Restratification of the Surface Mixed Layer with Submesoscale Lateral Density Gradients: Diagnosing the Importance of the Horizontal Dimension

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ABSTRACT

A depth-cycling towed conductivity–temperature–depth (CTD) and vessel-mounted acoustic Doppler current profiler (ADCP) were used to obtain four-dimensional measurements of the restratification of the surface mixed layer (SML) at a submesoscale lateral density gradient near the subtropical front. With the objective of studying the role of horizontal processes in restratification, the thermohaline and velocity fields were monitored for 33 h by 16 small-scale (≤15 km²) surveys centered on a drogued float. Daytime warming by insolation caused a unidirectional displacement of the initially vertical isopycnals toward increasing density. Across the entire SML (50-m vertical scale), solar insolation accounted for 60% of observed restratification, but over 10-m scales, the percentage decreased with depth from 80% at 25–35 m to ≥25% at 55–65 m. Below 35 m, stratification was enhanced by the vertically sheared horizontal advection of the lateral density gradient due to a near-inertial wave of ~100-m vertical wavelength that rotated anticyclonically at the inertial frequency. The phase and similar period (25.4 h) of the local inertial period to the diurnal cycle ensured constructive interference with isopycnal displacements due to insolation. Restratification by sheared advection matched that predicted due to vertically sheared inertial oscillations generated during the geostrophic adjustment of a density front, but direct wind forcing may also have generated the wave that was subsequently modified by interaction with mesoscale vorticity associated with a nearby large-scale front. By further including the effects of lateral uncompensated thermohaline inhomogeneity, the authors can account for 100% ± 20% of the observed $N^2$ during daytime restratification. No detectable restratification due to the slumping of horizontal density gradients under gravity alone was found.

1. Introduction

Lateral processes have frequently been invoked as potentially responsible for the excess restratification in the surface mixed layer (SML) that cannot be explained by solar insolation and vertical turbulent fluxes (Brainerd and Gregg 1993a, 1997; Caldwell et al. 1997). Typically only 60% ± 15% of observed restratification in the SML can be attributed to the vertical divergence of penetrative solar radiation, with a much smaller proportion due to vertical divergence of turbulent fluxes, leaving approximately 30% ± 10% unaccounted for and most likely due to lateral processes. The majority of observations has often been inhibited by the method of data acquisition, whereby repeated vertical profiles at a fixed location of CTD variables and velocity microstructure allow a quantification of the role played by one-dimensional vertical processes in restratifying the SML but are unable to provide a similar quantitative assessment of the importance of lateral processes. Similarly, models of the SML that resolve short temporal and spatial scales have typically been limited to the vertical dimension and the assumption of horizontal homogeneity (Niiler and Kraus 1977; Price et al. 1986), limiting their ability to model the diurnal cycle (Brainerd and Gregg 1993b). The processes proposed as potentially contributing to the additional restratification of the SML are lateral intrusions of uncompensated thermohaline variability (Brainerd and Gregg 1993a) and the relaxation of horizontal density gradients (Brainerd and Gregg 1997; Caldwell et al. 1997; Tandon and Garrett 1994, 1995) generated by, for example, localized atmospheric forcing (Soloviev and Lukas 1996; Soloviev et al. 2000; Wijesekera and Gregg...
The impact of near-inertial shear on restratification has not been observed before, although Franks (1995) modeled its effect on the distribution of phytoplankton in a similar context. The vertical wavelength of the shear supports the suggestions of Kunze and Sanford (1984) that sheared near-inertial waves are ubiquitous within the SML when forcing is suitable and the waves subsequently interact with the mesoscale field. The anticyclonic rotation with depth by 90°–180° over ~100-m vertical scales is further consistent with the high-order, short-vertical-wavelength baroclinic modes of Niiler and Kraus (1977). Such waves were further proposed by Shay et al. (1998) to be important to the deepening of the SML by lowering the Richardson number toward critical levels at the base of the SML, but the lack of sufficiently high–resolution measurements prohibited the formulation of a kinematical description of the three-dimensional evolution of such motions. The small horizontal scale and high temporal frequency of our observations address this shortcoming and permit an initial assessment of the importance of the three-dimensional kinematics associated with the near-inertial shear to restratification.

The paper is organized as follows: we first describe in section 2 the experimental details and the instrumentation used. In section 3, the prevailing meteorological conditions and the 1D vertical response of the SML are presented for the 33-h restratification period and the preceding 4 days, which established the initial environment for restratification. In section 4 the evolution of the SML is presented, beginning with the initial thermohaline fields and proceeding to the shear and its effect on the deformation of the density field. Section 5 assesses the relative roles played in the restratification by solar insolation, sheared advection by a near-inertial wave, and lateral inhomogeneities in the thermohaline fields. Section 6 discusses the implications of results for modeling, focusing on the predictability of restratification by near-inertial shear, the origin and modification of the near-inertial wave, and the broader implications for different contexts. Conclusions are drawn in section 7.

2. Experimental details

a. Study region and cruise strategy

Observations were made between 3 and 24 March 2004 during a cruise aboard the R/V Wecoma near the subtropical front in the northeast Pacific (Fig. 1). The second half of the cruise, from which the data presented in this paper are taken and on which we focus exclu-
sively henceforth, was conducted at 28°N between \( t = 76 \) and 84, where \( t \) refers to days elapsed in 2004 starting from 0 at 0000 UTC 1 January, in a region characterized by a large degree of uncompensated horizontal variability at small scales (\( \leq 10 \) km) (Hosegood et al. 2006). The principal focus of this paper is on the contribution of horizontal gradients to restratification and for which we employ a further subset of the observations at 28°N, referred to as the restratification period, that spans 33 h between \( t = 80.4 \) and 81.8. During this period, 16 surveys (groups 64–79) were completed sequentially, providing a time series of the 3D evolution of the SML.

Our strategy was to repeatedly sample the SML over a spatial area large enough to resolve restratification at submesoscales but small enough to be completed before the SML evolved significantly. This was achieved by completing consecutive surveys centered on a float drogued at 38 m that was advected by the mean flow, thereby ensuring that we sampled the same volume of water in a quasi-Lagrangian reference frame. The float maintained depth rather than following isopycnals and was thus more biased to inertial oscillations than Lagrangian density-following floats (e.g., D’Asaro et al. 1996) used in other studies. The surveys, referred to as groups, were composed of straight legs typically 5 km long at different orientations covering \( \sim 15 \) km\(^2\). As we searched for the optimum spatial coverage in a field advecting with mean and inertial speeds of 0.2 m s\(^{-1}\), the pattern varied, eventually settling on bow ties in which each of the four legs was recentered on a float.

b. Instrumentation

1) Oceanographic data

Our principal instrument was the Shallow Water Integrated Mapping System, a 300-kg towed body that depth cycles in a sawtooth pattern and whose primary sensor is a Sea-Bird CTD. Because we were mapping the SML and the region encroaching on the seasonal thermocline, a profiling depth range of 10–150 m was specified. Typical horizontal distances required to complete the downward and upward profiles at a tow speed of \( \sim 5 \) kt are 300 and 450 m, respectively, corresponding to a mean time of 2.6 min for each profile. The dissipation rate of turbulent kinetic energy, \( \epsilon \), is calculated from SWIMS data by computing Thorpe scales (Thorpe 1977), \( L_T \), which estimates \( \epsilon \) through the relationship \( L_\epsilon = 0.8 L_T \), where \( L_\epsilon = (\epsilon / N^3)^{1/2} \) is the Ozmidov scale.

