On Upwelling Along a Zonally-Orientated Coastline

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ABSTRACT

A common observation in major coastal upwelling regions is a reversal of the mean longshore current with depth during active upwelling. A numerical model and a simple analytical model applied to a two-layer ocean on a beta-plane indicate that this undercurrent cannot exist for upwelling off an east–west (zonal) coastline. The models are applied to the coastal upwelling occurring on the Campeche Bank during the period from March to September. The beta-plane solutions for upwelling along an east–west coastline resemble f-plane solutions for a meridional (N–S) coastline. In particular, the hypothesis explaining the undercurrent of Hurlburt and Thompson cannot hold for upwelling along a zonal coastline. The results in this paper form a new hypothesis which could be tested by measuring currents on the Campeche Bank during active upwelling.

1. Introduction

Except for Tapanes (1971) little theoretical work has been done on the upwelling along a zonally oriented coastline. Nearly all traditional upwelling studies have been applied to coasts aligned in the north–south direction such as off Oregon, California, Peru and West Africa. However, we have applied both a numerical and a simple analytical two-layer model to upwelling along an east–west coast such as the Campeche Bank off northern Yucatan. Easterly winds during the late spring and summer months are favorable for the production of coastal upwelling north of the Yucatan Peninsula. Cochrane (1969) has examined this region during the early summer periods and found that a cool stratum exists below a strong thermocline over virtually the entire bank. Furthermore, Cochrane (1966) has found anomalously cold water, which he attributes to upwelling, all along the northern Yucatan coast throughout the period from March to September. It will be shown that the dynamics of the upwelling along a zonally oriented coastline differs substantially from that along a meridional coast.

Hurlburt and Thompson (1973) have shown that the inclusion of the beta-plane approximation for upwelling along a north–south coast provides a mechanism for a longshore pressure gradient which, in turn, produces a current reversal with depth in the nearshore region. When similar dynamics are applied to an east–west coast, we will show that the mechanism which produces the sub-surface countercurrent for the north–south coast vanishes identically. Observations on the Campeche Bank during the upwelling season have yet to reveal the existence of a sub-surface countercurrent, and, from our work, we feel that such a nearshore deep countercurrent cannot exist in this region. Moreover, we will show that the beta-effect, which is very important to the upwelling dynamics along a meridional coast, is of minimal importance along an east–west coast, and that the resultant dynamics are similar to previous f-plane models of coastal upwelling.

2. The model

Consider a two-layer model of a rotating, stratified, hydrostatic fluid, the equations of motion for which have been vertically integrated over each layer. A right-handed coordinate system in which the positive x
and y axes point east and north, respectively, is used. The governing equations for this system are

\[
\begin{align*}
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} - f v_1 &= -\frac{\partial}{\partial x} \left( h_1 + h_2 + D \right) \\
&+ \frac{\tau_{1x} - \tau_{1y}}{\rho h_1} + A \left( \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} \right), \\
\frac{\partial v_1}{\partial t} + u_1 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_1}{\partial y} + f u_1 &= -\frac{\partial}{\partial y} \left( h_1 + h_2 + D \right) \\
&+ \frac{\tau_{1x} - \tau_{1y}}{\rho h_1} + A \left( \frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} \right),
\end{align*}
\]

(1)

\[
\begin{align*}
\frac{\partial h_1}{\partial t} + (h_1 u_1) \frac{\partial}{\partial x} (h_1 u_1) &= 0, \\
\frac{\partial h_2}{\partial t} + (h_2 u_2) \frac{\partial}{\partial x} (h_2 u_2) &= 0,
\end{align*}
\]

(3)

Integration of (7) over x from far offshore, x = -L, to any value of x gives

\[
\int_{-L}^{x} \beta u dx = g \left( \frac{\partial}{\partial y} \left( h_1 + h_2 + D \right) \right),
\]

(8)

if we set the north–south sea surface slope equal to zero far from the coast which follows from the imposition of no forcing or return flow in this region. Since the interior of our fluid will exhibit a Sverdrup balance and we are considering a region in which the wind stress curl is less than or equal to zero, then (8) reveals that the β-effect provides a mechanism for a north–south pressure gradient, the slope of which is downward from south to north. The result of the north–south sea surface slope is to produce a barotropic onshore flow whose magnitude is such that the geostrophically balanced onshore transport in the lower layer equals the offshore transport in the upper layer. Because of the kinematic boundary conditions at the coast, the onshore flow must become sub-geostrophic near the coast and result in a poleward current in the lower layer. Hence, the above discussion shows that for a meridional coast, inclusion of the β-effect in a two-layer model of coastal upwelling results in the presence of a north–south sea surface slope which, in turn, is responsible for a nearshore poleward upwelling.

