"Teddies" and the Origin of the Leeuwin Current

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ABSTRACT

The outflow from the Indonesian seas empties approximately 5–7 Sv of surface warm (and low salinity) Indonesian Throughflow water into the southern Indian Ocean (at roughly 12°S). Using a nonlinear 1½-layer model with a simple geometry consisting of a point source (of anomalous water) situated along a meridional wall on a β plane, the spreading of these waters is examined. An analytical solution is constructed with the aid of the "slowly varying" approach, and process-oriented numerical simulations are performed.

It is found that, immediately after emptying into the ocean, the outflow splits into two branches. One branch carries approximately 13% of the source mass flux and forms a chain of high amplitude anticyclonic eddies (lenses) immediately to the west of the source. These eddies drift westward and penetrate into the interior of the Indian Ocean. The second branch carries the remaining 87% of the mass flux via a coastal southward flowing current. Ultimately, this second branch separates from the coast and turns westward. (A detailed examination of this second branch separation is, nevertheless, beyond the scope of this study.)

It is suggested that the eddies recently observed to the west of the Island of Timor are a result of the above eddy generation process, which is not related to the classical eddy generation process associated with instabilities (i.e., the breakdown of a known steady solution). It is also suggested that this new nonlinear process explains why some of the Indonesian Throughflow water forms the source of the southward flowing coastal Leeuwin Current.

1. Introduction

Outflows situated along the eastern boundaries of the oceans (e.g., the Mediterranean) have a direct effect on the entire ocean into which they debouch. This is due to variation of the Coriolis parameter with latitude (β), which tends to force anomalous water to move toward the west and spread their content. By contrast, the immediate effect of outflows situated along the western boundaries of the oceans (e.g., the Red Sea or the Sea of Okhotsk) is usually confined to regions in the immediate vicinity of the boundary. The Indonesian outflow belongs to the first category and, hence, is expected to have an important effect on the entire Indian Ocean. In contrast to the well-studied Mediterranean outflow, which empties into the ocean at middepth (and, hence, is controlled by bottom topography), the Indonesian outflow debouches into the Indian Ocean at the surface (and, hence, is much less influenced by the bottom topography). Little has been said so far about the dynamics of such surface-trapped outflows on a β plane. We present here the first nonlinear analytical model for this kind of inviscid outflow that empties into the ocean along a meridional wall on a β plane.

a. Observational background

The Indonesian seas are the conduit for low salinity North Pacific water that are forced westward by the sea level difference between the Pacific and Indian Oceans [set up by the trade winds over the Pacific (Wyrtki 1987)]. Most of the highly variable transport measured at the exit passages (Fig. 1) takes place within the upper few hundred meters. The annual average surface trans-
port is roughly 5–7 Sv ($Sv = 10^6 \text{ m}^3 \text{ s}^{-1}$; Meyers et al. 1995) but there is a fairly important seasonal and interannual variability. Most of the throughflow enters the Indian Ocean via two passages (Fig. 1), Lombok Strait (~1 Sv) and the Timor Passage (~4–6 Sv). Recent measurements suggest, however, that Ombai Strait may at times also be an important conduit (Molcard et al. 2001). The eastern part of the Indian Ocean into which the Indonesian seas’ water debouches is not as quiescent as most other eastern basins because it contains the equator (which traps the energy). For a detailed description of the throughflow’s dynamics and the dynamics of the eastern Indian Ocean, the reader is referred to Fieux et al. (1994, 1996); Molcard et al. (1994); Meyers et al. (1995); Meyers (1996); Bray et al. (1997); Gordon and Susanto (1999); Gordon et al. (1999); Sprintall et al. (1999, 2000); and Chong et al. (2000).

Using altimeter data, anticyclonic eddies containing Indonesian Throughflow water (hereafter, referred to as “teddies” in analogy to “meddies,” which are eddies containing Mediterranean water) have been recently identified to the west of Timor at 12°–14°S (Feng and Wijffels 2002). They tend to be intraseasonally forced and are most common during the period July–September when the Indonesian Throughflow reaches a maximum. They propagate westward at 15–19 cm s$^{-1}$, have a radius of 100–150 km, and an orbital speed of 20–60 cm s$^{-1}$; they appear with a periodicity of 40–80 days. On the basis of our analytical model, we shall argue that these eddies are a result of $\beta$, which pushes the outflow westward and breaks it apart.

As far as the region to the south of the teddies is concerned, both drifter data (Quadfasel et al. 1996) and altimetric data (Morrow and Birol 1998; Birol and Morrow 1999) show that the variability off the western Australian coast is unusually large compared to other eastern boundary regions. Besides the aforementioned band of high eddy energy located along 12°–14°S (associated with the teddies), two bands of high variance have been observed farther to the south. The first is around 20°S between 110°E and the west Australian coastline, and the second extends across the entire Indian Ocean at 25°–30°S (Morrow and Birol 1998). The observation of these two bands of high energy is consistent with satellite SST observations of the Leeuwin Current, which show large meanders and westward propagating eddies (both anticyclonic and cyclonic) with time scales of 90 days (Pearce and Griffiths 1991).

b. Modeling background

Both analytical and numerical work has been done regarding the entrance to the Indonesian seas (e.g., Hirst
and Godfrey 1993; Nof 1995, 1996), the Indonesian seas themselves (e.g., Wajisowicz 1993a, 1993b, 1996; Pedlosky et al. 1997; Pratt and Pedlosky 1998) and the Indonesian–Pacific system, which constitutes water surrounding an “island” (e.g., Australia). The interested reader is referred to the special issue of the Journal of Geophysical Research (1996, Vol. 101) devoted to the throughflow.

Virtually nothing has been said so far on the formation of meddies. Furthermore, almost none of the ideas that have been put forward regarding the formation of meddies is applicable to meddies. Conventional wisdom has it that meddies are formed due to abrupt changes in the shape of the boundary (see, e.g., D’Asaro 1988a,b; Pichevin and Nof 1996). It has been suggested that, alternatively, changes in the topography (Cherubin et al. 2000; Stern 1999) or fluctuations in the transport (Nof 1991) may produce meddies. Finally, it has also been proposed that mixing on the continental shelves and a subsequent adjustment (McWilliams 1989) may form meddies. None of these concepts are applicable to meddies as (i) the Indonesian outflow is a surface outflow and, hence, is not topographically controlled and (ii) the observed fluctuations in the transport through the Indonesian seas take place over much a longer period than the meddies are generated. We shall also show that the classical instability processes which lead to the formation of eddies (via the breakdown of a known steady solution) is inapplicable to meddies as the system does not have a steady solution.

The manner in which water of anomalous density empties into an ocean has been of theoretical interest to oceanographers for decades. In particular, various attempts have been made to understand how the anomalous water is distributed once it debouches into the ocean (e.g., Takano 1954, 1955; Defant 1961, chapter 16; Nof 1978a,b; Garvine 1987, 1996, 2001; Chao and Boicourt 1986; O’Donnell 1990; Oey and Mellor 1993; Kourafalou et al. 1996; Yankovsky and Chapman 1997).

