

## Waves and the Air–Sea Momentum Budget: Implications for Ocean Circulation Modeling

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### ABSTRACT

The influence of waves on the mean flow is derived on a rotating earth in the form of interaction stresses and a mass flux in the averaged momentum balance and mass conservation equations, respectively, using Hasselmann’s formalism and keeping only the vertical component  $f$  of the Coriolis parameter. These stresses, easily computed from a spectral wave model, arise from both spatial gradients in the wave field and the bufferlike role of waves that store a small fraction of the air–sea momentum flux in the initial growth stages (young seas) and restore this momentum to the mean currents, atmosphere, or solid earth when wave energy is dissipated. The practical importance of these wave-induced stresses on the depth-integrated mean circulation is evaluated from wind-wave growth curves and a third-generation spectral wave model. In steady conditions, waves are shown to induce stresses opposed to the wind stress for wave growth stages that may represent up to 10% of the wind stress for short fetches. Assuming simple mean flow responses, wave-induced stresses shall translate into mean sea level variations, which are typically less than 1 mm in the middle of ocean basins but are much larger and significant in shallow areas like continental shelves. The present formulation is consistent with previous studies on wave-driven inertial oscillations and nearshore circulation, cases for which wave effects are known to be much stronger.

### 1. Introduction

Waves have a direct effect on the mean ocean circulation because they are nonlinear and cannot be averaged out of the equations of motion. Even if nonlinearity is generally weak at the scales carrying most of the wave energy, waves can transport or diffuse mass, momentum, and tracers. This is also true of the turbulence at subgrid scales of ocean circulation models (OCMs) that is generally parameterized in these models. Nevertheless waves have a behavior that is quite different from turbulence. In particular, they can radiate energy over very long distances at speeds,  $O(10 \text{ m s}^{-1})$ , much greater than those of mean currents. Wave effects therefore require a specific parameterization that is, to our knowledge, absent from all OCMs, except for recent attempts by Perrie et al. (2003). Furthermore, operational ocean circulation models rely heavily on satellite

altimetry to represent the ocean mesoscale accurately. This measurement of the sea surface height is closely related to depth-integrated ocean currents via the quasisgeostrophic equilibrium, but it is also contaminated by a still poorly understood bias, mostly associated with the local sea state. This “sea state bias” makes the sea surface height appear lower in the presence of waves by about 2%–4% of the significant wave height and is currently corrected using, at best, empirical algorithms (see, e.g., Gaspar et al. 1994; Chapron et al. 2001; Vandemark et al. 2002). This bias is thought to be related to the nonlinear wave geometry leading to a preferential sampling of wave troughs that are flatter and thus brighter to the altimeter than the wave crests. Known theory for this effect (Elfouhaily et al. 2001) still fail to account for the observed bias magnitude, and it is thus possible that a true change in the mean sea level is also caused by waves. Thus wave nonlinearities can have an impact on the ocean circulation itself and also on the interpretation of that circulation from remote sensing.

Leaving aside diffusion effects (for a discussion of

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these, see Herterich and Hasselmann 1982, Balk 2002) and indirect known effects such as the dependency of the wind stress on the wave age and swell (see, e.g., Janssen 1991; Donelan 1998; Drennan et al. 1999), the aim of the present paper is to provide a general parameterization of deterministic and direct wave effects in the equations of motion of the mean flow, uniformly valid from the surf zone to the global ocean, directly applicable using today's ocean circulation and wave numerical models. For our purpose, the relative importance of these processes is evaluated using a wave generation and propagation model. A more detailed investigation of the impact of waves on the mean flow, with the actual implementation of wave parameterizations in an OCM, is beyond the scope of the present paper and will be described elsewhere.

In and around the surf zone it is already clear from the many observations and theoretical work that interactions between waves and the mean flow, usually expressed as radiation stresses and Stokes drift, are the main forcing mechanisms for the mean ocean circulation (Longuet-Higgins and Stewart 1962; Longuet-Higgins 1970). On the continental shelf this question has already been addressed more or less directly by many authors, with mixed results. Among these, Lentz et al. (1999) provide unambiguous evidence of a significant contribution of waves to the mean flow momentum on the inner shelf, through cross-shore gradients in the radiation stresses that they computed to be comparable to the Coriolis force caused by alongshore currents.

In the deep ocean, the influence of waves on the mean circulation has also been widely investigated in terms of wave-driven flow or drift. Reviews were given by Huang (1979), Jenkins (1987, 1989), Xu and Bowen (1994), and more recently by Weber (2001). In short, the Lagrangian velocity known as Stokes drift cannot be applied to steady conditions in the deep ocean because the conservation of the absolute circulation forbids that a steady wave field produce a Lagrangian mean current (Ursell 1950). Hasselmann (1970) showed that this absence of Lagrangian mean velocity could be explained by the presence, in addition to the Lagrangian Stokes drift, of an Eulerian return flow (opposed to the Stokes drift), giving a Coriolis force in balance with a mean wave-induced stress. This stress, which we shall call the Hasselmann stress, is due to a small wave velocity component orthogonal to the wave propagation direction (hereafter called transversal component) that is driven by the Coriolis force applied on the wave field. Therefore, if only the Hasselmann stress and the Coriolis force are nonzero, the net velocity induced by the waves is zero. If vertical mixing is involved (see, e.g., Xu and Bowen 1994), the net velocity is not zero anymore. However, the depth-integrated mass transport remains null as the surface drift current shall be canceled by a return flow distributed over a larger depth. Observations confirming this theory are still missing (Gnanadesikan and Weller 1995), probably because of the weakness of

this return flow. Consequently, the coexistence of other forces is needed to yield a nonzero mean wave-induced mass transport.

Currents, and in particular inertial oscillations, can be forced by unsteady wave conditions (Hasselmann 1970). The Stokes drift may be significant also in areas where geostrophic equilibrium is not possible, in a narrow bay or on the shelf where friction upsets the (quasi-) geostrophic equilibrium.

Weber (1983) and Xu and Bowen (1994) further investigated the effect of viscosity that drives a vertically sheared flow. Since viscosity causes a dissipation of wave energy, this flow can be seen as a consequence of the transfer of wave momentum to the mean current, which we shall describe in section 2.

Waves have also been related to enhanced mixing at the ocean surface, both by turbulence injected down to about 15% of the wave height (Craig 1996; Donelan 1998) and Langmuir circulations, probably caused by the stretching of vorticity due to the Stokes drift vertical shear (Craig and Leibovich 1976; Leibovich 1983), that penetrate much deeper than the region where the wave motion is felt. These mixing effects are not considered here, as we focus on depth-integrated equations. However, the "vortex force" concept, used by Craik and Leibovich to explain Langmuir circulations, has effects relevant to the depth-integrated motion. This concept is the basis of McWilliams and Restrepo (1999) study of the impact of waves on the global ocean circulation. Separating the rotational transversal wave motion from the irrotational wave motion (the one on a nonrotating earth), they obtain an expression for the quasigeostrophic Eulerian surface mass flux [their Eq. (51) with inertial motions filtered out] in which an Eulerian current opposite to the Stokes drift is forced by the wave motion. For illustration purposes, these authors assume that the wave field is everywhere in equilibrium with the wind, taking the shape of a Pierson-Moskowitz (1964) spectrum. They estimate that at mid latitudes the Stokes drift can be as large as 40% of the Ekman transport. But, as they noted, this drift is largely cancelled by the wave-driven Eulerian flow and thus may have no impact on the transport of tracers or other quantities on a large scale, except for significant differences in the vertical profiles of the Eulerian and Lagrangian flows (see, e.g., Jenkins 1987; Xu and Bowen 1994).

Indeed, even if the Stokes layer is generally shallower than the mixed layer, the wave-driven Eulerian flow is easily redistributed over a large depth, leading to a net drift at the surface in the direction of wave propagation. Perrie et al. (2003) computed net wave effects at the surface in the Labrador Sea that are up to 40% of the Ekman transport. The McWilliams and Restrepo estimate of the Stokes drift is also relevant for inertial motions forced by variations in time of the wave field (Hasselmann 1970). Hasselmann showed that observed inertial oscillations in the Baltic Sea had the same order of magnitude as the oscillations that could be expected

from wave-induced forcing. The contribution of waves to inertial or other ageostrophic motions at mid latitudes should thus be of the same order as the wind contribution. McWilliams and Restrepo (1999) also evaluated mean surface wave-induced pressure, which may be hastily interpreted as a hydrostatic “mean sea level correction” (the title of their Figs. 3.a and 3.b) of the order of 10 cm. Unfortunately they do not comment, in their analysis, on another term that is generally of the same order (the wave contribution in their term  $\mathbf{R}_h$ ). As already mentioned, such a mean sea level change, if established, should be part of the sea-state-dependent biases for altimeter range measurements, with implications for the interpretation of satellite altimetry.

In order to provide an independent investigation of these results, we chose to clearly separate waves from the mean flow, following Hasselmann’s (1971) formalism. We only add viscosity and the Coriolis force in his derivation, keeping the results general enough so that the velocity and pressure fluctuations contain both waves and turbulence. Including viscosity allows a consistent account of stresses at the surface and bottom. It is also indicative of how turbulence can be represented using an eddy viscosity parameterization. Relating the momentum and mass equations to the wave energy or action balance equations used in current operational wave models, our approach provides a practical way of introducing wave effects in ocean circulation models. We originally took this approach without knowing about the pioneering work of Jenkins (1989) that shares the same practical aspects with some differences in the results.

