The Ocean Wave Height Variance Spectrum: Wavenumber Peak versus Frequency Peak

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ABSTRACT

Many authors assume that the frequency peak and the wavenumber peak of an ocean wave height variance spectrum are related by the ocean wave dispersion relationship. This note shows that this is not true and that the true relationship depends on the shape of the spectrum, thereby introducing an element of randomness into the relationship.

1. Spectral conversions

Ocean wave height variance spectra can be functions of either wavenumber or frequency. We refer to the first as the wavenumber spectrum and the second as the frequency spectrum. In a recent paper, I briefly mentioned that the peak of the frequency spectrum (f_p) cannot in general be related to the peak of the wavenumber spectrum (k_p) by the ocean wave dispersion relationship (Plant et al. 2005). Unfortunately, this fact does not seem to be widely recognized despite a general understanding of how to relate one spectrum to another. This has caused inaccuracies, even errors, in some papers relating the two types of spectral peaks (Walsh et al. 1989; Hwang 2004). In this brief note, I show that, except in the case in which the ocean wave spectrum is a delta function, the peak of the wavenumber spectrum is not related to the peak of the frequency spectrum by the ocean wave dispersion relation. In general, their relationship depends on the shape of the spectrum.

The relationship between the omnidirectional (i.e., integrated over all azimuth angles) frequency spectrum $F(f)$ and the omnidirectional wavenumber spectrum $F(k)$ is

$$\int F(k)k\,dk = \int F(f)\,df, \tag{1}$$

as dictated by the requirement that the total wave height variance must not depend on which independent variable is used. This equation immediately implies that

$$F(k) = F(f)c_g/(2\pi k). \tag{2}$$

2. The relationship between f_p and k_p

To illustrate how this equation affects the f_p–k_p relationship, I will use the Joint North Sea Wave Project (JONSWAP) spectrum (Hasselmann et al. 1973):

$$F(f) = \alpha g^2(2\pi)^{-4}f^{-5}\exp[-1.25(f_p/f)^4]\gamma^H, \tag{3}$$

where

$$H = \exp\{-0.5[(f - f_p)/\sigma f_p]^2\},$$

and will take σ to be 0.2. Since I am not interested in spectral magnitudes here, I will normalize both $F(k)$ and $F(f)$ to their peak values in the plots to follow. Furthermore, to keep $c_g/(2\pi k)$ on scale in these plots, I divide it by 2 times its value at f_p in the plots. Then the relationships among the three quantities are shown in Fig. 1, where $F(k)$ is plotted versus f and not versus k. The dispersion relation does relate these two independent variables, and therefore the abscissa could be easily converted to a more standard wavenumber axis. In these illustrations, I assume deep water.

Note that the peaks of $F(k)$ and $F(f)$ are not at the same frequency in either panel of Fig. 1 but that they are

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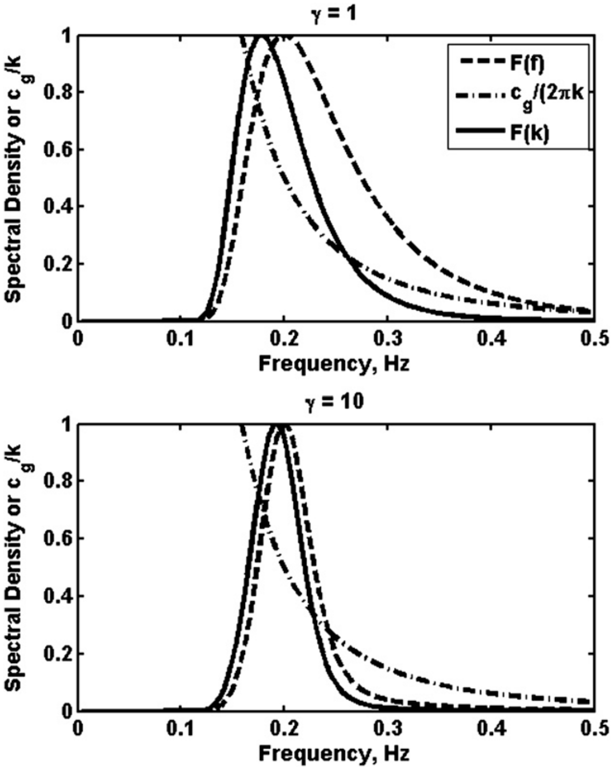

FIG. 1. Plots of the relationship between $F(k)$ and $F(f)$ for two different values of spectral widths specified by γ. They are related by $cg/(2\pi k)$, also shown.

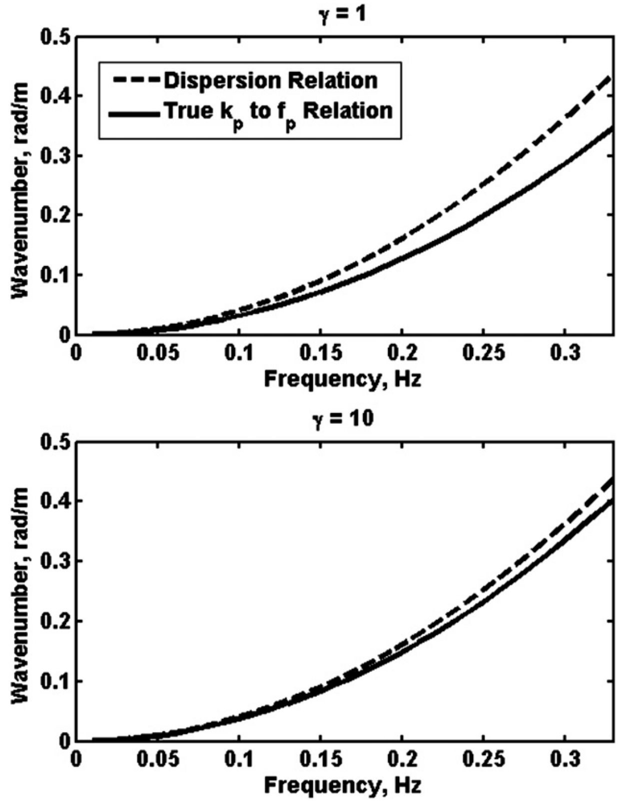

FIG. 2. The deep-water ocean wave dispersion relation and the true relation between k_p and f_p for two different spectral widths.

much closer together for the narrower spectrum. They are different because of the weighting applied to $F(f)$ by $c_g/(2\pi k)$ to obtain $F(k)$.

A value for k_p can be obtained from the plots in Fig. 1 by converting the frequency at which $F(k)$ maximizes to wavenumber via the deep-water dispersion relation $k = (2\pi f)^2/g$.

By letting f_p take on a range of values, the relationship between k_p and f_p shown in Fig. 2 is obtained for two values of γ. Also shown in Fig. 2 is the deep-water dispersion relation. This figure shows that 1) the relationship between k_p and f_p is not generally the dispersion relationship but depends on spectral shape and 2) k_p is closer to the value obtained from f_p via the dispersion relation for narrower spectra. In the case of a delta-function spectrum, and only then, f_p and k_p are related by the dispersion relation.

The dependence of the k_p–f_p relationship on spectral shape introduces a source of randomness into the relationship, causing the two variables to be at least somewhat decorrelated. This is in addition to the effects of currents and statistical uncertainty, which have not been considered here.

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