In everyday life, a source of anomalous water emptying into a large container tends to spread evenly in all directions. In the ocean, however, the earth’s rotation tends to confine the outflow to the coast (in the Kelvin wave sense) forming an alongshore current. The complications added by rotation do not end here, and recent analytical and numerical studies have shown that such an outflowing current on an $f$ plane can never be steady (see Fig. 2: Pichevin and Nof 1997; Nof and Pichevin 2001). Furthermore, numerical simulations and analytical work demonstrate that the outflow balloons in the sense that a forever-growing eddy is generated near the coast (Pichevin and Nof 1997; Fong 1998; Nof and Pichevin 2001). On this basis, it is expected that eddies would be periodically separated from an outflow on a $\beta$ plane, and here we present the first nonlinear analytical solution for this process. We also present numerical simulations which confirm our analytical calculations.

![Fig. 2. A schematic diagram of the steady configuration shown by Pichevin and Nof (1997) and Nof and Pichevin (2001) to be impossible on an $f$ plane. In this $f$-plane scenario, a steady inviscid outflow cannot exist because the alongshore momentum flux of the downstream current is not balanced. As a result of this impossibility, the eddy grows forever and the downstream current mass flux $q$ is smaller than the incoming mass flux $Q$ (see Fig. 3). We shall see that on a $\beta$ plane (Fig. 3), the eddy detaches periodically from the source because, once it reaches a large enough size, its westward drift exceeds its growth rate. For convenience, we first present the problem as if it were taking place in a northern hemisphere. Later, we will convert it to a southern hemisphere problem.](image)

**c. Present approach**

Consider the situation shown in Figs. 2 and 3. Our approach is to look at the eddy generation process as a slowly varying problem. This is based on the idea that the process involves two timescales, one fast and one slow. The fast timescale [$O(f^{-1})$, where $f$ is the Coriolis parameter] is associated with the time required for a particle to complete a single revolution within the eddy, whereas the slow timescale is the time associated with the gradual growth rate of the eddy and the period required for the generation of a single eddy. The eddy growth rate ($dR/dt$, where $R$ is the eddy’s radius) is slow because, once an eddy is getting established, its radius becomes larger than the downstream current scale so that its volume is also large (see Nof and Pichevin 2001). Furthermore, even if all the mass flux were to go into the eddies and the eddies were to osculate each other as they drift westward, it would still take a period of $O(\beta R)^{-1}$ to form each eddy; that is, the eddy generation periodicity is long compared to $f^{-1}$.

Since the only simple analytical solution for a lenslike eddy on an $f$ plane is the one for a zero potential vorticity...
Fig. 3a. A schematic diagram of the model under study. The “wiggly” arrows indicate the off-wall migration of the eddy $C_x(t)$; this results from both the growth of the eddy that forces itself away from the wall and from $\beta$. The thick dashed line indicates the integration path that will be used. We focus on “long” time in the sense that the process is slowly varying in time. The (immiscible) layer densities are $\rho$ and $(\rho + \Delta \rho)$.

Fig. 3b. The assumed (basic state) structure of the eddy’s edge during the formation process (i.e., prior to detachment). As shown, the basic state is taken to be tangential (at all times up to the detachment time) to the coast. This means that, without the shape distortion, the center would have migrated purely toward the west. It also means that $C_x = -dR/dt$, i.e., that the eddy pushes itself away from the wall in a manner that keeps it in constant contact with the wall until it detaches. This condition (implying that, prior to the detachment, the eddy is osculating the wall at all times) is easily justified because the eddy cannot grow unless it pushes itself away from the wall. It is clearly supported by the numerics.

Fig. 3c. The balance of the long-wall forces acting on the outflow. The along-wall Coriolis force $F_{cy}$ (resulting from the eddy center migration in the off-wall direction $C_x$) balances the jet force associated with the alongshore current, $F_l$, and the $\beta$-induced force, $F_{\beta}$.

Fig. 3d. As in Fig. 3c but for the offshore forces. Here, the Coriolis force $F_{cx}$ (resulting from the alongshore migration $C_y$, which will be later neglected) is pointing toward the wall. It is balanced by two forces: The first is the westward force associated with the source momentum flux $F_0$ [which, in contrast to the $y$ component, does not vanish because the source is not symmetrical with respect to the $x$ axis, i.e., the source velocity, $u$, obeys $v_x(x) \neq v_x(-x)$]. The second is the offshore pressure force associated with the nonzero thickness along the wall $F_w$. In contrast to the longshore balance shown in Fig. 3c (which will be used in our calculation), this offshore balance cannot be used because of the impossibility of calculating $F_w$. It is compensated for by an assumption on $C_y$ (which will be taken to be zero) and is shown here merely for completion.
(PV) eddy,\(^1\) we shall initially limit ourselves to zero PV outflows. We shall later construct analytical solutions for outflows and eddies with (relative) anticyclonic vorticity smaller than \(f\) (corresponding to nonuniform PV). We consider the inviscid shallow-water equations in a coordinates system traveling slowly away from the wall with the eddy’s center (section 2). We then consider the integrated balance of forces along the wall and neglect all terms of high order. After some fairly tedious algebra and some not-so-obvious scaling, we find a very simple analytical solution for the eddy’s increase in size (section 3). It shows that the eddy’s growth corresponds to a balance between the (longwall) momentum flux associated with the downstream current, the \(\beta\)-induced force, and the compensating Coriolis force associated with the migration of the eddy’s center away from the wall.

Using a numerical “reduced gravity” model (of the Bleck and Boudra type) we then show (section 4) that, as the analytical solution predicts, most of the outflow’s mass flux (87\%) goes into the downstream current. Encouraged by this comparison, we then extend our analytical theory to the cases where the eddy’s relative vorticity is smaller than \(f\). The associated numerical simulations are also in reasonable agreement with this solution. They show that, even though the small frictional effects accumulate over time to alter the PV, the inviscid solution is valid at each moment. The same cannot be said, however, with regard to our comparison to the observations because conventional wisdom has it that a transport much smaller than 87\% goes to the downstream Leeuwin Current (section 5). For example, Meyers et al. (1995) found that only \(~1\) Sv of surface flow (from the Indonesian seas) enters the Leeuwin Current. The remaining \(~4\)–\(~6\) Sv enter the South Equatorial Current in the southeast Indian Ocean. Our attempts to attribute the difference to the orientation of the coastline or to the advective currents have all failed miserably. They show that taking those effects into account increases rather than decreases the mass flux going into the coastal current. On this basis we shall argue that the difference is probably due to the tendency of eastern boundary current to separate from the coast due to \(\beta\) (section 5). The results are then summarized and discussed in section 6.

2. Formulation

This section describes the physics of the problem and the mathematical approach. For clarity, the solution is presented in two stages. First, by skipping the stage associated with the establishment of the initial eddy and using the slowly varying process approach, we set all derivatives with respect to time to zero. We then introduce a streamfunction and construct a perturbation scheme where the zeroth-order state is a radially symmetric \(f\)-plane eddy (Fig. 3b). Although we could follow the usual procedure of formally nondimensionalizing all the terms at once and then performing an expansion, it is much easier to retain the terms in dimensional form and examine their relative importance during each stage of the analysis. This is the manner in which the problem is presented below.

