Multiple Equilibria and Transitions in a Coupled Ocean–Atmosphere Box Model

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ABSTRACT

A six-box model is employed as a prototype of the coupled Atlantic ocean–atmosphere system. Ice dynamics are excluded. Numerical integration of this system shows that different thermohaline circulation patterns are possible under the same forcing conditions. They consist of a global thermal mode with oceanic poleward surface flow, a global saline mode with equatorward surface flow, and two intermediate modes that are combinations of the two global modes. The stability of the modern-day-like intermediate mode to finite amplitude freshwater flux perturbations in the high latitude North Atlantic (meant as a model of glacial melting) is explored. It is found that freshwater fluxes of the proper inferred magnitude are close to critical and can induce a transition of the coupled system to a saline mode. However, paleoclimatic data argues the last deglaciation was subcritical.

Further, working in a realistic subcritical parameter regime, the box model yields an unrealistic temperature record. This argues, in turn, that additional physics (e.g., sea ice effects) must be included to properly describe the fundamental mechanics of the last glacial retreat.

1. Introduction

In this paper, we examine multiple equilibria and climate in a simple representation of a coupled ocean–atmosphere system. A goal of this study is to assess the relevance of the physics captured by the model to the climate evolution during the last glacial retreat. A particular event of interest in that retreat is the so-called Younger Dryas cold anomaly.

We use the so-called box model approach to the problem. This was first used to examine the joint effect of temperature and salinity, and their respective feedbacks, on the thermohaline circulation in the pioneering work by Stommel (1961). Recently, several such “ocean-only” models have appeared (e.g., Rooth 1982; Huang et al. 1992) and have been used to examine a number of climate related questions. An interesting extension of these ideas was recently introduced by Nakamura et al. (1994, hereafter NSM). They included diagnostic parameterizations of atmospheric transports in their model, thereby admitting feedbacks within the coupled system to influence its behavior. The basic idea in all such models is to represent the media by a finite number of well-mixed boxes, each advectively exchanging salt and heat with their nearest neighbors. Further, the ocean and atmosphere can exchange properties across their common interface by means of specified laws. The difference between ocean-only and coupled models is that the atmospheric conditions are presumed fixed and known in the former, while in the latter, they are dependent upon the state of the system. Coupled models are thus, in principle, better models of the real climate system.

The novel feedbacks between the thermohaline circulation and the atmospheric transport processes, which coupled models include, are discussed by Marotzke (1994). For example, he identifies the so-called atmospheric eddy moisture transport feedback (EMT), which involves the adjustment of the ocean–atmosphere moisture exchange to transitions in the oceanic state. This feedback, which he argues is destabilizing, is absent from the ocean-only models and underscores the need for the study of coupled systems. We are, therefore, motivated to use a coupled box model in this study.

Such feedbacks have been modeled in NSM and Marotzke and Stone (1995, hereafter MS) in the context of a “semiactive” atmosphere, in which atmospheric transports depend diagnostically on the oceanic state. The model we use for this study is a minor modification of the above model. It extends geometry to a pole-to-pole configuration, allows for variable sea level, and computes atmospheric temperature as a prognostic variable (see also Marotzke and Pierce 1996). Also, a slightly different parameterization of an atmospheric latent heat transport is employed.

Despite the additional physics of the coupled system,
the model “climates” (equilibrium states) are analogous to those found in ocean-only pole-to-pole models. These equilibria represent a thermal mode with poleward surface flow, a saline mode with equatorward surface flow, and a pole-to-pole mode with surface flow in one direction only. The inclusion of a variable sea level does not significantly affect the model stationary states and their bifurcation properties.

The purpose of this study is to examine the stability of the pole-to-pole circulation to high-latitude freshwater input. Here we are motivated in part by paleoclimatic observations, which suggest the overturning cell in the North Atlantic weakened during the most recent glacial retreat. These records also suggest the occurrence of the well-known Younger Dryas event (see Broecker et al. 1986, 1988, 1989), that is, a cold anomaly during the otherwise increasing temperatures characteristic of the interglacial interval. The duration of this event is estimated to have been several hundreds to one thousand years.

