NOTES AND CORRESPONDENCE

Evidence for Hydraulically Controlled Outflow of Brackish Water from Holandsfjord, Norway

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ABSTRACT

A large number of historical vertical salinity profiles from the deep Holandsfjord are analyzed to investigate properties of the brackish surface layer. It is found that the mean salinity and mean thickness of the brackish layer in the fjord are almost horizontally homogeneous. By statistical regression it is shown that the freshwater content in the brackish layer, measured as the freshwater height H_{1f}, varies with the freshwater supply Q_f as $H_{1f} \sim Q_f^{2/3}$. This result is predicted by a simple two-layer model, with a thin active brackish surface layer upon a thick passive layer of seawater, subject to baroclinic hydraulic control in a contraction at the fjord mouth. The observed freshwater height, however, is ~50% greater than predicted. It is suggested that this, at least partly, is an effect of vertical stratification in the lower layer of seawater, focusing the estuarine compensation current toward the pycnocline.

1. Introduction

The possibility for two-layer, stationary baroclinic (internal) hydraulic control in the mouth of an estuary was first discussed and explored by Stommel and Farmer (1953). They showed that if the thickness of the upper layer is forced to change from the value in the estuary to the (smaller) value outside the mouth, the flow obeys the following condition:

$$\frac{u_{1m}^2}{g'H_{1m}} + \frac{u_{2m}^2}{g'H_{2m}} = 1. \tag{1}$$

Here $u_{1m(2m)}$, $H_{1m(2m)}$, and $\rho_{1(2)}$ are velocity, depth, and density of the upper (lower) layer in the mouth, respectively, and g' is defined by $g' = (\rho_2 - \rho_1)/\rho_0$, where ρ_0 is a reference density. The vertical section where Eq. (1) applies should be found in the mouth where there is a contraction in width and/or in depth (a sill). This section is called the control section. Equation (1) expresses a dynamical condition on the flow at the open boundary of the estuary (open boundary condition).

Stommel and Farmer (1953) applied Eq. (1) to the special case when the two layers have almost equal thickness in the mouth. They studied how the salinity S_1 in the estuary depends upon the salinity S_2 of the (external) seawater, the freshwater supply Q_f, and the physical dimensions of the mouth. For given values of S_2, Q_f, and depth and width of the mouth, they found that S_1 has a maximum that cannot be exceeded, no matter how great a supply of mixing energy the estuary is given. They called this the state of overmixing. The physical explanation of overmixing is of course that the maximum baroclinic transport capacity of the mouth is reached; see, for instance, Armi (1986) and Stigebrandt (1975, 1981). In a laboratory experiment Stigebrandt (1977) found that superposed barotropic fluctuations of sufficiently large amplitude increased the maximum baroclinic transport capacity of a short strait. In long straits frictional effects may reduce the maximum baroclinic transport capacity (Assaf and Hecht 1974). In addition it has been shown that Eq. (1) expresses the correct open boundary condition for steady two-layer flows in salt wedge rivers, for example, Turner (1973) and Officer (1976).

Based on the Froude-number-plane formulation in Armi (1986), Armi and Farmer (1986) and Farmer and

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Armi (1986) further developed the theory for maximal two-layer exchange through contractions and over sills and through a combination of a contraction and a sill. In the analysis they also included the effect of a barotropic flow. Maximal two-layer exchange over a sill was found to be fundamentally different from that through a contraction, and the Stommel and Farmer solution is not applicable to maximal two-layer exchange over a sill. Furthermore, they found that the transition to single-layer flow occurs at much lower speed for a barotropic component in one direction than in the other. The papers by Armi and Farmer also review previous work on maximal two-layer exchange. Dalziel (1991) developed a functional framework, which he considers to be a more flexible tool for handling hydraulic problems than the Froude-number-plane formulation. Theories for maximal two-layer exchange to geophysical flows have often been applied to the Strait of Gibraltar. According to Garrett et al. (1990), the flow in this strait switches between maximal exchange early in the year and submaximal later in the year. A number of recent papers on the physical oceanography of sea straits are collected in Pratt (1990).

The interest of the present paper is focused on flows through contractions in deep fjords. The thickness of the upper, brackish layer, sustained by local freshwater supply and wind mixing, is much less than the thickness of the lower layer. The internal Froude number of the lower layer is then small and the two-layer exchange through the contraction will be much less than maximal. The lower layer may in this case be regarded as passive and a deep sill may not control the active surface layer, for example, Carstens (1970), Long (1975), McClimans (1976), Stigebrandt (1975, 1981), and Armi (1986).

Carstens (1970) suggested that the outflow of brackish water from the Frier Fjord, Norway, is hydraulically controlled at the narrow mouth. This was supported by a few observations. An application to a laboratory fjord circulation experiment was made by McClimans (1976). In a frictional model for the circulation of brackish water in fjords of constant width Long (1975) applied the condition of hydraulic control of the brackish water at the fjord mouth. Pearson and Winter (1978) included bathymetric effects within the fjord in a similar model. An analytical two-layer model for stationary estuarine circulation in fjords subject to hydraulic control at the fjord mouth and wind-driven diapycnal mixing in the fjord was developed by Stigebrandt (1975, 1981). The model has been applied to many Norwegian fjords with quite realistic results; see Stigebrandt (1981) for an example. Most of the applications, however, are not published so that at the present time little systematic testing of the model has been undertaken. For a review of the dynamics of, and models for, estuarine circulation in fjords, see Farmer and Freeland (1983).

From our presentation above, one may conclude that baroclinic hydraulic control of thin surface layers has been observed in laboratory experiments, for example, McClimans (1976) and Armi (1986), and there are many indications for such controls in real fjords, for example, Carstens (1970) and Stigebrandt (1981). However, it seems that there is no experimental proof for the existence of baroclinic hydraulic controls of thin layers of brackish water in fjord mouths.