The reader is warned in advance that it may be difficult and painful to follow the mathematical analysis in detail. To alleviate some of this difficulty, it is useful to a priori introduce the governing equations that we are after. We are after two conservation relationships. The first is the (straightforward) integrated conservation of mass,

\[
\frac{dV}{dt} = Q - q,
\]

where \(V(t)\) is the eddy volume (slowly varying in time), \(Q\) is the steady outflow mass flux, and \(q\) is the mass flux of the downstream current. (For convenience, all variables are defined both in the text and in the appendix.) The second relationship that we seek is the not-so-simple conservation of long-wall momentum flux (or flow force),

\[
- \int \int_S C_s f h \, dx \, dy = \int_S \int h v^2 \, dx + \beta \int \int_S \psi \, dx \, dy,
\]

where \(\psi\) is a streamfunction, \(S\) is the eddy area, \(C_s\) the (slow) offshore migration of the eddy center (i.e., the point of maximum thickness), \(h\) the thickness, and \(v\) is the downstream current speed. The term on the left is the long-wall Coriolis force created by the (negative) off-wall migration of the eddy’s center (Fig. 3b) resulting from both \(\beta\) and the eddy growth (which forces it to push itself away from the wall). The terms on the right are the momentum flux of the downstream flow (i.e., the force created by the ejection of mass from the control volume) and the slowly increasing \(\beta\) force resulting from the fact that a particle senses a greater Coriolis force on the northern part of the eddy than on the southern part (for a northern hemisphere problem). This balance of forces along the wall is shown in Fig. 3c. Note that the first term on the right (i.e., the term containing the square of the velocity) is a nonlinear term that is not present in the usual quasigeostrophic calculations.

In contrast to the above balance, which plays a crucial role in our calculations, the complementary offshore balance of forces (shown in Fig. 3d) cannot be used because of the impossibility to compute the offshore pressure force; therefore, it will not be dealt with. This implies that \(C_s\), the eddy’s migration in the \(y\) direction, cannot be determined with our approach.

The above equation has two important limits. In the absence of \(\beta\) the equation reduces to a balance between

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\(^1\) Csanady (1979) derived an analytical solution for a finite potential vorticity lens, but the solution is not simple enough to be considered as a part of an involved perturbation expansion such as ours. Furthermore, it is inaccurate near the eddy’s rim (Flierl 1979).
the jet force and the Coriolis force; that is, this is the
forever growing eddy problem discussed by Pichevin
and Nof (1997) and Nof and Pichevin (2001). By
contrast, in the absence of a jet force (i.e., the absence
of a downstream current), the equation reduces to a “free”
balance between the Coriolis and the β-induced force.
This balance corresponds to freely propagating lenses
(Nof 1981). With the above presentation of the main
governing equations, the reader who is primarily inter-
rested in the results can now go directly to the solution
(3.3)–(3.7).

We now begin a detailed derivation of the above gov-
erning equations. First, as already mentioned, we note
that the problem involves a “fast” timescale (i.e., days)
and a “slow” timescale (i.e., weeks, months, or years).
The fast timescale is associated with the (geostrophic)
adjustment timescale and with the relatively short time
that it takes a particle to complete a single revolution
within the zero PV eddy. This fast timescale is also the
time that it takes a particle to get from the eddy to the
downstream current. By contrast, the slow timescale is
associated with β and the slow offshore migration of
the eddy’s center, which implies a large eddy’s radius
(R) compared to the downstream current scale.

The conceptual small parameter of our problem, $\varepsilon$, is
then the ratio between these short and long timescales
[i.e., $\varepsilon = \beta R / f_0$, which is typically $O(0.1)$] or, equiva-
ently, the ratio between the downstream mass flux $q$
(or the incoming mass flux $Q$) and the mass flux cir-
culating within the eddy. To see this more clearly, we
note that the volume of each eddy at any arbitrarily long
time [$([\beta R]^{-1} \gg f^{-1})$ is of the order of $Q/\beta R$ [the outflow
mass flux $Q$ times the eddy generation period ($\beta R$)]
so that the mass flux circulating in it is $O(Qf/\beta R)$ (i.e.,
its volume divided by the orbital timescale, $f^{-1}$). It
follows via the geostrophic mass flux relationship that
the eddy radius $[O(g Q/\beta R) f^{-1} f_0]$ is much greater than
the downstream current width, which is $O(g Q) f^{-1} f_0$.
Namely, if we wait a long enough time ($[\beta R f^{-1}]$)
after the outflow is first “turned on,” then the size of the
eddy is much greater than the downstream current.
It is mentioned here in passing that these scales may not be
relevant to the much smaller surface plumes, which are
dominated by frictional effects.

In what follows we shall consider the detailed conser-
vation of mass and momentum for the problem and ex-
amine the associated scales. We shall neglect all the
time-dependent terms in the (differential) momentum
and continuity equations a priori and, once the solution
is obtained, show that they are indeed small compared
to the smallest terms that were kept. This is the simplest
way to present our analysis as it is a simple matter to
examine the smallness of the neglected terms once the
analytical solution is obtained.

a. Conservation of mass

Assuming (and later verifying with our numerical ex-
periments) that the flow is geostrophic downstream
along the wall we find

$$\frac{d}{dt} \int_S h \, dx \, dy = Q - \frac{g' h_2^2}{2 f_0}, \quad (2.1)$$

where the left-hand side is the eddy volume rate of
change (which is very slow) and the right-hand side is
the difference between the steady incoming mass flux
$Q$ and the outgoing transport of the longshore current
$q$. Here, $h_B$ is the thickness at point B (Fig. 3a), $f_0$ is the
Coriolis parameter at the origin, $g'$ is the familiar “reduced
gravity,” and $t$ is time. As mentioned, since the time
associated with the volume change is long, one im-
mediately sees that the downstream current thickness is
small compared to the thickness of the eddy and that
its width is much smaller than the radius of the eddy.

b. Momentum flux

To examine the momentum-flux balance, we write the
nonlinear momentum equations (multiplied by $h$) and
the continuity equation in a coordinate system moving
with the eddy’s center away from the wall:

$$h \frac{\partial u}{\partial t} + h \frac{\partial C_s}{\partial t} + h u \frac{\partial u}{\partial x} + h u \frac{\partial u}{\partial y} - f (u + C_s) h + \frac{g'}{2} \frac{\partial}{\partial x} (h^2) = 0 \quad (2.2a)$$

$$h \frac{\partial v}{\partial t} + h \frac{\partial C_s}{\partial t} + h u \frac{\partial v}{\partial x} + h v \frac{\partial u}{\partial y} + f (u + C_s) h + \frac{g'}{2} \frac{\partial}{\partial y} (h^2) = 0 \quad (2.2b)$$

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (hv) = 0, \quad (2.3)$$

where, as before, the conventional notation is given in
both the text and in the appendix. Namely, $u$ and $v$ are
the horizontal velocity components (in the moving co-
ordinate system), and $C_s$ and $C_t$ are the time-dependent
migration rates in the $x$ and $y$ directions. Note that (2.2)–
(2.3) were obtained by applying the familiar transforma-
tion

$$y = \hat{y} - \int_0^t C_s(t) \, dt; \quad x = \hat{x} - \int_0^t C_t(t) \, dt; \quad t = \hat{t}$$

[where the variables with carets are associated with the
fixed coordinate system and the absence of a caret de-
notes the variables in the moving system] to the usual
time-dependent equations.

