

Note on the Vertical Velocity and Diffusive Salt Flux Induced by Evaporation and Precipitation

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ABSTRACT

Some (not all) of the oceanographic literature slightly miscalculates the vertical velocity (w) and diffusive salt flux induced by evaporation (E) and precipitation (P) at the sea surface. Short, simple, physical derivations are presented to show that, for a sea surface $h = h(x, y, t)$ varying in space and time, 1) $w = \mathbf{V}_H \cdot \nabla h + \partial h / \partial t + \rho_F(E - P) / \rho$, where \mathbf{V}_H is the horizontal component of the aggregate parcel velocity, and ρ_F and ρ are the densities of freshwater and surface seawater, respectively; and 2) the vertical diffusive salt flux at the sea surface (whether molecular or turbulent) is $-\rho_F S(E - P)$, where S is the surface salinity.

1. Introduction

In stationary conditions at a level sea surface, the vertical velocity (w) that is induced by evaporation (E) and precipitation (P) is nearly $(E - P)$. In recognition that the mass flux into (or out of) the atmosphere is of freshwater alone, attempts have been made to improve the representation: $w = (E - P) / (1 - S)$, where S is the mass-fraction salinity, at the sea surface (e.g., Schmitt et al. 1989); or $w = \rho_F(E - P) / [\rho(1 - S)]$, where ρ is the mass density of the surface seawater and ρ_F is that of the evaporated or precipitated freshwater, taken the same for both (e.g., Stern 1975, p. 209; Warren 2006; Schmitt 2008). These expressions are in fact *worse* representations [owing to the factor $(1 - S)^{-1}$] because the derivations confused the aggregate vertical velocity of a seawater parcel (w) with that of its freshwater component (w_W).

The upward movement of seawater carries salt as well as freshwater to the sea surface. Because little crosses the surface or accumulates there, it is necessarily diffused downward. The authors cited earlier included another spurious factor of $(1 - S)^{-1}$ in their expressions for this diffusive flux.

It may not have been widely noticed that both Griffies (2004, p. 39) and Müller (2006, p. 143) derived the exact expression for w , without restricting themselves to a stationary, level sea surface. Both authors, however,

glossed over the separate contributions of freshwater and salt transports to the total mass transport. Moreover, Griffies' result is a side benefit of a general treatment of mass and volume budgets, which is unnecessarily cumbersome for reaching just this single result, and Müller's derivation reaches through several sections of his book.

The purpose of this note is to give short, simple, direct derivations of the expressions for w and the diffusive salt flux at a moving, sloping sea surface. Further comment is made as well on other derivations. Given the observational uncertainty in values of $(E - P)$, this fastidiousness will not be significant for oceanographic practice, but the resulting clarity may be thought helpful.

2. Derivations

By definition, the aggregate velocity \mathbf{V} of a seawater parcel is its total momentum density divided by its total mass density:

$$\mathbf{V} \equiv \frac{\rho_W \mathbf{V}_W + \rho_S \mathbf{V}_S}{\rho_W + \rho_S}, \quad (1)$$

where \mathbf{V}_W and \mathbf{V}_S are the mean velocities of the freshwater and salt particles in the parcel, respectively, and $\rho_W + \rho_S = \rho$, with $\rho_W = \rho(1 - S)$ and $\rho_S = \rho S$ being the mean densities of the freshwater and salt in it, respectively (Landau and Lifshitz 1959, 219–220; Fofonoff 1985; Müller 2006, p. 34). The molecular diffusive fluxes of freshwater and salt are then, respectively, $\mathbf{F}_W = \rho_W(\mathbf{V}_W - \mathbf{V})$ and $\mathbf{F}_S = \rho_S(\mathbf{V}_S - \mathbf{V})$, and $\mathbf{F}_W = -\mathbf{F}_S$. (The density of the

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freshwater in the seawater parcel ρ_W is of course less than that of pure freshwater ρ_F at the same temperature and pressure.)

The flow of freshwater mass into (or from) the atmosphere across the sea surface is $\rho_F(E - P)$. To calculate the vertical flow of seawater that supplies (or accepts) it, resolve the aggregate velocity \mathbf{V} into two nonorthogonal components: a vector parallel to the sea surface, $h = h(x, y, t)$, and a vertical component. Relative to the moving sea surface, that component $w^* = w - \mathbf{V}_H \cdot \nabla h - \partial h / \partial t$, where w and \mathbf{V}_H are the conventional vertical and horizontal components of \mathbf{V} , respectively. This is the speed at which total mass is advected vertically across the sea surface. The corresponding speed of the freshwater transport is $w_W^* = w_W - \mathbf{V}_{WH} \cdot \nabla h - \partial h / \partial t$; and for the salt transport, $w_S^* = w_S - \mathbf{V}_{SH} \cdot \nabla h - \partial h / \partial t$. It follows that $\rho w^* = \rho_W w_W^* + \rho_S w_S^*$. Because in fact there is no significant transport of salt across the sea surface, the rate of vertical transport of freshwater mass across it, $\rho_W w_W^*$, is ρw^* and

$$\rho_W w_W^* = \rho \left(w - \mathbf{V}_H \cdot \nabla h - \frac{\partial h}{\partial t} \right) = \rho_F (E - P). \quad (2)$$

This amounts to observing that the total vertical transport of mass across the sea surface must be continuous between ocean and atmosphere, but distinguishing the freshwater transport shows how the two representations for w cited in section 1 erred; namely, by mistaking $\rho_W w$ for the correct $\rho_W w_W$ (no asterisks because the sea surface was both level and stationary there); the first took $\rho_F = \rho$ as well.