Now let us examine the same physical situation with the exception that Eqs. (1)–(3) are applied to an east–west coastline instead of the north–south alignment above. Again we neglect longshore variations in the velocity field and assume that the wind stress has a component in the longshore direction only, which in this case is in the negative x-direction. Differentiation of (2) with respect to x yields

\[
-\frac{\partial^2}{\partial x \partial y} (h_1 + h_2 + D) = 0.
\]

(9)

Hence, the longshore pressure gradient must vanish if, as in the previous case, we set the longshore pressure gradient equal to zero far from the coast. Thus, Eq. (9) states that the longshore pressure gradient must be identically zero everywhere.

Clearly, from our previous arguments, the absence of the east–west pressure gradient also results in the absence of the nearshore upwelling, where by upwelling we mean a longshore sub-surface flow directed opposite to the surface flow. Whereas the inclusion of the beta-plane approximation for the north–south coastline system induced a longshore pressure gradient and subsequently the nearshore upwelling, we see
that no such pressure gradient or resultant undercurrent may develop in the east–west coastline system even though the beta-effect has been included.

Application of these results to the upwelling observed off the northern coast of the Yucatan Peninsula indicates that no sub-surface countercurrent should be found in this region. J. D. Cochrane of Texas A & M University (personal communication) has informed us that he has yet to observe a counter undercurrent related to the upwelling regime on the Campeche Bank. On the other hand, the existence of the undercurrent for the north–south coastline system has been well documented (Wooster and Gilmartin, 1961; Wyrski, 1963; Pillsbury et al., 1970; Mooers et al., 1973).

3. Analytical solution

Using the results of the previous section, we apply a very simple two-layer analytical model to the upwelling driven by a longshore wind component along a zonally oriented coastline. If we neglect the advective terms and the nonlinear interfacial and bottom stress terms, linearize the continuity equation, neglect frictional effects and the slope of the shelf topography, and assume geostrophic motion in the longshore direction, the governing equations for this system become

\[
\begin{align*}
\frac{\partial u_1}{\partial t} - f_1u_1 &= \frac{\tau_{xx}}{\rho_1 H_1} \\
\frac{\partial h_1}{\partial t} + H_1 \frac{\partial u_1}{\partial y} &= 0 \\
\frac{\partial u_2}{\partial t} - f_2u_2 &= 0 \\
\frac{\partial h_2}{\partial t} + H_2 \frac{\partial u_2}{\partial y} &= 0 \\
\end{align*}
\]

(10)

where \( H_1 \) and \( H_2 \) are the initial heights of the first and second layers. The geostrophic restriction on the longshore flow eliminates the gravity waves from the problem. Also note that the east–west pressure gradient has been neglected which is consistent with the results of Section 2. A constant wind stress is applied impulsively to the system initially at rest. At the coast, boundary conditions are kinematic, and far from the coast, we merely prescribe that the north–south components of the velocity field remain bounded.

Following the technique of O’Brien (1973), we define

\[
v_1 = v_1' + V_B,
\]

where

\[
V_B = -\frac{\tau_{xx}}{f H_1}
\]

Elimination in favor of \( v_1' \) yields

\[
\frac{\partial}{\partial t} - g H \frac{\partial}{\partial y} = g H \frac{\partial}{\partial y} - \frac{\partial}{\partial t} \frac{\partial}{\partial y}, \quad (11)
\]

where we have neglected a term involving \( \beta^2 \). Utilizing a Taylor series expansion for \( f^2 \) (Geisler and Dickinson, 1972)

\[
f^2 = f_0 \left( 1 + \frac{2\beta y}{f_0} \right),
\]

we may express (11) as

\[
\frac{(1+\beta' y)v_1'}{y H^2} = -\gamma_1 \frac{\partial v_1'}{\partial y}, \quad (12)
\]

where

\[
\begin{align*}
\beta' &= \frac{2\beta}{f_0}, \\
\gamma_E^2 &= \frac{(H_1 + H_2)}{f_0^2}, \\
\gamma_T^2 &= \frac{g' H_1}{f_0^2 \left( 1 + H_1/H_2 \right)}.
\end{align*}
\]

The lengths \( \gamma_E \) and \( \gamma_T \) are the barotropic and baroclinic Rossby radii of deformation, respectively. Since the ratio of \( \gamma_T^2 \) to \( \gamma_E^2 \) is much less than 1, we may use boundary layer techniques to solve (12).