The existence of an internal hydraulic control can be verified in different ways. The most obvious one is to study the density and velocity fields in a vertical section perpendicular to and crossing the assumed control section. Approaching the mouth from the fjord, one expects the interface to rise due to acceleration toward the control section where Eq. (1) should apply. It may however be hard to verify from measurements that this equation applies since it is difficult to measure currents close to the sea surface and conditions are seldom stationary due to tides, changing winds, etc.

One may also deduce the existence of an internal hydraulic control from the hydrographic state of an estuary. In a deep fjord with a thin brackish layer one may, as suggested in the present paper, investigate the relationship between the freshwater content in the brackish layer and the freshwater supply. According to theory presented in section 2, the freshwater content should be independent of the rate of mixing in this type of fjord. This approach requires, however, many vertical salinity profiles (to diminish effects of random disturbances) and information about the freshwater supply during a certain time preceding the measurement of each salinity profile.

In the present paper we make a statistical analysis of a large number of vertical salinity profiles from Holandsfjord and Nordfjord, Norway. The analysis is specifically aimed at finding out if the states of the brackish water in these fjords are governed by internal hydraulic controls in the fjord mouths.

2. Theory

To set the framework for the forthcoming analysis of observational data we start with a discussion of stationary two-layer theory for brackish water in fjords. The thickness of the brackish layer is usually much less than the depth of the mouth and the depth of the lower layer may be regarded as passive. Equation (1) may then approximately be written

$$\frac{u_{1m}^2}{g'H_{1m}} = 1. \qquad (2)$$

Thus, the squared internal (densimetric) Froude number for the upper layer, $F_{d1}^2 = u_{1m}^2/g'H_{1m}$, equals unity in the control section. In fjords density variations are often almost exclusively determined by salinity variations. It is then a good approximation to use the following equation of state for brackish water:

$$\rho = \rho_f(1 + \beta S). \tag{3}$$

Here ρ_f is the density of freshwater and $\beta \approx 0.0008$ (psu^{-1}) is the salinity contraction coefficient.

The reduced gravity g' may then be written

$$g' = g\beta(S_2 - S_1), \tag{4}$$

where $S_1(S_2)$ is the salinity of the upper (lower) layer.

For a stationary state conservation of volume and salt is expressed by the so-called Knudsen's relationships

$$Q_1 = Q_f + Q_2 \tag{5}$$

$$Q_1 S_1 = Q_2 S_2, \tag{6}$$

where $Q_{1(2)}$ is the volume flow of the upper (lower) layer and Q_f is the freshwater supply.

Equations (5) and (6) together give

$$Q_1 = Q_f \frac{S_2}{S_2 - S_1}. \tag{7}$$

The relationship between the thicknesses of the brackish layer in the fjord H_1 and in the mouth H_{1m}, respectively, is determined by the dynamics of the flow from the fjord interior to the control section. Here we write

$$H_1 = \phi H_{1m}. \tag{8}$$

For wide fjords with narrow mouths (which in practice means that a fjord is more than about four times wider than the mouth), one finds that $\phi = 3/2$ if flows from the interior to the control section are frictionless (Carstens 1970). If one assumes that the momentum flux is conserved for the flow from the interior to the control section, one obtains $\phi = 3^{1/2}$ (cf. Stigebrandt 1981).

For our purposes it is convenient to introduce the height H_{1f} of freshwater in the fjord defined by

$$H_{1f} = H_1 \frac{S_2 - S_1}{S_2}. \tag{9}$$

Equations (8) and (9) give

$$H_{1m} = \frac{1}{\phi} \frac{S_2}{S_2 - S_1} H_{1f}. \tag{10}$$

Using Eqs. (7) and (10) and recognizing that $Q_1 = u_{1m} B_m H_{1m}$, where B_m is the width of the mouth, Eq. (2) may be written in the following way:

$$H_{1f} = \phi \left(\frac{1}{g\beta S_2 B_m^2} \right)^{1/3} Q_f^{2/3}. \tag{11}$$

Equation (11) shows that the freshwater height H_{1f} should be independent of the rate of mixing in the fjord as long as ϕ is independent of the mixing. Thus, the existence of an internal hydraulic control may be established without the need to consider mixing and therefore without the need for knowing wind speeds. It is thus sufficient to know the relevant freshwater supply for each observation of the stratification in the fjord.

As demonstrated above, a stationary model for the freshwater height H_{1f} in a fjord may be constructed from an assumption about baroclinic hydraulic control in the mouth, conservation of mass (Knuden's relationships), and a model describing the relationship between the thicknesses of the brackish layer within the fjord and in the mouth, respectively. A model for the thickness H_1 and salinity S_1 of the brackish layer requires, in addition, a model for entrainment of seawater into the brackish layer. Using the Kato and Phillips (1969) formula for the wind-driven entrainment flow Q_2, Stigebrandt (1975, 1981) derived the following expressions for H_1 and S_1:

$$H_1 = H_{1f} + \frac{N}{2Q_f g\beta S_2} \tag{12}$$

$$S_1 = \frac{S_2 N}{N + 2\phi\left[Q_f^5 \left(\frac{g\beta S_2}{B_m} \right)^2 \right]^{1/3}}, \tag{13}$$

where $N = CW^3A$ and $C \sim 2.5 \times 10^{-9}$ is an empirical constant, which contains the drag coefficient for airflow over water, the ratio between air and water densities, and another empirical constant often termed m_0. The latter describes the efficiency of turbulence with respect to diapycnic mixing; W is the wind speed and A is the surface area of the fjord. It should be noted that in models for many fjords it may be necessary to account for mixing between freshwater and seawater executed by rivers upon entering the fjord.