Equations for waves and the mean flow are derived and commented in section 2; a detailed comparison with the Lagrangian view of Weber and Melsom (1993), Craik and Leibovich (1976), and Jenkins (1989) is attempted in section 3. We then apply our equations to fetch limited wave conditions in section 4. Conclusions and perspectives for further applications are given in section 5.

## 2. Averaged momentum equations on a rotating earth

### a. General formalism

Following Hasselmann’s (1971) notation we use dummy Greek indices  $\alpha$  and  $\beta$  for the horizontal components  $x$  and  $y$ , denoted with indices 1 and 2. Indices  $i$  and  $j$  refer to Eulerian coordinates  $x$ ,  $y$ , and  $z$ , denoted with indices 1, 2, and 3. Nevertheless, our notation of momentum and stress differs from Hasselmann’s since we prefer to use dynamic rather than kinematic quantities. Kinematic quantities do not include the densities  $\rho_a$  and  $\rho_w$  of air and water and pose continuity problems at the free surface for example. We also warn the reader that we, like Hasselmann, take the variable  $p$  to be the pressure minus the hydrostatic equilibrium pressure,  $-gx_3$ ,

which allows the equations to be similar in the vertical and horizontal. Means are generally understood as averages over flow realizations, which, for random phase processes such as gravity waves is equivalent to a running average in time over several wave periods.

The mean horizontal momentum  $\overline{\mathbf{M}}$  is separated into a mean flow and a wave part,

$$\overline{\mathbf{M}} = \mathbf{M}^m + \mathbf{M}^w, \tag{1}$$

with

$$M_\alpha^m = \overline{\int_{-h}^{\zeta} \rho_w u_\alpha dz}, \quad \text{and} \tag{2}$$

$$M_\alpha^w = \overline{\int_{\zeta}^{\zeta} \rho_w u_\alpha dz}, \tag{3}$$

where  $\zeta(x, y)$  is the position of the free surface,  $(u_x, u_y)$  is the horizontal velocity vector and the overbar denotes an average over several wave periods and wavelengths. Hereinafter velocity fluctuations are treated as wave velocity only. Turbulence can be included in the present derivation by further separating these fluctuation into waves and turbulence, or representing it by an eddy viscosity in the mean flow equations.

On a rotating earth, the Coriolis force enters the wave momentum balance. Keeping only the vertical component  $f$  of the Coriolis parameter, the average of this force reduces to the vector product of  $\mathbf{f} = (0, 0, f)$ , oriented vertically, and the mass flux in the “surface layer,” between  $\zeta$  and  $\zeta$ . This surface layer mass flux is sometimes called the Stokes mass transport,  $\mathbf{M}^{st}$ , and is equal, by definition, to  $\mathbf{M}^w$ . Therefore this Coriolis force is

$$-\mathbf{f} \times \mathbf{M}^{st} = -\mathbf{f} \times \mathbf{M}^w. \tag{4}$$

Now, the Coriolis force also comes into the instantaneous wave momentum balance, imposing a (rotational) transversal component  $(u'_1, u'_2)$ , that is, perpendicular to the wave propagation direction on top of the usual (irrotational) wave velocity  $(u_1^w, u_2^w, u_3^w)$ . This transversal component  $\mathbf{u}'$  is an order  $f/\omega$  smaller than  $\mathbf{u}^w$  (see Xu and Bowen 1994), where  $\omega$  is the wave radian frequency. But more important, this component is in phase with the vertical velocity  $u_3^w$ .

The mean product  $(\langle u'_1 u'_3 \rangle, \langle u'_2 u'_3 \rangle)$  of horizontal and vertical velocity fluctuations (including wave motion) has the extra term  $(\langle u'_1 u_3^w \rangle, \langle u'_2 u_3^w \rangle)$ , which is equal to  $\mathbf{f} \times \mathbf{M}^w$  at  $z = 0$  (see Xu and Bowen 1994). For simplicity, we chose to remove this particular correlation term, which we call “Hasselmann stress”  $\mathbf{T}^H$ , from the stress  $\hat{T}^{int}$  defined by Hasselmann (1971). Thus the latter stress in a nonrotating frame, adding the surface viscous stresses, is mathematically equal to our interaction stress in a rotating frame, namely,

$$\hat{T}_\alpha^{\text{int}} = \left( \mu_w \frac{\partial \bar{u}_1}{\partial x_3} + T_{\alpha 3}^{\text{int}} - \frac{\partial \bar{\zeta}}{\partial x_\beta} T_{\alpha\beta}^{\text{int}} - \frac{\partial M_\beta^w}{\partial x_\beta} \bar{u}_\alpha \right)_{z=\bar{z}} - T_\alpha^H, \quad (5)$$

with

$$T_{ij}^{\text{int}} = \rho_w (\overline{u_3'^2} \delta_{ij} - \overline{u_i' u_j'}). \quad (6)$$

The Hasselmann stress  $\mathbf{T}^H$  is identical to the part of Craik and Leibovich (1976) vortex force that is due to planetary vorticity. Other effects on waves of the Coriolis force and the earth sphericity were derived by Backus (1962). They are a very weak (about  $10^{-6}$ ) relative change in the group speed and a small deviation of the wavenumber vector and propagation directions (on the order of  $10^{-3}$  to  $10^{-2}$  radians), both of which can be neglected here.

The wave momentum evolution equation in a non-rotating frame is given by Hasselmann [1971, Eq. (18)] and is not modified in a rotating frame because the depth-integrated Coriolis force acting on the surface layer (4) is equal to  $-\mathbf{f} \times \mathbf{M}^w$  and thus cancels the Hasselmann stress that we have removed from our definition of  $\hat{\mathbf{T}}^{\text{int}}$ , yielding

$$\frac{\partial \mathbf{M}^w}{\partial t} - \nabla \cdot \boldsymbol{\tau}^{\text{sl}} = \mathbf{T}^a - \hat{\mathbf{T}}^{\text{int}}, \quad (7)$$

with

$$\boldsymbol{\tau}^{\text{sl}} = - \overline{\int_{\bar{z}}^{\zeta} \delta_{\alpha\beta} p + \rho_w u_\alpha u_\beta dx_3} \quad (8)$$

and  $\mathbf{T}^a$  the usual wind stress, equal to the total atmosphere to ocean momentum flux [see Hasselmann (1971) and our appendix]. Although  $\mathbf{T}^a$  depends crucially on the sea state, we assume that it is a known forcing that may, in practice, come from a coupled ocean wave-atmosphere model such as the Integrated Forecasting System, including the wave model WAM-Cycle 4, which is operational at the European Centre for Medium-Range Weather Forecasts (ECMWF).

*b. Parameterization in terms of wave spectra and source terms*

Wave momentum is closely related to wave energy or action, which are the quantities used in operational wave models, based on a phase-averaged spectral energy balance (these models will be referred to as ‘‘WAM-type models’’). For weak currents, with velocities  $U$  much smaller than the phase speed, the energy balance equation (Gelci et al. 1957; see also the WAMDI Group 1988), used in WAM-type models, can be used instead of the action balance equation. Using a quasi-linear description of the wave field,  $\boldsymbol{\tau}^{\text{sl}} = E/2$ , and (7) can be rewritten from the energy balance equation for deep water waves,

$$\frac{\partial F}{\partial t} + \mathbf{C}_g \cdot \nabla_x F + \mathbf{C}_k \cdot \nabla_k F = S_{\text{in}} + S_{\text{ds}} + S_{\text{nl}}, \quad (9)$$

where  $\nabla_x$  and  $\nabla_k$  denote the horizontal gradients in the physical and spectral space, respectively;  $\mathbf{C}_g$  and  $\mathbf{C}_k$  are the wave energy advection vector velocities in physical space (the group speed) and wavenumber space; and  $F$  is the wave energy spectral density so that the wave energy  $E$  (in joules per square meter) is

$$E(\mathbf{x}, t) = \rho_w g \int F(\mathbf{k}, \mathbf{x}, t) d\mathbf{k}, \quad (10)$$

with  $\mathbf{x}$  and  $\mathbf{k}$  the horizontal position and wavenumber vectors. With this definition the mean wave momentum is

$$\mathbf{M}^w = \rho_w g \int \mathbf{k} F / (kC) d\mathbf{k}, \quad (11)$$

where  $C$  is the wave phase speed, a function of water depth and wavenumber magnitude  $k$ .

