A Multi-Layer Transient Model of Coastal Upwelling¹

RICHARD T. MCNIDER AND JAMES J. O'BRIEN

*Depts. of Meteorology and Oceanography and The Geophysical Fluid Dynamics Institute,
Florida State University, Tallahassee 32306*

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ABSTRACT

The transient development of wind-driven coastal upwelling in a rotating, stratified ocean is simulated using a four-layer numerical model. All longshore derivatives in the velocity field are neglected as are mixing processes between layers. The neglect of mixing effects necessarily limits the applicability of the model to short time scales of several days. The beta effect and longshore pressure gradients are included. The vertical structure of the nearshore upwelling zone is emphasized and offshore Ekman drift is found to be confined to the surface layer while onshore flow is evenly distributed in the lower three layers. An investigation of the baroclinic coastal current reveals an equatorward surface jet and a weaker poleward undercurrent in the β -plane solutions. The vertical structure of these coastal currents is dependent upon the stratification imposed. A pronounced tilt away from the coast with depth is observed in the coastal jet. A noticeable down-warping of the lowest interface is detected after the poleward undercurrent is established. Stream-function representation of the transverse circulation indicates propagating internal waves due to the impulsively imposed wind stress are trapped within the first 200 km of the coast.

1. Introduction

Coastal upwelling is a well-observed phenomenon along coastlines where an equatorward wind exists parallel to a coast. Under these conditions, offshore transport, as predicted by Ekman (1905), will develop in response to the Coriolis acceleration. To satisfy continuity requirements at the boundary, subsurface water must be brought to the surface in a region near the coast. This local phenomenon has a tremendous impact upon the marine biological production by providing a mechanism by which nutrient-rich subsurface waters can be brought to a level where sunlight is available. The combination of sunlight and nutrients is conducive to photosynthesis in small one-cell plants, phytoplankton. These plants provide a base for a complex food chain which can ultimately support large fish populations. Thus, upwelling areas are important fish-producing regions for the world fishing industry.

The cold upwelled water present at the surface also has a dramatic effect on the climate by cooling the air temperature and increasing the relative humidity. The humid Peruvian coastal deserts are prime examples of climate modification induced by upwelling processes. Smith (1968) provides an excellent review article on the basic dynamics and effects of upwelling.

The present investigation is concerned with the vertical structure of the upwelling circulation. Since the temperature and nutritive content of a water mass are

essentially dependent upon its depth of origin, a detailed description of the return flow is needed to understand the effect upwelling will exert on the environment. We will be specifically concerned with coastal upwelling induced by a steady wind blowing parallel to a north-south coastline. A multi-layer model patterned after the models of O'Brien and Hurlburt (1972), and Hurlburt and Thompson (1973) is utilized in the study of a simplified four-layer ocean. Simplifications include a hydrostatic fluid, constant density within the layers, and north-south symmetry in the velocity field. The wind stress forcing applied at the model surface is steady but not uniform across the model region and is applied impulsively at $t=0$. Longshore pressure gradients, the β -effect, and transfer of momentum between layers are included. Both internal and external gravity waves are present in the solutions and are excited by the impulsive start of the wind stress.

Dependent variables include layer thicknesses and horizontal velocity components within the layers. Vertical velocity can be calculated diagnostically from the above variables.

The model equations are integrated numerically for a wide, flat oceanic shelf of 200 m depth and a simulated Oregon coast case. Several different density profiles are used to investigate the effect of stratification on the strength of the upwelling and vertical structure of the longshore flow. A down-warping of isopycnals which reduces the effective depth of upwelling motions is found to be associated with a poleward flowing undercurrent.

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A previous simulation of coastal upwelling by O'Brien and Hurlburt (1972) produced a realistic width to the upwelling region at the coast and an equatorward surface jet which is observed in upwelling regions. Hurlburt and Thompson (1973), using a formulation which allowed inclusion of the β -effect and longshore pressure gradient, were able to produce a more realistic longshore velocity profile that included a poleward undercurrent. Both of the above models contained only two layers and thus little detail of the vertical structure was available. The present investigation is an extension of the above-mentioned work.

Analytical steady-state models such as those of Yoshida (1967) and Hsueh and Kenny (1972) include thermohaline mixing which has been neglected in our formulation. Omission of mixing processes may be a serious deficiency since this eliminates the possibility of any non-trivial steady state in the present model. However, the study of coastal upwelling on the event time scale (several days) does not require inclusion of thermohaline processes, since these mixing processes are only important on longer time scales.

Observational data which are essential for model comparison are unfortunately not complete. However, some of the best data available taken off the Oregon coast indicate a scale width for the upwelling zone of 15–25 km and vertical velocity of 30–60 m per month. Geostrophic currents for a two-week average (Mooers *et al.*, 1973) reveal a surface equatorward current of -20 cm sec^{-1} and a poleward undercurrent of 10 cm sec^{-1} at approximately 150 m depth. Other data along with further discussion of theoretical work will be cited in the course of the text where needed for comparison or documentation.

2. Model formulation

The brief discussion of the model equations and geometry used in this investigation follows O'Brien and Hurlburt (1972) and Hurlburt and Thompson (1973). A vertical vorticity equation permits the inclusion of the β -effect and longshore pressure gradient. For a detailed derivation of the equations of motion the reader should consult this earlier work.

TABLE 1. List of symbols and model constants.

f	$=10^{-4} \text{ sec}^{-1}$	Coriolis parameter
g	$=980 \text{ cm sec}^{-2}$	acceleration of gravity
ρ_1	$=1.0 \text{ g cm}^{-3}$	density of the top layer
A	$=10^6 \text{ cm}^2 \text{ sec}^{-1}$	coefficient of horizontal friction
c_I	$=10^{-5}$	internal stress coefficient in cgs units
c_B	$=10^{-3}$	bottom stress coefficient in cgs units
β	$=2 \times 10^{-13} \text{ cm}^{-1} \text{ sec}^{-1}$	variation of Coriolis parameter, [$\beta = df/dy = \text{constant}$]
u		x -directed component of motion
v		y -directed component of motion
w		z -directed component of motion
τ^{sy}		y -directed wind stress
τ^{sx}		x -directed wind stress
u_E		Ekman transport
h		layer thickness

