Wavenumber Spectrum in the Gulf Stream from Shipboard ADCP Observations and Comparison with Altimetry Measurements

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ABSTRACT

The wavenumber spectra for velocity and temperature in the Gulf Stream region are calculated from a decade (1994–2004) of shipboard acoustic Doppler current profiler (ADCP) measurements taken as part of the Oleander Project. The velocity and temperature spectra have comparable magnitude, in terms of the kinetic and potential energy, and both indicate a k^{-3} slope in the mesoscales. In contrast, the corresponding velocity spectrum determined from satellite altimetry sea surface heights yields a significantly higher energy level and a k^{-2} slope. The discrepancy between altimeter-derived and directly measured velocity spectra suggests that altimetric velocity probably is contaminated by noise in sea surface height measurement. Also, the k^{-3} slope, which appears to be in agreement with two-dimensional quasigeostrophic turbulence theory, does not support the contemporary surface quasigeostrophic theory. These results highlight large gaps in the current understanding of the nature of surface geostrophic turbulence.

1. Introduction

Mesoscale (10–100 km) eddies are dominant motions in the upper ocean. They arise primarily from baroclinic instability and scale to the baroclinic deformation radius (~30 km in midlatitudes). Energy is transferred through nonlinear turbulence interactions and is ultimately dissipated by viscosity. A fundamental question is how the equilibrium energy spectrum is established. In three-dimensional homogeneous isotropic turbulence, energy cascades to smaller scales following the Kolomogroff $k^{-5/3}$ law. In two-dimensional turbulence, due to energy and enstrophy (mean-square vorticity) constraints, energy is transferred to scales larger than the scale where energy is inserted (inverse cascade), whereas enstrophy is transferred in the opposite direction to the smaller scales (enstrophy cascade; Kraichnan 1967). A similarity argument leads to a $k^{-5/3}$ law for the inverse cascade and a k^{-3} law for the enstrophy cascade. In his seminal geostrophic turbulence study, Charney (1971) showed that the two-dimensional turbulence theory also holds for stratified quasigeostrophic (QG) flows. Hence, the direct cascade to smaller scales is also expected to follow a k^{-3} law. In addition, there is an equipartition between kinetic and potential energy.

The QG turbulence theory has been verified for the atmospheric synoptic scales based on wavenumber spectra determined from the global reanalyses and direct aircraft measurements (Nastrom and Gage 1985). In the atmosphere, the baroclinic deformation radius is ~1000 km, and the dominant scales are in the range of 1000–3000 km. The oceanographic observations, on the other hand, generally lack adequate spatial and temporal coverage, and velocity wavenumber spectra have been derived only indirectly from altimetric sea surface height (SSH) measurements. Stammer (1997), using the first 3 yr of data from the Ocean Topography Experiment [TOPEX/Poseidon (T/P)] mission, obtained a global SSH spectrum of $k^{-4.6}$. The corresponding surface slope (geostrophic velocity) spectrum of $k^{-2.6}$ appears to be consistent with the k^{-3} QG turbulence. However, the SSH spectrum in the Gulf Stream region is closer to a k^{-4} slope and to the corresponding velocity spectrum of k^{-2}. Le Traon et al. (2008), using all available altimeter measurements (T/P,

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Jason-1, *Geosat Follow-On*, and *Envisat*) in a 4-yr period (2002–05), showed that all oceanic high eddy activity areas have k^{-4} ($\sim k^{-11/3}$) slope. The inferred k^{-2} ($\sim k^{-5/3}$) velocity spectrum is significantly different from the QG spectrum. The k^{-2} velocity spectrum, on the other hand, is consistent with the surface quasigeostrophic theory (SQG), which assumes conservation of surface temperature variance (Blumen 1978; Held et al. 1995). Recent submesoscale eddy-resolving numerical model experiments also obtained a k^{-2} velocity spectrum at the surface (Lapeyre and Klein 2006; Klein et al. 2008; Capet et al. 2008a,b,c). While the evidence for SQG is strong, the conclusion of Le Traon et al. (2008) nevertheless is tentative. This is because the altimetric velocity estimate can be significantly affected by noise in altimeter surface height measurements.

In this note, we take advantage of the long-term (1994–2004) near-surface velocity and temperature observations from the Oleander Project (available online at http://www.po.gso.uri.edu/rafos/research/ole/index.html) to compute the wavenumber spectra in the Gulf Stream region. The directly measured wavenumber spectrum is also compared with the altimetric velocity spectrum. The implications for surface geostrophic turbulence are discussed.