The source terms  $S_{\text{in}}$ ,  $S_{\text{ds}}$ , and  $S_{\text{nl}}$  parameterize the transfer of energy from the atmosphere to the wave field, the dissipation of wave energy due to surface processes (water and air viscosity, whitecapping), and the redistribution of energy between wave components due to the wave nonlinearity, respectively. Essentially, (7) can be derived from the integration of (9) divided by the phase speed  $C$  over the spectral variable  $\mathbf{k}$ . This gives

$$T_\alpha^a - \hat{T}_\alpha^{\text{int}} = \int (S_{\text{in}} + S_{\text{ds}} + S_{\text{nl}}) \frac{k_\alpha}{kC} - \frac{C_{k,\beta}}{C} \left( \frac{\partial F}{\partial k_\beta} \right) d\mathbf{k} + \frac{\partial}{\partial x_\beta} \left[ E \left( \frac{1}{2} - \frac{Cgk_\alpha k_\beta}{Ck^2} \right) \right]. \quad (12)$$

By definition  $\mathbf{T}^a$  is the total wind stress (including the momentum flux from swell to the atmosphere). Accordingly the momentum lost by the waves  $\rho_w g \int S_{\text{ds}} / C d\mathbf{k}$  is entirely given to the mean ocean flow and not the atmosphere. If the integration is performed over the entire wavenumber range, from zero to infinity,  $S_{\text{nl}}$  can be left out. In practice, however, wave models have a finite frequency range so that  $S_{\text{nl}}$  must be kept in (12).

For unidirectional waves in deep water and without refraction,  $\mathbf{T}^a - \hat{\mathbf{T}}^{\text{int}}$  is more simply the sum of the source terms  $\int (S_{\text{in}} + S_{\text{ds}} + S_{\text{nl}}) \mathbf{k} / kC d\mathbf{k}$ . In shallow water, depth-induced refraction will contribute to a momentum flux to the bottom, and not the mean flow. Therefore the bottom stress  $\mathbf{T}^b$  must also be considered. Processes specific to shallow water can then be represented by additional source terms.

This quasi-linear description of the wave field, used in (9), only accounts for the energy of the freely propagating wave modes that generally carry most of the energy. One should not forget that all sorts of additional bound modes are necessary to satisfy exactly the bottom and surface boundary conditions (e.g., Stokes harmonics), and the evolution of the wave field given by the source terms is impossible without these modes. We assume here that their momenta and energies are neg-

ligible compared to those of the freely propagating modes.

*c. Horizontal momentum equations*

The averaged horizontal momentum equations are

$$\begin{aligned} \rho_w \left[ \frac{\partial \bar{u}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_1 \bar{u}_j) - f \bar{u}_2 \right] + \frac{\partial p^m}{\partial x_1} \\ = \mu_w \Delta \bar{u}_1 + \frac{\partial}{\partial x_j} T_{1j}^{\text{int}}, \quad -h \leq x_3 \leq \bar{\zeta}, \end{aligned} \quad (13)$$

and

$$\begin{aligned} \rho_w \left[ \frac{\partial \bar{u}_2}{\partial t} + \frac{\partial}{\partial x_j} (\bar{u}_2 \bar{u}_j) + f \bar{u}_1 \right] + \frac{\partial p^m}{\partial x_1} \\ = \mu_w \Delta \bar{u}_2 + \frac{\partial}{\partial x_j} T_{2j}^{\text{int}}, \quad -h \leq x_3 \leq \bar{\zeta}, \end{aligned} \quad (14)$$

with  $\mu_w$  the water dynamic viscosity.

The impact of waves in the horizontal mean flow momentum  $\mathbf{M}_\alpha^m$  is given by Hasselmann's (1971) Eq. (14), correcting for the apparent omission of the hydrostatic pressure in the bottom mean pressure term and a typographic mistake ( $\bar{p}^a \partial \bar{\zeta} / \partial x_3$ ; see our appendix for a derivation). These equations were also derived by Kudryavtsev (1994) for the coupling of surface wind waves with internal waves. A discussion of all the terms derived by Hasselmann can be found in his paper. We will here emphasize the practical parameterization of these terms.

By vertical integration of (13)–(14) we get (see the appendix),

$$\begin{aligned} \frac{\partial M_1^m}{\partial t} = & \left[ \frac{\partial \tau_{1\beta}^m}{\partial x_\beta} + f M_2^m + \bar{p}^a \frac{\partial \bar{\zeta}}{\partial x_1} \right. \\ & \left. + (p^m + gh)_{-h} \frac{\partial h}{\partial x_1} + T_1^a - T_1^b \right] \\ & + p_{-h}^w \frac{\partial h}{\partial x_1} + f M_2^w + \frac{\partial \tau_{1\beta}^{\text{int}}}{\partial x_\beta} + (\hat{T}_1^{\text{int}} - T_1^a) \end{aligned} \quad (15)$$

and

$$\begin{aligned} \frac{\partial M_2^m}{\partial t} = & \left[ \frac{\partial \tau_{2\beta}^m}{\partial x_\beta} - f M_1^m + \bar{p}^a \frac{\partial \bar{\zeta}}{\partial x_2} \right. \\ & \left. + (p^m + gh)_{-h} \frac{\partial h}{\partial x_2} + T_2^a - T_2^b \right] \\ & + p_{-h}^w \frac{\partial h}{\partial x_2} - f M_1^w + \frac{\partial \tau_{2\beta}^{\text{int}}}{\partial x_\beta} + (\hat{T}_2^{\text{int}} - T_2^a), \end{aligned} \quad (16)$$

where  $\tau^m$  is a horizontal tensor that contains mean momentum advection terms and mean-flow pressure gradients (including hydrostatic pressure) and viscous stresses,

$$\begin{aligned} \tau_{\alpha\beta}^m = & - \int_{-h}^{\bar{\zeta}} \rho_w (\bar{u}_\alpha \bar{u}_\beta) + \delta_{\alpha\beta} (p^m - \rho_w g z) \\ & + \mu_w \frac{\partial^2 \bar{u}_1}{\partial x_\beta \partial x_\beta} dx_3, \end{aligned} \quad (17)$$

$\mathbf{T}^a$  is the usual wind stress vector, and  $\mathbf{T}^b$  is the bottom stress vector, each equal to the total atmosphere to ocean and bottom to ocean momentum fluxes per unit horizontal surface. The last four terms in (15) and (16) represent the wave effects on the mean flow that are not represented in current ocean circulation models;  $p_{-h}^w (\partial h / \partial x_\alpha)$  can be neglected in deep water and will not be considered here. In steady quasigeostrophic conditions, the divergence of the Hasselmann stress  $\partial \mathbf{T}^H / \partial x_3 = -\mathbf{f} \times \mathbf{M}^w$  will drive a mean Eulerian transport that will exactly balance the Stokes drift giving a zero Lagrangian wave-induced transport. In other conditions, such as variations in time of the wave field, the Lagrangian wave-induced transport may not be balanced and waves may drive net mass transports (Hasselmann 1970).

In the terms before the last one in (15) and (16),

$$\tau_{\alpha\beta}^{\text{int}} = \int_{-h}^{\bar{\zeta}} T_{\alpha\beta}^{\text{int}} dz \quad (18)$$

is given by Hasselmann's (1971) Eqs. (16)–(17a). Using his sign convention and now using a linear wave approximation, one obtains (see the appendix)

$$\tau_{\alpha\beta}^{\text{int}} = \rho_w g \int_{\mathbf{k}} F(\mathbf{k}) \left[ \left( 1 - \frac{C_g}{C} \right) \delta_{\alpha\beta} - \frac{C_g}{C} \frac{k_\alpha k_\beta}{k^2} \right] d\mathbf{k}, \quad (19)$$

which gives, for monochromatic waves,

$$\tau_{\alpha\beta}^{\text{int}} = E \left[ \left( 1 - \frac{C_g}{C} \right) \delta_{\alpha\beta} - \frac{C_g}{C} \frac{k_\alpha k_\beta}{k^2} \right]. \quad (20)$$

This interaction stress is the sum of the nonisotropic wave momentum advected by the waves,  $-E(C_g/C)k_\alpha k_\beta/k^2$ , acting in the wave propagation direction, and the isotropic wave-added depth-integrated pressure term,  $E(1 - C_g/C)\delta_{\alpha\beta}$ .

The mean pressure added by the waves at the surface,

$$\bar{p}^w = -\rho_w \overline{\left( \frac{\partial \zeta^w}{\partial t} \right)^2}, \quad (21)$$

as given by McWilliams and Restrepo (1999) using linear wave theory, is included in this pressure part with the correct depth-integration. As derived here, there is no reason why this pressure would drive a hydrostatic change in mean sea level given by  $p^w/(\rho_w g)$ , which was suggested by the title of Fig. 3 in McWilliams and Restrepo (1999), because it is not hydrostatic and therefore must be integrated over depth to give the right stress that may or may not contribute to changes in the mean sea level. Besides, this term can be partially canceled by their other term  $\mathbf{R}_h$ .