Consider the governing equations for a multi-layer ocean using a right-handed Cartesian coordinate system with z increasing upward and x increasing to the east. The origin is chosen at the coast such that distances measured offshore are negative. In general form the vertically integrated equations are:

$$\begin{aligned} \frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} + v_j \frac{\partial u_j}{\partial y} \\ = f v_j - g \sum_{i=1}^n \frac{\partial h_i}{\partial x} - g \frac{\partial D}{\partial x} + g \sum_{i=1}^{j-1} \epsilon_{ij} \frac{\partial h_i}{\partial x} \\ + (\tau_j^{Tx} - \tau_j^{Bx}) / (\rho h_j) + A \left(\frac{\partial^2 u_j}{\partial x^2} + \frac{\partial^2 u_j}{\partial y^2} \right) \quad (1) \end{aligned}$$

$$\begin{aligned} \frac{\partial v_j}{\partial t} + u_j \frac{\partial v_j}{\partial x} + v_j \frac{\partial v_j}{\partial y} \\ = -f u_j - g \sum_{i=1}^n \frac{\partial h_i}{\partial y} - g \frac{\partial D}{\partial y} + g \sum_{i=1}^{j-1} \epsilon_{ij} \frac{\partial h_i}{\partial y} \\ + (\tau_j^{Ty} - \tau_j^{By}) / (\rho h_j) + A \left(\frac{\partial^2 v_j}{\partial x^2} + \frac{\partial^2 v_j}{\partial y^2} \right) \quad (2) \end{aligned}$$

$$\frac{\partial h_j}{\partial t} + \frac{\partial}{\partial x}(u_j h_j) + \frac{\partial}{\partial y}(v_j h_j) = 0 \quad (3)$$

$$j = 1(1)n$$

where:

$$\epsilon_{ij} = (\rho_j - \rho_i) / \rho_j$$

$$q = (u_j^2 + v_j^2)^{1/2}; \quad \bar{q}_j = (q_{j+1} + q_j) / 2$$

$$\tau_j^{Tx} = \rho c_I \bar{q}_j^T (u_{j-1} - u_j), \quad j \neq 1$$

$$\tau_j^{Bx} = \rho c_I \bar{q}_j^B (u_j - u_{j+1}), \quad j \neq n$$

$$\tau_j^{Ty} = \rho c_I \bar{q}_j^T (v_{j-1} - v_j), \quad j \neq 1$$

$$\tau_j^{By} = \rho c_I \bar{q}_j^B (v_j - v_{j+1}), \quad j \neq n$$

$$\tau_j^{Bx} = \rho c_B q_j u_j; \quad \tau_j^{By} = \rho c_B q_j v_j, \quad j = n$$

for which ρ is the density for the layer, c_I the interfacial stress constant, c_B the bottom stress constant, and n the number of layers.

The subscript j refers to the layer number with numbers increasing downward from the surface layer. Each layer is assumed to be homogeneous and the horizontal components u_j, v_j are assumed independent of depth within the layer.

The symbols τ_j^T and τ_j^B are the stresses at the top and bottom of each layer and are formulated such that stresses are continuous across the interfaces. Note that for $j=1$, τ_j^T becomes the wind stress at the sea surface and may be specified as a function of x and t . Bottom

topography is included through $D(x)$ which represents the height of the bottom above a deep reference level.

The model parameters ρ , g , f , A , c_I , c_B , β are specified initially and may be found in Table 1. We have chosen interfacial stress constants to be identical for all layers. Thompson and O'Brien (1973) have estimated that $c_I = 10^{-5}$ and $c_B = 10^{-3}$ in cgs units are realistic parameter values for upwelling on a continental shelf.

The time-dependent equations [(1)–(3)] represent a closed system with dependent variables u_j , v_j , h_j . Boundary conditions are $u_j, v_j = 0$ at the coast and in the far field. Layer thicknesses at the coast are calculated using a one-sided finite difference in the continuity equation. Initially the stratified fluid is at rest.

The curl of the momentum equations yields an equation for the vertical component of relative vorticity for each layer. If we assume that the velocity field is independent of y and that nonlinear terms that include longshore variations in y may be neglected in the vorticity and continuity equations, the equations for a multi-layer model ocean may be written

$$\frac{\partial u_j}{\partial t} + u_j \frac{\partial u_j}{\partial x} = f v_j - g \sum_{i=1}^n \frac{\partial h_i}{\partial x} - g \frac{\partial D}{\partial x} + g \sum_{i=1}^{j-1} \epsilon_{ij} \frac{\partial h_i}{\partial x} + (\tau_j^{Tx} - \tau_j^{Bx}) / (\rho h_j) + A \frac{\partial^2 u_j}{\partial x^2}, \quad (4)$$

$$\frac{\partial}{\partial t} \frac{\partial v_j}{\partial x} + \frac{\partial}{\partial x} \left(u_j \frac{\partial v_j}{\partial x} \right) + f \frac{\partial u_j}{\partial x} + \beta v_j = - \frac{\partial}{\partial x} [(\tau_j^{Ty} - \tau_j^{By}) / (\rho h_j)] + A \frac{\partial^3 v_j}{\partial x^3}, \quad (5)$$

$$\frac{\partial h_j}{\partial t} + \frac{\partial}{\partial x} (u_j h_j) = 0, \quad (6)$$

where

$$\beta = \frac{df}{dy} = \text{constant}.$$

An additional boundary condition is required by (5). Hurlburt and Thompson (1973) have demonstrated that for an inert, unforced far field it is consistent to assume that the longshore pressure gradient vanishes at some distance remote from the coast. This is equivalent to assuming

$$\frac{\partial^2 v_j}{\partial x^2} = 0 \quad (7)$$

at the computational boundary off shore.

A first integral in x of (5) and use of the y -momentum equation indicates the necessity of retaining the north-south pressure gradient in the model for $\beta \neq 0$, since

$$\int_{-L}^x \beta v_1 dx = g \sum_{i=1}^n \frac{\partial h_i}{\partial y}.$$

Yoshida (1967) has shown that idealized two-dimensional models may overestimate the intensity of upwelling circulations as compared to a three-dimensional model through neglect of longshore pressure gradients and variations in wind stress along the coast. O'Brien and Hurlburt (1972) have also demonstrated that f -plane solutions (no north-south pressure gradient) produce overly strong longshore velocities and fail to yield a poleward undercurrent. The present model formulation which retains the north-south pressure gradient permits the inclusion of an important three-dimensional effect in a two-dimensional model.

We will solve (4)–(6) for a four-layer system. The 12 equations are solved numerically using the economically efficient semi-implicit scheme developed by Kwizak and Robert (1971) and O'Brien and Hurlburt (1972). The implicit treatment of both the external and internal gravity waves permits a substantial increase in computational speed.