Relative to the Younger Dryas, there are several different deglaciation scenarios [see Ruddiman 1987a,b for a review]. In one of them (the so-called smooth deglaciation model), the most rapid change in the ice volume occurs during the Younger Dryas event. This scenario has been called into question by the sea level analysis performed by Fairbanks (1989). There it is argued that glacial retreat was marked by two distinct meltwater pulses of comparable strength. The melt water discharge rate in this latter scenario is minimal during Younger Dryas.

It is generally accepted that the North Atlantic deep-water production was reduced in the middle of the most recent glacial–interglacial transition, but the data do not adequately determine the timing of the Younger Dryas relative to the reduced production of deep water (Fairbanks 1989). Broecker et al. (1988, 1989) have recently proposed a scenario of the Younger Dryas event based on the smooth deglaciation model. They suggested that the increased discharge of glacial meltwater associated with the glacial retreat created a negative salinity anomaly in the North Atlantic. This in turn reduced the overturning rate, causing a decreased advective heat exchange between the equatorial and northern oceans. This resulted in colder conditions, which were necessarily associated with decreased North Atlantic deep-water formation. Important in this scenario is EMT feedback, originally proposed by Broecker et al. (1986) and discussed in NSM.

In the literature, there are numerous studies of the present-day-like overturning to perturbations in high latitude haline forcing. The models of differing complexity have been used, including box models (NSM; MS; Wang and Birchfield 1992; Birchfield et al. 1990; Tziperman et al. 1994), idealized 2D ocean-only models (e.g., Stocker and Wright 1991), idealized coupled models (Saravanan and McWilliams 1996; Rahmstorf 1994), and global coupled OAGCMs (Manabe and Stouffer 1995). The extreme sensitivity of the climates reproduced by the ocean-only models employing mixed boundary conditions (specified SST and freshwater flux at the ocean surface) to such perturbations is a characteristic of several past studies (e.g., Tziperman et al. 1994). It is possible that this behavior reflects the lack of an active atmosphere, although there is no guarantee that a coupled model is necessarily more stable than the uncoupled one (the EMT feedback of NSM is an example of a coupled system destabilizing feedback).

Thus, recent studies have employed models constraining the atmospheric hydrological cycle, including latitudinal and interocean water vapor transport (Stocker and Wright 1991; Wang and Birchfield 1992).

Wang and Birchfield have shown using an idealized three basin coupled ocean–atmosphere box model that different values of the net water vapor flux from the Atlantic to Pacific Ocean affect the stability characteristics of the system to perturbations in latitudinal moisture transport. Large values of interocean transport increase the sensitivity. The effects of interaction between Atlantic Ocean and atmospheric hydrological cycle (latitudinal moisture transport) were discussed by NSM (box model) and Saravanan and McWilliams (1996) (coupled 2D ocean–atmosphere model). Negative finite amplitude salinity perturbations applied to the North Atlantic region were used as a model of a glacial melt. The direct effect of such perturbations is in reducing the northward transport of heat and salt in the ocean (due to weakened overturning). This is accompanied by two atmospheric effects, that is, an increase in the heat and freshwater flux from the atmosphere into ocean (polar region). NSM found atmospheric freshwater flux feedback to be dominant, while Saravanan and McWilliams (1996) concluded that atmospheric heat transport effect is more important. (The reasons for the different results are not currently clear, but might be due to differences in model geometry.)

The same “pulse” experiments (with strong, short-lived freshwater additions) as those described above were conducted recently using more complex models by Rahmstorf (1994) and Manabe and Stouffer (1995). They found that such perturbations can induce rapid climate changes by affecting the thermohaline circulation of the ocean.

Here we systematically perform similar experiments using various freshwater pulses (differing in terms of both flux strength and duration) to see if analogous behaviors can be found in this much simpler box model. We find our pole-to-pole circulation is reasonably stable to freshwater input but that a sufficient volume of freshwater added to the high latitude ocean can trigger a transition to a saline mode. In the subcritical regimes, no internal oscillations were found in the system, the changes being led by glacial melt water input only, which decreases advective exchange between northern and equatorial boxes and causes lower atmospheric and oceanic temperatures and lower salinities in the northern
The coupled dynamics of such a single basin. (e.g., Broecker et al. 1986), and it is of interest to examine the coupled dynamics of such a single basin. This conﬁguration is chosen because the Atlantic sector of the World Ocean is believed to have undergone major changes during the last glacial–interglacial transition (e.g., Broecker et al. 1986), and it is of interest to examine the coupled dynamics of such a single basin.