Velocities were obtained by a vessel-mounted acoustic Doppler current profiler (VM-ADCP) as 2-min en-
seemle averages in 4-m vertical bins between 7 and 200 m. A number of $e$ profiles were obtained by a modular microstructure profiler (MMP), a loosely tethered instrument that measures turbulent shear velocities with a pair of airfoil probes, from which $e$ is calculated in the standard manner according to Osborn (1980). To avoid confusion with the small-scale groups conducted with SWIMS, we hereafter refer to the MMP groups of approximately 20 profiles explicitly as MMP groups.

2) Meteorological data

Meteorological data were acquired onboard the R/V Wecoma and version 2.6 of the COARE bulk algorithms (Bradley et al. 2000) was used to compute air–sea fluxes. Of principal interest to the mixing and restratification of the SML is the buoyancy flux, $J_b$:

$$J_b = c_p^{-1} g p^{-1} a Q + g p^{-1} b (E - P) S_{surf} (W \text{ kg}^{-1}),$$

(1)

where $Q = Q_{\text{shortwave}} + Q_{\text{longwave}} + Q_{\text{latent}} + Q_{\text{sensible}} + Q_{\text{rain}} (W \text{ m}^{-2})$ is the total heat flux, $c_p = 4000 \text{ J kg}^{-1} \text{ K}^{-1}$ is the specific heat of water, $g$ is the acceleration due to gravity, and $a$ and $b$ are the thermal and haline contraction coefficients computed from the daily mean SST and SSS measured by the intake at 3-m depth. The $E$ and $P$ are evaporation and precipitation, respectively, and $S_{surf}$ is the surface salinity. The $Q_{\text{rain}}$ is due to the difference in temperature between rain and the seawater. The wind speed influences the buoyancy flux through the sensible and latent heat fluxes:

$$Q_{\text{sensible}} = C_h U S T \rho_{air} C_p,$$

(2)

$$Q_{\text{latent}} = C_h U S q_{air} L,$$

(3)

where $U$ is the wind speed, $\rho_{air}$ is the density of air, $C_p$ is its heat capacity, $\Delta T$ is the difference between the air and sea temperature, and $C_h = 1 \times 10^{-3}$ is the transfer coefficient for $Q_{\text{sensible}}$. For the latent heat flux, $\Delta q$ is the difference between the surface humidity and saturation humidity, $L$ is the latent heat of evaporation and the transfer coefficient $C_h = 1.2 \times 10^{-3}$.

The comparative influences on mixing and convection are quantified by the Monin–Obukhov length scale:

$$L_m = \frac{-u_*^3}{\kappa J_b'},$$

(4)

where $J_b'$ is the surface buoyancy flux, $u_* = \sqrt{\tau/\rho_{air}}$ is the friction velocity, $\tau$ is the wind stress, $\rho_{air}$ is the density of air, and $k = 0.41$ is von Kármán’s constant. When $z/L \leq 1$, where $z$ is the depth, wind stress dominates turbulence production, and in the free convection case, $z/L \geq 1$, buoyancy dominates.

3. Meteorological forcing and the 1D response of the SML

a. Meteorological forcing

The meteorology was dominated by the passage of a storm during days 78 and 79 (Fig. 2a) and can thus be divided into four distinct periods (labeled A–D in Fig. 2) based on the relevant meteorological variables and the impact they had on the vertical structure of the SML. The pertinent aspects of each of these periods are discussed below.

A) Prior to the storm wind stress, $\tau$, remained $\leq 0.2$ N m$^{-2}$ and rainfall was negligible, except during day 77 when light showers contributed to maintaining $J_b \leq 1 \times 10^{-7}$ W kg$^{-1}$. After crossing an oceanic front which $T_{\text{sea}}$ dropped by $\geq 2^\circ$C during the transit from 30°N to 28°N, $\Delta T = T_{\text{sea}} - T_{\text{air}}$, where $T_{\text{air}}$ is the air temperature, remained approximately $2^\circ$C during nighttime, enabling a weakly positive $J_b$ during day 76. The $T_{\text{air}}$ exceeded $T_{\text{sea}}$ during the following daytime for the only time during the cruise, however, and in conjunction with the absence of significant destabilizing $\tau$ permitted a cruise minimum (ignoring rainfall) $J_b = -5 \times 10^{-7}$ W kg$^{-1}$.

B) Rainfall increased during day 78 to nearly 75 mm hr$^{-1}$ and $\tau$ increased slightly but remained $\leq 0.3$ N m$^{-2}$. Despite the rain falling during nighttime, when $\Delta T \sim 2^\circ$C, the stabilizing influence of the rainfall is identified by largely negative $J_b$, on day 78.

C) Wind stress $\tau$ peaked at $t = 79.6$ when the cessation of rainfall brought an increase of wind speed to 17.5 m s$^{-1}$ corresponding to $\tau = 0.64$ N m$^{-2}$. The increase in $\tau$ was accompanied by a drop in $T_{\text{air}}$ of $\geq 4^\circ$C, forcing the cruise maximum $J_b = 3.1 \times 10^{-7}$ W kg$^{-1}$.

D) Calm conditions with zero rainfall followed the storm and persisted through the restratification period (groups 64–79). Nighttime (destablizing) $J_b$ stayed $\geq 2 \times 10^{-7}$ W kg$^{-1}$ for the remainder of the cruise due to large $\Delta T$ despite negligible $\tau$. During daytime, $T_{\text{air}}$ increased by $\geq 2^\circ$C on days 80 and 81 ensuring that (stabilizing) $J_b$ approached $-5 \times 10^{-7}$ W kg$^{-1}$ during the restratification period itself when the weak $\tau$ promoted the stabilization of the SML.

b. Vertical response of the surface mixed layer

The 1D response of the SML is discussed below in terms of $\sigma_b$ (Fig. 3b), $N^2$ calculated over $\Delta z = 4$ m (Fig. 3c), and $\epsilon$ (Fig. 3d).
A) During the calm that preceded the storm, numerous horizontal density gradients were sampled within the SML. The SML base varied in depth between 20 and 80 m during the large-scale survey (right cross in Fig. 1) and the large star immediately following it at the beginning of observations at 28°N. Turbulent mixing was weak with due to light winds. The moderately destabilizing nighttime promoted weak convection that first had to penetrate the diurnal thermocline, revealed in MMP profiles at 10 m and thus above the range of SWIMS data. The increase in at t = 77.5 reinforced nighttime convection and elevated to 50 m, matching and thus indicative of wind-forced mixing.

B) The SML freshened due to the rainfall on day 78, decreasing from 24.625 kg m$^{-3}$ to 24.55 kg m$^{-3}$. A near-surface (±20 m), strongly stratified (N$^2$ ≥ 10$^{-4}$ s$^{-2}$) halocline formed that insulated the remainder of the SML from the destabilizing influences of nighttime convection, wind action, surface waves, and possibly the ship’s wake, restricting to depths ±10 m.