Last, the last term  $\hat{\mathbf{T}}^{\text{int}} - \mathbf{T}^a$  represents, essentially, the fraction of the wind stress that is gained (or lost) by the mean flow when it is released by (or stored in) the wave field. This term also can be computed from the source terms of a WAM-type wave model using the rather complex expression (12). A more practical set of equations is

$$\frac{\partial M_1^m}{\partial t} = \left[ \frac{\partial \tau_{1\beta}^m}{\partial x_\beta} + fM_2^m + \frac{p^a}{\rho} \frac{\partial \bar{\zeta}}{\partial x_1} + (p^m + gh)_{-h} \frac{\partial h}{\partial x_1} + T_1^a - T_1^b \right] + p_{-h}^w \frac{\partial h}{\partial x_1} + fM_2^w + \frac{\partial \tau_{1\beta}^{\text{rad}}}{\partial x_\beta} - \frac{\partial M_1^w}{\partial t}, \quad (22)$$

and

$$\frac{\partial M_2^m}{\partial t} = \left[ \frac{\partial \tau_{2\beta}^m}{\partial x_\beta} - fM_1^m + \frac{p^a}{\rho} \frac{\partial \bar{\zeta}}{\partial x_2} + (p^m + gh)_{-h} \frac{\partial h}{\partial x_2} + T_2^a - T_2^b \right] + p_{-h}^w \frac{\partial h}{\partial x_2} - fM_1^w + \frac{\partial \tau_{2\beta}^{\text{rad}}}{\partial x_\beta} - \frac{\partial M_2^w}{\partial t} \quad (23)$$

in which the rate of change of the wave momentum is simply subtracted from the total momentum rate of change, and the radiation stresses take their usual form, neglecting the mean current velocity relative to the phase speed of the waves (Phillips 1977):

$$\tau_{\alpha\beta}^{\text{rad}} = \rho_w g \int F(\mathbf{k}) \left[ \left( \frac{1}{2} - \frac{C_g}{C} \right) \delta_{\alpha\beta} - \frac{C_g}{C} \frac{k_\alpha k_\beta}{k^2} \right] d\mathbf{k}. \quad (24)$$

#### d. Mass conservation

The vertically integrated mass conservation equation is also given by Hasselmann's (1971) Eq. (21), and states that for general wave fields that are not uniform in space, the divergence of the Stokes transport  $\mathbf{M}^w$  must be taken into account:

$$\rho_w \frac{\partial \bar{\zeta}}{\partial t} + \frac{\partial M_\alpha^m}{\partial x_\alpha} = - \frac{\partial M_\alpha^w}{\partial x_\alpha}. \quad (25)$$

We thus have three equations, (22), (23), and (25), for three unknowns,  $M_1^m$ ,  $M_2^m$ , and  $\bar{\zeta}$ . These are the classic shallow-water equations with the addition of wave-forcing terms. It is important to realize that these equations are coupled and therefore the effects of wave mass fluxes cannot be dissociated from wave momentum fluxes.

Besides the term  $\nabla \cdot \tau^m$  in (22) and (23), (25) provides another connection between wave effects and mean sea level. These equations can represent well-known effects related to wave groups, such as the presence of bound long waves, also called infragravity waves for which the fluctuations in the surface mass transport is balanced

by undulations in the mean sea level (Longuet-Higgins and Stewart 1962). These effects undoubtedly occur for bound long waves with frequencies much less than the inertial frequency. If the wave field varies more slowly, the response of the mean circulation is not obvious and can be determined numerically by integration of (22)–(25). The significant amplitude of bound infragravity waves associated with wave groups of 3-m waves of 12-s period in 94 m of water was measured to be 5 cm and 14 cm for 9-m waves of about the same period, during the DUCK94 experiment on the North Carolina shelf (Herbers et al. 2000), that is, 1.5% of the significant wave height  $H_s$ .

In deeper water and for variations of the wave field over larger scale, it is expected that the variations of the mean water level will decrease as  $1/h$ , following the Longuet-Higgins and Stewart (1962) results for shallow water wave groups [their Eq. (3.36)]. In this case, mean water level changes are determined by a balance between the depth integrated hydrostatic pressure,  $\rho_w gh\bar{\zeta}$ , and the wave-added pressure that is present only in a shallow layer. However, it is true that the balance in our depth-integrated view cannot give the balance at all depth levels when things are discretized over the vertical. The waves stresses solely act close to the surface while a change in water level acts over the entire water depths. A balance at all depths may be possible in the presence of stratification.

### 3. Comparison with Lagrangian solutions

Most studies on wave interaction with the mean flow have been based on Lagrangian descriptions of the flow, following Pierson (1962); see also Ünlüata and Mei (1970) and Madsen (1978). This was motivated by the ability of Lagrangian coordinates to track the surface and the exact solution derived by Pollard (1970) for waves in a rotating frame, in the form of modified Gerstner waves. Recently Weber and Melsom (1993) and Jenkins (1986, 1987) have attempted to quantify realistically the effect of breaking waves. Although depth-integrated, the present momentum equation are consistent with Weber and Melsom's (1993) Eq. (15). Their term,  $-ifU_s$ , corresponds to our Hasselmann stress term. Their virtual wave stress term is part of our momentum buffer term  $\hat{\mathbf{T}}^{\text{int}} - \mathbf{T}^a$ . That term, as we described, is related to the integral of the net wave source terms including surface dissipation that represents both whitecapping and viscous dissipation. However, it is not clear that their stress  $\tau_0$  is identical to our wind to mean flow stress  $\hat{\mathbf{T}}^{\text{int}}$ . Somehow their stress  $\tau_0$  should exclude a wind to wave stress; otherwise the momentum given first to the waves and then to the mean flow by viscous wave damping would be counted twice.

Looking at the effect of wave breaking in a saturated sea, they consider the variations in Stokes drift as waves grow and break. From this point our description differs. First, in a saturated sea, our Stokes drift, computed for

random waves, would be constant because its fluctuations are averaged over the random realizations of the sea state. Second, we consider the wave momentum as well as the mass conservation. These wave momentum terms are missing in the Weber and Melsom (1993) derivation of the total velocity,  $W_L = W_E + W_S$  in their Eq. (25), which corresponds to our total mean momentum  $\bar{\mathbf{M}} = \mathbf{M}^m + \mathbf{M}^w$ . Indeed, they only consider the wave-induced mass flux and its variations in time [their Eq. (35)], omitting that momentum is stored in the wave field. Thus, for a wave field that grows or decays in time (but is uniform in space), they obtain a net drift  $W_L$  that is, for fast evolutions, to the right of the wave propagation direction for growing waves and to the left for decaying waves [their Eq. (34)]. In the present theory the total momentum does not change in this case so that the waves impose a stress in the direction of the wave propagation when they decay, giving their momentum to the mean flow, which probably result in a drift to the right due to quasigeostrophic equilibrium (the ocean reacts to this stress as it would to a wind stress) and a drift to the left when waves grow. This effect can be interpreted as a “negative wind stress.” The wind stress forcing is overestimated when one uses  $\mathbf{T}^a$  because a small fraction of the air–sea momentum flux is absorbed by the wave field. Therefore the present theory gives results in contradiction with Weber and Melsom (1993). We hold that this difference is caused by their hypothesis that  $O(\bar{v}^2)$  terms are negligible in their Eq. (7), which seems inconsistent with keeping  $\{\bar{w}\tilde{u}\}_{z=0}$  in their Eq. (17) and the Stokes drift  $U_s$  in their Eq. (21). These hypotheses would essentially render our wave–current interaction stresses  $T^{\text{int}}$  and  $\tau^{\text{int}}$  equal to zero so that total momentum would no longer be conserved.

Another approach, used by McWilliams and Restrepo (1999), directly introduces wave effects through a vortex force that is the vector product of the Stokes drift and the total vorticity. As already noted, the part of this vortex force due to the planetary vorticity is equal to the divergence of the Hasselmann stress. A more thorough comparison is beyond the scope of the present paper. The connection between integrated and vertically distributed momentum equations is the subject of ongoing work with a generalized Lagrangian mean formalism (see, e.g., Leibovich 1980) or other coordinates (Mellor 2003). One term that is clearly missing in the present formalism is the vertical shear that gives an upward vortex force. This may come from neglecting the effect of vertical current shear on wave kinematics and the vertical shear that should give a second-order term in the expression of  $\tau^{\text{sl}}$ .

For depth-integrated equations, Garrett (1976) showed that for a uniform current on the scale of the Stokes depth  $1/(2k_p)$ , where  $k_p$  is the dominant wavenumber, the radiation stresses can be transformed into the vortex force plus the gradient of a modified pressure [ $\pi$  term in Leibovich’s (1980) Eq. (10a)] by including the mean current effect on the radiation stresses. Mean

currents contribute to the wave radiation stresses by modifying the wave momentum, and the term  $U_\alpha M_\beta^w + U_\beta M_\alpha^w$  must be added to our expression of  $\tau_{\alpha\beta}^{\text{rad}}$  (Phillips 1977). The results of Garrett are therefore recovered from the present analysis by this simple extension. Garrett’s (1976) derivation yields the term  $U_\alpha \partial M_\beta^w / \partial x_\beta$  in the  $\alpha$  component of the momentum equations that does not appear in the Craik Leibovich (CL) equations because the wave field is assumed homogeneous (see Smith 1980 and Holm 1996 for discussions of this hypothesis).

This derivation by Garrett (1976) has a dynamically consistent interpretation of the vortex force as the compensation for the change in wave momentum due to wave refraction by horizontal current shears. In this situation wave momentum is exchanged with mean flow momentum; this effect is thus analogous to the “remote recoil” described by Bülher and McIntyre (2003). This consistency is lost in all other derivations of CL equations that assume a uniform wave field. In these, momentum is not conserved locally because the vortex force appears like a momentum source for the mean flow while the wave field has a constant momentum.