There are six boxes, three of them being oceanic and three atmospheric. The ocean is assumed to occupy $fw = 1/6$ of the globe, roughly in keeping with Atlantic sector, and the rest is considered as land. The boundaries of the equatorial box are arbitrarily taken as 30°S and 30°N, due in part to the observed structure of the atmospheric eddy transports, which we parameterize here. Specifically the eddy transports, particularly of heat, are observed to dominate the atmospheric transport in the Northern Hemisphere at roughly 30°N (Peixoto and Oort 1992).

Polar boxes extend to the poles. The ocean initial depth is $H_o = 5000 \text{ m}$, and the height of the atmosphere is $H_\text{a} = 8000 \text{ m}$. The surface areas of the oceanic boxes are $S_i$ (i = 1, 2, 3). Perfect mixing is assumed within each box (oceanic and atmospheric), so the oceanic boxes are characterized by their temperature $T_\text{c}$ and salinity $S_\text{c}$, where $i$ is the number of the box (i = 1, 2, 3).

Atmospheric boxes have temperatures $T_\text{a}$. The quantities $q_i$ (i = 1, 2, 3, 4) represent volume ﬂuxes between the oceanic boxes (we apply the Boussinesque approximation, so that mass and volume ﬂuxes are interchangeable). This exchange is accomplished by means of two tubes connecting adjacent boxes. We assume that the free surface of each box is subject to instantaneous leveling, so that if freshwater volume ﬂuxes $Fw_i$ were added to the ith box, the depth of the ocean $\bar{H}$ would change according to

$$\bar{H} = (\bar{F}_{W1} + \bar{F}_{W2} + \bar{F}_{W3})/(\Sigma_1 + \Sigma_2 + \Sigma_3),$$

where an overdot denotes a time derivative.

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where an overdot denotes a time derivative.

The fluid motion in the lower tube balances the horizontal pressure gradient and linear friction. Assuming a linear equation of state for seawater, we obtain

$$q_1 = k(H/H_o)[\beta(S_1 - S_2) - \alpha(T_i - T_j)]$$

(2.2a)

and

$$q_2 = k(H/H_o)[\beta(S_2 - S_3) - \alpha(T_2 - T_3)],$$

(2.2b)

where gravitational acceleration $g = 9.8 \text{ m s}^{-2}$, $\alpha = 2 \times 10^{-4} \text{ (C)}^{-1}$, $\beta = 8 \times 10^{-4} \text{ psu}^{-1}$, and $k = 2.77 \times 10^6 \text{ m}^3 \text{ s}^{-1} \text{ (MS)}$.

Oceanic mass conservation gives

$$\bar{H}\Sigma_1 = PE_i\Sigma_1 - q_1 - q_2 + \bar{F}_{W1},$$

(2.3a)

and

$$\bar{H}\Sigma_3 = PE_i\Sigma_3 + q_3 + q_4 + \bar{F}_{W3},$$

(2.3b)

Here $PE_i$ (i = 1, 2, 3) has the dimensions of velocity and is the net freshwater forcing for a given oceanic box, including evaporation, precipitation, and river runoff. $\bar{F}_{W3}$ (i = 1, 2, 3) are external water volume fluxes (e.g., from a glacial source). After explaining atmospheric and land hydrological cycle, we will see that only these latter $\bar{F}_{W3}$ fluxes can actually change oceanic volume.