C) The halocline deepened and weakened with increasing as followed the base of the actively mixing layer to which overturning penetrated. Vertical turbulent entrainment arising from shear instabilities likely eroded the base of the halocline further as critical Richardson numbers, Ri = N$^2$/(∂u/∂z)$^2$ ≤ 0.25, were achieved. By t = 79.9, the SML was vertically homogenous and extended to ~75 m. At this point, the SML conditions in which the restratification that we study below took place had been established. Density increased to 24.58 kg m$^{-3}$ and cruise maximum was achieved due to large and . As t dropped at the end of day 79, mixing ceased during daytime but resumed with the following nighttime as convection resumed. Significant lateral density gradients were still present near the float. During group 63, increased suddenly from 24.575 to 24.63 kg m$^{-3}$ as the ship crossed a front before dropping back to 24.6 kg m$^{-3}$ at which it remained within ±0.02 kg m$^{-3}$ for the remainder of the record;

D) The L$_o$ decreased to ±10 m as t became negligible midway through day 80 and convection dominated vertical mixing, in direct contrast to the wind-dominated mixing regime during the preceding days. Stratification had been eradicated by the earlier storm and nighttime convective overturning with reached the top of the seasonal thermocline at 70 m. Mixing continued until

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**FIG. 2.** Meteorological variables measured aboard the R/V Wecoma at 28°N; (a) rainrate (mm h$^{-1}$) and wind stress, τ (N m$^{-1}$), (b) air and sea temperature, and (c) buoyancy flux, J$_b$ (W kg$^{-1}$). The times of each group are indicated along the top of (a) and the restratification period, between groups 64 and 79, is highlighted. Negative J$_b$ corresponding to stabilizing conditions are shaded in (c).
group 69, $t = 80.82$, approximately 1.9 h after $J_b$ became stabilizing. The lag is double the period of a convective eddy, $(D^2/J_b)^{1/3} = 0.9$ h, where $D$ is the nighttime SML depth and $J_b$ is the nighttime buoyancy flux (Brainerd and Gregg 1993b). On the resumption of nighttime overturning on day 81, 5.3 h passed before $\sigma_\theta$ below 15 m increased above daytime levels. The time for overturning to extend to the seasonal thermocline also took considerably longer than the previous nighttime (Fig. 3c).

4. 4D observations of the SML

We focus our attention here on groups 64–79, $t = 80.41–81.80$, for which we have an uninterrupted series of groups illustrating the mixing and restratification of the SML throughout the diurnal cycle. The restratification became a 4D process due to the presence of a weak lateral density gradient, responsible for the vertical banding over time of $\sigma_\theta$ in Fig. 3 when the data are plotted as a linear time series. Given the dominant impact of the diurnal cycle on the observations, $T_0 (t = 80.73)$ and $T_1 (t = 81.11)$ mark when $J_b$ turned negative and positive at the beginning and end of daytime during the surveys we are considering. For brevity we refer only to the density and velocity fields from groups 64, 68, 70, 73, 76, and 78, as these are representative of the SML evolution through all the stages of the diurnal cycle.

a. Density field

Data during each group are gridded into volumes of a Lagrangian coordinate system centered on the float position. An inherent assumption is that the thermohaline and velocity fields did not change significantly during the 2–3 h required to complete each group. Figure 4 shows the density evolution during representative groups.

- **GROUP 64** ($T_0 \sim 7.7$ h, i.e., 7.7 h before sunrise): In the middle of nighttime the vertical isopycnals were aligned in an approximately southwest to northeast direction, defining a horizontal density gradient within the SML with density decreasing toward the northeast. Convection mixed the SML to $\sim 70$ m, where the base of the SML was apparent as a sharp transition in isopycnal inclination from vertical to horizontal.
GROUP 68 ($T_o + 0.3$ h): Just after the onset of daytime and the cessation of convection, the isopycnals began to tilt away from the vertical. Smaller than previous bowties, group 68 only resolved the 24.6, 24.605, and 24.61 isopycnals but nonetheless indicated that during the 8 h since group 64, the isopycnal surfaces moved $\sim$1.5 km northeast toward lighter water as convection cooled the SML. The first signs of tilting of isopycnals are evident as the displacement of surface water toward the southwest (toward increasing density) while in the lower half of the SML the 24.605 and 24.61 isopycnals displayed a slight movement to the northeast.

GROUP 70 ($T_o + 5.2$): The entire SML had been restratified. The 24.61 kg m$^{-3}$ isopycnal extended across the entire base of the SML but detached from the thermocline at $\Delta x \sim -2$ km and rose to $\sim$40 m at the western edge of the survey area.

GROUP 73 ($T_1 + 1.9$ h, i.e., 1.9 h after sunset): The magnitudes of isopycnal displacement at each depth, and thus their angles with respect to the vertical, increased throughout the daytime. This is exemplified by the 24.59 kg m$^{-3}$ isopycnal that was initially $\sim$1 km to the east of the float at the surface, but by the beginning of nighttime outcropped 1 km to the west of the float. An eastward flow at the base of the SML was implied by the continued displacement of the densest water during groups 70 → 74. Initially, during

Fig. 4. Observed displacement ($\Delta x, \Delta y$) of isopycnal surfaces relative to the float (at $x, y = 0$) for selected groups during the restratification period. Isopycnals are plotted between 24.59 → 24.61 kg m$^{-3}$ at 0.005 kg m$^{-3}$ increments (labeled in group 64). The black arrow at the surface indicates north, the green circles and black diamonds the beginning and end of the groups, respectively, and the red dots indicate the position of each SWIMS profile between. The time of each group is indicated at the bottom of each panel.
group 64 the 24.605 kg m$^{-3}$ isopycnal intersected the thermocline at $\Delta x \sim -2.5$ km, but by group 73 the intersection had moved east to $\Delta x \sim 2$ km, a total displacement of $\sim 4.5$ km. Entrainment of dense thermocline water may have had a contributory effect, but the weak winds would diminish the impact according to Brainerd and Gregg (1993a).

- **GROUP 76**: The effects of nighttime convection became apparent as the upper half of the SML had been vertically mixed and isopycnal surfaces had returned to vertical. The data presented in Fig. 4 are taken from SWIMS, which only reached 15-m depth and did not resolve the diurnal thermocline that convection had to erode before the vertical mixing driven by a surface buoyancy flux could mix the more weakly stratified layer beneath. It was therefore not until group 75, and more noticeably during group 76 ($T_1 + 8.4$ h), that the uppermost surface waters became homogenized, while the lower half of the SML remained stratified. The depth to which convective mixing (wind stress was negligible) penetrated during the deepening of the active mixed layer was uniform in the horizontal.

- **GROUP 78**: At the end of nighttime on day 81, the SML was vertically homogenous and all isopycnals were vertical. The 24.60 kg m$^{-3}$ isopycnal was oriented SE–NW as during group 64 the previous nighttime but was positioned where it had intersected the thermocline during group 76, despite outcropping at the surface $\sim 2$ km to the west of the float. Thus the combined effects of the restratification during daytime and the ensuing nighttime convective mixing had the effect of translating the isopycnals to the northeast (toward decreasing density) relative to the previous nighttime.

b. **Velocity field**

Following the storm, velocities within the SML were dominated by the superposition of a near-inertial wave and a steady eastward flow, both of amplitude 0.2 m s$^{-1}$. Although the velocities associated with both components were almost slablike, we examine here the relationship between the residual velocity and the isopycnal displacements following subtraction of the depth-mean velocity within the SML.

To determine an appropriate depth range within which to calculate the depth-mean velocity while not including contaminating velocity signatures from the underlying thermocline (increased shear exists $\sim 10$ m above the base of the SML when defined by a $\sigma_0$ threshold), we define a lower limit as the maximum depth, SML$_{\text{shear}}$, at which shear-squared $S^2 = (\partial u/\partial z)^2 + (\partial v/\partial z)^2$ remained $\leq 10^{-5}$ s$^{-2}$. The depth mean for each velocity component is then computed at each time step between 10 m and SML$_{\text{shear}}$ and extracted from the total velocity. We henceforth consider only the residual in each velocity component, $U$ and $V$.

