994).

b. Choice of appropriate variable

To exploit the recorded time series at the six gauges, a physical quantity is needed that allows an objective comparison of the incoming, reflected, and transmitted waves both in one experiment and between different experiments. When the waveform is (roughly) persistent, the amplitude, scaled with its initial value, is often used as a comparative quantity. In our case, however, amplitude comparison is inappropriate for the following reasons: (i) dispersive effects, which contribute to a broadening and shallowing of the wave, are considered as dissipative whereas they are not and (ii) waveforms may change considerably when the waves are scattered by topographic obstructions.

For this reason it appears more meaningful to use as a comparing quantity the potential energy density per unit width

$$E_{ps}(t) = \frac{1}{2} (\rho_2 - \rho_1) \int_{x_1}^{x_2} \xi^2(x, t) \, dx, \quad (3.1)$$

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henceforth called the potential energy for brevity, where $g$ is the gravity constant and $\xi(x, t)$ for fixed time the distribution of the (vertical) interface displacement along the channel; $x_1$ and $x_2$ are locations to the left and right of a wave train, where $\xi$ values are negligibly small. This also guarantees that contributions from earlier, reflected, or later, approaching, waves contribute negligible values to the potential energy of a wave packet. For solitons with small amplitudes (as compared to the smaller layer thickness), the potential and kinetic energies are numerically nearly equal (see Bogucki and Garret 1992); for other waveforms, they are proportional to one another. Clearly, since $E_{pot}$ is determined by integration of snapshots of the long-channel–interface distribution, the waveform is accordingly taken into account.
obstruction, it is expedient to subtract from the data the damping due to (i) wall friction and (ii) friction at the fluid interface. A separate study has shown that both amplitude and potential energy of a solitary wave decay exponentially with position (see Maurer et al. 1995). Figure 6 illustrates the feature; in semilogarithmic representation, it shows the amplitude and energy at the six gauge positions (of one forward and one backward moving wave) normalized to the respective values at the first passage of gauge 1. Evidently, linear fits approximate the data points fairly well. The negative inclination of the straight lines, that is, the logarithmic decrement through the amplitude points, is smaller than that through the energy points; however, the ratio is only about 1.5 and not 2, as is the case for the SECH-profile of a pure soliton. Let the logarithmic decrement of the amplitude be \( \Lambda \) (m\(^{-1}\)) and let \( x_0 \) be the position of the reference (first) gauge. Then the amplitude-based, damping-corrected interface displacement would be \( \zeta' = \zeta \exp[x - x_0 \Lambda] \), and the emerging soliton-like wave \( \zeta' \) would have constant amplitude. The energy, however, would grow with time, which is unphysical. Obviously, the amplitude decay must be due to both damping and dispersion, which is the primary cause for the aforementioned decrement ratio of 1.5 instead of 2. We must in this reduction take account only of damping. If we now suppose that this discrepancy is caused merely by the increase of the amplitude decrement due to dispersion and the waves in the experiment have a shape close to the SECH-profile, then it follows that damping correction of the amplitudes should be performed with one-half of the energy decrement; that is,

\[
\Lambda = \frac{1}{2} \ln \frac{E_1}{E_2},
\]

where \( E_1(x_1) \) and \( E_2(x_2) \) are two energies on the energy interpolation line of Fig. 6. With this definition of \( \Lambda \), the damping-corrected energy curve is constant, while

c. Elimination of damping

To quantify the fission of a solitary wave and energy dissipation due to vortex formation at the topographic

FIG. 5. Development of the wave from a soliton within a ramp-plateau combination. The geometry of the obstruction is defined and the interface displacement is measured at 17 equidistant positions. The measurements are obtained from 17 experiments performed under identical conditions [reproduced from Schuster (1992) with permission].

FIG. 6. Typical example of the attenuation of the amplitude and energy of a solitary wave, normalized with respect to its value at the first gauge, plotted against position. The symbols show values at the six gauge positions for a forward- and backward-moving wave in a constant-depth layer system.
the corrected amplitude curve is still slightly falling (Figs. 7a,b).