The wave–mean flow interaction caused by  $\tau^{\text{rad}}$  in stationary conditions can take place without dissipation in the case of wave refraction by a current shear as described by Garrett (1976). This effect is not included in Jenkins’ (1989) equations who parameterized the wave effects on the mean flow only through wave growth and dissipation source terms. In practice the application of Jenkins’ (1989) Eq. (7) on the beach, with a proper finite depth dispersion relation for wave effect, would drive an alongshore current and wave setup that would be too weak because the driving term would have only an integrated wave breaking source term equal to  $\partial E / \partial x$ , thus missing the extra  $(\partial E / \partial x) / 2$  that accounts for the wave momentum advection to give the correct radiation stress derived by Longuet-Higgins and Stewart (see also Phillips 1977). In his derivation, Hasselmann (1971) checked that his equations were consistent with the Longuet-Higgins and Stewart radiation stresses applied to deep water. Consequently, while more powerful than our equations by fully taking into account the three-dimensional nature of the problem, it appears that the Jenkins (1987, 1989) theory misses effects of wave field gradients. This flaw may not be too much of a problem for large-scale situations where wave fields are generally thought to be fairly uniform, but wave gradients may be relevant to coastal situations such as examined below.

#### 4. Wave effects in steady fetch-limited conditions

As mentioned, the wave-induced terms that enter the mean-flow momentum and mass conservation equations (15)–(25) can be estimated from spectral wave model calculations. WAM-type models rely on parameterizations of the source terms (see Komen et al. 1984) that differ between models. However, they are usually cal-

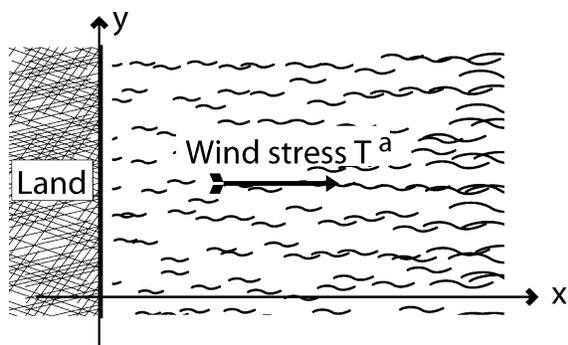


FIG. 1. Schematic of the fetch-limited wave growth situation.

ibrated against observations among which the fetch-limited cases (see the review by Kahma and Calkoen 1992) are probably the most important. In such cases, observations are given of local wave generation with unambiguous initial conditions (the wave height starts from zero at the coast). Details in the evolution of the wave spectrum may be sensitive to the particular parameterization chosen in the wave model, but wave height and period “growth curves” computed by the models are tuned to these observations. Consequently, the stresses computed here for fetch-limited conditions, that are integrals over the entire wave spectrum, are not likely to differ significantly with the choice of a third generation wave model provided it has been tuned, with the exception of wave directional spreading effects that are discussed below.

We therefore chose to use our own model CREST (Ardhuin et al. 2001) with a multistep ray advection scheme with one step only (Ardhuin and Herbers 2003,

manuscript submitted to *J. Atmos. Oceanic Technol.*). The physics of wave generation and nonlinear evolution were added in CREST following the simple parameterization of WAM Cycle 3 (the WAMDI Group 1988), including the adapted Snyder et al.’s (1981) wind generation source term  $S_{in}$ , Hasselmann and Hasselmann’s (1985) discrete interaction approximation (DIA) parameterization of wave–wave interactions, and Komen et al.’s (1984) tuned adaptation of Hasselmann’s (1974) description of wave energy dissipation due to white-capping. The FORTRAN code for the source terms was actually adapted from version 2.22 of Wavewatch III, using the wave growth limiter and fractional step schemes described by Tolman (1992, 2002).

The model was run in steady state in a one-dimensional (transect) configuration representing alongshore-uniform conditions (along the  $y$  axis) with a constant depth  $H = 100$  m. The wind was prescribed directly as a uniform (in space) and constant (in time) friction velocity  $U^* = (T^a/\rho_a)^{1/2}$  with a direction along the  $x$  axis (Fig. 1). The spatial resolution used was 500 m with a time step of 120 s and a frequency range 0.041–0.7 Hz, with exponentially spaced frequency using a 10% increment from one frequency to the next, and a  $15^\circ$  resolution. This spectral grid is a standard WAM grid, imposed by the DIA parameterization. The reproduction of Kahma and Calkoen’s (1992) growth curve was satisfactory (see Fig. 2) with the standard coefficients of WAM-Cycle 3 parameterizations, which were therefore kept unchanged. On this flat bottom, the wave mean bottom pressure term vanishes, and we computed three terms from the wave spectra and source terms: the total and net wave momentum intakes (Fig. 3) and wave–

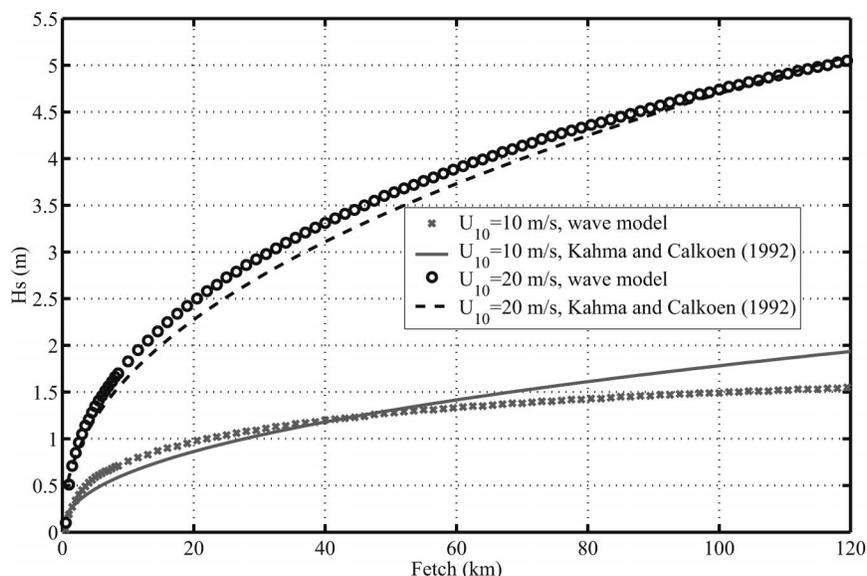


FIG. 2. Wave height growth curves obtained with the CREST model, using WAM Cycle 3 parameterizations for  $U^* = 0.38$  and  $U^* = 0.917$  corresponding to  $U_{10} = 10$  and  $20$  m  $s^{-1}$ , respectively.

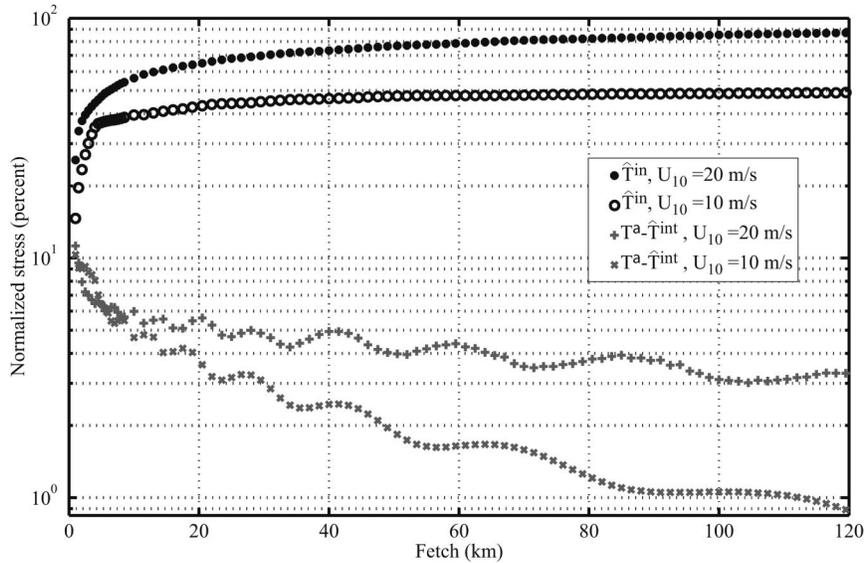


FIG. 3. Percentage of the wind stress represented by the stress,  $T_{in} = \int S_{in} k_x / kC \, dk$ , imparted to the wave field and the net gain of momentum  $\int (S_{in} + S_{ds} + S_{nl}) k_x / kC \, dk$ , by the wave field following the wave propagation.

current radiation stresses. The effects of the wave-induced mass flux are not considered here.

*a. Momentum storage in the wave field*

The portion  $T^{in} = \int k S_{in} / (kC) \, dk$  of the wind stress  $T^a$  that is transmitted to the wave field grows with the inverse wave age  $U^*/C$  and fetch  $x$ . While the steepness of the dominant waves decreases with fetch, the sea surface roughness increases toward offshore with the height and wave spectrum broadens. Accordingly, the wind stress is increasingly supported by form drag over the waves (Fig. 3).