Equation (12) shows that the thickness of the brackish layer may be expressed as the sum of the freshwater thickness H_{1f}, which is influenced by the hydraulic control, and a "mixing" thickness proportional to the Monin–Obukhov length that is proportional to the ratio between the energy flux from the wind and the buoyancy flux due to supply of freshwater; see Stigebrandt (1981). The model predicts that mixing thickness is independent of the hydraulic control.

3. Observations from Holandsfjord and Nordfjord

Holandsfjord and Nordfjord cut into high (~1 km) mountains just north of the Arctic Circle at the coast of the Norwegian Sea; see the map in Fig. 1. The horizontal surface area of Holandsfjord (Nordfjord) is 23.2 (8.9) km² and the greatest depth is 170 (250) m. The mouth of Nordfjord is situated at Enganeset where the width at the sea surface is approximately 375 m and the greatest depth is about 110 m. Holandsfjord has a narrow section (ca. 850 m) at Kopskjær where the depth is about 115 m. The width is about 1100 m at Kalvskjær where there is a sill with greatest depth 45 m. The wide and deep Skarsfjord outside Holandsfjord has two openings to the sea. The major opening of width 1500 m is seen in Fig. 1; the other, situated northeast of Kalvskjær, is ca. 450 m wide. The tidal ampli-

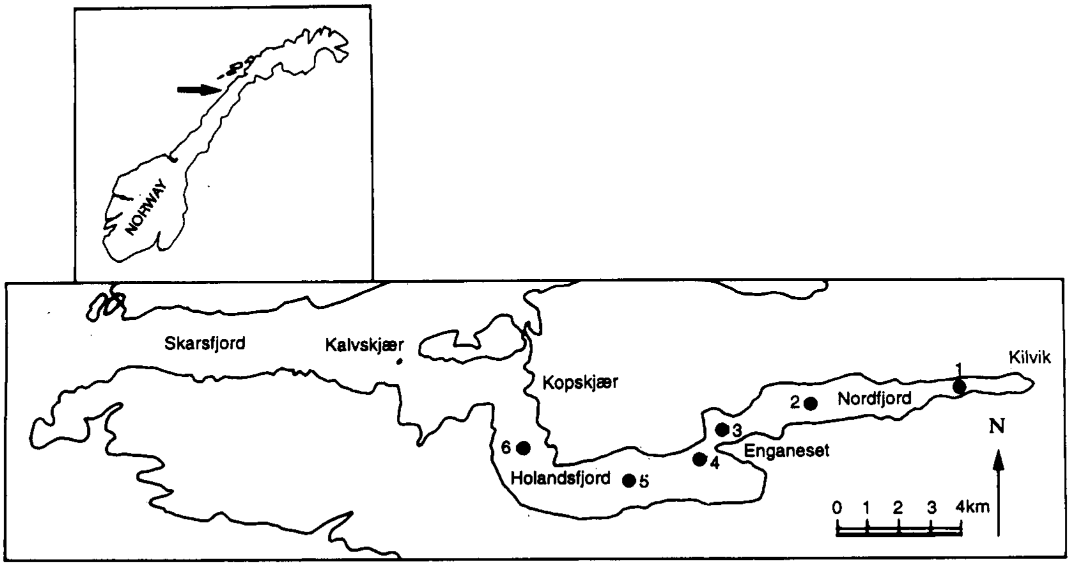

FIG. 1. Map showing the location of the measurement sites in Holandsfjord and Nordfjord.

tude (semidiurnal) is typically 1 m, implying maximal tidal barotropic velocities in the fjord less than 0.05 $m\ s^{-1}$.

The freshwater supplies to Nordfjord and Holandsfjord come from local runoff from land and from parts of Svartisen, the second largest glacier in Scandinavia. Statkraft (formerly NVE, the Norwegian Water and Electricity Board) has computed the daily runoff to the two fjords for the years 1977–1991. Annual means of the freshwater supply through runoff are 10.7 and 6.2 $m^3\ s^{-1}$ for Holandsfjord and Nordfjord, respectively. Precipitation directly on the fjords supplies additionally about 1.4 and 0.5 $m^3\ s^{-1}$ (annual means). There are large annual variations in the freshwater supply, with the maximum in summer and minimum in winter; see Fig. 2a for the mean annual cycles of the runoff from land for the period 1977–1991. In connection with heavy rainfall there are quite large short-term variations in runoff; compare Fig. 2b showing the estimated runoff in 1991. The uncertainty of the computed runoff from land is estimated to be 10–15%. The uncertainty of the computed time for flow maxima and minima is estimated to be 1–2 days (J.-P. Magnell 1993, personal communication).

In the period 1976–1990 Statkraft ran a field program where vertical profiles of salinity and temperature were obtained in the two fjords. The measurements were taken by a local observer equipped with a TSsonde (Electronic Switchgear). The accuracy of the salinity measurements are probably better than 0.5 psu. Observational depths are 0.5, 1, 1.5, 2, 3, . . . , 9, 10, 12, 15, 17, 20 m. However, due to wind action the boat may drift and this may occasionally cause some error in the depths of the TS readings. The six positions for the measurements are shown in Fig. 1. The number of profiles during each month of the year at the different stations is given in Table 1. As can be seen, stations 3 and 4 have been measured quite frequently due to an initial assumption that there should be a hydraulic control at Enganeset. Stations 1 and 2 in the inner parts of Nordfjord have been visited fairly seldom in winter due to ice cover. No measurements were taken in Skarsfjord outside the mouth of Holandsfjord. However, the freshwater supply to this fjord is quite small, and due to the two connections with the open sea the brackish layer in Skarsfjord is expected to be appreciably thinner than in Holandsfjord.

The annual cycles of monthly mean temperature and salinity in the upper 20 m of the water column at station 4 are shown in Figs. 3 and 4, respectively. There is a strong freshwater signal close to the sea surface with salinity minimum in July when the freshwater supply is greatest. The freshwater content in the water column is quite small in winter when the freshwater supply is small.