Figure 5 illustrates the relation between the sheared residual velocities and the orientation of the isopycnals during the six groups shown in Fig. 4. Panels represent west-to-east (left) and south-to-north (right) sections of $\sigma_0$ with the $U$ and $V$ residual velocity components, respectively, overlain. The paths through the float were chosen because every group passed the float in the north–south and east–west direction and thus interpolation errors are negligible.

The $U$, $V$ were in the direction of isopycnal displacement during daytime from group 68 until group 73 ($T_1 + 1.9$ h, i.e., 1.9 h after sunset), when both components rotated to a direction that assisted in the destabilization of the stratification. A weak westerly component remained at the surface but by group 76 had been eradicated, presumably by the homogenization of momentum in the upper half of the SML by nighttime convection. It should be noted, however, that isopycnals were oriented from northwest to southeast (Fig. 4) and the impact of advection on isopycnal displacement is dependent on the angle between the velocity vector and isopycnal. Largest isopycnal displacements are forced by northeast and southwest flows, whereas northwest and southeast flows are along isopycnals and have no impact on isopycnal displacement. On this basis, forcing during group 69 had the greatest impact on restratification and during group 74 on destabilizing the SML if we assume a constant orientation of the isopycnals in the horizontal.

The largest velocities in both components were observed between groups 70 and 73 when stratification was greatest and $U$, $V$ were in the direction of isopycnal displacement. Throughout the observations, maximum residual velocities were observed below the 24.605 kg m$^{-3}$ isopycnal and were at least twice those at the surface. During groups 64, 68, 76, and 78 when convective mixing of both density and momentum was either active or recent, $U$, $V$ were typically weaker ($\leq 3$ cm s$^{-1}$) and had a less coherent vertical structure than groups 70 and 73 when a 180° phase difference between surface velocities and those at the base of the SML was observed. The $U$ component during group 73 and $V$ component during group 76 do not exhibit the 180° phase difference, however, and their vertical structure is indicative of clockwise rotation with depth of the residual velocity vector during groups 73 and 76. Residual velocities were minimal at a mid-SML depth of $\sim 40–45$ m and coincided with an apparent pivot point for isopycnal displacement during restratification, illustrated in
The magnitude of isopycnal displacement achieved by residual shear at each depth is calculated by constructing a group mean profile of vertical shear. The shear components in each depth bin are averaged over the duration of the group to provide a single representative shear profile. Each depth bin is then integrated.
over the group duration to provide the total displacement (Fig. 6). The velocity is assumed to be nondivergent over the horizontal extent of the survey area. Displacements at the surface (8 m) are typically ~250 m, approximately half the ~500 m displacements near the SML base (72 m). Confirming the observed velocity minima in Fig. 5, advective displacements are approximately zero between 40 and 50 m.

The inertial rotation of the shear is confirmed by backrotating the integrated residual velocity vector by

\[
\bar{U} = \bar{u} + i\bar{v} = (u + iv)e^{i(f_o(t - t_o))},
\]

where \( u, v \) are the east and north components of horizontal displacement, respectively, \( f_o \) is the local inertial frequency \((6.28 \times 10^{-2} \text{ s}^{-1})\), \( t \) is time, and \( t_o \) is the midpoint of group 64. For purely inertial shear, the backrotated vectors in Figs. 6c,d would be orientated in the same direction as group 64 at all depths. This is the case at the surface and toward the base of the SML during all groups with some deviation at the surface during groups 76 and 78 that may be due to the coarse time resolution of the approximation, whereby \( t \) is defined simply as the midpoint of each group. All groups demonstrate a 180° phase difference between the surface and base of the SML. During groups 73 to 76, and to a lesser extent group 70, when stratification was well established, displacement vectors rotated clockwise with depth with a minimum in velocity at 45 m. In contrast, the shear vectors during the weakly stratified groups maintained a constant heading but decayed in amplitude toward ~45 m, below which amplitude steadily increased toward the SML base in the opposite direction.

5. Mechanisms for restratification of the SML: 4D observations versus model simulations

In this section we quantify the proportion of observed restratification due to solar insolation, vertically
sheared advection due to a near-inertial oscillation, and spatial inhomogeneities in the thermohaline field passively advected into the study region. It is hereafter understood that restratification by the thermohaline inhomogeneity is due to passive advection and that the shear arises from the near-inertial oscillation alone. Beginning with a simple diagnostic model whose initial conditions match those of the SML, we restratify the model SML by first heating the SML with the observed incoming shortwave solar radiation measured aboard the R/V Wecoma. Second, the lateral density gradient is differentially advected with depth by the observed shear measured by the VM-ADCP, and third we account for a salinity front that was advected into the sampling domain. The total restratification is assumed to be a linear supposition of each of the processes, all of whose forcing we directly measured.

a. SML model configuration

The initial state of the model density field is defined according to the observed SML during group 64. A vertically homogenous SML extends to 75 m above a thermocline with constant $N^2$. A lateral density gradient of $2.5 \times 10^{-6}$ kg m$^{-4}$ linearly decreases $\sigma_\theta$ from the southwest to the northeast. The model dimensions are $8 \, \text{km} \times 8 \, \text{km}$ in the horizontal and $100 \, \text{m}$ in the vertical, with corresponding resolution of 100 and 1 m, respectively. Figure 7 compares a slice of the model at the surface with the observed density field during group 64. During the model comparisons the following assumptions are made:

- heating by insolation is horizontally uniform;
- residual velocities, and thus group-average vertical shear, are horizontally uniform within each group, as are the resulting displacement profiles; and
- the total stratification is a linear supposition of the stratification caused by each of the different mechanisms.

These assumptions constrain the model to be relatively simplistic but are also quite realistic when considering the small size of the survey areas across which little spatial variability would be expected unless submesoscale dynamics have a significant impact on the evolution of the SML. We calculate $N^2$ due to solar insolation, $N^2_{\text{solar}}$, to sheared advection, $N^2_{\text{shear}}$, and to the salinity front, $N^2_{\text{salinity}}$, and we sequentially sum their effects to compare with the observed stratification, $N^2_{\text{obs}}$. The relative significance of each process is calculated as a percentage of $N^2_{\text{obs}}$. Here $N^2$ for all cases is
computed over 10-m vertical intervals in addition to a mean SML value computed between 15 and 65 m ($\delta z = 50$ m), referred to hereafter as the 50 m $N^2$.

b. Modeled restratification comparison with observations

1) SOLAR INSOLATION

SML restratification by insolation is achieved by the divergence of the shortwave penetrative solar radiation, $Q_{\text{shortwave}}$, one of five components of the total surface heat flux, $Q$ in Eq. (1). The vertical distribution of $Q_{\text{shortwave}}$ throughout the SML at each time step is calculated by multiplying the time series of surface incoming $Q_{\text{shortwave}}$ with the depth-dependent transmission function, determined by Carter Ohlmann with a SeaWIFS Profiling Multichannel Radiometer (SPMR). After a period $\Delta t$, the temperature following warming at each depth, $T_{\text{sml}}^+$, is given by

$$T_{\text{sml}}^+ = T_{\text{sml}} + \frac{\Delta t}{\rho_o C_p h_{\text{sml}}} \left[ Q_{\text{sw}}(0) - Q_{\text{sw}}(h_{\text{sml}}^+) \right], \quad (6)$$

where $T_{\text{sml}}$ is the initial temperature at each depth, $\rho_o$ is the reference density of seawater, $C_p$ is its specific heat at constant pressure, $Q_{\text{sw}}(0)$ is the penetrative component of the heat flux, and $Q_{\text{sw}}(h_{\text{sml}}^+)$ is that part of it escaping the base of the volume. The change in temperature due to warming by solar insolation is then applied to the linearized equation of state for seawater to obtain the corresponding density change over depth.