As individual waves grow, they also break more frequently so that the fraction  $R$  of the wind stress that is locally stored in the wave field, and added to the wave momentum, decreases with wave development;  $R$  is largest for young windseas and short fetches (Fig. 3), with a maximum of about  $R = 10\%$ . This fraction can be estimated more simply from the wave growth curve

$$m_0 = a(xg/U_{10}^2)^b U_{10}^4 / g^2, \tag{26}$$

where  $a$  and  $b$  are empirical coefficients. We use a representative group speed  $C_g$  and phase speed  $C$  for the entire spectrum so that the energy flux is  $C_g \rho_w g m_0$  and the wave momentum is  $\rho_w g m_0 / C$ . Taking the wind stress  $T^a$  as  $\rho_a C_d U_{10}^2$ , with  $\rho_a$  the density of air, the momentum balance of the wave field (11) can be rewritten as

$$\rho_w g \frac{d(C_g m_0 / C)}{dx} = R \rho_a C_d U_{10}^2. \tag{27}$$

In deep water,  $C_g = C/2$ , and the ratio  $R = \hat{T}^{int} / T^a$  is obtained by replacing the growth curve expression (26) in (27):

$$R = \frac{ab \rho_w}{C_d \rho_a} \left( \frac{xg}{U_{10}^2} \right)^{b-1}. \tag{28}$$

Taking  $a = 5 \times 10^{-7}$ ,  $b = -0.9$ , and  $C_d = 1.3 \times 10^{-3}$ ,  $R = 15\%$  for  $x = 0$  and goes to zero like  $(xg/U_{10}^2)^{b-1}$ .

The wave field becomes more “transparent” to the air–sea momentum flux as the waves get more developed because the momentum given to the waves is immediately lost to the current through whitecapping. This view of the air–sea momentum balance is consistent with Mitsuyasu’s (1985) conclusions, who determined this ratio  $R$  as a function of the wave steepness. The values we find here are probably overestimated for short fetches (Donelan 1998 observed a maximum value  $R = 4\%$ ), essentially because we use a constant value for  $C_d$ . It is now firmly established that for a given wind speed  $C_d$  can increase by a factor 3 for very young windseas (i.e., short fetches).

Also, if the total source term balance  $S_{in} + S_{nl} + S_{dis}$  is rather well constrained by observations, there is still a wide variety of parameterizations for the different source terms. In particular, values of  $S_{in}$  and thus  $T^{in}$  have been proposed about 4 times smaller than what is presented in Fig. 3 (Burgers and Makin 1993; Tolman and Chalikov 1996).

*b. Overall wave-induced effect*

We have seen that, for fetch-limited wave growth, the two terms,  $T^a - \hat{T}^{int}$  and the divergence of the interaction stress  $\nabla \cdot \tau^{int}$ , act in opposite direction. However, since we look at stationary conditions, we also know that the total right-hand side of the surface layer mo-

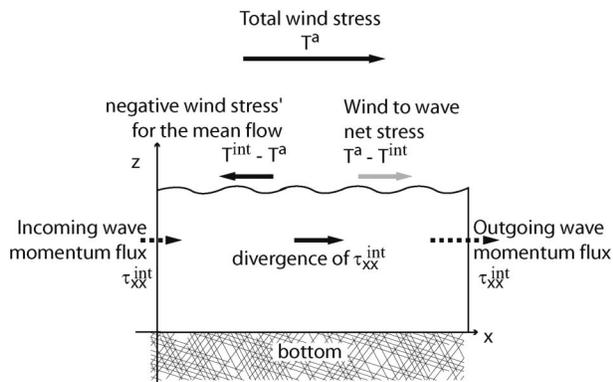


FIG. 4. Vertically integrated momentum balance for the wave field (shaded arrow) that gains momentum from the wind and wind and wave effects on the mean flow momentum (black solid arrows). The mean flow is accelerated offshore by the total wind stress  $T^a$  and the divergence of the interaction stress  $\tau^{int}$ , and onshore by the correction  $\hat{T}^{int} - T^a$  on the wind stress (overestimated by  $T^a$ ). In stationary conditions these two terms add up to give the divergence of the radiation stress  $\tau^{rad}$ .

mentum equation [(7)] is zero. The total wave forcing is thus

$$T^a - \hat{T}^{int} + \nabla \cdot \tau^{int} = \nabla \cdot (\tau^{sl} + \tau^{int}) = \nabla \cdot \tau^{rad}. \quad (29)$$

As commented by Hasselmann (1971), stationary conditions yield the usual radiation stress formulation. In deep water this stress acts against the wave propagation direction, as summarized in Fig. 4. This total stress (here a force per unit surface) can be as large as 10% of the wind stress for short fetches and decreases away from the coast, staying above 3% out to 30 km for  $U_{10} = 10 \text{ m s}^{-1}$  and out to 100 km for  $U_{10} = 20 \text{ m s}^{-1}$  (Fig. 5).

This total stress is only a function of the gradients of the total wave height and the directional spread of the wave spectrum. Although we may trust third-generation wave models for the growth curves of wave heights and peak periods, there is a real shortage of directional wave data to calibrate the wave directional distribution predicted by models, and the wave directional spread is generally not considered in the tuning of numerical wave models. Indeed today's wave models have different directional properties. In particular WAM Cycle 4, operational at ECMWF, presents a distribution similar to Wavewatch III, operational at the National Centers for Environmental Prediction, but has different source term parameterizations. The WAM Cycle 3 parameterization used here probably gives an upper bound on the wave directional spread and on  $\tau_{xx}^{rad}$ .

Assuming that the adjustment is only between the hydrostatic pressure induced by the mean sea level,  $p^m = p^a + \rho_w g w(\zeta - z)$ , and the radiation stress, we can write

$$(\rho_w g h + \bar{\zeta}) \frac{\partial \bar{\zeta}}{\partial x} = \frac{\partial \tau_{xx}^{rad}}{\partial x}. \quad (30)$$

For small ratios  $\bar{\zeta}/h$  this leads to

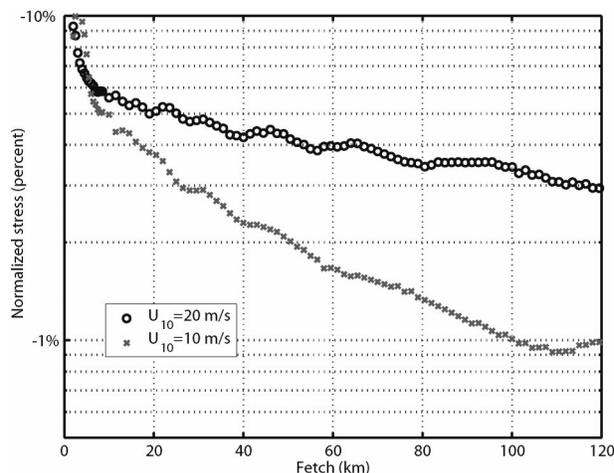


FIG. 5. Total wave-induced stresses  $\nabla \cdot \tau^{rad}$  for stationary fetch-limited wave growth as a percentage of the wind stress  $T^a$ .

$$\bar{\zeta} = -m_\theta \frac{(H_s/4)^2}{2h}, \quad (31)$$

with the directional moment  $m_\theta$  defined by

$$m_\theta = \frac{\int_k \int_\theta F(\mathbf{k}) \cos^2 \theta k \, dk \, d\theta}{\int_k \int_\theta F(\mathbf{k}) k \, dk \, d\theta}. \quad (32)$$

In the present calculations  $m_\theta$  is very close to 0.7 and the results for a water depth  $h = 100 \text{ m}$  are given in Fig. 6, with a maximum setdown along the fetch of 12 mm for 5-m waves. Of course on a real coastline the depth is variable, increasing from zero at the coast, so that the actual relative setup at the coast is larger. This

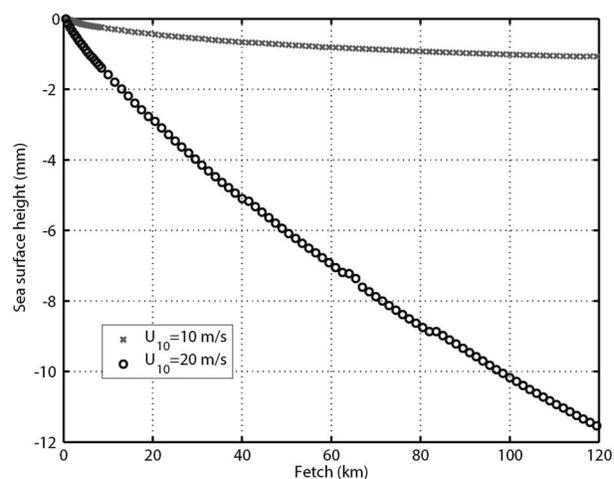


FIG. 6. Wave setup at the coast in fetch-limited wave growth conditions, assuming a balance between hydrostatic pressure and the divergence of the interaction stresses  $\tau^{rad}$  and using a water depth  $h = 100 \text{ m}$ .

is also increased by the fact that  $\tau_{xx}^{\text{rad}}$  goes to  $-3E/2$  in the limit of small dimensionless depth  $kh$ , compared to  $-E/2$  in deep water. The actual balance of forces between radiation stresses and the mean flow adjustment probably involves a reduction in the Ekman transport together with a setdown along the fetch (a situation opposite to that of shoaling waves described by Lentz et al. 1999).