Oceanic heat equations are

$$\Sigma_1HT_1 = |q_1|(T_2 - T_1) + Q_1/\rho_w c_w,$$

(2.4a)

$$\Sigma_3HT_3 = |q_3|(T_2 - T_3) + Q_3/\rho_w c_w,$$

(2.4b)

and

$$\Sigma_2HT_2 = |q_2|(T_2 - T_3) - |q_3|(T_2 - T_3) + Q_2/\rho_w c_w,$$

(2.4c)
where \( \rho_w = 1000 \text{ kg m}^{-3} \), \( c_w = 4000 \text{ J/(kg °C)} \) are air density and heat capacity, and \( Q_i \) (\( i = 1, 2, 3 \)) are the total heat flux to an oceanic box (see below).

Finally, the oceanic salt equations can be written as

\[
\Sigma_i H \dot{S}_1 = \text{SADV}_1 - \Sigma_i S_i H, \quad (2.5a)
\]

\[
\Sigma_i H \dot{S}_3 = \text{SADV}_2 - \Sigma_i S_i H, \quad (2.5b)
\]

\[
\Sigma_i H \dot{S}_2 = -\text{SADV}_1 - \text{SADV}_2 - \Sigma_2 S_2 H. \quad (2.5c)
\]

The advective salt transports \( \text{SADV}_i \) (\( i = 1, 2 \)) depend on the direction of oceanic volume fluxes. If, for example, \( q_i > 0 \) and \( q_2 < 0 \) then \( \text{SADV}_1 = -q_i S_1 - q_2 S_2 \). Analogous expressions when the directions of the fluxes are different from the above example are obvious. Note that total salt in the system is conserved. The mean oceanic salinity in the initial state is 35 psu.

The ocean model described by (2.1)–(2.5) is forced by the heat and freshwater fluxes through ocean–atmosphere interface. These fluxes are the functions of both oceanic and atmospheric states. As mentioned before, the prognostic atmospheric variables are the atmospheric temperatures \( T_a, i = 1, 2, 3 \). Knowing those and the oceanic temperatures, we can find the heat fluxes to the ocean as

\[
Q_i = [F_i + A + B(T_a - \overline{T})] - \sigma \overline{T}^4 - 4\sigma \overline{T}^3 (T_a - \overline{T}) + \lambda(T_a - T_l) \quad (2.6a,b,c)
\]

Here \( T_a, T_l (i = 1, 2, 3) \) are the atmospheric and oceanic temperatures in kelvins. The terms proportional to Boltzmann constant \( \sigma = 5.7 \times 10^{-8} \text{ W K}^{-4} \) describe the longwave radiation loss by the ocean. The Stefan–Boltzmann formula has been linearized about \( \overline{T} = 273 \text{ K} \). The last term on the right-hand side of (2.6) is the sensible and latent heat exchange between ocean and atmosphere, with \( \lambda = 50 \text{ W m}^{-2} \text{ °C}^{-1} \), \( F_i = F_i = 1639 \text{ W m}^{-2} \), and \( F_3 = 3168 \text{ W m}^{-2} \) are the net shortwave radiation forcings at the top of the atmosphere. Atmospheric longwave radiation is expressed as a linear function of atmospheric temperature with the coefficients \( A = 212 \text{ W m}^{-2} \) and \( B = 1.7 \text{ W m}^{-2} \text{ °C}^{-1} \) (NSM). Atmosphere radiates both up and down; the downward flux is absorbed by the ocean as seen in (2.6).

Neglecting land heat capacity and heat conductivity and assuming the atmosphere to be transparent to shortwave radiation, the atmospheric equations are

\[
(p_c c_H \Sigma_i/fw) \overline{T}_{a_1} = Lw_1 + H_{sw_1} + H_{land_1} + H_{dis_1}, \quad (2.7a)
\]

\[
(p_c c_H \Sigma_i/fw) \overline{T}_{a_2} = Lw_2 + H_{sw_2} + H_{land_2} + H_{dis_2}, \quad (2.7b)
\]

\[
(p_c c_H \Sigma_i/fw) \overline{T}_{a_3} = Lw_3 + H_{sw_3} + H_{land_3} - H_{dis_3}, \quad (2.7c)
\]

where \( \rho_a = 1.27 \text{ kg m}^{-3} \), \( c_a = 1004 \text{ J/(kg °C)} \) are air density and heat capacity. The expressions for \( Lw_i \), \( H_{sw_i} \), and \( H_{land_i} \) are

\[
Lw_i = -2(\Sigma_i/fw) [A + B(T_a - \overline{T})], \quad (2.8)
\]

\[
H_{sw_i} = -\Sigma_i [-\sigma \overline{T}^4 - 4\sigma \overline{T}^3 (T_a - \overline{T}) + \lambda(T_a - T_l)], \quad (2.9)
\]

\[
H_{land_i} = (\Sigma_i/fw)(1 - fw)[F_i + A + B(T_a - \overline{T})]. \quad (2.10)
\]