Obviously, this procedure is only meaningful as long as the waveform approximately equals the SECH-profile. How large the errors might become, if the procedure is applied to measurements with topographic obstructions leading to substantial waveform transformations, is difficult to estimate. However, it turned out that the sum of the corrected energies of the reflected and transmitted waves never exceeded the energy of the approaching wave. Thus, it is certainly not unphysical, and this demonstrates that in the worst case the damping is not completely filtered out; this does not represent a deterioration in comparison to the untreated case.

4. Reflection and transmission at a triangular obstruction

Most experiments have been performed for a ratio of the two layers \( r_H = 3 \); that is, the upper layer is three times as large as the lower layer, and the soliton displacement is upward into the upper layer. A few experiments were also performed for \( r_H = 1/3 \), the reverse case, for which the soliton-like interface displacement is downward toward the obstruction. We first report results for \( r_H = 3 \).

**a. Comparison between reflected and transmitted signals**

Consider a typical experiment, see Fig. 8, in which a soliton-like disturbance of the interface is generated by the wave machine. As time proceeds it will pass the equidistant gauges (distance 1.45 m each), be reflected, and pass the gauges in reverse order. In Fig. 8 the time series of the interface displacements (cm) at gauges 1 to 6 (from top to bottom) are plotted against time (s). The obstruction is positioned between gauges 2 and 3 with degree of blocking \( B = 1 \). The signal generated by the wave machine is not an exact soliton because, as explained in connection with Fig. 3, it is accompanied by small trailing oscillations. The straight lines help interpret the propagation of and separation into various transmitted and reflected signals of the major initial wave crest.

In Fig. 9 the signal of the approaching and reflected wave at gauge 2, scaled with the maximum amplitude
at gauge 1, against six different values of degrees of blocking is displayed in enlarged form. The signals of the transmitted waves at gauge 4 are also shown. Experiments were performed for the conditions shown in Table 1. A constant Froude number Fr = 0.88 means that the particle speeds are the same in all experiments. It is clear that the amplitude of the reflected wave is larger and the wavelength smaller when the degree of blocking is greater. For the transmitted waves this behavior is (obviously) reverse. We emphasize that the position of gauge 4 is beyond the region of direct influence of a possible gyre formed at the topographic obstruction.

For practical reasons, experiments had to be performed with triangles having constant side lengths $s$ but different heights. This led to different steepness of the obstructions at different degrees of blocking; consequently, shape was (unfortunately) not preserved. To study the influence of the steepness of the obstruction on the fission of the wave, we have plotted in Figs. 10a,b the difference between the normalized energy of the transmitted and reflected waves; that is,

$$\Delta E = \frac{E_{\text{max(gauge4)}} - E_{\text{eff(gauge2)}}}{E_{\text{approach(gauge2)}}},$$

(4.1)

versus the degree of blocking for three different side lengths of the triangle. Figures 10a and 10b are for Fr = 0.44 and Fr = 0.88, respectively. We emphasize that in computing the potential energies we have performed the integration not only over the major peak but equally over all trailing oscillations.

The two diagrams of Figs. 10a,b do not disclose a dependence of the fission of the wave upon the side length of the triangle. Apparently, experimental errors are too large to achieve this separation. (We have also plotted the results with the wedge angle $\alpha$ as a parameter instead of the side length of the triangle with no more conclusive inference.) Noteworthy, however, is that the condition $\Delta E = 0$, that is, for which the energies of the reflected and transmitted signals are the same, occurs at $B = 1$ for Fr = 0.44 but at $B = 1.1$ for Fr = 0.88. The reason for the difference is that the amplitude of the approaching soliton depends on the Froude number, $a = a(\text{Fr})$. The effective degree of blocking $B_{\text{eff}} = h/[1 + (a*)^2]H_2$ accounts for this; corroboration is provided in Fig. 10c, which shows $\Delta E$ plotted versus $B_{\text{eff}}$ for both Froude numbers. All experimental points lie now in a single, common band.