*c. Extension for wave setdown in nonstationary conditions*

The balance obtained in (31) can be extended to nonstationary conditions. For uniform conditions in the  $y$  direction and wave propagating in the  $x$  direction, variations on the scale of a weather system can be seen as an extension of Longuet-Higgins and Stewart's results [1962, Eq. (3.26)] for wave groups long compared with the depth (please remember the opposite sign convention for  $\tau^{\text{rad}}$  between Hasselmann and Longuet-Higgins and Stewart)

$$\bar{\zeta} = \frac{1}{\rho_w(gh - c^2)} \tau_{xx}^{\text{rad}}, \tag{33}$$

where  $c$  is the velocity of the wave height pattern (storm or wave group). This solution can be generalized from a linearization of Eqs. (22), (23), and (25) for the variables  $M_x^m$ ,  $M_y^m$ , and  $\bar{\zeta}$  forced by a propagating wave energy perturbation of the form  $Ee^{i(Kx - \Omega t)}$ , proportional to the significant wave height  $H_s$  squared, giving

$$\begin{aligned} \bar{\zeta} &= \frac{\tau_{xx}^{\text{rad}}}{\rho_w[gh - (\Omega^2 - f^2)/K^2]} \\ &= -m_0 \frac{g(H_s/4)^2}{2[gh - (\Omega^2 - f^2)/K^2]}, \end{aligned} \tag{34}$$

where the second equality is only valid for deep water waves. This last result shows that the bound long waves forced by waves in a moving storm are only weakly modified by the earth rotation and thus have very small amplitudes, on the order of  $H_s^2/h$ .

**5. Conclusions and perspectives**

We have established a framework for studying all wave-current interactions by a straightforward extension of Hasselmann's (1971) formalism describing mass and momentum conservation in a rotating frame. This theory is uniformly valid, from global-scale ocean circulation to the nearshore, and is expected to be consistent with other derivations that use Lagrangian coordinates and/or separate turbulence and wave motions (e.g., Jenkins 1986; Weber 2001; Groeneweg and Klopman 1998). Our practical applications invoke linear wave theory to compute second-order quantities, which has been shown to be a robust assumption, even in the surf zone (e.g., Thornton and Krapohl 1974). However,

its validity in the present context still needs to be assessed.

The effects of waves on the mean flow can thus be computed using existing operational wave models in which the decomposition of wave evolution into source terms can easily be transformed in wave momentum changes. Care should be given in separating the three possible recipients of the wave momentum: the atmosphere, the mean ocean circulation, or the bottom. For instance, the dependency of the wind stress on wave age and swell strength and direction should also further affect both source terms  $S_{in}$  and  $S_{ds}$ . As proposed, the wave energy balance equation can be adequately modified to express a depth-induced breaking source term, following common practice (e.g., Booij et al. 1999), to explicitly represent phenomena such as wave setup and setdown and alongshore currents in the nearshore.

As discussed, wave effects can be expected from theory at scales ranging from the wave group to the ocean basin, including infragravity waves, setup at the coast for offshore winds, and inertial oscillations, all with a magnitude that scales as the square of the wave height. The combination of large-scale infragravity motions forced by a divergence in the wave mass transport and the gradients of the interaction stresses may yield a setdown under areas where waves are larger. This is likely to be much smaller than the values of the "mean sea level correction" suggested by McWilliams and Restrepo (1999), with 0.5 mm in the deep ( $h = 5000$  m) ocean for important (from 0 to 10 m significant wave height) over large scale [larger than  $O(10$  km)] variations in the wave field, according to our Eq. (31) or (34). Essentially, the wave pressure and momentum acts in a very thin surface layer that cannot balance a large surface elevation  $\zeta$  resulting in a depth integrated hydrostatic pressure of the order of  $\rho_w g h \zeta$ .

This further confirms the general idea that most of the measured altimeter sea state bias shall be solely attributed to the local sea surface geometry and instrumental probing characteristics. However, while small, the expected sea level adjustments predicted here may still be of concern in the context of corrections applied to precision altimetry. These variations in water level will be systematic over preferred ocean regions. Increasing as  $1/h$  toward shallow areas, sea level changes become significant, with the well-known wave setdown, where many tide gauges used for altimeter validation are situated.

Last, on smaller scales (e.g., groups of about 10 waves), wave height modulations will drive infragravity waves of larger amplitude because they do not reach the bottom [Longuet-Higgins and Stewart (1962) Eq. (3.29)]. These effects should certainly be taken into account for planned high-resolution satellite altimeters and the use of altimetric measurements close to the coasts. These infragravity motions may still contribute to the electromagnetic bias of large-footprint altimeters because of a correlation of low water elevations (the long wave troughs) with short wave steepness.

Using a standard wave model, the wave-induced stresses for the mean flow were calculated to be as large as 10% of the wind stress for stationary conditions and short fetches. Ignoring this effect probably introduces regional biases in the forcing of ocean circulation models with wind stresses. These biases are still small compared to the known variation in wind stress induced by changes in the sea state such as swell and wave age (Drennan et al. 1999, 2003).

In the future, the actual implementation of the present theory in an ocean circulation model will require the transformation of molecular viscosity into a variable turbulent viscosity, following common practice. For fine vertical resolution, resolving the Stokes depth  $1/(2k)$ , an extension in three dimensions will also be needed that will have to be consistent with present results. That extension will likely benefit from the work of Jenkins (1987, 1989) and Mellor (2003). Determination of the eddy viscosity profile is a major challenge in this respect. Such a parameterization should ideally include effects of wave breaking and Langmuir circulation, as proposed by, for instance, Jenkins (1987, 1989) and McWilliams and Sullivan (2001).

The practical advantage of using a wave model to take into account the (surface) wave effects described here can also be exploited to compute surface drift, turbulent fluxes of kinetic energy at the surface, and the associated mixing. This combination of wave and circulation modeling can thus be most fruitful for a three-dimensional ocean model, while demanding a more rigorous validation of individual wave energy source terms and wave spectral moments that can be computed from today’s wave forecasting models.

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APPENDIX

**Derivation of the Mean Flow Momentum Equations**

Following the notation of section 1, we derive the equation for  $M_1^m$ . The derivation of the equation for  $M_2^m$  is similar. We start from the Navier–Stokes equations on a rotating earth (“*f* plane”):

$$\rho_w \frac{\partial u_1}{\partial t} + \rho_w \frac{\partial}{\partial x_j}(u_1 u_j) + \frac{\partial}{\partial x_1} p - f \rho_w u_2 = \mu_w \Delta u_1, \quad -h \leq x_3 \leq \zeta, \quad (A1)$$

$$\frac{\partial u_j}{\partial x_j} = 0, \quad -h \leq x_3 \leq \zeta \quad (A2)$$

$$p - g\zeta = p^a, \quad x_3 = \zeta \quad (A3)$$

$$\frac{\partial \zeta}{\partial t} + u_\alpha \frac{\partial \zeta}{\partial x_\alpha} - u_3 = 0, \quad x_3 = \zeta \quad (A4)$$

and

$$u_\alpha \frac{\partial h}{\partial x_\alpha} + u_3 = 0, \quad x_3 = -h. \quad (A5)$$

In the surface boundary conditions (A2)–(A5) we have not specified the horizontal stress, essentially because it results from a coupling of the air and water flow around the interface so that the pressure and horizontal stress fluctuations at the surface cannot be prescribed.

We thus assume that the mean stress is known and is carried by small-scale viscous and pressure stresses on a surface that is not horizontal. General boundary conditions and the different mechanisms of air–sea momentum transfer are given and discussed by Jenkins (1992). For a correct separation of wind to wave and wind to mean flow momentum, the wave growth must be attributed to both the correlation of air pressure and water elevation at the scale of the waves and the modulation of the tangential stress along the wave profile. The rest of the wind stress is then given to the ocean. In the same way, the wave dissipation is the result of viscous stresses [the virtual wave stress in Weber (1983); see also Xu and Bowen (1994) for an Eulerian description] and wave breaking that we group in  $\mathbf{T}^a - \hat{\mathbf{T}}^{\text{int}}$ . Therefore our end result should be coherent with a careful accounting of the exact dynamic boundary condition at the surface.