The effect of the vertically dependent heating is to displace the initially vertical isopycnals westward (toward negative $\Delta x$) in the same direction as observed in Fig. 4 (Fig. 8). Displacement decreases with depth such that after 11 h isopycnals at the surface have been displaced nearly 4 km from their initial position, but $\approx 500$ m at 70 m (Fig. 9). Displacements due to solar warming are west in the direction of increasing density, giving the appearance of slumping isopycnals. The isopycnal displacements are due to a change in the local temperature–salinity (T–S) relationship, however, and not horizontal advection as would be the case for slumping.

Large fluctuations in the 50-m $N_{\text{obs}}^2$ prior to restratification reflect convective mixing during nighttime but also the spatial variability in stratification during the small-scale surveys (Fig. 10b). With the onset of daytime warming, convective overturning is shut down (Fig. 3d) and restratification is apparent as the growth in $N_{\text{obs}}^2$ at all depths. Consistent with predicted restratification by insolation, $N_{\text{solar}}^2$, the growth in $N_{\text{obs}}^2$ is largest and most rapid between 15 and 25 m, increasing from $3.16 \times 10^{-7}$ to $5.01 \times 10^{-6}$ s$^{-2}$ over 12 h. In contrast, $N_{\text{obs}}^2$ between 45 and 55 m increases by less than an order of magnitude but exhibits large fluctuations over time (note that while the data are presented as a time series the ship is constantly moving around the drogue in a bowtie pattern and thus variability may be temporal or spatial). When fluctuations in $N_{\text{obs}}^2$ between 45 and 55 m peak, the estimated $N_{\text{solar}}^2$ is half a decade smaller between 45 and 55 m than between 15 and 25 m, but during the troughs the estimate almost matches $N_{\text{obs}}^2$. As expected for the decay of solar penetration with depth, $N_{\text{solar}}^2$ is more than a factor of 5 less at 45–55 m than at 15–25 m. At $t = 80.995$, however, $N_{\text{obs}}^2$ at both 15–25 m and 45–55 m are equal and correspond to one of the peaks in $N_{\text{obs}}^2$ at 45–55 m. Thus additional factors elevate $N_{\text{obs}}^2$ in the lower half of the SML beyond that expected for insolation alone. As would be expected for convective mixing, the nighttime drop in $N_{\text{obs}}^2$ at 15–25 m occurs before that at 45–55 m as the vertical mixing takes time to penetrate downward. The increase in $N_{\text{obs}}^2$ at $t = 81.3$ during nighttime is not consistent with any aspect of the diurnal cycle, however, and implies a spatial anomaly in the thermohaline field. Between 45 and 55 m, $N_{\text{obs}}^2$ only drops significantly at $t = 81.6$, just a few hours before the following sunrise.

The proportion of restratification that can be accounted for by solar insolation alone, $N_{\text{solar}}^2/N_{\text{obs}}^2$, thus decreases with depth (Fig. 10c) from $\approx 90\% \pm 20\%$ between 15 and 25 m to $\approx 50\%$ between 55 and 65 m. Between 15 and 65 m, $N_{\text{solar}}^2/N_{\text{obs}}^2 \sim 65\% \pm 20\%$. The peaks in $N_{\text{obs}}^2$ represent troughs in $N_{\text{solar}}^2/N_{\text{obs}}^2$. The amplitude of the fluctuations is $\approx 20\%$ between 15 and 65 m but $\approx 40\%$ between 45 and 55 m. The fluctuations in $N_{\text{solar}}^2/N_{\text{obs}}^2$ are largely in phase at all depths except for the latter part of the daytime when fluctuations at 55–65 m occur at slightly different times to those at shallower depths.

2) DIFFERENTIAL HORIZONTAL ADVECTION BY VERTICAL SHEAR: INERTIAL STIRRING

Isopycnal displacements due to insolation are in the direction of increasing density only (i.e., southwestward). Observed isopycnal displacements in the lower half of the SML are directed northeastward, however, bringing dense water beneath light water and enhancing stratification above that due to insolation alone. The restratification caused by the sheared residual velocities is computed during each group by differentially advecting with depth the horizontal density gradient by the distance determined by integrating the group-mean velocity profiles in Fig. 6. Formally, the new density at each time step is given as
\[
\rho(z) = \rho_0(z) + \int_0^{t} u(z) \, dt \cdot \nabla \rho,
\]

where \( \rho_0 \) is the density from the previous time step. The decrease in \( \rho \) due to solar warming at each depth can be added to the right-hand side of Eq. (7) to provide the total change in density.

The resulting density field predicted by the model when shear-driven advection of the preexisting lateral density gradient and heating by solar penetrative radiation are taken into account is in good qualitative agreement with observations (Fig. 11). In contrast to the unidirectional displacement produced by solar warming, inclusion of the advective component drives a northeastward flow of dense (\( \sigma_b \approx 24.595 \text{ kg m}^{-3} \)) water in the lower half of the SML. The phase of the inertial
shear ensures that the surface advective displacements during daytime are in the same direction as isopycnal displacements due to solar insolation. During groups 70 and 73, the modeled density field tends to slightly overestimate the surface density in the northeast while underestimating $\sigma_\theta$ in the lower half of the SML in the south and west. The modeled density field does not change significantly between groups 70 and 73 as the heat flux due to insolation is reduced to zero during nighttime and the phase of the inertial shear causes displacements along isopycnals, with no effect on $\Delta \varphi$.

The underestimation of $\sigma_\theta$ toward the base of the SML is more pronounced during group 73 in which isopycnals are more horizontal in the observations than the model. By group 76 the effects of nighttime convective cooling and the resultant vertical mixing are apparent in the observations. The focus of the paper is on the restratification rather than vertical mixing, and we do not include the effects of convection in the model. Vertical turbulent entrainment is also neglected as previous studies (Brainerd and Gregg 1993a) indicate it to be negligible when $\varpi$ is weak.

To compute the effect of advection on stratification, the density changes at each depth were interpolated to match the time series of insolation and the resulting stratification, $N^2_{\text{shear}}$, added to $N^2_{\text{solar}}$ (Fig. 12). The smoothly varying inertial rotation of the shear ensures that the interpolation does not introduce significant error. As residual velocity magnitudes were largest at the base of the SML where the effects of insolation were smallest, the largest impact on $N^2$ in the model is in the lower half of the SML. The most significant aspects of the comparison between the modeled and observed $N^2$ when advection is added to solar effects are as follows:

- $(N^2_{\text{solar}} + N^2_{\text{shear}})/N^2_{\text{obs}}$ in the lower half of the SML $(\approx 35 \text{ m})$ is increased by $\sim 40\%$, reaching $100\%$ at $t = 80.87$ between 55 and 65 m at the beginning of daytime restratification.
- The cumulative growth of $N^2_{\text{solar}}$ is added to an initial value of $2.2 \times 10^{-7}$ in (b), indicative of the value of $N^2_{\text{obs}}$ between 15 and 65 m at the beginning of daytime restratification.
- Percentages in (c) are only calculated for daytime, delineated by the vertical dashed lines between which $J_b \leq 0$. 

Fig. 10. (a) Buoyancy flux, $J_b$, (groups indicated along top axis), (b) observed (obs) and modeled (solar) buoyancy frequency squared, $N^2_{\text{obs}}$ and $N^2_{\text{solar}}$, respectively, between depth intervals indicated, and (c) the percentage of $N^2_{\text{obs}}$ attributable to $N^2_{\text{solar}}$. The cumulative growth of $N^2_{\text{solar}}$ is added to an initial value of $2.2 \times 10^{-7}$ in (b), indicative of the value of $N^2_{\text{obs}}$ between 15 and 65 m at the beginning of daytime restratification. Percentages in (c) are only calculated for daytime, delineated by the vertical dashed lines between which $J_b \leq 0$. 