Four comments should be made regarding (2.1)–(2.3).
First, in the moving coordinate system the wall appears
to be slowly moving away from the eddy so that the
wall boundary condition is

$$u = -C_s \quad \text{at} \quad x = x_{wall} - \int_0^t C_s \, dt.$$
Eq. (2.2a) will not be used because it is impossible to calculate the pressure exerted on the outflow by the wall with the method that we shall use. Hence, \( C_i \) will not be computed and (2.2a) is given here merely for completeness.

Third, the condition \( C_i = -dR/dt \) (implying that the eddy’s basic state is osculating the wall at all times) will be used to close the problem. The condition is plausible because the eddy cannot grow unless it pushes itself away from the wall. A very similar condition was used by Nof (1999) and by Nof and Pichevin (2001). We shall see later that it is clearly satisfied by the numerics. Fourth, as in Nof (1988), Pichevin and Nof (1997), Nof and Pichevin (1999), and Nof and Pichevin (2001), we shall neglect the source contribution to the longshore momentum flux. This is done on the ground that, at the source, the source cannot influence the longshore component of the flow because it is small. This is in principle possible because of the momentum equation \((2.2b)\) with the time dependent terms neglected. After the solution is obtained we shall see that this approximation is also decent. As mentioned, inline with the slowly varying approximation, all terms that explicitly include derivatives with respect to time in both the momentum and continuity equations are neglected. After the solution is obtained we shall show that the neglected terms are indeed small.

With these important simplifications, the y-momentum equation \((2.2b)\) with the time dependent terms neglected is now integrated over the area bounded by the thick dashed line shown in Fig. 3a noting that, outside the eddy, \( h = 0 \). Using the approximated continuity equation \([i.e., (2.3)\) with the time dependent terms neglected], one finds

\[
\iint_S \left[ \frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (h\nu^2) \right] dx \, dy
\]

\[
+ \iint_S (f_0 + \beta y)(u + C_i) h \, dx \, dy
\]

\[
+ \frac{g'}{2} \iint_S \frac{\partial}{\partial y} (h^2) \, dx \, dy = 0. \tag{2.4}
\]

Note that, since in our coordinate system the wall is moving slowly away from the eddy, the integration area \( S \) is a (weak) function of time. This movement of the wall has no direct bearing on our eddy momentum calculation because all of the time-dependent terms are ignored.

Next, we define the streamfunction \( \psi \) to be \( \psi = \frac{\partial \psi}{\partial y} = -uh \), \( \frac{\partial \psi}{\partial x} = \psi \) and rewrite (2.4) as

\[
\iint_S \left[ \frac{\partial}{\partial x} (hu) + \frac{\partial}{\partial y} (h\nu^2) \right] dx \, dy
\]

\[
- \iint_S (f_0 + \beta y) \psi \, dx \, dy
\]

\[
+ \iint_S C_i (f_0 + \beta y) h \, dx \, dy
\]

\[
+ \frac{g'}{2} \iint_S \frac{\partial}{\partial y} (h^2) \, dx \, dy = 0. \tag{2.5}
\]

Application of Stokes’ theorem (which, for our problem, is just a special case of Green’s theorem) to (2.5) gives

\[
\oint_\phi huv \, dx - \oint_\phi (h\nu^2 + g'h^2/2 - f\psi) \, dx
\]

\[
+ C_i \iint_S fh \, dx \, dy + \beta \iint_S \psi \, dx \, dy = 0, \tag{2.6}
\]

where \( \psi \) is the boundary of \( S \) (i.e., ABCDA shown in Fig. 3a) and the arrowed circles indicate counterclockwise integration. [Note that in the derivation of (2.6) we also used the condition \( \psi = \partial (y\psi)/\partial y = \partial (y\psi)/\partial y - \psi \) and defined \( \psi \) to be zero along the boundary \((h = 0)\).] With the exception of section \( BC \) (Fig. 3a), at least one of the three variables \( h, u \) and \( v \) vanish along the boundary. Also, with the slowly varying approach, \( \psi \) can be taken to be a constant along the boundary.

In view of these, (2.6) can be written as

\[
- \iint_B \left( h\nu^2 + \frac{g'h^2}{2} - f\psi \right) \, dx + C_i \iint_S fh \, dx \, dy
\]

\[
+ \beta \iint_S \psi \, dx \, dy = 0. \tag{2.7}
\]

Application of Bernoulli to the front \((h = 0)\) implies that the speed along the outer edge of the downstream
current must be approximately equal to the orbital speed along the eddy’s periphery. However, as shown in Nof and Pichevin (2001) and discussed earlier in this document, both the thickness and width of the downstream current are small compared to the thickness and radius of the eddy. Namely, within the downstream current \( v \sim O(g'H)^{1/2} \) (where \( H \) is the eddy’s depth scale) but \( h < H \) and, hence, the last two terms within the first integral of (2.7) are small compared to the first and can be neglected. Note that the last two terms in that first integral would have dropped out of the problem even if these were not small as geostrophy implies that they cancel other out [see, e.g., Pichevin and Nof (1997)]. In view of this, (2.7) can be ultimately written as

\[
- \int h^2 \ dx + C_s \int f h \ dx \ dy + \beta \int \psi \ dx \ dy = 0. \tag{2.8}
\]

Equation (2.8) represents a balance of three longshore forces (Fig. 3c). The first is a long-wall force associated with the downstream current (\( F_\beta \) in Fig. 3c). It is analogous to the force produced by a jet. The second is an integrated Coriolis force resulting from the offshore migration of the eddy’s center (\( F_\psi \), in Fig. 3c). This migration results from both \( \beta \) and the eddy is in constant contact with the wall so that by growing it pushes itself away from the wall. The third is a southward \( \beta \) force resulting from the fact that as a particle circulates in a clockwise manner within the eddy it senses a larger Coriolis force on the north side than it senses on the south side (\( F_y \) in Fig. 3c).

When a similar treatment is given to (2.2a), one finds a balance between three offshore forces (Fig. 3d). Because of the impossibility to compute the offshore pressure force exerted on the outflow by the nonzero thicknesses along the wall (\( F_y \), in Fig. 3d), the balance will not be used and is mentioned here, in passing, merely for completion. Our inability to use (2.2a) implies that we will not be able to compute \( C_y \) with our approach.

3. Solution

To obtain the solution to the problem, we now introduce the perturbation scheme

\[
u = \overline{\nu} + \nu' + \cdots; \ h = \overline{h} + h' + \cdots \tag{3.1}
\]

where the overbars denote association with a radially symmetric \( f \)-plane eddy barely touching the wall (Fig. 3b) and the primes denote distortions introduced by \( \beta \) and the wall. This “small distortion approximation” is analogous to that used in Nof and Pichevin (1999). The introduction of this perturbation scheme implies that our problem is not only slowly varying in time but also that, at any given moment, the eddy’s shape is not very far from the circle shown schematically in Fig. 3b.