4. Observed and theoretical freshwater heights

The freshwater height is computed from the observed salinity profiles using the expression

$$H_{lf} = \int_0^{D_{ref}} \frac{S_{ref} - S(z)}{S_{ref}} dz. \qquad (14)$$

Here $S(z)$ is the salinity at the depth z and $S_{ref} = S(D_{ref})$, where D_{ref} is a depth of reference, situated below the brackish surface layer due to local runoff. For the computations we put $D_{ref} = 7$ m, which is deeper than the locally generated brackish layer, possibly with exceptions for periods with strong wind mixing and small freshwater supplies. These computations

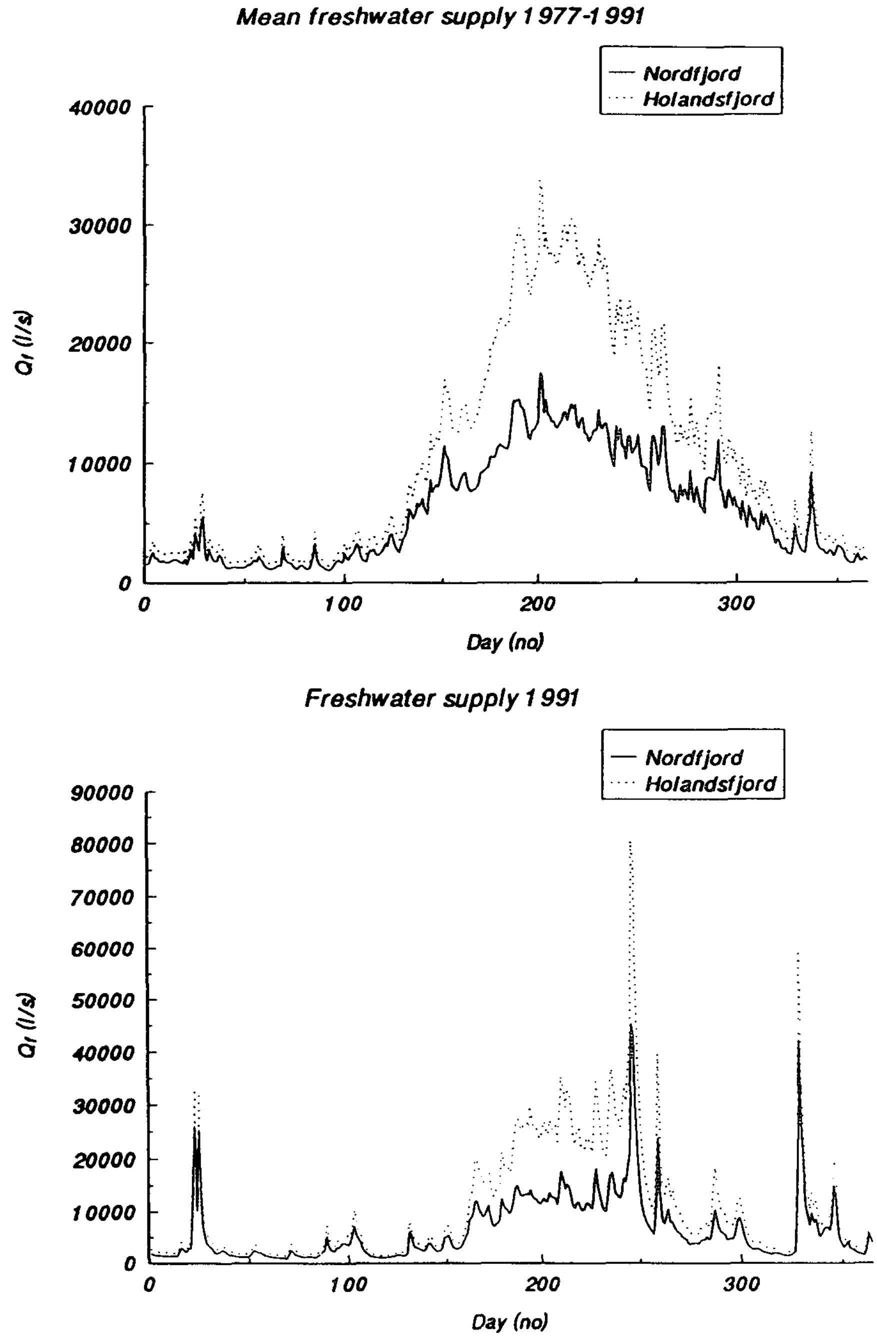

FIG. 2. Computed runoff in liters per second to Holandsfjord (solid line) and Nordfjord (dotted line): (a) the mean annual cycle 1977–91 and (b) the year 1991.

are not particularly sensitive to the choice of D_{ref} as long as this is in the interval 5–15 m. If the depth of reference is taken much deeper than this, one may obtain significant contributions to H_{1f} from the intermediary layer when the latter is strongly stratified with respect to salinity.

To obtain a measure of the observed salinity and thickness of the brackish water one may construct an equivalent two-layer stratification. It is assumed that the constructed and the observed stratifications are equivalent if they (i) contain the same amount of freshwater and (ii) have the same potential energy (PE);

TABLE 1. Number of T–S measurements at the different stations in Nordfjord and Holandsfjord obtained by NVE for the period Dec 1976–Mar 1990.

	Station					
Month	1	2	3	4	5	6
Jan	4	6	26	28	12	10
Feb	4	7	28	30	12	13
Mar	3	7	23	22	9	8
Apr	7	9	22	24	10	10
May	10	10	22	23	11	9
Jun	10	11	26	25	11	10
Jul	12	12	27	27	11	11
Aug	9	10	27	27	9	10
Sep	8	9	25	24	8	7
Oct	5	5	19	18	6	6
Nov	7	7	24	23	8	8
Dec	2	5	27	28	10	9

compare Stigebrandt (1987) and Farmer and Freeland (1983).