Here \( Lw_i \) is the longwave radiation loss by the \( i \)th atmospheric box, and \( H_{sw_i} \) is the heat lost by the ocean and absorbed by the atmosphere. This includes ocean longwave radiation and sensible and latent heat exchange between the ocean and atmosphere. Here \( H_{land_i} \) is the heat input from the land. In writing (2.10) the land heat equation was implicitly used.

The last terms on the right-hand side of each of (2.7) describe atmospheric sensible and latent heat transport:

\[
H_{a_i} = H_{sem} + H_{lat}, \quad (2.11)
\]

where \( i = 1, 2 \). Here

\[
H_{sem} = 2\pi H_o a e \cos 30^\circ C_1 |T_{a_1} - T_{a_2}|^n (T_{a_2} - T_{a_1}), \quad (2.12)
\]

\[
H_{lat} = 2\pi H_o a e \cos 30^\circ C_1 |T_{a_2} - T_{a_3}|^n \times (qs(T_{a_3}) - qs(T_{a_2})). \quad (2.13)
\]

Here

\[
q_s(T) = 0.622 \frac{2.53 \times 10^{11}}{287.04 \rho_a T} \exp \left[ -\frac{5420}{T} \right]. \quad (2.14)
\]

is the Clapeyron–Clausius equation for the saturation specific humidity of the air at temperature \( T \) and \( a_o = 6400 \text{ km} \) is the radius of the earth. We choose \( n = 1 \), and tune the model to have realistic atmospheric heat transport (e.g., NSM) in standard equilibria (see below). This gives

\[
C_1 = 15 \text{ W m}^{-2} \text{ °C}^{-2},
\]

\[
C_2 = 2 \times 10^4 \text{ W m}^{-2} \text{ °C}^{-1}.
\]

We assume no storage of moisture in the atmosphere or on land, and the evaporation from the land is neglected. Moisture can be transported meridionally by means of atmospheric fluxes and rivers on land:

\[
F_{w_i} = F_{rivers} + F_{atmosphere}, \quad i = 1, 2. \quad (2.15)
\]

\[
F_{atmosphere} = \frac{L}{\rho_o} H_{lat}, \quad (2.16)
\]

where \( L = 2.5 \times 10^4 \text{ J kg}^{-1} \) is the latent heat of vaporization of water. We introduce the river transport as

\[
F_{rivers} = fw(A_i - fw^{-1}) F_{atmosphere}, \quad (2.17)
\]

where \( A_i \) is the “Atlantic factor” (NSM). For example, with \( f_w = \frac{v}{c_o} \) and \( A_i = 3 \), \( F_{rivers} = -\frac{1}{2} F_{atmosphere} \), implying half the total poleward atmospheric transport returns.
equatorward via rivers. Obviously, half the $F_{\text{atmosphere}}$ ends up in the polar box. If $A_i = f w^{-1}$ then the river transport is zero and moisture is carried to the poles by the atmosphere only. $A_i = 1$ corresponds to the situation where the atmospheric poleward moisture transport over land is exactly compensated by equatorward river flux. We typically use $A_i = 3$.

The net oceanic haline forcing is then

$$PE_i \Sigma_1 = F_{W_1}, \quad (2.18a)$$

$$PE_i \Sigma_3 = F_{W_2}, \quad (2.18b)$$

$$PE_i \Sigma_1 + PE_i \Sigma_2 + PE_i \Sigma_3 = 0. \quad (2.18c)$$

The model is now complete. For further discussion about different aspects of the model, the reader is referred to NSM, MS, Marotzke (1994), and Marotzke and Pierce (1997).