For the internal scale where \( y = \gamma_E \xi \), Eq. (12) reduces to

\[
\psi' = \psi' \xi. \quad (13)
\]

Since this equation does not contain beta, the internal mode will be identical to that of the \( f \)-plane solution. The solution for the external scale is found by letting \( y = \gamma_T \xi \); Eq. (12) then reduces to

\[
(1+\gamma_E \beta' \xi)\psi' = \psi' \xi. \quad (14)
\]

If we make the transformation of variables

\[
\eta = (1+\gamma_E \beta' \xi)/(\gamma_T \beta'),
\]

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The f-plane solutions for the east–west coastline case are

\[
\begin{align*}
    u_1 &= -tfV_B \left[ \frac{H_2}{H_1+H_2} \exp(-y/\gamma_1) \right. \\
    & \quad \left. + \frac{H_1}{(H_1+H_2) \text{Ai}[\alpha(1+2\beta y/\gamma_0)]} \right] \\
    u_2 &= tfV_B \left[ \frac{H_1}{H_1+H_2} \exp(-y/\gamma_1) \right. \\
    & \quad \left. - \frac{H_1}{(H_1+H_2) \text{Ai}[\alpha(1+2\beta y/\gamma_0)]} \right].
\end{align*}
\]  

(15)

In comparing (14) and (15) we see that the internal or baroclinic parts are identical for both the \(\beta\)- and f-plane solutions. However, there appear to be considerable differences between the external or barotropic parts of these solutions. These differences become negligible, though, when the terms are actually evaluated. Table 1 reveals the similarity between the \(\beta\)- and f-plane solutions for the barotropic mode; at various distances from the coast the functions \(\text{Ai}[\alpha(1+2\beta y/\gamma_0)]/\text{Ai}(\alpha)\) and \(\exp(-y/\gamma_0)\) are tabulated. The close agreement between these functions allows us to state that for upwelling off an east–west oriented coastline, we find essentially no differences between the f- and \(\beta\)-plane solutions. This situation contrasts sharply with previous numerical and analytical studies of upwelling off a north–south aligned coast where the inclusion of the \(\beta\) effect led to substantial differences between the \(\beta\)- and f-plane solutions. The most prominent difference has already been discussed, i.e., the existence of the longshore pressure gradient and, consequently, the nearshore undercurrent in the lower layer when the \(\beta\)-plane approximation is employed.

Furthermore, for the N–S coast, the inclusion of the \(\beta\)-effect requires a Sverdrup balance in the longshore flow outside the coastal boundary layers. For typical values of the wind stress curl, the velocities determined by the Sverdrup balance are negligible compared to those of the coastal jet and undercurrent which have a wide scale governed by the baroclinic Rossby radius of deformation. Hence, for the \(\beta\)-plane solutions appreciable longshore velocities exist only on the baroclinic scale, whereas appreciable longshore velocities exist also on the barotropic Rossby radius of deformation scale for the f-plane solutions. For a more complete discussion of the f-plane analytical solutions the reader is referred to Gill (1973) and O’Brien (1973).

The essential reason for the absence of the above differences between the \(\beta\)- and f-plane solutions for the zonal coastline system is that the interior longshore flow is no longer constrained by Sverdrup dynamics thereby reducing the effect of including \(\beta\). Although the \(\beta\)-effect is not the only mechanism capable of

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**Table 1. Comparison of barotropic scales for \(\beta\)-plane and f-plane solutions.**

<table>
<thead>
<tr>
<th>(y) (km)</th>
<th>(\text{Ai}(\eta)/\text{Ai}(\alpha))</th>
<th>(\exp(-y/\gamma_B))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>0.1</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>0.5</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>1.0</td>
<td>0.998</td>
<td>0.998</td>
</tr>
<tr>
<td>2.0</td>
<td>0.995</td>
<td>0.995</td>
</tr>
<tr>
<td>6.0</td>
<td>0.986</td>
<td>0.987</td>
</tr>
<tr>
<td>10.0</td>
<td>0.978</td>
<td>0.978</td>
</tr>
<tr>
<td>20.0</td>
<td>0.954</td>
<td>0.956</td>
</tr>
<tr>
<td>60.0</td>
<td>0.868</td>
<td>0.875</td>
</tr>
<tr>
<td>100.0</td>
<td>0.790</td>
<td>0.799</td>
</tr>
<tr>
<td>200.0</td>
<td>0.621</td>
<td>0.640</td>
</tr>
<tr>
<td>600.0</td>
<td>0.231</td>
<td>0.262</td>
</tr>
<tr>
<td>1000.0</td>
<td>0.080</td>
<td>0.107</td>
</tr>
</tbody>
</table>
Table 2. Model parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>( 5.4 \times 10^{-4} ) sec(^{-1} )</td>
<td>( H_t )</td>
<td>( 4 \times 10^6 ) cm</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( 2 \times 10^{-3} ) cm(^2) sec(^{-1} )</td>
<td>( H_t )</td>
<td>( 12 \times 10^6 ) cm</td>
</tr>
<tr>
<td>( g )</td>
<td>( 10^3 ) cm sec(^{-2} )</td>
<td>( \Delta x )</td>
<td>( 4 ) km</td>
</tr>
<tr>
<td>( k' )</td>
<td>( 2 ) cm sec(^{-2} )</td>
<td>( \Delta t )</td>
<td>( 1 ) hr</td>
</tr>
<tr>
<td>( A )</td>
<td>( 10^8 ) cm(^2) sec(^{-1} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

eliminating the barotropic mode, we do not believe that other mechanisms which would establish an east–west pressure gradient are strongly active in the Gulf of Mexico. Hence, any actual undercurrent on the Campeche Bank will be very weak if it exists at all.