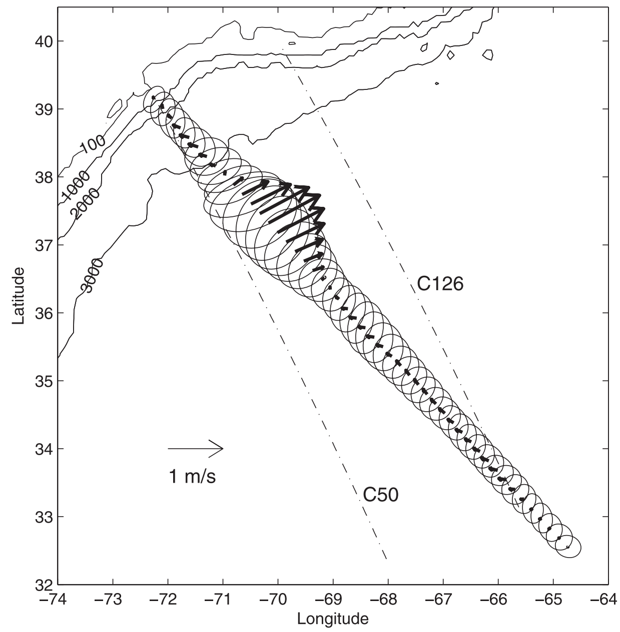

FIG. 1. Mean current and velocity ellipses from *Oleander* ADCP measurements at 30-m depth (plotted every 20 km). The depth contours (solid lines) and T/P tracks are marked (dashed–dotted lines).

2. Analysis

In this study, 10 years (1994–2004) of shipboard acoustic Doppler current profiler (ADCP) observations from the Oleander Project are used to calculate the wavenumber spectra for near-surface velocity and temperature in the Gulf Stream region. From 1994 through 2004, a 150-kHz Teledyne RD Instruments narrowband ADCP, mounted on the M/V *Oleander* measured upper-ocean velocity during the container ship's weekly round-trip between New York Harbor and Bermuda. Data are averaged every 5 min, resulting in a horizontal resolution of approximately 2.4 km. In the vertical, the bin size ranges between 8 and 16 m in the open ocean. The sea surface temperature is measured at the instrument level, about 5 m below the surface.

The *Oleander* observations have previously been used to study interannual variations of the Gulf Stream surface transport (Rossby et al. 2005), seasonal variations of the shelf/slope jet in Middle Atlantic Bight (Flagg et al. 2006), and Gulf Stream warm-core rings (Wei et al. 2008). In this study, we focus on the open ocean to be compatible with altimetry measurements. We use only transects of at least 90% good observations to ensure high data quality. For each ~1000-km-long transect, the data are interpolated onto a 2-km grid using a Barnes filter (Daley 1991) with a correlation length of 2 km (the filter has no impact on scales >10 km); if the data gap is too large to allow for meaningful interpolation, then the transect is discarded. This strict screening process results in a total of 213 transects (out of >700 potential transects) spread over the 10-yr period. Mean current and velocity ellipses along the *Oleander* track are shown in Fig. 1. They are similar to those found in Rossby et al. (2005). Near the Gulf Stream axis, the ellipses are polarized in the downstream (zonal) direction, reflecting the effects of Gulf Steam meanders, whereas away from the Gulf Stream, the ellipses indicate a more isotropic eddy field.

To calculate the wavenumber spectrum, the time (ensemble) mean is first removed from each transect to be compatible with the altimetry that does not measure the mean sea surface height. Including the mean velocity profile slightly increases the energy level at large scales but has no effect on the spectral shape. For each transect (of velocity and temperature), the spatial mean and linear trend are removed, the residuals are multiplied by a cosine bell (Hanning) window, and a fast Fourier transform (FFT) is applied. The spectrum is the sum of the squares of the Fourier components at each wavelength averaged over all transects. There are about 240 wavenumber estimates at a resolution of $\sim 10^{-3}$ cycle per kilometer (cpkm). To determine the number of degrees of freedom (DOF), we use a conservative estimate counting only transects (~120) that were taken at least