The potential energy of the measured stratification is

$$\mathrm{PE} = g\rho_f\beta \int_0^{D_{\mathrm{ref}}} (S_{\mathrm{ref}} - S(z))z dz. \qquad (15)$$

The potential energy of the equivalent two-layer stratification should be equal to PE; that is,

$$g\rho_f\beta(S_{\mathrm{ref}} - S_1)\frac{H_1^2}{2} = \mathrm{PE}. \qquad (16)$$

The freshwater content of the equivalent two-layer stratification should be equal to H_{1f}:

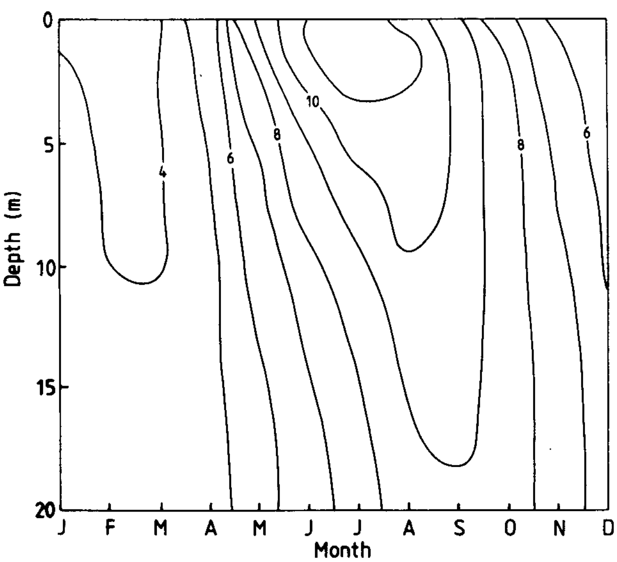

FIG. 3. Annual mean cycle of temperature at station 4 in Holandsfjord.

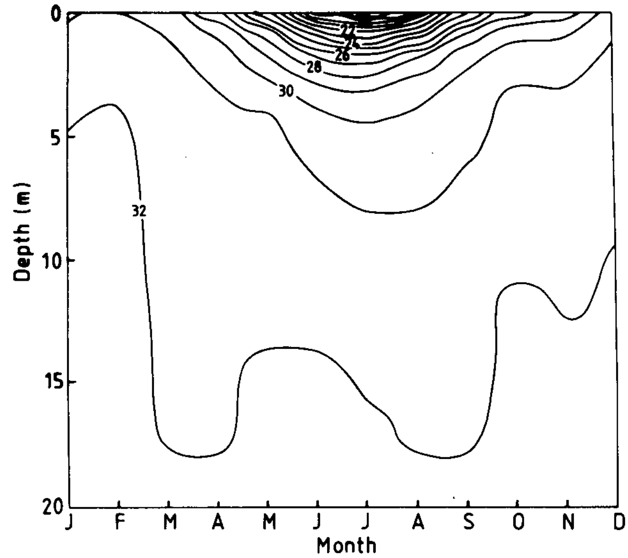

FIG. 4. Annual mean cycle of salinity at station 4 in Holandsfjord.

$$H_1 \frac{S_{\mathrm{ref}} - S_1}{S_{\mathrm{ref}}} = H_{1f}. \qquad (17)$$

From Eqs. (16) and (17) S_1 and H_1 can be expressed in terms of H_{1f} and PE in the following way:

$$S_1 = S_{\mathrm{ref}} - \frac{g\rho_f\beta H_{1f}^2 S_{\mathrm{ref}}^2}{2\mathrm{PE}} \qquad (18)$$

$$H_1 = \frac{2\mathrm{PE}}{g\rho_f\beta H_{1f} S_{\mathrm{ref}}}. \qquad (19)$$

We computed H_{1f}, PE, S_1, and H_1 for all observed profiles from all stations. The mean values of these quantities and of S_{ref} are given in Table 2. The mean thickness of the brackish layer is about constant in the whole fjord, while the mean salinity increases slightly toward the mouth (Table 2). These results show that the fjords may be considered as one fjord that is rather well mixed horizontally. The mean freshwater height H_{1f} is nearly constant from the outer Nordfjord (sta 3) and through Holandsfjord (sta 4–6). The mean reference salinity S_{ref} is practically constant in the fjords.

TABLE 2. Analysis of NVE's measurements in Nordfjord/Holandsfjord. Mean values of S_1, H_1, S_{ref}, and H_{1f} with $D_{\mathrm{ref}} = 7$ m.

Station	S_1 (psu)	H_1 (m)	S_{ref} (psu)	H_{1f} (m)
1	24.84	3.21	31.20	0.47
2	26.21	3.45	31.44	0.47
3	27.85	3.59	31.59	0.35
4	28.34	3.52	31.62	0.30
5	28.12	3.30	31.66	0.32
6	28.28	3.74	31.65	0.32

When interpreting the results in Table 2, one should remember that there are fewer observations, in particular in winter, from the inner stations in Nordfjord (sta 1 and 2).

Stations 3 and 4 on each side of Enganeset have the greatest numbers of observations. These stations have almost equal thicknesses of the brackish layer and approximately equal freshwater heights. From this observation one may conclude that, in general, there is no internal hydraulic control in this strait.

To find out how well Eq. (11) describes the freshwater height in the fjords as a function of runoff from land we have done a regression analysis of observed H_{1f} versus Q_f using the function

$$H_{1f} = aQ_f^b. \quad (20)$$

For the regression analysis we used for Q_f the sum of the freshwater supplies to Nordfjord and Holandsfjord. This is obviously correct for Holandsfjord. We also used this Q_f for Nordfjord since we found above that the brackish layer in the two fjords may be considered as one layer that is rather homogeneous horizontally. The residence time for freshwater in the surface layer in a fjord of area A is $\tau_f = AH_{1f}/Q_f$ and may be computed using Eq. (11). For average freshwater supply τ_f is about one week for Nordfjord plus Holandsfjord. Accordingly, we used the mean value of Q_f for the seven days preceding the hydrographical measurements.