**b. Persistence of the transmitted wave**

It is to be expected that with increasing blocking, the solitary wave will be transformed and, finally, will decay into a dispersed wave train. One is tempted to refer to this behavior as instability. However, the transition from a compact wave hump into a dispersed wave train is not an instability phenomenon per se; it is the breakup of the wave and the formation of a gyre by the obstruction that is the instability phenomenon, which may have occurred prior to gauge 4. To analyze this phenomenon we study in this section the damping characteristics of the transmitted wave. To this end, the logarithmic decrement of the transmitted energy between gauges 3, 4, and 5 is determined as a function of blocking. Figures 11a,b,c, display the results; error bars are omitted so as not to overload the figures; however, errors are as large as 25%. (A similar analysis for the reflected wave is not meaningful since at gauge 1 the reflected signal was superimposed on the upcoming back reflection from the wave generator).

For small obstructions, that is, degrees of blocking $B \leq 0.8$, the damping effectively vanishes owing to its numerical elimination (see §3). For increasing degrees of blocking it grows considerably up to a certain max-

<table>
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<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Froude number $F_{\text{max}}$</td>
<td>0.44, 0.88</td>
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<tr>
<td>Side length of triangle, s (cm)</td>
<td>12.4, 24.8, 37.0</td>
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<tr>
<td>Layer ratio $r_s = H_1/H_2$</td>
<td>$\frac{3}{1}$, $\frac{1}{3}$</td>
</tr>
<tr>
<td>Degree of blocking $B$</td>
<td>0.6, 0.1, 1.8</td>
</tr>
</tbody>
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**Table 1.** Parameter values for which experiments have been performed.
blocking do they decay into a dispersed wave train ($B = 0.9 - 1.5$); at even larger degrees of blocking, however, form preservation is recovered. At these large $B$ values the wave does not really pass the obstruction. Overshooting of lower layer fluid across the sill leads to a diving jet at the lee side of the obstruction, which generates the “transmitted” signal that is actually newly formed (this was already discussed in connection with Fig. 1).

c. Contribution of the leading wave signal to the transmitted wave

So far, the form preservation of the transmitted wave was explained by means of the damping properties. To

![Diagram](image)

Fig. 10. (a) Normalized energy difference $\Delta E$ defined in (4.1) of the transmitted and reflected wave for experiments with (a) $Fr = 0.44$ and (b) $Fr = 0.88$. (c) Normalized energy difference $\Delta E$ defined in (4.1) of the transmitted and reflected wave plotted against the effective degree of blocking $B_{eff} = h[(1 + a^*)H_2]$, combining panels (a) and (b) (amplitude $a = 1.3$ cm for Fr = 0.44, and $a = 2$ cm for Fr = 0.88).

![Diagram](image)

Fig. 11. Logarithmic decrement of the energy of the transmitted wave as determined from time series at gauges 4 and 5 for (a) $Fr = 0.88$ plotted against $B$, (b) $Fr = 0.44$ plotted against $B$, and (c) $Fr = 0.88$ and Fr = 0.44 plotted against $B_{eff}$.

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Fig. 12. Ratio of the energy of the main first transmitted wave peak to the total energy of the transmitted wave (first peak plus oscillations) for (a) Fr = 0.88 plotted against $B$, (b) Fr = 0.44 plotted against $B$, and (c) both Froude numbers, plotted against $B_{\text{eff}}$. All results are for $r_n = 3$.

Substantiate the interpretation that the strong increase of the damping behind the obstruction is indeed the result of the transition from the solitary wave to a dispersed wave train, we compare in Fig. 12 the energy of the main first transmitted wave peak to the total transmitted energy.

From the graphs of Fig. 12 it can be inferred that for degrees of blocking $B \leq 0.85$, irrespective of the Froude numbers, almost 100% of the total transmitted energy is carried in the transmitted "soliton" (first peak). This result does not seem to depend much on the height of the wave nor on the particle speed. Only for larger values of $B$ that lead to a strong decay of the soliton energy do the aforementioned ratios fall more sharply for small Froude numbers and reach their minimum at smaller $B$ than corresponding ratios at large Froude number. When plotting the ratios against $B_{\text{eff}}$ (panel a) the difference again becomes smaller. A comparison of the results in Fig. 12 with those of Fig. 11 is interesting: The degrees of blocking at which the maximal damping arises (Fig. 11) are the same as those at which the ratio of the energy of the transmitted soliton to the total energy of the transmitted wave (Fig. 12) is minimal. Also, in both figure sets damping seems to be maximal for those triangular obstructions with smallest slope angle.