3. Applicable time scales

Recent intensive observational studies off the Oregon coast indicate at least two time scales are required to adequately describe coastal upwelling processes. These are a seasonal time scale of many months and an event time scale of several days. The seasonal time scale essentially encompasses the formation of large-scale climatological features favorable to upwelling and the average adjustment of the ocean to these features. The event time scale involves the short-term breakdown and reestablishment of the seasonal features.

The present investigation is concerned with a simple model describing upwelling processes on the event time scale. Results will be shown after only 5–10 days of model spin-up.

Although thermohaline mixing may be important for completely describing the upwelling phenomena and an associated steady state, we neglect mixing. This can be justified in a sense by consideration of the following scaling argument.

For example, the exchange of density between two upper layers can be written (O'Brien, 1967)

$$\frac{d\epsilon_{12}}{dt} = \frac{-K_s \epsilon_{12} q_s}{h},$$

where $\epsilon_{12} = (\rho_2 - \rho_1) / \rho_2$, $q_s = [(u_2 - u_1)^2 + (v_2 - v_1)^2]^{1/2}$, and K_s is a constant bulk exchange coefficient. The above equation simply relates the mass flux through the interface to the velocity shear and density contrast of the layers.

A suitable value for K_s is difficult to estimate. A good first approximation might be $K_s = c_I = 10^{-5}$, the internal momentum stress coefficient. Pike (1971) formulates a vertical mixing process at the thermocline dependent upon turbulent stress at the surface and velocity shear across the thermocline. A value for $K_s = 10^{-5}$ was chosen

so that velocity shear contribution and wind stress contribution to thermocline mixing would be of equal magnitude.

Using $K_s = 10^{-5}$ and scaling the above equation with appropriate model values ($q_s = 25 \text{ cm sec}^{-1}$ and $h = 2500 \text{ cm}$), we find the time scale for mixing processes to become important to be approximately 100 days. A larger value for K_s would reduce this time scale; however, solutions on the order of 5–10 days should still remain valid.

Observations in upwelling zones also seem to indicate that mixing processes may not be a dominant feature on the several day time scale since sharp temperature gradients and frontal zones remain well defined for several days (O'Brien and Tamura, 1972).

4. Model display

The numerical solutions to (4)–(6) yield profiles of u_j , v_j and h_j as a function of x and t . Since the velocity is independent of y , it should be possible to define a streamfunction in the x - z plane. However, u_j represents an average over the layer and the vertical velocity w_j is defined only at the interface between layers and at the sea surface. In order to display the solutions in a pictorial form which is more easily understood, we elect to treat the velocity field as continuous in z . The u_j and v_j are regarded as estimates of the velocity components at the center of each layer. Using linear interpolation between layers and linear extrapolation above mid-depth of the surface layer, it is possible to generate a *continuous* profile of u . Similarly, we can interpolate linearly between the five w 's to obtain a *continuous* profile for w . If we assume that this modified u , w field satisfies

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0, \quad (8)$$

we may define a streamfunction ψ where

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = \eta,$$

and η is the y component of the vorticity vector $\nabla \times \mathbf{v}$.

The vertical velocity at any interface j is

$$w_j(x, t) = \sum_{i=j}^n \frac{d}{dt} h_i + \frac{d}{dt} D, \quad (9)$$

where w_1 is vertical movement of the sea surface and D the topography height above a deep reference level. The vertical velocity w_B at some point above the bottom is

$$w_B = \frac{dD}{dt}. \quad (10)$$

The streamfunction ψ is zero at the coast, along the bottom, and at the far-field boundary. Along the

surface ψ does not vanish. We calculate ψ at the free surface from

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{\partial w}{\partial x}, \quad (11)$$

with boundary conditions $\psi = 0$ at the coast and in the far field. [Eq. (11) has to be used instead of $\psi_x = -w$ to insure that ψ matches both horizontal boundary conditions.] A finite-difference analogue of this equation is solved using the tri-diagonal variant of Gaussian elimination. The interior solution for ψ is found using a standard over-relaxation procedure. Contours of ψ in the x - z plane contain all the information of the u_j and h_j profiles. This procedure should be regarded only as a convenient method to display the solutions for the four-layer model.

We may also interpolate v_j to obtain a continuous profile of v . Contours of the longshore velocity are a convenient way to display the model solutions. No new information content has been introduced by the interpolation. Between layers, the gradients are linear. One may easily construct the computed v_j from the contoured x - z distributions of the longshore velocity.

5. Results

Many model cases have been run exploring various vertical density profiles, layer thicknesses, wind stress and bottom topography. We have singled out three cases with which to explain the essential physics and results of the model. We will mention additional cases where applicable to better focus on a particular result.

a. Flat-shelf case

We shall first consider the circulation on a wide flat oceanic shelf. The fluid depth is taken to be 200 m with layer thicknesses $h_1 = h_2 = 25 \text{ m}$, $h_3 = 50 \text{ m}$, $h_4 = 100 \text{ m}$ (see Fig. 1). The east-west model width is 800 km; however, we shall generally only be concerned with solutions near the coast ($< 200 \text{ km}$). The wind stress imposed for this case is given in Fig. 1. Three distinct regions may be identified: a constant wind stress region inward of 200 km, a region with wind stress curl 200–700 km, and an unforced outer region 700–800 km.

Model stratification will be expressed in terms of density contrasts across the interfaces in σ_t units. The vertical density profile for this case is constructed so that there is a density difference of $2.0 \times 10^{-3} \text{ gm cm}^{-3}$ between the first and second layers and differences of $0.5 \times 10^{-3} \text{ gm cm}^{-3}$ between the remaining layers. The corresponding $\Delta \sigma_t$ units are 2.0, 0.5 and 0.5.

All model cases were run with the fluid initially at rest at $t = 0$. The wind stress was applied impulsively and maintained until the interface between the first and second layers reached the sea surface. (Hereafter, we shall refer to this uppermost internal interface as the first interface, and so forth.)

FIG. 1. Initial conditions (left) $u=v=0$, $h_1=h_2=25$ m, $h_3=50$ m, $h_4=500$ m. Wind stress components (right) as a function of distance offshore; τ_{sy} is 0.5 dyn cm^{-2} at the coast; τ_{sx} is zero everywhere.