4. Numerical solution

Eqs. (1)–(6), after neglecting the longshore variations in the velocity field and the longshore derivatives of the pressure field in the continuity equations, are solved numerically for three cases: 1) a \( \beta \)-plane with a north–south coastline, 2) a \( \beta \)-plane with an east–west coastline, and 3) an \( f \)-plane. The latter is independent of coastline orientation. The coordinate system used in each case is the appropriate system as described in Section 2 with all model parameters remaining unchanged for each case, i.e., only the coastline orientation is altered. Latitude 22N was chosen, corresponding to the latitude of the North Yucatan coast. A steady wind stress directed in the longshore direction only (Fig. 1) is applied to a model region 1200 km in width. Initially, the system is at rest, and the boundary conditions are kinematic and no slip at both the coast and the far wall. Furthermore, as discussed in Section 2, the longshore pressure gradient at the far boundary is assumed to be zero. Table 2 is a list of the model parameters.

For the numerical solutions it was possible to use the model of O’Brien and Hurlburt (1972) and the variant by Hurlburt and Thompson (1973) with only minor modification. These models employ a highly efficient, semi-implicit numerical scheme patterned after Kwizak and Robert (1971), but which is sufficiently different that it is discussed in detail by O’Brien and Hurlburt (1972). The equations are finite differenced using leapfrog for the time-derivative terms, quadratic averaging for the advective terms, and lagged-in-time diffusive terms.

The resultant longshore velocity profiles after 10 days are shown in Figs. 2 and 3; although the model width is 1200 km, only the profiles within 200 km of the coast are shown. A superposition of the \( \beta \)-plane solutions for the meridional and zonal coastlines is shown in Fig. 2. As expected from the discussions in Sections 2 and 3, the meridional solution, which possesses a well-defined undercurrent, contrasts sharply with the zonal coastline solution in which no undercurrent is present. Furthermore, although the magnitude and width scale of the coastal jet are approximately equal in both cases [O(50 cm sec\(^{-1} \)) and O(20 km), respectively], the barotropic mode, which has a width scale governed by the barotropic radius of deformation, is present only for the east–west coastline system. A comparison of the \( f \)-plane solution in Fig. 3 with the zonal coastline, \( \beta \)-plane solution reveals the close agreement between these solutions.

Although the height field is not shown, the depth of the upper layer at the coast has decreased from an initial depth of 40 m to 9 m after 10 days for the \( \beta \)-plane, E–W coastline case. The width scale of the upwelling is the same as the scale of the coastal jet, that is O(20 km).

5. Critique

The upwelling occurring off the northern coast of the Yucatan Peninsula has been highly idealized by the models presented in this paper. In particular, the
analytical solution is presented only in the spirit of revealing the essential differences between the upwelling occurring along different coastline orientations. Although the numerical model includes many more physical mechanisms than the analytical model, it, too, is a somewhat idealized simulation of upwelling occurring off Yucatan. In particular, it would be desirable not only to develop a three-dimensional model of the upwelling in this region, but also to include the effects of thermodynamics and the ability of the model to continue running after the interface has surfaced. All these facets of the upwelling problem are presently being examined.

6. Summary and conclusions

In order to examine the upwelling occurring along a zonally oriented coastline, both numerical and simple analytical models were applied to this region. It was shown that even though both models included the $\beta$-plane approximation, the resultant dynamics closely resembled previous $f$-plane solutions. This result contrasted sharply with previous models of coastal upwelling along a north–south oriented coastline where it was shown that the inclusion of the $\beta$-plane approximation altered the solutions substantially. In particular, whereas the $\beta$-effect for upwelling along a meridional coast induced a longshore pressure gradient and a subsurface countercurrent, it was shown that no such pressure gradient or upwelling could develop for upwelling along a zonal coast. Furthermore, it was shown that the width scale for appreciable longshore velocities associated with upwelling along an east–west coastline is governed by the barotropic Rossby radius of deformation rather than only the baroclinic scale as is the case for upwelling along a meridional coast.

In conclusion we feel that the influence of the $\beta$-effect on the dynamics of coastal upwelling is dependent on the orientation of the coastline, and that this influence is minimal for a coastline whose orientation is in the east–west direction. This is a hypothesis which can be tested by collecting current records on the Campeche Bank during the upwelling season.

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