Fig. 9. Cumulative horizontal isopycnal displacements, $\Delta x$, over time due to warming by insolation. The surface heat flux is constant over the model domain, as is the transmission profile used. Therefore, displacements at each depth for a given time are the same at all horizontal positions.
as the enhanced stratification that was observed in the lower half of the SML is accounted for in the model, the 50-m $\left( N_{\text{solar}}^2 + N_{\text{shear}}^2 \right)/N_{\text{obs}}^2$ increases from 50% to 80% when only insolation was included to $\sim 100\% \pm 20\%$ before $t = 80.93$ and $\sim 70\% \pm 10\%$ thereafter;

- surface values of $\left( N_{\text{solar}}^2 + N_{\text{shear}}^2 \right)/N_{\text{obs}}^2$ ($\leq 35$ m) do not differ significantly with the addition of advection, typically increasing by $\leq 10\%$;

- the effect of shear on restratification is most pronounced when $t \sim 80.8-80.95$ (during group 70), after which the shear is orientated along isopycnals and

FIG. 11. (left-hand column) Density field predicted by advection isopycnals by inertial shear within the SML and heating by the solar penetrative radiation, and (right-hand column) observations for the respective group.
thus has no effect on stratification. Thereafter the shear acts to destabilize the SML.

3) Uncompensated thermo-haline variability

At approximately $t = 80.95$ the percentage of observed stratification accounted for by the model with the effects of insolation and shear included dropped by $\approx 40\%$ at 10-m scales below 35 m (Fig. 12). The SML-average $(N_{solar}^2 + N_{shear}^2)/N_{obs}^2$ was reduced from 100% to 60%–80% and exhibited quasi-periodic fluctuations of 20% that increased to $\approx 40\%$ at 10-m length scales between 35 and 55 m. The fluctuations were caused by the periodic sampling of a largely uncompensated salinity front whose origin can be traced back to the rainfall on day 78. The front, initially located 5 km west of the survey area during group 64, remained outside the sampling area until group 70 when the phase of the inertial shear advected the lower half of the front eastward inside the western edge of the float-centered survey region (Fig. 13). The vertically sheared residual velocities advected the lower half of the front farther east while rotating clockwise. By group 78 the shear had rotated to a westerly heading and advected the front back outside the survey area.

The shear tilted the salinity front, thereby generating a vertical gradient in salinity, which, not being balanced by temperature, contributed to $N^2$. As is apparent in Fig. 13, the inclined front was constrained at the western extremity of the sampling area and was thus only periodically sampled. To investigate whether the front was responsible for the fluctuations due to its periodic sampling and overall reduction in $(N_{solar}^2 + N_{shear}^2)/N_{obs}^2$ below 35 m, we approximate the contribution made by the vertical salinity gradient to $N_{obs}^2$ and add it to $(N_{solar}^2 + N_{shear}^2)$.

The relative effects of temperature and salinity variations on the density perturbation, $\rho - \rho_o$, were assessed using the linearized equation of state

$$\rho = \rho_o \left[ 1 - \alpha_o (\theta - \theta_o) + \beta (S - S_o) \right],$$

where $\theta$ and $S$ are potential temperature and salinity, respectively, $\alpha_o$, $\beta_o$ are their respective expansion coefficients, and the subscript designates mean values computed for each group duration. The temperature and salinity terms represent the perturbations in the group-mean density due to spatial variability in $\theta$, $S$.

The impact of the salinity front on $N_{obs}^2$ during groups 64–79 was estimated by calculating the density perturbation due to the salinity term in Eq. (8) on the premise that $R_{ss} = \beta(S - S_o)/\alpha(\theta - \theta_o) = 2$ in the sheared front (Fig. 14d). This will slightly overestimate the contribution of the salinity anomaly to $N^2$ as some of its contribution to $\rho - \rho_o$ will be balanced by temperature (Fig. 14c) but whose inclusion complicates the interpretation because of the temperature variability associated with the thermocline depth. This is discussed below. Here, $N^2$ due to the salinity front $N_{salinity}^2$ is then calculated using $\partial [\rho_o \beta_o (S - S_o)]/\partial z$. The inclination of the salinity front mainly influenced $N^2$ below 35 m, and so its contribution to the observed stratification and the sum of all three effects $(N_{solar}^2 + N_{shear}^2 + N_{salinity}^2)$ is
only shown below 35 m. The 50-m \( N^2 \) is defined as the sum of \( N^2_{\text{salinity}} \) between 35 and 65 m and \( N^2_{\text{solar/shear}} \) between 15 and 65 m.

The final comparisons between the modeled and observed \( N^2 \) indicate that at the 50-m scale between 15 and 65 m, the combined effects of insolation, shear-driven advection, and the inclusion of the effects of the salinity front, deformed by shear, predict \( N^2_{\text{obs}} \) to within 20% (Fig. 15). At 10-m scales, however, the proportion of \( N^2_{\text{obs}} \) explained by the model varies with depth, with the modeled \( N^2 \) overestimating \( N^2_{\text{obs}} \) between 35 and 55 m, except for briefly at \( t = 80.975 \), and underestimating it between 55 and 65 m. By accounting for the uncompensated variability in salinity, the periodic fluctuations in the ratio of modeled to observed \( N^2 \) are reduced to an extent between 35 and 45 m and 45 and 55 m but are still apparent between 55 and 65 m. In the following section this will be shown to be due to the variability in the depth of the thermocline that toward the south of the survey area is \( \geq 10 \) shallower than to the west of the surveys and thus increases \( N^2_{\text{obs}} \) between 55 and 65 m.

6. Implications for modeling

Our four-dimensional observations demonstrated the qualitative effect of a lateral density gradient on the restratification of the SML by insolation and the impact
this can potentially have on the misinterpretation of measurements that do not resolve the horizontal dimension. Quantitatively, restratification by insolation was consistent with previous estimates, but the identification of the impact of near-inertial shear on the vertical structure of the SMLs containing lateral density structure offers hope for improving SML predictions, particularly in the lower half where the effect of insolation is reduced. Here we discuss the implications of our results for modeling the SML beyond the traditional 1D representations, focusing on parameterizing restratification by near-inertial shear, its potential generation mechanisms and subsequent interaction with both the mesoscale fields and the diurnal cycle, and implications for submesoscale dynamics within the SML.

a. Restrification by near-inertial shear

Tandon and Garrett (1994, 1995) derive a metric for the restratification of an SML containing a horizontal density gradient by the geostrophic adjustment of initially vertical isopycnals. Allowing for time dependence, a fluid initially at rest with vertical isopycnals above a strongly stratified thermocline experiences inertial oscillations (Rhines 1988) that cause the isopycnals to pivot about their average position:

Fig. 14. (a) Temperature and (b) salinity contribution to the perturbation density as given in Eq. (8), (c) their sum, and (d) the ratio of the salinity to the temperature contribution, \( R_{S/T} \), during group 73. The spatial scale and coordinates are the same as in Fig. 13. In (d), red shading corresponds to regions dominated by salinity and blue to temperature-dominated regions. Note that in (d) the eastern portion of the survey is dominated by salinity, but the net effect of thermohaline perturbations in (c) is very small and uniform in the vertical.