It is worth pointing out again at this stage of the presentation that, given our slowly varying approach, our eddy does not have to conserve potential vorticity on the long timescale but must conserve potential vorticity on the short timescale. That is to say, at each moment in time, the eddy’s potential vorticity must be equal to the downstream current potential vorticity. However, frictional effects, which at each given moment are small and negligible, can accumulate over the long time period to become important and alter the PV. We shall return to this important aspect momentarily.

a. Approximated conservation of mass

By substituting (3.1) into (2.1) and keeping only the highest-order terms we find

\[
\frac{d}{dt} \overline{h} \ dx \ dy = Q - q. \tag{3.2a}
\]

Focusing, for the moment, on a zero PV outflow whose PV is not altered during the slow growth process, we get

\[
\frac{16 \pi f^3 R_y^3}{g'} \frac{dR_y}{dt} = Q - q. \tag{3.2b}
\]

In deriving (3.2b) it has been taken into account that the volume of a zero PV eddy is \( 4 \pi f^3 R_y^3 g' \) [because \( h = H(t) - f^2 r^2/8 g' \) where \( H(t) \) is the eddy’s central depth (see, e.g., Nof 1981)]. Here, \( R_y \) is the time-dependent Rossby radius, \( [g'H(t)]^{1/2} f \).

b. Alongshore momentum flux

Substituting (3.1) into (2.8), neglecting products of the perturbations (and keeping in mind that, although within the downstream current the speed is high, the width of the current is small), we get

\[
- \int_0^L h'(\overline{\tau}) \ dx + C_s \int \int f_0 \overline{\tau} \ dx \ dy + \beta \int \overline{\tau} \ dx \ dy = 0, \tag{3.3a}
\]

where \( \overline{S} \) is the area that the basic eddy occupies (Fig. 3b), and \( L \) is the (small) downstream current width. Note that, along the eddy’s edge, the velocity is \( fR/2 \) [where \( R \) is the eddy radius] so that the downstream current speed along the edge is also \( \overline{\tau} = fR/2 \). This is so because \( h = 0 \) along the edge so that the Bernoulli implies that the velocity along the edge of the eddy is the same as the velocity along the edge of the downstream current. Furthermore, since the downstream current is very narrow compared to the eddy, the shear across it can only produce small variations in the velocity and, consequently, our perturbation scheme implies that its velocity should be taken to be uniform across it (i.e., \( \overline{\tau} = fR/2 \) is the uniform speed for the downstream current). It is important to realize that this is also true for the nonzero PV case; that is, the velocity of the downstream
current should be taken to be uniform even in the finite or variable PV case.

In view of this, one of the \( \tau \) constituting the square of the velocity in the first integral of (3.3a) can be taken outside the integral and (3.3a) can be rewritten as

\[
\frac{fRq}{2} + C_s \int \int \frac{fH}{\tau} dx \, dy + \beta \int \int \frac{\psi}{\tau} dx \, dy = 0,
\]

where \( q = \int_t^\infty \frac{fH}{\tau} dx \).

Taking again into account the known velocity and depth profiles of a radially symmetric zero PV eddy, we find that (3.3b) takes the form

\[
\frac{\sqrt{\gamma}}{8\pi} q' q - \sqrt{\gamma} R^2 \frac{dR_f}{dt} + \frac{\beta}{3} \frac{R_f}{f} = 0. \tag{3.4}
\]

Note that the condition \( C_s = -dR/dt \) has also been used. As mentioned, this involves the plausible assumption that the eddy progresses away from the wall at the same rate that its radius is increased; that is, it is in touch with the wall until its detachment, which, as we shall see, occurs when its \( \beta \)-induced westward speed exceeds its growth rate.

c. Solution for a zero potential vorticity outflow

Elimination of \( q \) between (3.2b) and (3.4) gives a single differential equation for the temporal eddy’s Rossby radius \( R_d \),

\[
-3 \sqrt{\gamma} R^2 f \frac{dR_f}{dt} + \frac{\sqrt{\gamma} q' Q}{8\pi} + \frac{\beta R_f}{3 f} = 0. \tag{3.5}
\]

This equation can be solved for all \( t \) but is not expected to be valid during the first part of the eddy generation process because it takes time to establish a decent-sized eddy (required for our slowly varying approach). It is certainly valid, however, during the detachment (because the eddy is well established by that time), which is what we are really after.

In other words, it is a simple matter to find the solution of \( R_d \) as a function of time from (3.5) but this general solution is not very useful as the detachment time is not known a priori nor is it known when is the earliest time that (3.5) is valid. It is much more useful to introduce the detachment condition and then find the solution of (3.5) only for this particular time as this solution will give us all the information that we are looking for. For this reason it is the detachment time that we shall focus on.

The solution obeying the detachment condition stating that the eddy’s westward drift, \( (2/3) \beta R^2 \) (according to Nof 1981), just exceeds its growth rate \( dR/dt \); that is,

\[
2 \sqrt{\gamma} \frac{dR_f}{dt} = \frac{2}{3} \beta R_f^2 \quad \text{at } t = T,
\]

is found from (3.5) to be

\[
R_d = \left( \frac{g' Q 3 \sqrt{\gamma}}{\beta f^2 16\pi} \right)^{1/5},
\]

which can also be expressed as

\[
\frac{R_d}{R_f} = \left[ \frac{3 \sqrt{\gamma}}{16\pi (\beta R_f f)} \right]^{1/5}, \tag{3.6}
\]

where \( R_d \) is the deformation radius. The periodicity \( T \) will be determined shortly.

Note that the logic behind the above detachment condition [i.e., \( (2/3) \beta R_f = dR/dt \)] is as follows. The wall-induced westward speed of the gradually growing eddy \( (C_s) \) equals \( dR/dt \) because the eddy must push itself away from the wall to accommodate its growth. When this westward speed first matches the “free” open ocean \( \beta \)-induced speed \( [(2/3) \beta R_f^2] \), then the eddy is said to be at the moment of detachment because it corresponds to the state where the force of the downstream current is zero.

To compute the fractions of the outflow that go into the eddies and the downstream current, it is assumed here (and later verified with our numerical simulations) that the eddies are not only osculating the wall as they are produced but are also osculating each other as they drift westward. Together with (3.6) this condition gives the ratio of the mass flux going into the eddies, \( Q_e \), to the incoming mass flux, \( Q \), to be

\[
Q_e/Q = 1/8, \tag{3.7}
\]

implying that \( (7/8)Q \) goes into the downstream current. The corresponding periodicity \( T \) is

\[
T = \frac{4 \sqrt{\gamma} R_f}{(2/3) \beta R_f^2} = 6 \sqrt{\gamma} (\beta R_f)^{-1}. \tag{3.8}
\]

It should perhaps be stressed again that the fact that our solution holds only for the detachment time is not really an issue, as all the desired information (e.g., ratio of mass fluxes, eddy size, and the eddy shedding period) can be determined from this solution.