The coefficients a and b in Eq. (20) were determined using the method of least squares; the results are presented in Table 3. The coefficients a and b have quite similar values for stations 3 and 4 implying, again, that in general there should be no internal hydraulic control at Enganeset. The inner parts of Nordfjord have greater freshwater thicknesses than station 3. This may be due to the local freshwater supply in combination with the small width of the fjord. Another factor of importance is westerly winds. These may cause convergence and

TABLE 3. Results from the regression analysis of observed H_{1f} vs Q_f using the equation $H_{1f} = aQ_f^b$. The standard errors of the coefficients a and b are given by the ± figures. Here "3–6 all" means all data from stations 3–6; "3–6 aver" means that data from stations 3–6 taken the same day have been averaged before the regression analysis. "3 + 4 + 5 + 6 aver" implies that only averages based on all four stations are used.

Station	a	b
1	0.083 ± 0.017	0.62 ± 0.07
2	0.074 ± 0.011	0.62 ± 0.05
3	0.043 ± 0.004	0.72 ± 0.04
4	0.038 ± 0.004	0.72 ± 0.04
5	0.059 ± 0.010	0.58 ± 0.06
6	0.053 ± 0.009	0.63 ± 0.06
3–6 all	0.043 ± 0.003	0.69 ± 0.02
3–6 aver	0.039 ± 0.004	0.73 ± 0.04
3 + 4 + 5 + 6 aver	0.040 ± 0.006	0.77 ± 0.06

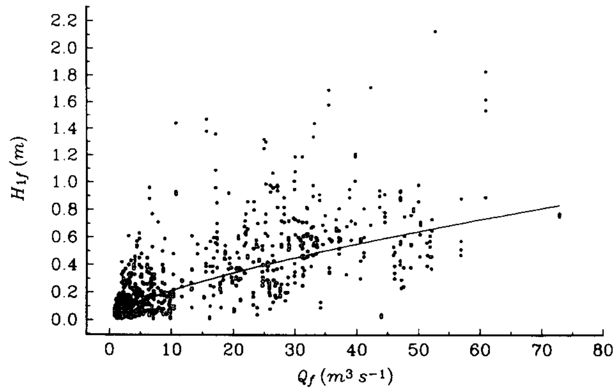

FIG. 5. Plot of H_{1f} vs Q_f. Dots show observational points from stations 3–6 and the line is the regression curve $H_{1f} = 0.043Q_f^{0.69}$.

thickening of the brackish surface layer in the inner reaches of this fjord. Regression analysis applied to all data from stations 3–6 gives $a = 0.043 \pm 0.003$ and $b = 0.69 \pm 0.02$ ("3–6 all" in Table 3); see also Fig. 5. The squared correlation coefficient R^2 for this curve is $R^2 = 0.51$. Computations using Q_f averaged over 5 days instead of 7 gave the same result as presented above. However, with Q_f averaged over 10 days we get a slight change in the coefficient b and a slightly greater standard error ($b = 0.71 \pm 0.03$).

In the present analysis we did not have the opportunity to also take the precipitation directly on the sea surface into account, which may be of importance, particularly in periods of heavy rainfall. The uncertainty in the analysis, expressed by the size of the standard errors of the coefficients a and b in Eq. (20) (see Table 3), are due to several factors. The brackish layer is usually not in steady state due to the often rapidly varying freshwater supply (cf. Fig. 2). Internal waves and wind-driven advection will at times lead to horizontal gradients of the freshwater height in the fjords. As discussed before, there may be some measurement errors in the observed salinity profiles. The assumption of a fixed reference depth $D_{\rm ref}$, used in our computations of the freshwater thickness, may occasionally be inappropriate, as further discussed below. The computed freshwater supplies by runoff have some errors; see section 3. These factors are probably responsible for much of the scatter found in the observations; see Fig. 5.

In order to reduce the scatter due to possible horizontal gradients of the freshwater height in the fjord, vertical profiles obtained on the same day from stations 3 to 6 were averaged with equal weight. The resulting averages are based on 1–4 profiles, and regression analysis applied to this dataset gave $a = 0.039 \pm 0.004$ and $b = 0.73 \pm 0.04$, with $R^2 = 0.57$ ("3–6 aver" in Table 3). If we only accept averages based on four profiles, that is, from stations 3, 4, 5 and 6, the regression analysis gives $a = 0.040 \pm 0.006$ and $b = 0.77 \pm 0.06$ with $R^2 = 0.62$ ("3 + 4 + 5 + 6 aver" in Table

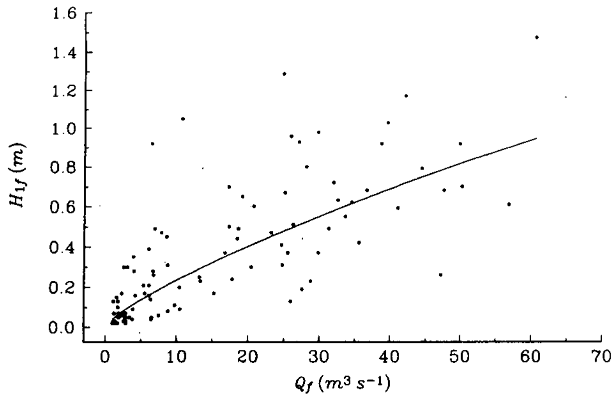

FIG. 6. Plot of H_{1f} vs Q_f. Dots show horizontally averaged observations from stations 3 to 6 obtained on the same day. The line is the regression curve $H_{1f} = 0.040Q_f^{0.76}$.