Fig. 13. (a) Normalized energy difference $\Delta E$ defined in (4.1) of the transmitted and reflected wave. (b) Logarithmic decrement of the energy of the transmitted wave obtained with time series at gauges 4 and 5. (c) Ratio of the energy of the main first transmitted wave peak to the total energy of the transmitted wave (first peak plus oscillations). All results are for $r_n = 1/3$ and $Fr = 0.88$. 
d. Results for $r_{ih} = 1/3$

Measurements for a two-layered fluid system with a salt layer 15 cm high and a freshwater layer 5 cm thick were difficult to perform because it was far more difficult to produce two layers with a sharp interface. Experiments were only performed for $Fr = 0.88$ and triangles with side length $s = 24.8$ cm.

Results are very similar to those for a layering $r_{ih} = 3$. We display the normalized difference $\Delta E$ (defined in (3.3)) of the transmitted and reflected energy (Fig. 13a), the logarithmic decrement of the transmitted wave (Fig. 13b, see also Fig. 11), and the ratio of the "soliton" (first hump) energy of the transmitted wave to that of the entire transmitted signal (Fig. 13c), all plotted against $B$. Here the energy of the transmitted wave equals that of the reflected wave at $B = 0.85$, a considerable smaller value than for $r_{ih} = 3$, where it is $B = 1$ (Fig. 10). Of course this is so because the soliton forms here as a trough that makes an obstruction already effective at smaller values of $B$.

5. Annihilation of wave energy by vortex formation at the obstruction

For solitons that form as crests (when the upper layer is thicker than the lower layer), the largest gyres are
formed on the lee side of the obstruction (see Fig. 1); when they form as troughs, the largest gyres are generally formed on the luv side (see Fig. 14). A precursory process to vortex formation is the generation of a near-wall jet on the lee side; that is, fluid from the upper layer intrudes in the form of a relatively narrow jet more or less deeply into the lower layer; see Fig. 1 (left). Only when the blocking is further increased is a large gyre formed, Fig. 1 (right). It is clear that these are localized turbulent structures, which are efficient dissipation mechanisms, and they are obviously not even approximately contained in the theoretical models of nonlinear large amplitude internal waves. It is also apparent that the geometry of the obstruction influences the amount of generated turbulence. For instance, if the vertex of the triangle in Fig. 14 were smoothed by rounding it off, less turbulence will be formed, and the corresponding dissipation would be less. It is clear that the analysis of our experiments can only be a rough estimate of real phenomena, yielding an order of magnitude of this localized dissipation mechanism.

A measure of the energy loss in such turbulent eddy sources is the sum of the energies of the transmitted and reflected waves. The difference of this sum and the energy of the approaching wave is then the amount of energy dissipated at the obstruction. Clearly, in this argument, dissipation due to wall and interface friction is ignored (it is eliminated in the results as explained in § 3.) Figure 15 displays the normalized energy loss

\[ \Delta E_{\text{loss}} = \frac{E_{\text{trans}} - E_{\text{refl}}}{E_{\text{approach}}} \]  

plotted against \( B \). Considering the relatively large errors, a dependence of the energy loss on slope angle of the obstruction or the particle speed (Froude number) cannot clearly be discerned. Layer ratios \( r_H = 3 \) lead to maximum energy losses within this obstruction-bound gyre of approximately 40%, and the degree of blocking at which this arises depends on the particle speed. For a layer ratio of \( r_H = 1/3 \), the maximum loss is 55% and at considerably smaller values of the degree of blocking, as seen in Fig. 16. Note also that \( \Delta E_{\text{loss}} \) from measurements is negative for two narrow bands of degrees of blocking (Fig. 15a, \( B \approx 0.6 \), and Fig. 16, \( B \approx 0.6 \) for \( r_H = 3 \) and \( B \approx 1 \) for \( r_H = 1/3 \)). Of course, this is physically unrealistic but provides an indication that the experiments must be interpreted with some care. ‘Fortunately,’ error bars are (except for one experimental point) large enough that the physics is not violated.