Fig. 15. Ratio of the sum of the stratification due to solar insolation, advection, and horizontal inhomogeneity in the salinity field during daytime, to the observed stratification, \( N^2_{obs} \). Only depths of 35–65 m are shown because the salinity anomaly was constrained below 35 m.
They suggest, as an extension to their work, a study of turbulent dissipation and eradicates the restratification to the diurnal cycle when nighttime convection drives effects, therefore precluding the extension of their model shear. Tandon and Garrett (1994) ignore dissipative overturning overcomes any stratifying influence of the horizontal density structure of the SML in terms of wavelength and magnitude and can thus be applied to Eq. (10) over large regions.

The state of the SML prior to storm mixing may be important to the eventual adjusted state. Initial conditions corresponding to Case III of Tandon and Garrett (1995) specify a front in thermal wind balance that is homogenized by the impulsive mixing event. Following adjustment, the thermocline is differentially displaced in the vertical on either side of the initial position of the horizontal density gradient as horizontal divergence causes an upward motion on the side of the inertial circle to which the horizontal velocity vectors are directed. This is in qualitative agreement with the observed thermocline shoaling at the southern edge of the survey area between groups 70 and 77 (t ~ 80.9–81.6) when the residual velocity at the SML base was directed southward. The upward advection of the thermocline may explain the underestimation of $N^2$ between 55 and 65 m in our model that assumes the position of the thermocline to remain constant at 75 m. The decrease in depth of the thermocline in the observations by a mechanism not accounted for in the model (no horizontal divergence and thus vertical motion is allowed in our model) enhances $N^2_{\text{obs}}$ above 65 m and subsequently reduces the ratio of modeled to observed $N^2$.

### b. Origin and modification of near-inertial shear

The predicted restratification in Eq. (10) closely matches that achieved by shear in the observations, yet mechanisms other than geostrophic adjustment may generate sheared near-inertial oscillations. The storm that homogenized the SML on day 79, setting the initial conditions for the geostrophic adjustment problem above, may also have directly generated the wave. The meteorological data indicate that the storm was a cold front, typhified by a sudden increase in wind stress as wind speed exceeded 17.5 m s$^{-1}$, rapid clockwise veering of the wind vector over ≤1 day, and a large drop in air temperature (Fig. 2). Such properties make cold

\[ \xi = x - \frac{b_x(z + H/2)}{f^2} (1 - \cos ft), \]  
\[ N^2 = \frac{b_x^2(1 - \cos ft)}{f^2}, \]  
with the assumption that $b_x$ is constant, consistent with the assumption made in our diagnostic model in the previous section. Equation (10) accurately predicts the maximum observed restratification of the SML due to shear ($N^2_{\text{shear}}$) at $t = 80.875$ (Fig. 16). The addition of $N^2 = 2.2 \times 10^{-7}$ observed at the beginning of daytime provides an artificially high estimate of $N^2 = b_x^2(1 - \cos ft)/f^2$ during nighttime in Fig. 16 when convective overturning overcomes any stratifying influence of the shear. Tandon and Garrett (1994) ignore dissipative effects, therefore precluding the extension of their model to the diurnal cycle when nighttime convection drives turbulent dissipation and eradicates the restratification. They suggest, as an extension to their work, a study of the effects of the diurnal cycle on the restratification achieved by the geostrophic adjustment. The results presented here provide observational evidence of the limiting effects of nighttime convection. Furthermore, the accurate predictions of Eq. (10) suggest that the restratification of the SML in regions of lateral density variability may be more accurately modeled given information of the magnitude of density gradients at given horizontal scales and knowledge of the near-inertial oceanic response to meteorological forcing. The latter is within current capabilities, while observations such as those in Hosegood et al. (2006) define the lateral density structure of the SML in terms of wavelength and magnitude and can thus be applied to Eq. (10) over large regions.

FIG. 16. The $N^2$ between 15 and 65 m due to shear ($N^2_{\text{shear}}$ above) and $N^2$ predicted by Eq. (10) (Tandon and Garrett 1994) for the time-dependent geostrophic adjustment of a horizontal density gradient. The phase of the geostrophic adjustment has been matched to the observed shear so that maximum $N^2$ occurs when the shear is directed exactly across isopycnals, corresponding to group 69. The observed initial $N^2$ of $2.2 \times 10^{-7}$ at the beginning of the daytime is added to both values of $N^2$, and therefore the geostrophic adjustment solution is maintained higher than the model predicts during nighttime ($J_b > 0$ and shaded in the figure) when $N^2 \to 0$. Note that the phase of the inertial shear in the studied case is such that restratification by inertial shear occurs during daytime and therefore constructively interferes with the diurnal cycle.
fronts efficient generators of near-inertial waves (Hobbs et al. 1980), with short horizontal scales given the requirement for wind forcing that the horizontal scale of meteorological forcing and oceanic response be similar. We lack the spatial data to elaborate on the horizontal scale of the near-inertial wave, however, and can only illustrate the suitability of the meteorological conditions for wind forcing as a potential alternative generation mechanism.

The present study was conducted at the subtropical front because of its perceived influence on dynamics. During observations in the same region, Kunze and Sanford (1984) observed an energetic, downwind-propagating, near-inertial wave group of vertical wavelength ~100 m whose properties were modified by interaction with the mesoscale vorticity associated with the alongfront jet. Gradients in vorticity occur where baroclinic alongfront jets flow along geostrophically balanced fronts in thermal wind balance, as typically found in frontal regions such as the STF. Near-inertial waves originating inside the negative vorticity trough on the warm side of the front are unable to propagate away and undergo several changes to satisfy the dispersion relation

\[ w_o = w - (k \cdot V) = f_{\text{eff}} + \frac{N^2_{\text{eff}} k_H^2}{2f k_z^2}, \]  

where the effective Coriolis frequency

\[ f_{\text{eff}} = f + \frac{1}{2} \left( \frac{\partial V}{\partial x} - \frac{\partial U}{\partial y} \right) \]  

is the planetary vorticity plus half the geostrophic vorticity, \( \zeta \). The across-front wavenumber decreases, and as it propagates downward it encounters an increasing \( f_{\text{eff}} \) due to the weakening with depth of the geostrophic vorticity. The vertical wavelength is reduced and the wave is amplified due to conservation of action flux, both modifications enhancing wave shear. Of paramount importance is the origin of the waves inside the negative vorticity trough, as waves beginning outside and propagating toward the trough will be reflected by the turning points on either side of the trough. Van Meurs (1998) also found the evolution of near-inertial currents to be strongly influenced by mesoscale vorticity, in particular, the rapid decay of near-inertial currents in regions of high-vorticity gradients and development of near-inertial shear due to vertical-mode separation, whereby lower modes propagate away faster. The amplitude of the near-inertial oscillation in our observations, whose vertical wavelength is estimated as ~120 m from the group-mean residual profiles and progressive vector plots, decayed rapidly from \( \geq 0.2 \) to 0.125 m s\(^{-1}\) over the four inertial periods until the end of observations at \( t = 83.8 \).

During the large-scale surveys (one north–south and one east–west leg, each of a 100-km length passing through the float) conducted during days 76 and 83 before and after the small-scale surveys, a partially compensated front was traversed ~10 km north of the float position. Eastward velocities in the alongfront jet were elevated by up to 0.3 m s\(^{-1}\) within ~\( \pm 20 \) km of the front, with negligible contribution from \( V \). The float was deployed on the warm side of the front where \( U \geq 0.2 \) m s\(^{-1}\) above background levels due to the jet and remained steady throughout the small-scale surveys. The local relative vorticity and effective Coriolis frequency \( f_{\text{eff}} \) were thus calculated for the north–south leg of the survey on day 83 from the \( \partial U/\partial y \) component of the relative vorticity only, assuming \( \partial V/\partial x \) was negligible in the vicinity of the front. A model inertial velocity of 0.125 m s\(^{-1}\) amplitude was extracted from \( U \) by fitting the model phase to observations to account for the temporal change in \( U \) during the north–south leg (Fig. 17).