The solutions after 5.5 days for this first model case are shown in Figs. 2–5. The diagrams show velocity and height profiles shoreward of 100 km (Fig. 2), stream-functions for half the model region (Figs. 3 and 4), and the longshore flow field (Fig. 5).

Offshore transport as predicted by classical Ekman dynamics is confined to the upper layer. Onshore flow required by continuity is essentially evenly distributed throughout the lower three layers. This is the first important result of the multi-layer model.

Longshore velocity profiles (Fig. 2, right) show a fairly strong equatorward jet in the upper layer of -35 cm sec^{-1} at 8 km. Horizontal friction in the model requires the velocity to vanish at the coast. In the absence of horizontal friction the jet minimum will occur at $x=0$. The longshore velocity in the second layer is weaker with a minimum of -5 cm sec^{-1} at approximately 12 km. This offshore tilt of the jet will be discussed subsequently. The lower two layers exhibit a poleward flowing countercurrent as observed by

Mooers *et al.* (1973), in upwelling regions with maximum velocity in the bottom layer.

Time plots from the model in Fig. 6 show the evolution of the longshore velocity components. The strong equatorward jet in the upper layer begins forming immediately and begins to penetrate into the second layer after 2.5 days. In the lowest layer the poleward flowing undercurrent appears near the coast after 1.8 days model time while positive velocities occur in the third layer a day later.

The accompanying height profiles (Fig. 2, left) show the position of the interfaces at 5.5 days. At the coast all three interfaces have moved upward from their initial positions in response to the one-sided divergence at the coast. However, the displacement is less with increasing depth; e.g., the third interface has risen only about one-third the distance of the first interface. This corresponds to a mean vertical velocity at the interfaces of $w \approx 5 \times 10^{-3} \text{ cm sec}^{-1}$ in the surface layer and $w \approx 1.5 \times 10^{-3} \text{ cm sec}^{-1}$ in the bottom layer.

FIG. 2. Height profiles (left) after 5.5 model days. Upwelling is indicated at the coast by the upturning of the interfaces. Longshore velocity profiles (right) are shown also after 5.5 days. Both diagrams are for the β -model with imposed stratification $\Delta\sigma_t^*$ of 2.0, 0.5, 0.5. Layer numbers are given in the figure.

FIG. 3. Streamfunction after 0.4 day (left) and 0.6 day (right) model time. Distortion of the streamlines by an internal gravity wave can be observed near the coast. The main circulation is cyclonic.

Although vertical velocities are smaller near the bottom, a noticeable increase in the scale width of the upwelling region is observed with increasing depth. This width change and flattening of the interfaces near the coast will be related to the longshore velocity profiles subsequently.

Not shown in the accompanying diagrams is the offshore region 200–700 km, where the interfaces drop below their initial positions. This downwelling can be attributed to simple Ekman pumping induced by the negative curl in the wind stress. The unforced region beyond the downwelling region (700–800 km) is characterized by essentially no displacement of the interfaces and near zero velocities.

Due to the impulsive starting of the wind stress the solutions exhibit a strong inertial character in early stages of spin-up. In fact, the cross-stream flow as shown in the streamfunction diagrams of Figs. 3 and 4 is clearly dominated by these oscillations which can be seen by an increase and decrease in streamlines. Recent oceanographic literature (e.g., Mooers, 1970) indicates

strong inertial oscillations in nature. The inertial period for the model is 17.44 hr. The sudden application of wind stress curl in the model also excites small-amplitude Rossby waves of $1\text{--}2\text{ cm sec}^{-1}$ in the longshore velocity field.

Propagating internal gravity waves are shown as small closed circulations in Figs. 3 and 4. The internal waves and inertial oscillations can be seen more clearly in time plots of the u -velocity components. In Fig. 7 x - t contours for the top layer and a time series at 18 km show that the inertial oscillations decay rapidly in time as the system reaches geostrophic equilibrium. Internal-inertia gravity waves can also be seen propagating seaward at a 30° angle on the x - t plots. The physical behavior and importance of these waves will be discussed later.

The strength of the upwelling produced depends to a large extent upon the imposed stratification. A large density contrast between the two upper layers inhibits vertical motion, thus reducing the magnitude of the upwelling. If the lower layers are more stably stratified

FIG. 4. Streamfunction after 0.8 day (left) and 1.0 day (right) model time. The propagating internal gravity wave is now a closed circulation. An inertial oscillation can be seen by an increase and decrease of streamlines.

FIG. 5. Longshore flow field contoured in the x - z plane after 5.5 days. A strong equatorward surface jet and poleward undercurrent can be observed. Contour interval is 2 cm sec^{-1} . Values >0 are shown by dotted line.

than the upper layers, then the strength of the upwelling is necessarily greater in the upper levels. Hsueh and Kenny (1972) have shown a similar relationship (for a continuously stratified fluid).

We also notice the lowest interface is considerably flattened in comparison to the interfaces above. This

can be attributed to the geostrophic adjustment of the layer to the poleward undercurrent. The importance of this down-warping will be discussed subsequently for a case which displays more dramatically this divergence of the interfaces.

As mentioned above, the width of the upwelling region at the coast increases with depth showing a very noticeable change between the first and second interfaces. This width difference results in a thinning of the second layer at approximately 12 km. Blumsack (1972) has shown, using boundary-layer analysis techniques for a continuously stratified fluid, that the upwelling region below a surface Ekman layer spreads laterally with increasing depth.

The longshore flow is balanced geostrophically in all layers and is thus directly related to the height profiles mentioned above. Hurlburt and Thompson (1973) show that the β -effect can produce a poleward undercurrent by inducing a north-south pressure gradient. The onshore flow in the lower layers is balanced by this longshore pressure gradient except near the coastal boundary. The kinematic boundary condition at the coast requires a breakdown in the onshore geostrophy. The resulting cross-gradient flow gives a northward component of motion allowing a poleward undercurrent

FIG. 6. Longshore flow field for all four layers contoured as a function of distance from shore and time. Contour interval is 2 cm sec^{-1} for the top layer and 1 cm sec^{-1} for the remaining lower layers. Dotted lines indicate negative values.

to develop. A reviewer observed that the βv term in the vorticity equation provides only a negligible contribution near the coast. It is, however, the integral of βv across the model region which provides for the north-south pressure gradient in the model and thus allows a poleward undercurrent to exist.