d. Solution for a finite potential vorticity outflow

It is a simple matter to extend these results to eddies whose vorticity is less than \( f \), that is, to those cases where the PV is either nonzero to begin with or is nonzero due to a gradual, slow accumulation of frictional effects. To do so, we take the eddy orbital speed to be

\[
v_o = -\frac{\alpha}{2} f r, \tag{3.9}
\]

where \( \alpha \), the vorticity coefficient is smaller than unity, that is, \( \alpha < 1 \). This velocity distribution corresponds to nonuniform PV, but this does not prevent us from finding a solution because, as mentioned, with our perturbation scheme, the speed of the narrow downstream current is taken to be uniform across all potential vorticities.
The familiar momentum equation gives the eddy volume $V$,

$$V = \alpha(2 - \alpha)\pi f^2 R_f^3/16g',$$

(3.10)

where $R_f$, the (final) eddy radius, is related to the final Rossby radius $R_{ed}$ via

$$R_f = \frac{2\sqrt{2}R_{ed}}{[(2 - \alpha)\alpha]^{1/2}}.$$  

(3.11)

Note that, for $\alpha \ll 1$, (3.11) reduces to

$$R_f = 2R_{ed}/\alpha^{1/2}.  \tag{3.11a}$$

By following the procedure employed earlier for the zero PV case, we find the momentum budget equation for the linear velocity profile to be

$$\frac{\beta f^5}{3(2 - \alpha)^{1/2}}R_f^3 + \frac{g'f\sqrt{2}}{8\pi} Q - \frac{\sqrt{2}}{\alpha^2(2 - \alpha)} R_{ed} \frac{dR_{ed}}{dt} = 0,$$

which, with the drift speed of

$$C = \frac{(2/3)\beta R_f^3}{2 - \alpha},$$

and the detachment condition

$$\frac{dR_{ed}}{dt} = \left(\frac{\alpha}{2 - \alpha}\right)^{1/2} \beta R_f^3 3\sqrt{2},$$

gives the final eddy Rossby radius

$$R_{ed} = \left[\frac{3\alpha^{1/2}(2 - \alpha)^{1/2} g' Q}{8\pi\beta \sqrt{2} f^2}\right]^{1/5}.  \tag{3.12a}$$

Note that, for $\alpha \ll 1$, (3.12a) gives the eddy radius

$$R_f = \left(\frac{24g' Q}{\alpha^2 f^2 \pi \beta}\right)^{1/5}  \tag{3.12b}$$

and

$$R_{ed} = \left(\frac{24g' Q}{\alpha^2 f^2 \pi \beta}\right)^{1/5}.$$  

(3.12b)

Although the radius for $\alpha \ll 1$ is larger than that of the zero PV case ($\alpha = 1$), the eddies are shallower than the zero PV eddies and, consequently, drift westward more slowly. It turns out that the ratio of the mass flux going into the eddy $Q_f$ to the incoming mass flux $Q$ remains the same as before (i.e., it is 1/8). The periodicity is

$$T = \frac{12\sqrt{2}(1 - \alpha/2)(\beta R_{ed})^{-1}}{[(2 - \alpha)\alpha]^{1/2}} = 12\left[\frac{2f^2(2 - \alpha)}{3\alpha^2 \beta^4 g' Q}\right]^{1/5},  \tag{3.13}$$

which, for $\alpha \ll 1$, gives

$$T = 12/(\alpha^{1/2} \beta R_{ed})  \tag{3.13a}$$

e. **Offshore momentum flux**

When a similar integration treatment is given to (2.2a), one finds a balance between three offshore forces (Fig. 3d). It can be written as

$$-\int_a^u h u^2\,dy - \frac{g'f}{2} \int_a^u h^2\,dy + C_f \int_s f h\,dx\,dy = 0.  \tag{3.14}$$

The first term is the (now) nonzero momentum flux of the source ($F_w$ in Fig. 3d). It results from the asymmetry of the source relative to the $x$ axis, which is very different from the previously discussed symmetry relative to the $y$ axis; that is, here, $v$, the source speed obeys $v(x) \neq v(-x)$ whereas, in the alongshore momentum-flux case, it obeys $v(x) = v(-x)$. The second term is the impossible-to-compute offshore pressure force exerted on the outflow by the nonzero thickness along the wall ($F_m$ in Fig. 3d). The third is the Coriolis force resulting from the alongshore migration $C_f$ ($F_m$ in Fig. 3d). Because of the impossibility of calculating the pressure exerted on the outflow by the wall, (3.14) will not be used. It means that our solution will not give any information on $C_f$. Our numerical simulations will show, however, that it is small compared to $C_f$.

4. **Numerical simulations**

To further analyze the validity of our assumptions (e.g., that the flow is parallel to the wall downstream), we shall now present numerical simulations and quantitatively analyze the results.

a. **Numerical model description**

We used the Bleck and Boudra (1986) reduced-gravity isopycnic model with a passive lower layer and employed the Orllanski (1976) second-order radiation condition for the open boundary to the south. We found that this condition is satisfactory because the downstream streamlines were not disturbed when they crossed the boundary. To speed up the experiments (which make our runs more economical) and reduce the effect of friction, we used a magnified value for $\beta$ in most of our experiments. Specifically, we performed two kinds of experiments. The first kind were those with an intense zero potential vorticity outflow, whereas the second group were those with a weak varying potential vorticity. Within each group the results were very similar to each other, and, consequently, we present here only one experiment of each group. Since each run provides numerous data points, we believe that this presentation is adequate. As is typical for these kinds of experiments, our wall was slippery, and we took the vorticity to be zero next to it.

2 Experiments with no-slip walls were also conducted; they are virtually indistinguishable from the free-slip experiments.
The runs that we present were conducted with a (magnified) $\beta$ of $10^{-10}$ m$^{-1}$ s$^{-1}$ and a relatively high resolution corresponding to $\Delta x = \Delta y = 5.4$ km. For numerical stability, we chose an eddy viscosity of 360 m$^2$ s$^{-1}$; the time step was 5 min and the upstream thickness along the right wall was 450 m. These choices are certainly adequate for a Rossby radius of 30 km (corresponding to a $g'$ of $2 \times 10^{-3}$ m s$^{-2}$ and $f_0 = 10^{-4}$ s$^{-1}$). Furthermore, these choices always allowed for at least ten grid points across the downstream current, which is also adequate. Our mass flux was always 20 Sv, and we used a feeding channel instead of a point source. We chose the channel width so that the upper-layer thickness was zero along the left wall; the thickness along the right wall was 450 m. We ran both the zero PV and the finite PV experiment for a long enough time (100 days) so that even the zero PV experiment ultimately had its potential vorticity altered by friction. This enabled us to obtain data for nonzero PV outflow even from the “zero PV” experiment.

b. Results

The results are shown in Figs. 4–8. All show a decent agreement with the theory despite the fact that the error in the perturbation expansion is relatively large [$O(\varepsilon^2)$] where $\varepsilon$, the ratio between the downstream current width and the eddy radius, was roughly 0.4).

Figure 4 shows that, as the theory predicts, a chain of eddies is indeed established. Within the newly formed eddy the flow separates from the wall to the right of the source but this has no bearing on our solution, even in the limit of no viscosity which may involve velocity discontinuities because the equations that are used must still be satisfied.