A few additional remarks on incorporation of the above findings into theoretical models might be appropriate: modeling of the dissipation mechanisms that account for the results summarized in Fig. 16 is required. This can, perhaps, be achieved by adding a resistance force in the momentum balance, proportional to the interface displacement squared with a ‘drag coefficient’ that depends on \( B \) in such a way that the graphs of Figs. 15 or 16 are followed. However, a resistive force proportional to the squared difference of the upper and lower layer velocities is equally plausible. Presently we have no evidence as to whether any of these proposals is appropriate. It is, however, clear that an exact description of the initial conditions for the waveform beyond such obstructions can hardly be achieved. The theoretical models must necessarily be less ambitious: exact matching of waveforms and phases can no longer be the goal; one must be satisfied if the energetics and perhaps the phase of the dominant signal are properly taken into account.

6. Conclusions

The fission of an internal wave of a two-layer fluid by a triangular obstruction and the ‘‘stability,’’ that is, form preservation, of the transmitted wave, as well as the energy annihilated or dissipated by the vortex due to the obstruction were studied. To obtain a physically objective determination of these notions, the time series of the interface displacement at fixed gauge locations were freed from wall and internal dissipation by renormalizing them with the logarithmic decrement of energy damping. Scaling arguments have shown that results depend in principle on the depth ratio of the layers \( r_H \), the degree of blocking of the obstruction \( B \), its shape (i.e., the slope angle of the triangle), and the Froude number, that is, the speed of the particles relative to the interface phase speed. The dependence of the potential energy of the reflected and/or the transmitted wave and the dissipation on the degree of blocking turned out, in general, to be more pronounced in comparison to other parameters.

For the separation of the approaching wave into a reflected and transmitted component, the dependence on Froude number and slope angle could not be clearly discerned. The form preservation of the transmitted wave was studied by two different methods: First, the logarithmic decrement of the energy far distant beyond the obstruction was functionally related to the degree of blocking, and it was shown that for \( B \) larger than 0.8 to 0.9 a compact, solitary wave was transformed into a dispersed wave train with larger damping. Second, to support this argument, the energy of the first peak of the transmitted wave was compared with the energy of the total transmitted wave, and it was found that the contribution of the first peak to the total transmitted energy sharply decreased for \( B \approx 0.85 \). A dependence of the instability threshold on Froude number and slope angle of the obstruction was not very pronounced; further scrutiny would be needed to substantiate this. A spectral analysis of the transmitted signal using FFTs (not shown here) also revealed that for \( B \approx 0.8 \) the spectral distribution appeared as a compact crest with a single maximum at \( 2 \times 10^{-2} \) Hz. With increasing \( B \), several modes formed that are representative of the dispersed wave train.
energies of the total transmitted and reflected waves. The experimental results are again fraught with errors too large to disclose a clear dependence on Froude number and slope angle, but maximum losses of energy seem to depend on the layer ratios. For a 3:1 layering (upper layer three times as large as lower layer) this loss is about 30% to 40%; for 1:3 layering it is 55% and occurs at smaller $B$. In both cases $B \geqslant 0.6$ is practically ineffective as an energy dissipator.

The above results are experimental and have been obtained for an obstruction of triangular shape with a relatively sharp vertex. Natural sills are more rounded and thus are less prone to generating vortices at their lee or luv sides. In other words, the above results are likely those of a worst case scenario. It follows that in lakes (or the ocean), sill depths and thermocline depths need to be rather close to each other to be effective as energy dissipators.

The above analysis obviously only touches upon a limited question of its significance in physical limnology or oceanography. The theoretical realization remains an important unknown problem. However, there are also experimental questions that need be further scrutinized. Such a problem is shown in Fig. 17, which shows a ramp–plateau combination reminiscent of the edge of a shelf region. For $B = 1$, an approaching internal wave can cause density undercurrents that are very efficient as turbulent diffusion mechanisms. In such situations the two-layer models of weakly nonlinear waves must be replaced by well-understood density current models.

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