Multiplying (2) by S shows that, because $\rho_S w_S^* = 0$ at the sea surface,

$$F_{SV} - \mathbf{F}_{SH} \cdot \nabla h = -\rho_F S (E - P), \quad (3)$$

where F_{SV} and \mathbf{F}_{SH} are the vertical and horizontal components of the molecular salt flux, respectively. Close to the sea surface, turbulence is often suppressed, so the vertical molecular flux (3) carries salt away from (or toward) the surface, to return that advected by the aggregate flow. Because Stern (1975), Schmitt et al., (1989), Warren (2006), and Schmitt (2008) incorrectly divided $(E - P)$ by $(1 - S)$ in their expressions for w , they made the same mistake with the diffusive salt flux. [Müller (2006) did not explicitly relate the molecular flux to $(E - P)$; Griffies (2004, p. 88) took $\mathbf{V}_S = \mathbf{V}_W = \mathbf{V}$ at the outset, and so did not actually treat the molecular flux at all.]

However, more than a centimeter or so below the sea surface, turbulence is fully developed (Kraus and Businger 1994, chapter 5.1). It prevents the sharp gradients required

for the molecular fluxes, and the turbulent motions carry the diffusive salt flux.

To calculate the relation corresponding to (3) immediately below the molecular boundary layer, consider a thin layer bounded above by the sea surface, $h(x, y, t)$, and below by a surface, $b(x, y, t)$, near the base of the boundary layer, and spaced a uniform vertical distance from the sea surface. The Reynolds-averaged salt-conservation equation is

$$\begin{aligned} \frac{\partial(\rho \bar{S})}{\partial t} + \nabla_H \cdot (\rho \bar{S} \bar{\mathbf{V}}_H) + \frac{\partial}{\partial z} (\rho \bar{S} \bar{w}) \\ = -\nabla_H \cdot (\rho \overline{S' \mathbf{V}'_H}) - \frac{\partial}{\partial z} (\rho \overline{S' w'}) - \nabla_H \cdot \bar{\mathbf{F}}_{SH} - \frac{\partial \bar{F}_{SV}}{\partial z}, \end{aligned} \quad (4)$$

where (as usual) the overbar means some suitable time averaging, the prime means departure from the mean, and variation in seawater density is ignored. Let subscripts b and h denote values at $z = b$ and $z = h$, and define w_b^* and w_h^* as pertaining to the surfaces $z = b(x, y, t)$ and $h(x, y, t)$, respectively. Integrating (4) from $z = b$ to h and using (i) the fact from (2) that $\rho \bar{S}_h \bar{w}_h^* = \rho_F \bar{S}_h (\bar{E} - \bar{P})$, (ii) Eq. (3), and (iii) the specifications that $w' = 0$ at $z = h$ and $\mathbf{F} = 0$ at $z = b$ shows that

$$\begin{aligned} \frac{\partial}{\partial t} \int_b^h \rho \bar{S} dz + \nabla_H \cdot \int_b^h \rho \bar{S} \bar{\mathbf{V}}_H dz - \rho \bar{S}_b \bar{w}_b^* \\ = -\nabla_H \cdot \int_b^h \rho \overline{S' \mathbf{V}'_H} dz - \rho \overline{S'_b \mathbf{V}'_{Hb}} \cdot \nabla b + \rho \overline{S'_b w'_b} \\ - \nabla_H \cdot \int_b^h \mathbf{F}_{SH} dz. \end{aligned} \quad (5)$$

Assume that the integration interval $(h - b)$ is so small that the integrals make no significant contribution to (5) and that, in effect, $\bar{\mathbf{V}}_h = \bar{\mathbf{V}}_b$ so $(\bar{w}_h^* = \bar{w}_b^*)$. Then, because $\nabla b = \nabla h$,

$$\rho \overline{S'_b w'_b} - \rho \overline{S'_b \mathbf{V}'_{Hb}} \cdot \nabla h = -\rho_F \bar{S}_b (\bar{E} - \bar{P}), \quad (6)$$

which makes sense and is to be compared to (3).

Molecular boundary layers are too thin to be sustained when the sea surface is pelted by raindrops, or torn into spindrift, or when static instability leads to overturning so (6) then applies to the agitated surface itself (b becomes h). With practical averaging, the correction term for surface slope is unlikely ever to be relevant, especially when rough seas blur the sea surface. And of course the vertical turbulent flux is usually parameterized by an eddy diffusivity (e.g., Müller 2006, p. 129).

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