In some sense, Fig. 5 is the “backbone” of our analytical–numerical comparison as it shows the momentum balance corresponding to (2.8). We see that, although the fluxes vary with time, at each moment the integrated forces balance each other. Namely, Fig. 5 illustrates that there are no unaccounted forces and that, despite that frictional effects accumulate over time to become an important effect, at each moment in time the inviscid balance of forces is an excellent approximation to the problem.

Note that we show here (in Fig. 5) the values only for the period covering the formation of the first eddy as it is difficult to interpret the results for a longer integration period. This difficulty is due to the accumulation of fluid within the integration box [resulting from both downstream meanders (not present in Fig. 4) and the addition of eddies to the box]. Such accumulation also causes the accumulation of momentum flux which masks the periodicity in the long-period results. [Note, however, that the expected periodicity is, of course, clearly apparent in the mass–flux plots (Fig. 7b).]

Figure 6 shows that, on the long-time scale, the PV is (very gradually) altered by the small frictional forces. The alteration in time is slightly larger in the zero PV case (upper panel) than in the finite PV case because the initial velocities and, hence, the initial frictional effects are larger. Since our slowly varying approach merely requires the PV to be conserved at each moment
Fig. 5. A comparison of the three terms in (2.8). In this comparison the solid line is the northward Coriolis force corresponding to the westward migration associated with the second term. The dashed line is the absolute value of the southward (negative) force due to both $\beta$ and the momentum flux (i.e., the first and third terms). Note that the lines are almost the same, in excellent agreement with our analytical derivation and the “slowly varying” assumption [i.e., it verifies (2.8)]. Note that we present here only the results for the first eddy because the interpretation of a longer record is quite difficult (due to the accumulation of fluid in the integration box).

(i.e., on the short-time scale), this small frictional effect which accumulates over the long time to become important does not invalidate the theory.

Figure 7 shows a comparison of the analytical and numerical mass fluxes for the zero and finite PV outflows. The periodicity of the mass fluxes is clearly apparent here, and we see that the agreement of the downstream current mass flux (Fig. 7a) is good in both cases. The agreement of the eddy fluxes (Fig. 7) is not as good but this is to be expected because the (non-dimensional) fluxes are relatively small. As shown in Fig. 8a, the agreement between the analytics and the numerical values of the eddy mean radius is also reasonable. As mentioned, the differences between the numerics and the analytics are primarily due to the “osculating assumption” and the fact that the vorticity is changing in time. In the zero PV experiment ($\alpha = 1$, initially), the numerical radius is not very far from the analytical detachment value corresponding to $\alpha = 0.17$ (Fig. 8a, upper panel). Similarly, in the finite PV experiment, the numerical radius is not far from the analytical value corresponding to $\alpha = 0.13$ (Fig. 8a, lower panel). The same can be said of the eddy migration (Fig. 8b).

The agreement of the periodicities is also reasonable. While the analytics suggest a periodicity of about 103 days for the zero PV flow and 120 days for the finite PV flow [taking again the appropriate $\alpha$ of 0.17 and 0.13 and using (3.13a)], the numerics suggest a periodicity of about 120 days and 230 days, respectively. Note that the “osculating assumption” may introduce a factor of 2 difference between the analytical and numerical values as the actual eddies have space between them (see, e.g., Pichevin et al. 1999). In addition, because the numerical eddies did not have a linear distribution of the orbital velocity (assumed in the analytics) there was also a factor of 2 difference between the analytically predicted propagation rate and the numerical propagation. [This difference disappears when the integrated formula, which does not assume a particular velocity profile (Nof 1981), is used.]

We also ran an experiment (not shown) where we doubled the eddy viscosity. We found that the mass flux of the downstream current decreased. That is, the frictional effect (which causes a decreased downstream current) explains why the theoretical current transport is larger than the actual numerical values (Fig. 4). The results of the second set of experiments (weak, finite PV outflow) are in better agreement with the numerics than the zero potential vorticity case because the velocities are smaller so that the frictional forces are smaller too.

c. Limitations

As is frequently the case, both the analytical and the numerical model have their limitations. The three most important weaknesses of the analytical solution result from the slowly varying assumption, that we did not find the complete first-order solution, and the use of a 1½-layer model. (Note that the latter limitation is also present in the numerics.) We shall take these three issues one by one.

The first assumption eliminates the contribution of the time-dependent terms to the alongshore momentum flux. The assumption has been used successfully before (Nof and Pichevin 2001) and is valid as long as the eddy radius is much larger than the downstream current. The second limitation can be important because the complete
Fig. 6a. The (numerical) nondimensional mean vorticity (averaged over the first eddy) as a function of time. The parameter \( a \) is the relative vorticity nondimensionalized with the Coriolis parameter. Note that, due to the length of our integration and the accumulated effect of friction, the mean vorticity decreases significantly even in the zero PV outflow case (upper panel). The reduction is a bit less dramatic in the finite PV case (lower panel) because the initial velocities are smaller so that the frictional forces are smaller too. Since the difference between the zero PV experiment and the finite PV experiment is not that significant, it follows that the accumulated role of friction is very important. (The finite PV outflow case corresponds to a PV depth of 500 m, i.e., an upstream relative vorticity of 0.1 f near the right bank.) To minimize the effect of the thin eddy edge on our computations, this calculation as well as those shown hereafter were done by averaging the calculated property over the part of the eddy whose nondimensional depth is greater than 0.5.

Fig. 6b. The mean nondimensional potential vorticity \( PV^* = \left( \xi / (f + 1) / (h/Hp) \right) \) as a function of time for the first eddy. Note that the difference between the upper and lower panels is not that significant indicating again that the accumulated effect of friction is important. Recall, however, that the instantaneous role of friction is small and negligible (Fig. 5) because a long time is required for it to become important.

5. Summary and discussion

The results of our theory can be summarized as follows:

1) The inviscid eddy-formation process involves two timescales, a fast-time scale (i.e., the orbital timescale) and a slow-time scale (i.e., the time associated with its growth, the resulting offshore displacement of its center, and its final detachment). Alternatively, the problem can be thought of as involving two speeds, a fast orbital speed (of the fluid within the eddy) and a slow offshore migration of the eddy center.

2) The general analytical solution (3.6)–(3.13) corresponds to a balance between three alongshore forces: the “jet” force resulting from the downstream current, the integrated Coriolis force resulting from the offshore movement of the eddy center, and the \( \beta \)-induced force (Fig. 3).

3) Both intense and weak outflows (with a linear orbital velocity profile) are associated with approximately 13% of the outflow mass flux \( Q \) going into the eddies.
and the remaining 87% going into the downstream current. This is in reasonable agreement with the numerics (Fig. 7).

4) The size of an intense eddy is $2\sqrt{2} \times \left[\frac{(g'Q/Bf^2)}{(3\sqrt{2}/16\pi)}\right]^{1/5}$ and that of a weak eddy is $(24g'Qa\beta^2\pi)\beta^{1/5}$. Similarly, the periodicity of an intense eddy is $6\sqrt{2}\left[\frac{(g'Q/Bf^2)}{(3\sqrt{2}/16\pi)}\right]^{1/5}$, and that of a weak eddy is $12(3\alpha^2g'Q/4f^2\pi)^{1/5}$.