To further explain the mechanics of the undercurrent consider a simplified vorticity equation in which friction and nonlinear terms are neglected:

$$\frac{\partial}{\partial t} \left(\frac{\partial v}{\partial x} \right) = -f \frac{\partial u}{\partial x} - \beta v.$$

As indicated above βv can be neglected through use of scale analysis in the upwelling zone; in the lower layers the kinematic boundary condition at the coast forces $\partial u / \partial x < 0$. In this region there is growth of cyclonic vorticity induced by $\partial u / \partial x$ as indicated in the vorticity equation above. This type of cyclonic vorticity growth is necessary but not sufficient for the formation of a poleward undercurrent. In order for a true poleward flowing current to develop the above-mentioned positive vorticity growth must take place within a realistic velocity field.

Fig. 8 shows model solutions on an f -plane ($\beta=0$); note that even though cyclonic vorticity growth takes place in the lower three layers no poleward flow develops. This is because the cyclonic vorticity is imbedded within a strong barotropic equatorward flow field. O'Brien (1973) has shown analytically for a simplified two-layer model that imposition of a constant north-south pressure gradient eliminates the barotropic velocity component identically. The removal of the barotropic component allows the cyclonic vorticity growth in the lower layers to produce a poleward flowing undercurrent. In the present model the north-south pressure gradient is retained through βv and the barotropic mode is reduced so that the positive vorticity growth can produce poleward flow.

The tilt of the equatorward jet with depth (mentioned earlier) is an interesting feature of the four-layer model. To clarify our discussion of the tilt let us consider in place of the beta model an f -plane model in which no poleward undercurrent can exist. The solutions for the f -plane case may be found in Fig. 8. The $\Delta \sigma_t$ stratification imposed is 0.5, 2.0, 0.5, with a constant wind stress near the coast of -1.0 dyn cm^{-2} . The dominant feature in the longshore profiles is the strong equatorward jet in the upper layer extending into the second layer with a slight tilt offshore. If we look at the accompanying height profiles we again see the sharp increase in the width of the upwelling zone between the first and second layers.

O'Brien and Hurlburt (1972) have explained the presence of the longshore jet in the upper layer using conservation of potential vorticity. Consider the following equations where we have neglected horizontal

FIG. 7. Contours of top layer u -component (top) as a function of distance from shore and time. Contour interval is 1 cm sec^{-1} . Time plot (bottom) at 18 km of u -velocity components for the four layers.

friction and interfacial stresses, and $\text{curl}_z \vec{\tau} = 0$:

$$\frac{\partial v_1}{\partial t} + u_1 \frac{\partial v_1}{\partial x} + f u_1 = \tau^{xy} / \rho H_1, \quad (12)$$

$$\frac{\partial v_j}{\partial t} + u_j \frac{\partial v_j}{\partial x} + f u_j = 0, \quad j = 2(1)4, \quad (13)$$

$$\frac{\partial h_j}{\partial t} + \frac{\partial}{\partial x} (h_j u_j) = 0, \quad j = 1(1)4. \quad (14)$$

We may form the following potential vorticity equation

FIG. 8. Height profiles (left) and velocity profiles (right) after 1.8 days. Both diagrams are for f -plane model with constant wind stress near the coast of 1 dyn cm^{-2} and stratification of 0.5, 2.0, 0.5. Layer numbers are given in the figure.

for each layer:

$$\frac{d}{dt} \left(\frac{\partial v_j}{\partial x} + f \right) = 0, \quad j=1(1)4, \quad (15)$$

from which, after integrating once along a trajectory and using the initial conditions, we obtain

$$\frac{\partial v_j}{\partial x} = f \left(\frac{h_j - h_{j0}}{h_{j0}} \right), \quad (16)$$

with h_{j0} the initial thickness of a layer j at $t=0$.

Upwelling at the coast implies $h_{10} > h_1$ so v_1 must have a relative minimum at the coast. Horizontal friction which brings the longshore flow to zero at the boundary will displace the minimum away from the shore. Thus, a longshore equatorward jet is predicted in the upper layer with the strongest flow slightly off shore. For a two-layer model this equatorward jet is necessarily confined to the upper layer.

In the second layer we find in the upwelling region that $h_{20} \leq h_2$ everywhere except in the vicinity of 12 km where the second layer is thinned in response to the width difference of the upwelling zone between the two layers. This thinning of the layer implies a change of sign of the vorticity and an inflection point in the velocity field. The point of inflection is a relative minimum giving an equatorward jet in the second layer near the region of maximum thinning. Thus, conservation of vorticity predicts an equatorward jet extending at least through the second layer with the jet axis tilted offshore.

Stratification plays an important role in the structure of the longshore coastal currents since certain density profiles tend to enhance the features responsible for the formation of the jet. A vertical density profile in which the biggest contrast occurs between the first and second layers inhibits the formation of a minimum in the second

layer by reducing the thinning effect mentioned above. Stratification more conducive to upwelling increases the thinning and strengthens the jet in the second layer. The increased thinning in this case could be attributed simply to the greater upward movement of the second interface. However, analysis of two-layer systems indicate the width scale of the upwelling zone is determined by the baroclinic Rossby radius of deformation,

$$\lambda = \left[\frac{g \epsilon H_1 H_2}{f^2 (H_1 + H_2)} \right]^{1/2}.$$

In a multi-layer system more than one internal scale exists and the size of the internal horizontal scale lengths is dependent on the stratification and the layer thicknesses.

The same type of argument is valid for the β -plane model, thus explaining the tilt in the jet axis as seen in Fig. 2. The formation of the poleward undercurrent in the model complicates the argument in the lower layers. However, we can develop certain relations which predict fairly well the structure of the longshore velocity field in the β -plane model.

Let us return to the β -plane solutions discussed earlier and shown in Fig. 2. An interesting feature of the beta model is that the vertical integral of the longshore transport is approximately zero near the shore where the wind stress is constant. This can be verified directly from the solutions or derived using a diagnostic analysis of the model equations (see Hurlburt and Thompson, 1973). We want to use this fact and geostrophy in the longshore flow to derive relations governing the velocity profile near the coast.