3); see also Fig. 6. However, although the exponent b is not far from the expected value, Fig. 6 shows that the freshwater heights are underestimated at higher freshwater supplies. Linear, polynomial, logarithmic, and exponential regression curves all gave inferior fits to this dataset with R^2 values in the range 0.49–0.51.

To further refine the analysis, we used the time series of the freshwater supply for the period 1977–91 to compute the freshwater thickness in the fjord from the simple model presented earlier. In the model the uniform freshwater thickness in the fjord changes only as a result of imbalance between freshwater supply by runoff and removal by outflow through the hydraulic control according to

$$\frac{dH_{1f}}{dt} = \frac{1}{A}\left[Q_f - cB_m\left(\frac{g\beta S_{\mathrm{ref}}}{\phi^3}\right)^{1/2} H_{1f}^{3/2}\right]. \quad (21)$$

For the model integration we used $S_{\mathrm{ref}} = 31.6$ psu (from Table 2), $g = 9.82$ m s^{-2}, $\beta = 0.0008$ psu^{-1}, and $B_m = c850$ m, where c is a factor reducing the effective width of the mouth due to flow contraction. The contraction factor c is assumed to be equal to 0.75 (e.g., Stigebrandt 1977). We varied ϕ in the model runs to get a good fit between computed and observed freshwater thicknesses. It was found that $\phi = 2.75$ gives a good fit as shown below. In Fig. 7 we present the observed versus computed freshwater thicknesses for the days when there are horizontal averages based on four vertical profiles. This figure demonstrates two features that are discussed in the following. First, for low freshwater supplies the model tends to give larger freshwater thicknesses than observed. The explanation for this is probably that, when the freshwater supply is small, the buoyancy of the surface layer in the fjord is also small. Wind mixing may then easily create a mixed layer of greater thickness than 7 m, the depth D_{ref} used as representative of the ''pure'' seawater in the estimate of the freshwater height from salinity observations. Thus, it is likely that we underestimate the amount of freshwater in the surface layer of the fjord at low freshwater supplies. Second, the model results show some scatter due to the unsteady freshwater supply. In order to resolve the problem of possible deep surface layers at occasions with small freshwater supply and the accompanying difficulty to estimate the freshwater content from salinity profiles, we plotted model results and observations only for freshwater supplies greater than 10 m^3 s^{-1}. The two regression curves are quite close and have a values of 0.06 and 0.05 and b values of 0.68 and 0.71 for model and observations, respectively (Fig. 8). Due to the large scatter the regression curve for the observational data is of course quite uncertain in this case.

The value of the coefficient b in the last analysis above is quite close to the theoretical value 0.67. This suggests that the brackish layer in Holandsfjord and Nordfjord is hydraulically controlled in a section seaward of station 6, probably at Kopskjær. The theoretical value of the coefficient a depends on the values of ϕ and B_m and may be computed from Eq. (11). This equation and thereby the theoretical value of the coefficient a requires that Eq. (2) is valid. With $\phi = 1.7$ and $B_m = 0.75 \cdot 850$ m, one obtains from theory $a = 0.037$. However, to explain the observed freshwater thicknesses ($a \sim 0.05$–0.06), we were forced to use a ϕ value about 50% higher. A possible explanation for the high empirical ϕ value is discussed below.

There may also be errors in the model assumptions. One possible error is connected to the assumption of a two-layer flow in the mouth including a homogeneous lower layer of seawater. The lower layer is often stratified; compare Figs. 3 and 4. This implies that the water participating in the estuarine compensation current, forced into the fjord because of entrainment of seawater into the brackish water, will not be evenly distributed over the depth of the lower layer. Instead it will be concentrated toward the upper part of this layer,

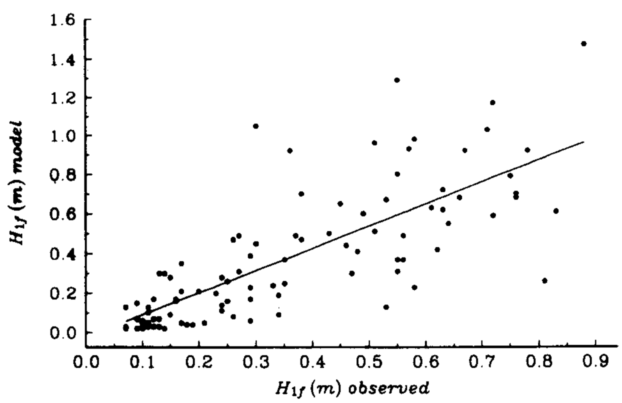

FIG. 7. Observed H_{1f} vs model computed H_{1f} (ϕ = 2.75). Observations are horizontal averages from stations 3, 4, 5, and 6.