5) The main differences between the numerics and the analytics are due to three aspects: First, although the frictional effects are small, the eddy generation period is long so that they can accumulate over time and become important. This is primarily apparent in the alteration of vorticity (which can be partly accounted for in the analytics). Second, the numerical eddies are not necessarily osculating each other as they move westward (as assumed in the analytics). This may introduce an error of $O(10\%)$. Third, the downstream current is at times subject to frontal instabilities (not present in Fig. 4) which shed fluid westward. This can also introduce an error of $O(10\%)$.

Our results can be applied to the penetration of the Indonesian Throughflow into the Indian Ocean. Taking the incoming mass flux $Q$ to be approximately 5 Sv, $g'$ to be $2 \times 10^{-2}$ m s$^{-2}$, the vorticity coefficient ($\alpha$) to be unity, $f$ to be $0.3 \times 10^{-4}$ s$^{-1}$, and $\beta$ to be $2 \times 10^{-11}$ m s$^{-1}$, we find with the aid of (3.6) that the eddy Rossby radius is about 55 km. [Note that the adopted high vorticity (i.e., $\alpha = 1$) is based on the observed high speeds at relatively small radii.] These correspond to a radius of approximately 150 km and a westward drift speed of about 4 cm s$^{-1}$. The radius seems to be in agreement with the observations but the westward drift is smaller.
than the observed speed (15–19 cm s\(^{-1}\)) probably due to advection or a barotropic component. This is not unusual as most oceanic rings (e.g., Gulf Stream rings, Agulhas rings) drift faster than their theoretical speed. This is usually attributed to advection by the surrounding flow. (It should probably be mentioned here in passing that the only known exceptions to this are Loop Current rings that drift westward at the theoretically predicted speed because there are no significant advective currents in the Gulf of Mexico.) The presence of a barotropic component is also a real possibility as recently conducted deep casts of absolute velocity (from lowered ADCP measurements) suggest that the eddies may indeed have a barotropic component to their circulation (Sprintall et al. 2001).

The above discrepancy between the modeled drift...
speed and the observations raises a potentially important issue, as it could affect the outcome of the detachment condition that we used (i.e., the match between the free $\beta$-induced drift and the eddy growth rate). Taking into account that a westward advection immediately to the west of the oceanic wall [i.e., within a distance of $O(R_w)$] is unlikely to lead us to conclude that the discrepancy only exists in the region situated some distance to the west of the wall. This implies that the results of the models would probably not be affected significantly by it. An additional possibility is that the eddies change their structure as they evolve altering their original lenslike structure to a more convoluted vertical form.

Evidence for the lenslike characteristics of the original eddies comes from a number of recent (and ongoing) observational studies. Altimetric data first confirmed their present near $13^\circ$S. WOCE CTD hydrographic sections directly transected the eddies, clearly sampling coherent lenses of warm, relatively fresh water derived from the Indonesian interior seas. The ongoing frequently repeated XBT section IX1 between Western Australia and Java remains probably our best possibility for eddy-detection (through thermal characterization). The ship has recently been outfitted with an ADCP, and XCTDs are also planned for deployment. An experiment that would enable us to detect the change in eddy propagation speed (in response to vertical structure changes) would require some combination of altimetric and hydrographic data. Since the mesoscale features have a clear sea-surface height signal (see Bray et al. 1997; Morrow and Birol 1998; Feng and Wijffels 2002) their horizontal scale and pathway could be determined from these data. The newly deployed profiling ($T$–$S$) ARGO float data and the XBT/XCTD and ADCP transects would also offer some information about the subsurface structure.

For the earlier mentioned numerical values, the analytically predicted periodicity (3.13) is 90 days, which is not very far from the observed values (40–80 days). In contrast to this agreement, the partition of mass flux between the eddies and the downstream current is not at all in agreement with the observations. According to both the theory and the numerics most of the mass flux should go into the current, whereas the observations suggest that most of it goes into the eddies. Attempts to resolve this issue by examining variations in the positions of the coasts and outflow and the inclinations of advective currents all showed that these are probably not the causes of the discrepancy. We do not really know the reason for the discrepancy but, as shown below, we speculate that it may have something to do with a detachment of the coastal current.

An important question that we left unanswered from a theoretical point of view is what happens to the noneddies branch that hugs the coast and proceeds southward carrying 7/8 of the original outflow mass flux $Q$. Obviously, it cannot continue forever, as $\beta$ tends to force it westward. Ultimately, this branching current must separate from the coast and penetrate into the interior. To see this, consider the situation shown in Fig. 9, presented here from a southern hemisphere perspective. The geostrophic transport of the southward flowing branch $Q_s$ is given by

$$Q_s = \frac{7}{8} Q = \frac{g^2 h_w^2}{2f},$$

where $h_w$ is the near-wall thickness. The Bernoulli integral along the wall can be written as

$$v^2_w/2 + g^2 h_w = B,$$

where $B$ is a constant (which can be determined from the upstream conditions). As one proceeds southward from the source, the absolute value of $f$ increases so that by (5.1), the thickness $h_w$ must increase as well.

Since the Bernoulli integral (5.2) must remain a constant, an increased $h_w$ means a decreased near-wall velocity $v_w$. But there is a limit on how much $v_w$ can decrease because $v_z^2$ can never go below zero. Ultimately, $v_w$ goes to zero along the wall at which point the branching current must separate from the coast and penetrate into the interior. Determination of the location where this separation occurs depends on the initial conditions and the value of $B$ and is left as a subject of further investigation. Note, however, that a boundary current with a very small near-wall speed will separate almost immediately after beginning to flow southward. It is speculated here that it is because of this separation that the discrepancy only exists in the region situated some distance to the west of the wall. This implies that the results of the models would probably not be affected significantly by it.
of the second branch that most of the outflow does not go into the Leeuwin Current but rather turns westward.

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APPENDIX

Variables Defined

| $C_x$ | alongshore migration of the eddy center |
| $C_y$ | offshore migration of the eddy center |
| $f$ | Coriolis parameter |
| $f_{w}$ | along-wall Coriolis force |
| $f_{o}$ | Coriolis parameter at the origin |
| $g$ | reduced gravity |
| $H$ | eddy maximum thickness |
| $h$ | thickness |
| $h_{w}$ | near-wall thickness |
| $L$ | downstream current width |
| $q$ | downstream mass flux |
| $Q$ | incoming mass flux |
| $Q_{o}$ | mass flux going into the eddy |
| $R$ | eddy radius |
| $R_{t}$ | time-dependent Rossby radius |
| $R_{f}$ | deformation radius |
| $R_{f}$ | final eddy radius |
| $S$ | eddy area |
| $t$ | time |
| $v$ | downstream current speed |
| $v_{r}$ | near-wall velocity |
| $V$ | eddy volume |
| $v_{h}$ | mean orbital flow of an eddy |
| $\alpha$ | vorticity coefficient |
| $\beta$ | variation of the Coriolis parameter with latitude |
| $\varepsilon$ | ratio between the downstream current width and the eddy radius |
| $\phi$ | boundary of eddy area $S$ |
| $\psi$ | usual streamfunction ($\partial \psi / \partial y = -uh; \partial \psi / \partial x = vH$) |

REFERENCES


——, 1978b: On geostrophic adjustment in sea straits and wide es-