We will simplify our analysis by considering a three-layer system in which the net longshore transport is zero so that

$$v_1 h_1 + v_2 h_2 + v_3 h_3 = 0. \quad (17)$$

The procedure we will follow can be carried out for the four-layer model. However, the final results and their

discussion can more easily be presented using only three layers. Assuming geostrophy in (4) we obtain

$$f(v_1 - v_2) = g\epsilon_{12}h_{1x}, \quad (18)$$

$$f(v_2 - v_3) = -g\epsilon_{12}h_{1x} + g\epsilon_{13}h_{1x} + g\epsilon_{23}h_{2x}. \quad (19)$$

However,

$$\epsilon_{13} = \epsilon_{12} + \epsilon_{23};$$

therefore

$$f(v_2 - v_3) = g\epsilon_{23}(h_{1x} + h_{2x}). \quad (20)$$

We can solve (17), (18) and (20) for the three longshore velocities to obtain

$$v_1 = \frac{g\epsilon_{12}(h_2 + h_3)h_{1x}}{fH_T} + \frac{g\epsilon_{23}h_3(h_1 + h_2)_x}{fH_T}, \quad (21)$$

$$v_2 = \frac{-g\epsilon_{12}h_1h_{1x}}{fH_T} + \frac{g\epsilon_{23}h_3(h_1 + h_2)_x}{fH_T}, \quad (22)$$

$$v_3 = \frac{-g\epsilon_{12}h_1h_{1x}}{fH_T} - \frac{g\epsilon_{23}(h_1 + h_2)(h_1 + h_2)_x}{fH_T}, \quad (23)$$

where

$$H_T = h_1 + h_2 + h_3. \quad (24)$$

These relations can be used to explain qualitatively the velocity regime we would expect in the three-layer system. If upwelling is present then $h_{1x} < 0$ and $(h_1 + h_2)_x < 0$ except when down-warping of the second interface occurs with a strong poleward countercurrent. Also from our solutions we see generally that in absolute magnitude $|h_{1x}| \geq |(h_1 + h_2)_x|$. With this in mind we note in (21) that v_1 should be negative (e.g., equatorward flow). Eq. (23) predicts a poleward flow in the bottom layer since both terms would make a positive contribution to the longshore velocity in the layer. In the second layer [Eq. (22)] we see the terms are of opposite sign; thus, we expect small velocities in this layer. However, a strong density contrast between layers 1 and 2 coupled with a widening of the upwelling zone with depth would predict equatorward flow near the edge of the upwelling zone and poleward flow near the coast. Fig. 2 for the four-layer model indicates this type of behavior in the second layer with equatorward flow at approximately 20 km and near zero velocity at the coast.

b. Flat-shelf case: Wind stress curl at the coast

The model geometry for this case is the same as discussed previously with a flat shelf and layer thicknesses of 25, 25, 50 and 100 m. The problem has been changed, however, by introducing curl in the wind stress in the coastal region (see Fig. 9). The inclusion of the positive curl may more realistically depict the wind stress distribution by parameterizing the frictional effects of the adjacent land. Little actual wind data are available for upwelling regions with which to determine a realistic offshore stress profile. Some data

FIG. 9. Wind stress components as a function of x with curl near the coast. τ_{xy} is shown by solid line and τ_{xz} is zero everywhere.

from the Peruvian coast do indicate a positive curl in the wind stress (Compania Administradora del Guano, 1960).

We will consider two different density profiles in this section but emphasize the effect of the new wind stress distribution.

The solutions (Fig. 10) are for the same $\Delta\sigma_i$ stratification as discussed earlier for the beta model: 2.0, 0.5, 0.5. Although the positive wind stress curl is more conducive to upwelling through Ekman suction, the actual upwelling at the coast is decreased. The decreased upwelling may be explained by the reduction of the one-sided divergence at the coast in accordance with the smaller wind stress at the boundary. The first interface reaches the sea surface 10.5 days after the winds are first applied. This time scale compares with 5.5 days for the case of no wind stress curl. Ekman suction due to the curl at the coast does increase the width of the upwelling zone.

The longshore velocity profiles in Fig. 10 show that the equatorward surface current has been reduced from -35 cm sec^{-1} after 5.5 days in the previous case to -20 cm sec^{-1} after 10 days for the present case. At the same time the poleward undercurrent is found to be much stronger with a positive velocity in the lower layer of 5 cm sec^{-1} . We also note that the flow is poleward in all three lower layers in contrast to the equatorward velocity found in the second layer for the case with no curl. The marked change in the longshore velocities can be explained using modified Sverdrup dynamics. Through a detailed scale analysis Hurlburt and Thompson (1973) show that a Sverdrup balance does not hold in the upwelling zone. Nevertheless, the positive wind stress curl in the upwelling zone does contribute a positive Sverdrup component to the flow. This positive component would thus tend to increase the poleward flow in the lower layers and decrease the equatorward flow near the surface. Fig. 13 illustrates the increased poleward undercurrent due to the positive Sverdrup component.

FIG. 10. Height profiles (left) after 10.0 days model time. Down-warping can be noted in the lower interface at the coast. Velocity profiles (right) also shown after 10.0 days indicate a strong poleward undercurrent. Layer numbers are given in the figure.

A notable feature of the height profiles found in this multi-layer model is the down-warping of the lower interface at the coast. This can be seen as a slight bulging shape to the third interface in Fig. 10. As we mentioned before, the longshore flow is essentially in geostrophic balance with the baroclinic and barotropic east-west pressure gradients. The drop in the third interface is simply the geostrophic adjustment of the height field to the poleward undercurrent. The bulge or convex shape to the interface seems to be characteristic of the shape of isopycnals observed at intermediate depths of upwelling regions. Yoshida (1967) gives some good examples of this structure of the isopycnals as observed off the California coast.

The drop of the interface at the coast reduces the effective depth of the actual upwelling motions. In this way the poleward undercurrent plays a significant role in determining the depth at which upward movement of water can occur.

The transverse circulation as given by the streamfunctions in Figs. 11 and 12 shows basically the same pattern we found for the previous case with no curl. The

main cyclonic circulation is still present although we note again the widening of the upwelling zone. Superimposed on the flow are the effects of several of the propagating internal waves. The amplitudes of the waves are not as large for this case because the wind stress is less at the coast. A further discussion of the internal waves will be found in Section 6.

Using the same wind stress profile (Fig. 9) the model was run with a stratification $\Delta\sigma_t$ of 0.5, 2.0, 0.5. The model solutions are found in Fig. 14. Since the fluid is less stably stratified in the upper two layers than before, we expect higher vertical velocities and a more rapid rise of the first interface. This agrees with our solutions as the first interface surfaces after 7.8 days compared to 10.5 days for the more stably stratified case. This indicates the strong effect stratification exerts on the magnitude of the upwelling motions.