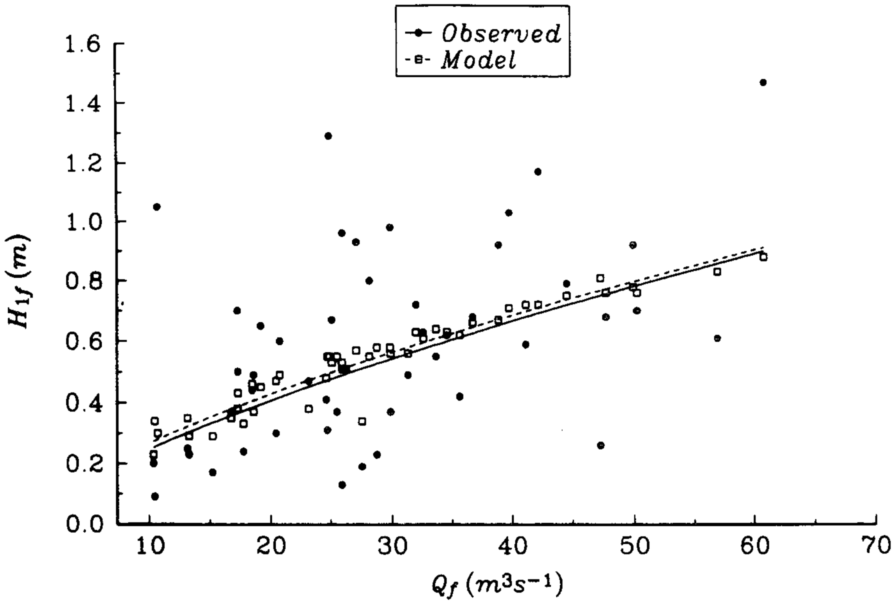

FIG. 8. Plot of H_{1f} vs Q_f for $Q_f > 10$ m^3 s^{-1}. Dots show horizontally averaged observations from stations 3, 4, 5 and 6 and unfilled squares show data from the model run with $\phi = 2.75$. The equations of the regression lines may be found in the text.

a phenomenon often observed (e.g., McAlister et al. 1959). The velocity of the inflowing compensation current may then be much higher than if the lower layer was homogeneous. It is then questionable if it really is a good approximation to set the squared Froude number of the outgoing brackish current equal to unity as done in Eq. (2). The squared Froude number for the upper layer should rather be less than unity in this case. If so, this may at least partly explain why the empirical ϕ value estimated above is greater than the theoretical value from the dynamics of two homogeneous layers.

The theoretical a value computed for hydraulic control at Kopskjær should not be relevant if there is a control in Skarsfjord that ''drowns'' the assumed control at Kopskjær. Since the two openings to the sea have a combined width of ~2000 m, and there is only a small local freshwater supply (see section 3), the thickness of the brackish layer in Skarsfjord should be about one-half of that in Holandsfjord. Thus, there is no reason to believe that the hydraulic control of the surface layer in Holandsfjord is situated seaward off Kopskjær.

The estuarine circulation will possibly interact with other time-dependent circulation modes in the mouth. Effects of interaction between estuarine circulation and tidal currents in a fjord mouth were investigated by Stigebrandt (1977). From a laboratory experiment it was found that the net effect of the tidal current upon the baroclinic transport is quite weak as long as the tidal current amplitude is less than the amplitude of the estuarine currents. Although the experiment concerned the case of maximal two-layer exchange, it seems reasonable that this result also applies to submaximal two-layer exchange. Thus, we do not believe that the interaction between the estuarine circulation and the weak tidal currents will have any significant influence upon our results. The interaction between the time-dependent so-called intermediary circulation, driven by a fluctuating density field outside the fjord mouth (see Stigebrandt 1990), and estuarine circulation is possibly important but has never been investigated thoroughly.

5. Concluding remarks

Our analysis suggests that the freshwater height H_{1f} in Holandsfjord and Nordfjord is hydraulically controlled in some section seaward off station 6, probably at Kopskjær. The empirical functional dependence between H_{1f} and Q_f is quite close to the theoretical prediction $H_{1f} \sim Q_f^{2/3}$. However, the observed freshwater heights are about 50% greater than predicted by theory. This is believed to be at least partly due to stratification in the seawater beneath the brackish layer, which should lead to a concentration toward the halocline of the estuarine compensation current. The internal Froude number for the upper layer should thereby be less than 1, and Eq. (2) may not be completely relevant. Since the thickness of the active part of the lower layer is unknown, we have not tried to quantify this effect. However, if the estuarine compensation current is only one to two times thicker than the surface layer in the control section, this may explain the observed thickening of the freshwater height.

The analysis shows that usually there is no hydraulic control at Enganeset. This is easily explained by the relatively large freshwater supply to Holandsfjord, that is, to the fjord outside Enganeset. For a hydraulic control to occur, it is required that the brackish layer be substantially thinner outside than inside the control section. Using Eq. (11), we thus conclude that the existence of a hydraulic control at Enganeset requires that the ratio Q_{fN}/B_{mN} be substantially greater than Q_{fN+H}/B_{mH}. Here Q_{fN} and Q_{fN+H} are the freshwater supplies to Nordfjord and to Holandsfjord plus Nordfjord, respectively. Here B_{mN} and B_{mH} are the widths of the mouths of Nordfjord and Holandsfjord, respectively. Substituting in numbers given earlier we obtain the ratios 0.017 and 0.019 for Nordfjord and Holandsfjord, respectively. This shows that the freshwater height due to a hydraulic control in the mouth of Holandsfjord, and the total freshwater supply landward of this, actually should be greater than that due to a hydraulic control in the mouth of Nordfjord at Enganeset and the freshwater supply to this fjord. Thus, there should be no hydraulic control at Enganeset since this should be "drowned" by the hydraulic control at Kopskjær. However, a recently accomplished regulation of the runoff to Nordfjord, Holandsfjord, and adjacent fjords (mainly Glomfjord) has lead to a decreased runoff to Holandsfjord and a greatly increased runoff to Nordfjord, such that $Q_{fN} \sim 50$ and $Q_{fN+H} \sim 55$ m^3 s^{-1} as annual averages. Thereby Q_{fN}/B_{mN} has changed to 0.13 for Nordfjord and Q_{fN+H}/B_{mH} has changed to 0.065 for Holandsfjord. This should be a sufficient difference to establish baroclinic hydraulic control at Enganeset. A few vertical salinity profiles obtained at stations 3 and 4 after the regulation actually indicate that there now is a hydraulic control at Enganeset. This will be reported in the future when more data are available.

Equation (2) has a great potential for use as the dynamical open boundary condition in simple circulation models for the brackish water in deep fjords. Because of this and of purely theoretical reasons, it is of interest to further investigate the modification of baroclinic hydraulic controls (in essentially two-layered systems) by weak stratification in the layers, by interaction with time-dependent intermediary circulation, and by other factors.

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