Comments on "Multiple Equilibrium States in Combined Thermal and Saline Circulation"

JOSEPH EGGER

Meteorologisches Institut, Universität München, Munich, Germany

14 September 1994 and 18 May 1995

An interesting analysis of the structure and stability of multiple equilibria of the thermohaline circulation in two-dimensional box models is presented in Huang et al. (1992, HLS). In particular, HLS consider a 2 × 2 box model and reduce this model to a two-box model. A fairly complete structural analysis of this two-box model is carried out. I suspect, however, that these results of HLS are affected by an error. The two-box model turns out to be trivial when this error is rectified.

The authors introduce the variables u^+, u^- according to Fig. 1. These are related to the conventional directional flow velocities u_1, u_3 by

$$u^+ = u_1, \tag{1}$$

$$u^- = \delta u_3, \tag{2}$$

[see also (5) and (6) of HLS]. Thus, u^+ and u^- are proportional to meridional volume fluxes and $u^+ = u^-$ in the absence of precipitation [see (7) of HLS]. Of course, (2) implies $u^- = 0$ for $\delta = 0$. HLS invoke a linear damping law for momentum and relate both u^+ and u^- to the pressure differences via

$$u^+ = + \frac{c(P_1 - P_2)}{L} \tag{3}$$

$$u^- = - \frac{c(P_3 - P_4)}{L}, \tag{4}$$

[see (2) of HLS except for the signs; P_i pressure; c constant]. While (3) is correct, HLS appear to have overlooked the fact that u^- is not a velocity. Therefore, (4) must be replaced by

$$u^-/\delta = -c(P_3 - P_4)/L. \tag{5}$$

This error turns out to have serious consequences. According to (4), HLS obtain a finite volume flux u^- even for a vanishing depth of the lower layer and devote section 2b of their paper to an analysis of this situation. A finite flux requires an infinite velocity for $\delta = 0$. However, velocities are bound by the hydrostatic relationship and must be finite. Therefore, (4) leads to physically inconsistent results.

In what follows, we briefly present the corrected formulas for the two-box model. HLS arrive with (3) and (4) at the nondimensional equation

$$u^+ = -\frac{p}{2} + A[T_1 - T_2 + \delta(T_3 - T_4)] - B[S_1 - S_2 + \delta(S_3 - S_4)], \tag{6}$$

where the constants A, B do not depend on the fractional depth δ [see (7) of HLS]. However, one finds with (5) the corrected values (index c)

$$A_c = A2\delta/(1 + \delta)$$

$$B_c = B2\delta/(1 + \delta), \tag{7}$$

which reduce to $A_c = A$, $B_c = B$ for $\delta = 1$. Moreover, the term $-p/2$ in (6) must be replaced by $-p/(1 + \delta)$. Now, if $\delta \to 0$ as in the two-box model of HLS,

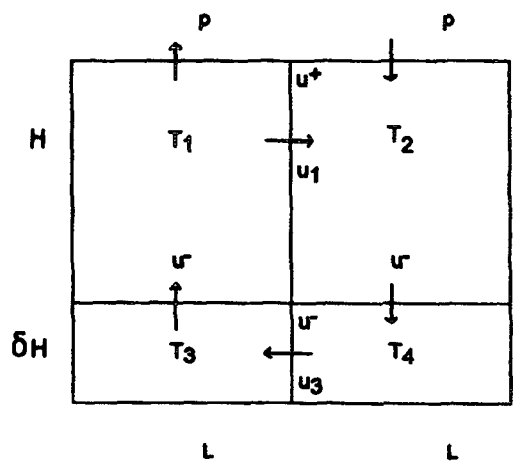

FIG. 1. The 2 × 2 box model of HLS where the directional flow velocities u_1, u_3 are added. All other symbols as in HLS: T_i temperature, p evaporation.

Corresponding author address: Prof. Joseph Egger, Meteorologisches Institut der Universität München, Arbeitsgruppe für Theoretische Meteorologie, Theresienstr. 37, 80333 Munich, Germany.

one has $u^+ = -p$, $u^- = 0$, and there is no circulation in the model except for the flow induced by precipitation. HLS, on the other hand, arrive via (6) at an erroneous circulation for $\delta = 0$, where $u^- \neq 0$. It appears, therefore, that the results of HLS obtained for the two-box model are invalid. Our correction makes good sense. A layer of vanishing depth is not a "pipe" (HLS), but can be simply discarded from the model. On the other hand, the error outlined above does not affect those results of HLS, where $\delta = 1$.

REFERENCES

Huang, R.-X., J. Luyten, and H. Stommel, 1992: Multiple equilibrium states in combined thermal and saline circulation. *J. Phys. Oceanogr.*, **22,** 231–246.