The longshore velocity field also shows a slight change due to the new stratification (compare Figs. 10 and 14). The equatorward surface jet is substantially reduced in the present case and, unlike the previous case, the jet extends into the second layer. If we use geostrophy in

FIG. 11. Streamfunction after 1.0 day (left) and 2.0 days (right). Several internal waves can be seen superimposed on the flow.

FIG. 12. Streamfunction after 3.0 days (left) and 4.0 days (right). The wavelength of the internal waves is observed to decrease as the waves move offshore.

the longshore flow, i.e.,

$$f(v_1 - v_2) = g\epsilon_{12}h_{1x}, \quad (25)$$

$$f(v_2 - v_3) = g\epsilon_{23}(h_1 + h_2)_x, \quad (26)$$

we can qualitatively explain the effect of stratification. Neglecting for the moment change in the slopes of the interfaces due to stratification, we find from (25) that, for a larger density contrast between the first and second layers, we expect a larger vertical shear in the longshore velocities. This concurs with our results for the $\Delta\sigma_t$ stratification of 2.0, 0.5, 0.5 (Fig. 10). Likewise, we would expect a smaller shear for the 0.5, 2.0, 0.5 case (Fig. 14). However, this is not a direct relationship since we note that a change in stratification from 2.0 $\Delta\sigma_t$ to 0.5 $\Delta\sigma_t$ between layers one and two does not increase the shear by a factor of 4. Instead the imposed stratification also determines the width of the upwelling zone thus affecting the x -derivatives of the layer depths. This can be observed in the height profiles of Fig. 10 and Fig. 14 in which we see the slope of the interface is approximately twice as large for the 0.5, 2.0, 0.5 case. If we apply this correction for h_{1x} in (25) in determining the shear for the two different stratifications, we would expect the shear to be larger by a factor of 2 (rather than a factor of 4) for the 2.0, 0.5, 0.5 case. This agrees well with the model velocities shown in Figs. 10 and 14.

c. Oregon coast simulation

The last case we want to consider is a region with topography similar to that found off Newport, Ore. This area is of interest because data collected by Oregon State University over many years can be used to compare with model results. Also a large-scale field experiment was carried out in the summer of 1972 to provide details of the upwelling structure and wind data. The geometry used in the model is shown in Fig. 15. Present model requirements dictate that the topography be confined entirely to the lower layer. Sielecki and Wurtele (1971) have shown that the equations can be

integrated for a case in which an interface intersects the topography. This is accomplished by extrapolating in time to predict the position of an interface as it moves up the topography. While experiments employing this technique have been undertaken, results thus far have been unsatisfactory.

The wind stress used is constant at the coast (e.g., no curl; see Fig. 1). The model solutions for this case may be found in Fig. 16 with only the upper right-hand corner of the model region depicted in the height profiles. Since the upper layer is only 20 m thick, the first interface surfaces after 4.5 days. The longshore velocity is equatorward in all four layers with a strong surface jet. No poleward undercurrent is observed. This is not in agreement with observations.

We can explain why a countercurrent fails to develop in this model. As we mentioned in a previous section, the poleward undercurrent forms in response to a breakdown of geostrophy in the onshore flow near the coast leading to subgeostrophic flow. In the present case we expect the countercurrent to form over the sharply sloping shelf. However, over the sloping shelf the fluid

FIG. 13. Longshore flow field contoured in the x - z plane after 10.0 days for the case with wind stress curl at the coast. Contour interval is 2 cm sec⁻¹. Values >0 are shown by dotted lines.

FIG. 14. Height profiles (left) and velocity profiles (right) after 7.5 days model time. The stratification imposed is 0.5, 2.0, 0.5 and wind has curl at the coast.

in the lowest layer is being accelerated in response to thinning of the layer by the topography. Thus, instead of onshore subgeostrophic flow which yields a countercurrent we have a supergeostrophic flow which inhibits the formation of a countercurrent. This deficiency in the model could be eliminated by permitting the lower interfaces to intersect the topography and would allow a countercurrent to develop to the left of the shelf topography.

6. Internal gravity waves

We have shown that time-dependent phenomena such as inertial oscillations and internal gravity waves are prominent features of the solutions. In this section we want to investigate the production and behavior of the propagating internal gravity waves in the model. The internal and external waves are an integral part of the geostrophic adjustment process in the model. Wind data from Oregon indicate strong diurnal variations in wind stress. These impulses may continually excite large inertial oscillations making the time-dependent features even more prominent in the real ocean than in

the present model. The streamfunctions in Figs. 3, 4, 11 and 12 show that the internal waves are generated at the coast and slowly propagate out to sea. The waves decay rapidly as they move offshore and their apparent wavelength is appreciably shortened. The investigation of these waves is similar to the geostrophic adjustment problem discussed by Rossby (1938), Cahn (1945), and others.

The internal waves are excited and propagate along the first internal interface. Below this interface the waves are barotropic and behave in accordance with continuity requirements. Let us consider the following simple linear model equations for a two-layer system:

$$\frac{\partial u_1}{\partial t} = f v_1 - g(h_1 + h_2)_x, \quad (27)$$

$$\frac{\partial v_1}{\partial t} = -f u_1 + \tau^{sy}/(\rho H_1), \quad (28)$$

$$\frac{\partial u_2}{\partial t} = f v_2 - g(h_1 + h_2)_x + g \epsilon_{12} h_{1x}, \quad (29)$$

$$\frac{\partial v_2}{\partial t} = -f u_2, \quad (30)$$

$$\frac{\partial h_j}{\partial t} = -H_j \frac{\partial u_j}{\partial x}, \quad j=1(1)2, \quad (31)$$

where H_j is the initial layer thickness. If we assume the flow is negligible in the lower layer (this can be justified by allowing $h_1 \ll h_2$ so that return flow supplying the upwelling motions is extremely small), Eq. (29) becomes

$$g(h_1 + h_2)_x = g \epsilon_{12} h_{1x}. \quad (32)$$

It is also convenient to divide the velocity into a constant Ekman drift component and a perturbation component, e.g.,

$$u = \bar{u} + u', \quad \text{where} \quad \bar{u} = \tau^{sy}/(f H_1) = u_E. \quad (33)$$

FIG. 15. Model geometry and bottom topography for the Oregon coast simulation. Layer thicknesses are $h_1=20$ m, $h_2=25$ m, $h_3=25$ m, $h_4=930$ m. The $\Delta\sigma_t$ stratification imposed is 2.0, 0.5, 0.5.