The float was located within a large (~40 km) region of negative vorticity to the south of the front that reduced \( f_{\text{eff}} \) to 0.9f (Fig. 17). To the north of the jet, positive vorticity increased \( f_{\text{eff}} \) to \( \geq 1.1f \). Therefore, assuming local generation, justified by the in situ meteorological observations as discussed above, the near-inertial oscillation we observed may have been trapped inside the negative vorticity trough on the southern side of the front and modified in the manner described by Kunze and Sanford (1984). This explains the short vertical wavelength and (assumed) horizontal scale, the near-inertial shear (Kunze and Sanford 1984), and the rapid decay of near-inertial energy (Van Meurs 1998).

c. Application to broader contexts

The observations discussed in this paper represent specific instances of processes that under different environmental conditions may have greatly different effects on the SML structure.

- The latitude is of essential importance to the relative influence of the diurnal cycle and inertial motions. At 30°N the period of the diurnal cycle matches that of inertial motions and, depending on the phase of the inertial currents, the restratification effects can either constructively (matching phase) or destructively (180° phase difference) interfere with each other. Due to the phase of the inertial currents in the present case, advection reinforced isopycnal displacements due to solar insolation, but the difference in period between the inertial and diurnal cycles at 28°N...
means that the effects would become out of phase within a few days. In a similar manner, the presence of preexisting currents may influence the net effect of the inertial currents generated by a storm as previously noted by D’Asaro (1985). If the storm-generated wave has a phase that matches that of the preexisting currents, the net effect of advection on restratification can be enhanced, and vice versa.

- The total effect of inertial stirring and solar warming on the restratification of the SML is dependent on the water clarity and near-inertial vertical wavelength. The percentage of solar radiation at 6-m depth that reached 35 m, approximately where residual velocities were zero, dropped to 25%. At the base of the SML, where residual velocities were greatest, only 5% of the surface insolation penetrated, and the proportional importance of shear advection was considerably greater than the effects of insolation. A change in the vertical wavelength of the shear, and thus the depth at which residual velocities peak and tend to zero, or a change in the clarity of the water that dictates the depth to which insolation effects penetrate would introduce destructive interference at some depth within the SML regardless of the phase of the inertial and diurnal cycles. SMLs of different depths may be subject to additional dynamics, specifically density driven flows in deeper SMLs.

- By assuming a constant lateral density gradient and horizontally nondivergent flow, we neglected the possibility of vertical velocity playing a role in the dynamics. We are thus unable to directly compare our findings to those of Mahadevan and Tandon (2006), who attempt to diagnose the origin of strong vertical velocities arising from ageostrophic motion at a front in modeling results. The mean profiles of vertical shear constructed in the earlier section assumed that horizontal velocity was nondivergent throughout each group, yet the individual profiles suggested that a degree of divergence may have occurred and would indeed have been required if vertical advection of the thermocline occurred as discussed above. Divergence in \( U, V \) could be indicative of nonhydrostatic vertical velocities that represent the signatures of submesoscale vertical motion at fronts studied by Mahadevan (2006) and Mahadevan and Tandon (2006). Large differences were found between model results when horizontal resolution was decreased from 1 km to 500 m. The horizontal distance between profiles in the present field study was at least 350 m, and it is thus possible that we did not resolve the narrow regions of up- and downwelling associated with the ageostrophic motion in the model of Mahadevan (2006).

- The submesoscale frontal structures in Mahadevan (2006) result from instability of the frontal region when subjected to wind forcing. The wind during the storm on day 79 may have generated the frontal region that we observed throughout this paper by destabilizing the large-scale frontal region of the STF. Although inertial oscillations are not a significant part of the model solution of Mahadevan (2006), the ageostrophic conditions that lead to the development of submesoscale lateral density gradients may have,
in our case, initiated the conditions for geostrophic adjustment of the lateral gradients, which, applying the principles of Tandon and Garrett (1994), generate near-inertial oscillations. We note, however, that the shear in our observations contributes to an increase in relative vorticity, promoting ageostrophic conditions and the possible development of mixed layer instabilities (Boccaletti et al. 2007) as the Rossby increases toward \( O(1) \);

7. Conclusions

Four-dimensional observations of an \(~15 \text{ km}^2\) area of the surface mixed layer (SML) during 33 h found 60\% of daytime restratification to be attributable to insolation at the 50-m vertical scale of the SML. By observing the evolution of the three-dimensional density field, however, we were able to 1) detect the presence of a small-scale lateral density gradient within the SML, 2) observe its impact on the structure of the SML during restratification, and 3) identify and quantify the impact of two additional mechanisms that accounted for the remaining \(~40\%\) of restratification. The major findings of this research are as follows:

- insolation accounted for 60\% of the observed restratification during daytime at the 50-m vertical scale of the SML. At 10-m vertical scales, the percentage decayed with depth from nearly 100\% at the surface to \(~25\%\) toward the base of the SML at 55–65 m;
- due to the presence of a lateral density gradient, the vertical attenuation of insolation caused a unidirectional horizontal displacement of isopycnals in the direction of increasing density. The magnitude of horizontal displacement decayed with depth at a rate dependent on the vertical penetration of shortwave solar radiation. The three-dimensional isopycnal displacements thus took on the appearance of gravitational slumping but had no dynamic signature, and instead occurred as a result of the modification of the local \( T–S \) relationship;
- the SML was vertically sheared at near-inertial frequency, identified in the residual velocity after extracting the depth-mean velocity within the SML. The shear rotated clockwise at the inertial frequency and exhibited a 180\° phase difference between the surface and the bottom of the SML, where the magnitude of the residual velocity component was at least twice that at the surface and in the opposite direction. The residual current reversed about a middepth minimum in residual velocity at \(~40 \text{ m}\);
- isopycnals were advected horizontally by the shear and pivoted about the middepth residual minimum. Because of the near match between the local inertial period and the diurnal cycle, the phase of the near-inertial oscillation, and the orientation of the lateral density gradient, advection reinforced and enhanced restratification achieved by insolation. In contrast to the unidirectional displacement of insolation, however, the reversal of the residual velocity at middepth advected dense water in the opposite direction below 35 m, elevating \( N^2 \) in the lower half of the SML above that achievable by insolation. The proportion of observed restratification attributable to advection increased with depth from \(~10\%\) at the surface where insolation was dominant to \(~40\%\) near the SML base where insolation was weak and shear was strongest;
- the restratification attributable to shear matched that predicted for a vertically sheared near-inertial oscillation arising from the geostrophic adjustment of a lateral density gradient. The oscillation results from allowing for time dependence in the solution of the geostrophic adjustment process, but in the present case it may also have been directly generated by a storm 2 days earlier. The oscillation subsequently interacted with the mesoscale vorticity associated with the large-scale frontal region nearby. Having been generated in the negative vorticity trough of the alongfront jet on the southern warm side of the front, the oscillation was amplified and reduced in vertical wavelength as it attempted to propagate outside the trough; and
- the remaining \(~20\%\) of observed restratification below 35 m was attributed to an uncompensated salinity front that due to the shear was tilted in the vertical and advected into the sampling region. Its impact on restratification was spatially nonuniform and localized due to its confinement at the edge of the sampling region.

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