By averaging ( $\rho_w$  and  $\mu_w$  are assumed to be uniform in space) we get

$$\rho_w \frac{\partial \bar{u}_1}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_1 \bar{u}_j) + \frac{\partial}{\partial x_1} p^m - f \bar{u}_2 = \mu_w \Delta \bar{u}_1 + \frac{\partial}{\partial x_j} T_{ij}^{\text{int}}, \quad -h \leq x_3 \leq \bar{\zeta}. \quad (A6)$$

Now we compute the evolution of the mean momentum:

$$\frac{\partial M_1^m}{\partial t} = \frac{\partial}{\partial t} \int_{-h}^{\bar{\zeta}} \bar{u}_1 dx_3 = \int_{-h}^{\bar{\zeta}} \frac{\partial}{\partial t} \bar{u}_1 dx_3 + \frac{\partial \bar{\zeta}}{\partial t} \bar{u}_1(\bar{\zeta}). \quad (A7)$$

Replacing (A1) in (A7) and using

$$\begin{aligned} & \int_{-h}^{\bar{\zeta}} -\frac{\partial}{\partial x_1} p^m dx_3 \\ &= \int_{-h}^{\bar{\zeta}} -\frac{\partial}{\partial x_1} (p^m - \rho_w g z) dx_3 \\ &= \frac{\partial}{\partial x_1} \int_{-h}^{\bar{\zeta}} -(p^m - \rho_w g z) dx_3 + [p^m(\bar{\zeta}) - g\bar{\zeta}] \frac{\partial \bar{\zeta}}{\partial x_1} \\ & \quad + [p^m(-h) + \rho_w g h] \frac{\partial h}{\partial x_1}, \end{aligned} \tag{A8}$$

we get

$$\begin{aligned} \frac{\partial M_1^m}{\partial t} &= \int_{-h}^{\bar{\zeta}} -\frac{\partial}{\partial x_j} (\bar{u}_1 \bar{u}_j) - \frac{\partial}{\partial x_1} p^m + f \bar{u}_2 + \mu_w \Delta \bar{u}_1 \\ & \quad + \frac{\partial}{\partial x_j} T_{ij}^{\text{int}} dx_3 + \frac{\partial \bar{\zeta}}{\partial t} \bar{u}_1(\bar{\zeta}) \\ &= \frac{\partial}{\partial x_\beta} \int_{-h}^{\bar{\zeta}} -(\bar{u}_1 \bar{u}_\beta) - \delta_{1\beta} (p^m - g z) dx_3 \\ & \quad + \bar{u}_1(\bar{\zeta}) \bar{u}_\beta(\bar{\zeta}) \frac{\partial \bar{\zeta}}{\partial x_\beta} + \bar{u}_1(-h) \bar{u}_\beta(-h) \frac{\partial h}{\partial x_\beta} \\ & \quad + [p^m(\bar{\zeta}) - g\bar{\zeta}] \frac{\partial \bar{\zeta}}{\partial x_1} + [p^m(-h) + gh] \frac{\partial h}{\partial x_1} \\ & \quad + \int_{-h}^{\bar{\zeta}} -\frac{\partial}{\partial x_3} (\bar{u}_1 \bar{u}_3) dx_3 + \int_{-h}^{\bar{\zeta}} f \bar{u}_2 dx_3 \\ & \quad + \mu_w \frac{\partial \bar{u}_1}{\partial x_3} \Big|_{\bar{\zeta}} - \mu_w \frac{\partial \bar{u}_1}{\partial x_3} \Big|_{-h} + \mu_w \int_{-h}^{\bar{\zeta}} \frac{\partial^2 \bar{u}_1}{\partial x_\beta \partial x_\beta} dx_3 \\ & \quad + \int_{-h}^{\bar{\zeta}} \frac{\partial}{\partial x_j} T_{ij}^{\text{int}} dx_3 + \frac{\partial \bar{\zeta}}{\partial t} \bar{u}_1(\bar{\zeta}). \end{aligned} \tag{A9}$$

Grouping the terms in  $u_1(\bar{\zeta})$ ,  $u_1(-h)$ , and using Hasselmann's definitions with the added viscous stresses,

$$\tau_{\alpha\beta}^m = - \int_{-h}^{\bar{\zeta}} (\bar{u}_\alpha \bar{u}_\beta) + \delta_{\alpha\beta} (p^m - g z) + \mu_w \frac{\partial^2 \bar{u}_1}{\partial x_\beta \partial x_\beta} dx_3, \tag{A10}$$

and

$$\tau_{\alpha\beta}^{\text{int}} = \int_{-h}^{\bar{\zeta}} T_{\alpha\beta}^{\text{int}} dx_3, \tag{A11}$$

we get

$$\begin{aligned} \frac{\partial M_1^m}{\partial t} &= \frac{\partial}{\partial x_\beta} \tau_{1\beta}^m + \bar{u}_1(\bar{\zeta}) \left[ \frac{\partial \bar{\zeta}}{\partial t} + \bar{u}_\beta(\bar{\zeta}) \frac{\partial \bar{\zeta}}{\partial x_\beta} - \bar{u}_3(\bar{\zeta}) \right] \\ & \quad + \bar{u}_1(-h) \left[ \bar{u}_\beta(-h) \frac{\partial h}{\partial x_\beta} + \bar{u}_3(-h) \right] \\ & \quad + [p^m(\bar{\zeta}) - g\bar{\zeta}] \frac{\partial \bar{\zeta}}{\partial x_1} + [p^m(-h) + gh] \frac{\partial h}{\partial x_1} \end{aligned}$$

$$\begin{aligned} & + f M_2^m - T_1^b + \frac{\partial}{\partial x_\beta} \tau_{1\beta}^{\text{int}} + \mu_w \frac{\partial \bar{u}_1}{\partial x_3} \Big|_{\bar{\zeta}} - \frac{\partial \bar{\zeta}}{\partial x_\beta} T_{1\beta}^{\text{int}} \\ & - \frac{\partial h}{\partial x_\beta} T_{1\beta}^{\text{int}}(-h) + T_{13}^{\text{int}}(\bar{\zeta}) - T_{13}^{\text{int}}(-h). \end{aligned} \tag{A12}$$

This simplifies by using the boundary conditions (A3)–(A5):

$$\begin{aligned} \frac{\partial M_1^m}{\partial t} &= \left\{ \frac{\partial}{\partial x_\beta} \tau_{1\beta}^m + \bar{p}^a \frac{\partial \bar{\zeta}}{\partial x_1} + [p^m(\bar{h}) + gh] \frac{\partial h}{\partial x_1} \right. \\ & \quad \left. + f M_2^m + T_1^a - T_1^b \right\} - T_1^a + \frac{\partial}{\partial x_\beta} \tau_{1\beta}^{\text{int}} \\ & \quad + \mu_w \frac{\partial \bar{u}_1}{\partial x_3} \Big|_{\bar{\zeta}} + T_{13}^{\text{int}}(\bar{\zeta}) - \frac{\partial \bar{\zeta}}{\partial x_\beta} T_{1\beta}^{\text{int}}(\bar{\zeta}) \\ & \quad - \bar{u}_1(\bar{\zeta}) \frac{\partial M_\beta^w}{\partial x_\beta} - \frac{\partial h}{\partial x_\beta} T_{1\beta}^{\text{int}}(-h) - T_{13}^{\text{int}}(-h), \end{aligned} \tag{A13}$$

which is almost Hasselmann's Eq. (14) with the addition of the horizontal Coriolis force, either because  $u = 0$  at the bottom or assuming linear wave theory at the top of the bottom boundary layer in order to remove the extra bottom terms. The only real differences are the hydrostatic pressure in the bottom pressure term, probably an omission on the part of Hasselmann, and the clear typographic mistake  $\bar{u}_\beta$  instead of  $\bar{u}_\alpha$  in the definition of  $\hat{T}_\alpha^{\text{int}}$ .

This equation yields our Eq. (13), that  $\hat{\mathbf{T}}^{\text{int}}$  is redefined by (5) in order to separate the Coriolis-wave term, which we call the Hasselmann stress, that drives the Eulerian return flow in steady open ocean conditions.

Using linear wave theory the wave-added stresses can be determined at first order. The derivation of  $\hat{\mathbf{T}}^{\text{int}} - \mathbf{T}^a$  from the source terms of a WAM-type model is given in section 2, and the stress  $\tau^{\text{int}}$  can be computed from the definition (18) and linear wave theory applied to random waves:

$$\overline{(u_3^w)^2} = \int \omega^2(k) \frac{\sinh^2(kz + kh)}{\sinh^2(kh)} F(\mathbf{k}) d\mathbf{k}, \tag{A14}$$

and

$$\overline{u_\alpha^w u_\beta^w} = \int \omega^2(k) \frac{k_\alpha k_\beta \cosh^2(kz + kh)}{k^2 \sinh^2(kh)} F(\mathbf{k}) d\mathbf{k}, \tag{A15}$$

with

$$k = |\mathbf{k}| \quad \text{and} \tag{A16}$$

$$\omega(k) = gk \tanh(kh). \tag{A17}$$

The nonhydrostatic pressure term in  $\tau^{\text{int}}$  is therefore

$$\begin{aligned} & \rho_w \int_{-h}^{\bar{\zeta}} \overline{(u_3^w)^2} dz \\ &= \rho_w \int_{\mathbf{k}} gk \frac{\tanh(kh)}{\sinh^2(kh)} F(\mathbf{k}) \int_{-h}^{\bar{\zeta}} \sinh^2(kz + kh) dz d\mathbf{k} \end{aligned}$$

$$\begin{aligned}
&= \rho_w \int_{\mathbf{k}} \frac{2gk}{\sinh(2kh)} F(\mathbf{k}) \\
&\quad \times \int_{-h}^{\bar{z}} 0.5[\cosh(2kz + 2kh) - 1] dz d\mathbf{k} \\
&= \rho_w g \int_{\mathbf{k}} \frac{k}{2k \sinh(2kh)} F(\mathbf{k}) [\sinh(2kh) - 2kH] d\mathbf{k} \\
&= \rho_w g \int_{\mathbf{k}} F \left[ \frac{1}{2} - \frac{kH}{\sinh(2kh)} \right] d\mathbf{k}. \quad (\text{A18})
\end{aligned}$$

Using linear wave theory expressions for  $C_g$  and  $C$ , (A18) is equivalent to the term with  $\delta_{\alpha\beta}$  in (19). The other term is given in the same way by integrating (A15) instead of (A14) in (A18).

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