FIG. 16. Height profiles (left) and velocity profiles (right) after 4.5 days for the Oregon coast simulation.

Using (31) and (32), we can rewrite Eqs. (27), (28), (31) as

$$\frac{\partial u'}{\partial t} + g\epsilon_{12}h_x = fv, \quad (34)$$

$$\frac{\partial v}{\partial t} = -fu', \quad (35)$$

$$\frac{\partial h}{\partial t} + H_1 \frac{\partial u'}{\partial t} = 0, \quad (36)$$

(the subscripts having been dropped).

The initial and boundary conditions are:

$$\left. \begin{aligned} u(0,t) &= 0, & u'(x,0) &= 0 \\ \bar{u}(0,t) &= u_E, & v(x,0) &= 0 \\ u'(0,t) &= -\tau^{sy}/(fH_1), & h(x,0) &= 0 \\ u'(-\infty,t) &= 0 \end{aligned} \right\}. \quad (37)$$

If we eliminate variables in (34)–(36) in favor of u' , we obtain

$$u_{tt}' - g\epsilon_{12}H_1 u_{xx}' + f^2 u' = 0, \quad (38)$$

with conditions

$$\left. \begin{aligned} u'(0,t) &= -u_E, & u'(x,0) &= 0 \\ u'(0,t) &= 0, & \frac{\partial u'}{\partial t}(x,0) &= 0 \end{aligned} \right\}.$$

We normalize the coordinates in (38) using

$$\alpha = -x[f/(g\epsilon_{12}H_1)^{1/2}] \quad \text{and} \quad \eta = tf \quad (39)$$

to obtain

$$u_{\eta\eta}' + u' - u_{\alpha\alpha}' = 0. \quad (40)$$

Laplace transform methods allow us to find a solution to (40) in transform space of the form

$$U(\alpha,s) = -\frac{u_E}{s} \exp[-(s^2+1)^{1/2}\alpha], \quad (41)$$

where s is the transform variable. The inverse transform to the above equation can be found in appropriate tables (Chemical Rubber Company, 1964)

$$u'(\alpha,\eta) = u_E \int_{\alpha}^{\eta} \frac{\alpha}{(\eta'^2 - \alpha^2)^{1/2}} J_1[(\eta'^2 - \alpha^2)^{1/2}] d\eta' - u_E, \quad (42)$$

with limits on the transformation

$$\left. \begin{aligned} f(s) &= e^{-\alpha s} - \exp[-\alpha(s^2+1)^{1/2}] \rightarrow F(\eta) = 0, \quad \text{for } 0 < \eta < \alpha \\ f(s) &= \frac{e^{-\alpha s}}{s} \rightarrow F(\eta) = 0, \quad \text{for } 0 < \eta < \alpha \end{aligned} \right\}.$$

We cannot evaluate directly the integral found in (42). However, we can obtain an asymptotic expansion valid for $\eta > 1$ given by

$$u'(\alpha,\eta) = u_E \left\{ 1 - e^{-\alpha} - \frac{\alpha}{\eta} J_0(\eta^2 - \alpha^2) - \frac{\alpha(\eta^2 - \alpha^2)^{1/2}}{\eta^3} J_1[(\eta^2 - \alpha^2)^{1/2}] + \dots \right\} - u_E, \quad (43)$$

where J_0 and J_1 are Bessel functions of the first kind.

To determine the behavior of the solution this expression was evaluated numerically in space and time. The results are shown in Fig. 17.

The solutions behave in a manner similar to those of Cahn (1945) for a slightly different geometry. As in the model solutions the waves have a length of 40–60 km near the coast and the wavelength decreases offshore. If we took several *snapshots* of the solution in time we would find peaks forming near the coast. The peaks move offshore at the internal gravity wave speed $(g\epsilon_{12}H_1)^{1/2}$ while the amplitude slowly decreases. Fig. 17 also gives the solution in time from which we see the wave period is approximately the inertial period. We also note that successive waves passing a point have increasingly smaller amplitudes. The damping is not

FIG. 17. Perturbation velocities in the u field. Solutions (left) are shown as a function of x after 48 hr. The coast is now on the left with several waves visible. The solutions are also shown as a function of t (right) for a point 40 km from the coast. Strong damping of waves can be noted.

due to friction (which we have completely neglected in the problem). Instead, the amplitude of the waves gradually becomes smaller as the energy of the oscillation excited by the impulsive start of the wind is radiated into a wider band of fluid.

The presence of internal gravity waves in the real ocean is well documented (Defant, 1961; Yoshida and Mao, 1957). In an upwelling zone because of the manner in which the waves are damped they may exert some type of cumulative effect. An understanding of the actual effects of the waves would require extensive momentum budget considerations which have not been made to date.

7. Critique

We have attempted to study some details of the vertical structure of the upwelling circulation. Certain points should be noted that have been neglected in our investigation.

A major drawback to the present model is that integration must cease when the upper interface reaches the sea surface. More realistically, we should allow the upper interface to behave as a surface front and investigate its behavior under the surface wind stress. For a circular basin Stuart (1971) has indicated one method which may be helpful in implementing this scheme. A second problem (already mentioned) is the inability of the computer program to allow interfaces to intersect the sloping boundaries. These two problems are related in that both deal with cases in which the initial number of layers is reduced at some point in the model region.

Thermohaline mixing between layers has also been disregarded. We have thus implicitly assumed the time scale for interfacial mixing is greater than that for the vertical transport processes.

All model cases, so far, have dealt with the fluid initially at rest, thus omitting all effects of the general ocean circulation. The geostrophic general circulation could be included by a judicious choice of interfacial and sea surface slopes in the model. Tidal interaction (Mooers, 1970) with the upwelling process should also be investigated.

8. Summary

A four-layer numerical model of a stratified ocean on a β -plane has been used to investigate the vertical structure of the wind-driven upwelling circulation in its evolving stages. Over a flat shelf the return flow supplying the upwelling motions is found to be evenly distributed throughout the lower three layers. The width of the upwelling zone spreads laterally with increasing depth and the strength of the vertical motion is reduced.

In the longshore flow an equatorward surface jet and a poleward undercurrent are the dominant features. The equatorward jet axis tilts offshore with depth. The depth at which the poleward undercurrent forms depends upon the stratification and wind stress imposed. Down-warping of the interfaces is noted in response to the poleward undercurrent. Propagating internal gravity waves excited at the coast by the impulsive start of the wind stress are trapped within 200 km of the coast.

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