The system is integrated numerically. With the standard set of parameters the model has four stable equilibria (Tables 1a,b). They are the global thermal mode with poleward surface flow, the global saline mode with equatorward surface flow, and two asymmetric intermediate modes, which are superpositions of the global modes. In Tables 1a,b only one such mode with surface flow everywhere northward (to box 3) is listed since the other one can be obtained from this by symmetry. Such equilibria are also found given reasonable variations in model parameters. In the next section we will explore the stability of the intermediate mode in Tables 1a,b to finite amplitude freshwater perturbations applied to the northern box.

### 3. Intermediate mode stability experiment

The modern Atlantic is thought to be in a thermohaline pattern most like our intermediate mode. An interesting question involves the effect on such a mode of a freshwater discharge into the high latitude North Atlantic. This may be considered as a crude model of the glacial melt, which is thought to have occurred during the most recent glacial–interglacial transition (roughly 10 kyr: e.g., Broecker et al. 1986, 1988, 1989; Fairbanks 1989). The answer to this question bears on our understanding of the (perceived) climate variability characteristic of this era. To address this issue, an intermediate mode stability experiment was performed. Mathematically, we are exploring the stability of our modern-looking intermediate mode to finite amplitude freshwater perturbations. To accomplish this, a freshwater source ($F_{W_1}$) is included in the equations. With this comes two new adjustable parameters to the model. These are $F_{W_1}$, the freshwater flux rate, and $t_{\text{flush}}$, the timescale over which the flux is assumed to occur. Several experiments were conducted corresponding to differing flux rates and flux duration times. Due to admitted crudity of the model, we interpret our results in terms of orders of magnitude only when considering data.

We begin the discussion with pulse experiments characterized by short duration times $t_{\text{flux}} \ll t_{\text{flush}} \sim 350$ yr, where $t_{\text{flush}} = H_{i,2}/q_3$, is the northern box flushing time.

Depending on the value of the flux rate, one of the following scenarios takes place. The direct effect of the freshwater pulse is in creating a negative salinity anomaly, and, consequently, negative density anomaly in the northern part of the ocean. This anomaly results in weakened thermohaline overturning. At small values of the flux rate the system, somewhat perturbed, still recovers to the intermediate mode state. During the evolution, the largest changes in model climate (temperatures and salinities) occur in the northern part of the ocean and atmosphere, the rest of the system being affected much less. This is in agreement with other studies (e.g., Manabe and Stouffer 1995). The final state of the system is slightly modified due to the now different ocean depth. A 2% change in depth induces a similar magnitude (2%) change in the salinity field. The temperature field is relatively unaffected (maximum changes are around 0.3%). On the other hand, if a pulse is strong enough to create a sufficient density anomaly, the intermediate mode transitions to a saline mode with the overturning being reversed.

The threshold amount of additional water separating the two scenarios corresponds to approximately a 100-m sea level rise, like that thought to occur during the last glacial retreat (cf. Fairbanks 1989). In Fig. 2, the evolution of the system near the critical point is shown.

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### Table 1a. Standard equilibria set (symbols are defined in the text).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Thermal</th>
<th>Saline</th>
<th>Intermediate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 (^\circ \text{C})$</td>
<td>5.13</td>
<td>3.86</td>
<td>3.55</td>
</tr>
<tr>
<td>$T_2 (^\circ \text{C})$</td>
<td>31.41</td>
<td>32.68</td>
<td>32.07</td>
</tr>
<tr>
<td>$T_3 (^\circ \text{C})$</td>
<td>5.13</td>
<td>3.86</td>
<td>5.4</td>
</tr>
<tr>
<td>$S_1 (\text{psu})$</td>
<td>33.8</td>
<td>30.8</td>
<td>29.34</td>
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<tr>
<td>$S_2 (\text{psu})$</td>
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<td>30.02</td>
<td>30.68</td>
<td>30.37</td>
</tr>
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<td>$T_3 (^\circ \text{C})$</td>
<td>3.34</td>
<td>2.06</td>
<td>3.64</td>
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</table>

### Table 1b. Fluxes for the standard equilibria set (symbols are defined in the text).

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<th>Variable</th>
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<th>Intermediate</th>
</tr>
</thead>
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<td>-2.28</td>
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<tr>
<td>$q_2 (\text{Sv})$</td>
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<td>-9.66</td>
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<tr>
<td>$F_{W_1} (\text{Sv})$</td>
<td>0.58</td>
<td>0.65</td>
<td>0.64</td>
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<tr>
<td>$F_{W_2} (\text{Sv})$</td>
<td>0.58</td>
<td>0.65</td>
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</tr>
<tr>
<td>$PE_i (\text{m yr}^{-1})$</td>
<td>0.86</td>
<td>0.95</td>
<td>0.93</td>
</tr>
<tr>
<td>$PE_i (\text{m yr}^{-1})$</td>
<td>-0.86</td>
<td>-0.95</td>
<td>-0.9</td>
</tr>
<tr>
<td>$H_m (\text{PW})$</td>
<td>2.98</td>
<td>3.28</td>
<td>3.28</td>
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<td>3.17</td>
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<td>2.99</td>
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<td>-0.28</td>
<td>-1.03</td>
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</table>
Fig. 2. Intermediate mode stability experiment with the additional flux \( F_w \), applied to the northern (3rd) oceanic box at \( t = 10 \) yr for the time period \( t_{\text{flux}} \), causing the sea level change of \( \Delta H \) (\( t_{\text{flux}} = 1 \) mo, \( \Delta H = 101.7 \) m). Time series of (a) \( T_3 \), (b) \( S_3 \), (c) \( q_3 \), and (d) \( T_{\text{a3}} \).

A short freshwater pulse is applied to the system at \( t = 10 \) yr. In this particular example (\( t_{\text{flux}} = 1 \) mo, \( \Delta H = 101.7 \) m) the transition occurs at roughly 3000 years. First, the system finds itself in a quasi-equilibrium state with very small growth rates (the portion of Fig. 2 between 500 and 1500 yr). This state is characterized by weak northern overturning cell, while the circulation pattern in the southern part of the ocean is largely unaffected. The closer the added watermass is to the critical value, the longer the system stays in the state described above. Presumably, at some critical watermass, the system stays there forever. We have not numerically found the value of this critical mass, but we appear to have bounded it. The evidence for this is that we find for a pulse stronger than a certain value, an exponential transition to a saline mode eventually occurs (as in Fig. 2). Small differences in the pulse near this critical regime determine both if the system flips to the saline mode and how long it stays near the quasiequilibrium.

One can argue that the quasi equilibrium above is like the unstable “slow thermal mode” in Stommel (1961). We have three oceanic boxes in our system rather than two, but the dynamics and climatology of the southern part of the ocean changes very little in the course of the stability experiment, which makes the analogy with Stommel’s problem more transparent.

The range of flux values (for a given \( t_{\text{flux}} \)), where we see the behavior displayed in Fig. 2, is very small. As a rule, a 1% beyond critical flux gives a smooth transition to a saline mode, whereas a 1% weaker flux is not enough to push the system to the quasi-equilibrium state discussed above. Such sensitivity suggests our near-critical transition behavior may not survive in models of greater complexity; however, further work is needed to resolve the issue.

Although the total oceanic volume change in the pulse experiments is realistic, the flux rates are huge due to short duration times. Further, the model transitions are
monotonic, and oscillations like those found in the Younger Dryas proxy data are not observed. In the subcritical regimes, the characteristic time during which the system returns to a previous state seems to depend solely on the additional flux strength. The more the system is perturbed, the longer it takes for a perturbation to disappear. It is not clear if this is the case in more complete models because they have not been so tested (Saravanan and McWilliams 1996; Rahmstorf 1994; Manabe and Stouffer 1995). Nonetheless, we argue that the recovery time does not represent an internal timescale of the model. The experiments with more realistic flux rates are, in any case, of interest.

The analysis of Fairbanks (1989) yields the value of $0.1 \pm 0.4$ Sv ($10^6$ m$^3$ s$^{-1}$) for maximum glacial discharge rates. Increasing $t_{\text{max}}$ to 1000 yr ($\gg t_{\text{flush}} \approx 350$ yr) reduces threshold value of the flux so that total critical sea level rise (proportional to $F_{W_3} t_{\text{max}}$) changes very little. For $t_{\text{max}} = 1000$ yr this value is about $\Delta H \sim 125$ m. This, in fact, supports the use of pulse experiments for stability analyses of glacial melt in more complicated climate models.

Since it is generally accepted that the oceanic overturning slowed, but did not stop during the last deglaciation, we can assume the most reasonable scenario for our model in a subcritical one. Here the system behaves as follows [see Fig. 3 ($t_{\text{max}} = 1000$ yr, $F_{W_3} = 0.3$ Sv)]: after turning on the additional flux, the system goes to a state where the circulation is in a slowly evolving equilibrium with changes induced by the dilution of the salinity. Eliminating the additional freshwater flux breaks the balance and the system relaxes back to the pole-to-pole mode. The intermediate state described above is characterized by colder conditions in the northern part of the ocean due to reduced overturning rate.

Our model yields a sequence of events reminiscent of those proposed by Broecker et al. (1988, 1989) in their theory of the Younger Dryas event. In particular, our near-critical meltwater discharge rate ($\sim 120$ m of sea level rise per 1 kyr) is similar to that which would...
have occurred under the smooth deglaciation scenario. We also find the extreme temperature conditions are in phase with the weakest overturning, which is itself in phase with the meltrate. The temperature differences of about 1°C occur during the adjustment (Fig. 3). We feel this is significant because the Younger Dryas is itself identified as a cold climatic anomaly. Indeed, estimates of temperature differences during the anomaly are often as high as 5°C. Our value of 1°C is smaller than that, but one needs to remember that this represents the volume averaged temperature difference over the volume of the northern box. If the major changes occurred in the northmost part of the Atlantic region (which is, presumably, the case), their amplitude might be much larger than volume-averaged value.

On the other hand, our model is at odds with sea level. The most complete record of glacio–eustatic sea level is given in Fairbanks (1989) and consists of two distinct meltwater peaks. It further appears that the minimum in overturning occurred roughly between those peaks. The timing of the cold anomaly is less certain, but occurred at some point during the meltwater extreme. We have conducted experiments with Fairbanks meltwater history (not shown) and find the results to be straightforward. Under such forcing, we obtain two cold extrema, occurring in phase with the meltwater record, and separated by a warm period in between. There is no evidence for such a temperature record in paleoclimatic data. Rather, we are led to conclude the physics of Younger Dryas is more complicated than the subcritical response to glacial melt water discharge that is captured by our model. We speculate that sea ice in particular [see Jayne and Marotzke (1998, manuscript submitted to J. Climate) for the extension of NSM to include sea ice] may have a role to play, as one is naturally led to the idea that the Younger Dryas represents some internal mode of oscillation.

4. Conclusions

A simple six-box model describing the long-term climate tendencies of the Atlantic ocean–atmosphere system has been used to comment on the possible evolution of the North Atlantic subject to a glacial melt.

The model is a straightforward extension of MS to include pole-to-pole geometry, variable sea level, and explicit energy balance atmosphere model. The model proved to have the same types of linearly stable equilibria as do many uncoupled models (Stommel 1961; Huang et al. 1992). The equilibria are represented by a global thermal mode with poleward surface flow, a global saline mode with equatorward surface flow, and two pole-to-pole intermediate modes. The stability of the modernlike intermediate mode to finite perturbations of freshwater flux at the high latitudes was considered. These may be thought of as a model of glacial melt, which occurred during the last glacial retreat. It was shown that the intermediate mode was stable to small melt rates, but stronger melt rates could induce a transition. Although our model was subcritical when forced by reasonable melt rates, no phase lag between overturning strength and melt rate was found. This is seemingly at odds with what is known about sea level during the last glacial retreat. In particular, the appearance of one cold anomaly in the presence of two meltwater peaks suggests that feedbacks other than those strictly between the ocean and the atmosphere were essential to the Younger Dryas. The results of this study show that the simulations of the last deglaciation using realistic glacial water melt rates (not pulse experiments) in more complete models might be of interest.

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