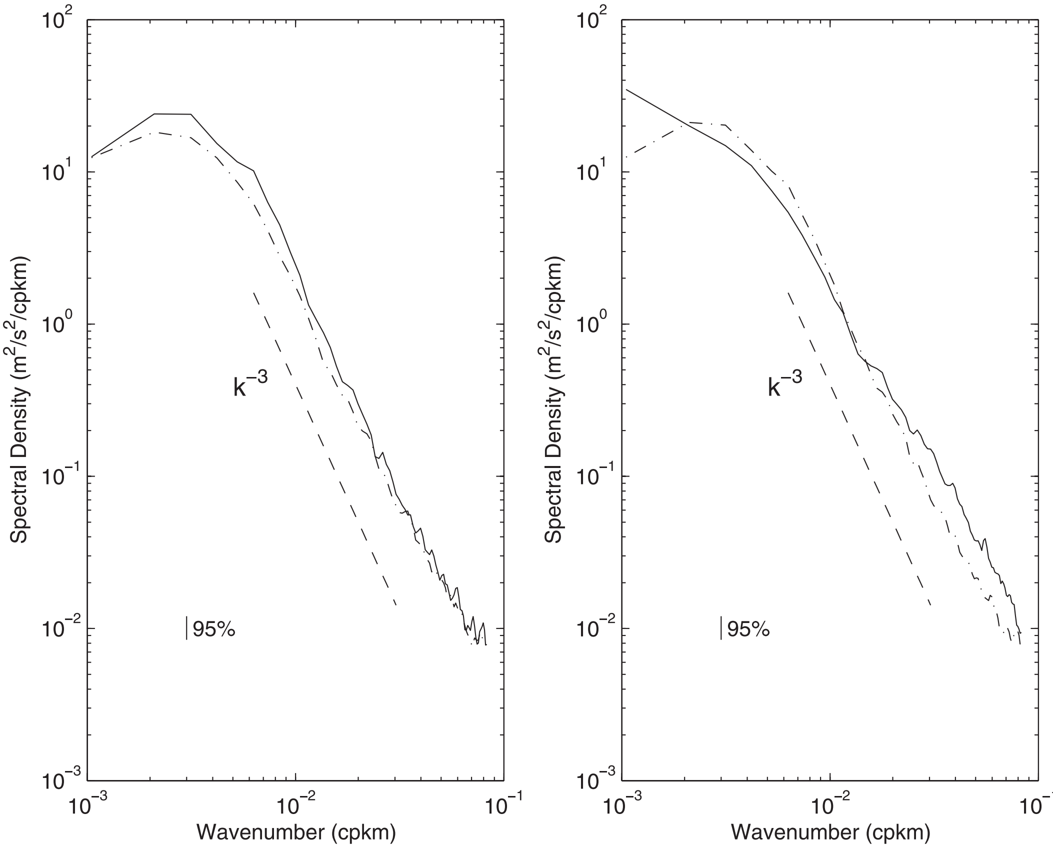

FIG. 2. (left) Zonal (solid) and meridional (dashed–dotted) velocity spectra and (right) potential energy (solid) and kinetic energy (dashed–dotted) spectra from the *Oleander* observations. Dashed lines indicate a −3 slope. The 95% confidence interval is marked.

two weeks apart. This leads to 240 DOF. We also assume the speed of the vessel (16 kt) is much greater than the speed of along-track movement of disturbances.

The temperature spectrum is converted into the potential energy spectrum as follows. In QG theory, the potential energy (ignoring the ½ factor) is given by

$$\mathrm{PE} = \frac{1}{N^2}\langle b^2\rangle \approx \alpha\langle T^2\rangle, \qquad (1)$$

where N^2 is the mean Brunt–Väisälä frequency, b is the buoyancy ($=g\rho'/\rho_o$), and α is an empirical constant relating temperature and buoyancy. Because the density change is affected by both temperature (T) and salinity (S), we need to assume that the T/S relation is not affected by eddy stirring (seasonal heating is largely eliminated as a result of removing the spatial mean of each transect in the spectrum calculation). In the Gulf Stream, temperatures change (in the vertical) by about 4°C over the upper 100 m, and densities by about 2 kg m^{-3}. If we use this ratio, which takes into consideration the salinity contribution, α is about 0.1 (in MKS units). While this estimate is crude, it is sufficient for this study because buoyancy is expected to mainly depend on the temperature.

Figure 2 shows the estimated zonal and meridional velocity spectra plus the kinetic and potential energy spectra. The velocity spectra are calculated from ADCP measurements 30 m below the surface; the spectra are basically identical for velocities at 60 or 90 m. Two features stand out in the velocity spectra. First, the velocity component spectra are comparable; the zonal velocity is slightly larger at scales >100 km. Second, the velocity spectra show a distinct linear slope of −3 over the entire mesoscale range, O(10–100 km). In the potential energy (temperature) spectrum, the same −3 slope also seems to hold, though there is an offset of energy level at about 50-km wavelength. Alternatively, if the slope of the potential energy spectrum is fitted over the entire mesoscale range, the slope would be about −2.3. Perhaps, most importantly, the kinetic energy and potential energy spectra are remarkably similar over the entire wavenumber range (considering the uncertainty in potential energy estimate). Taken together, three results emerge: a k^{-3} law for velocity

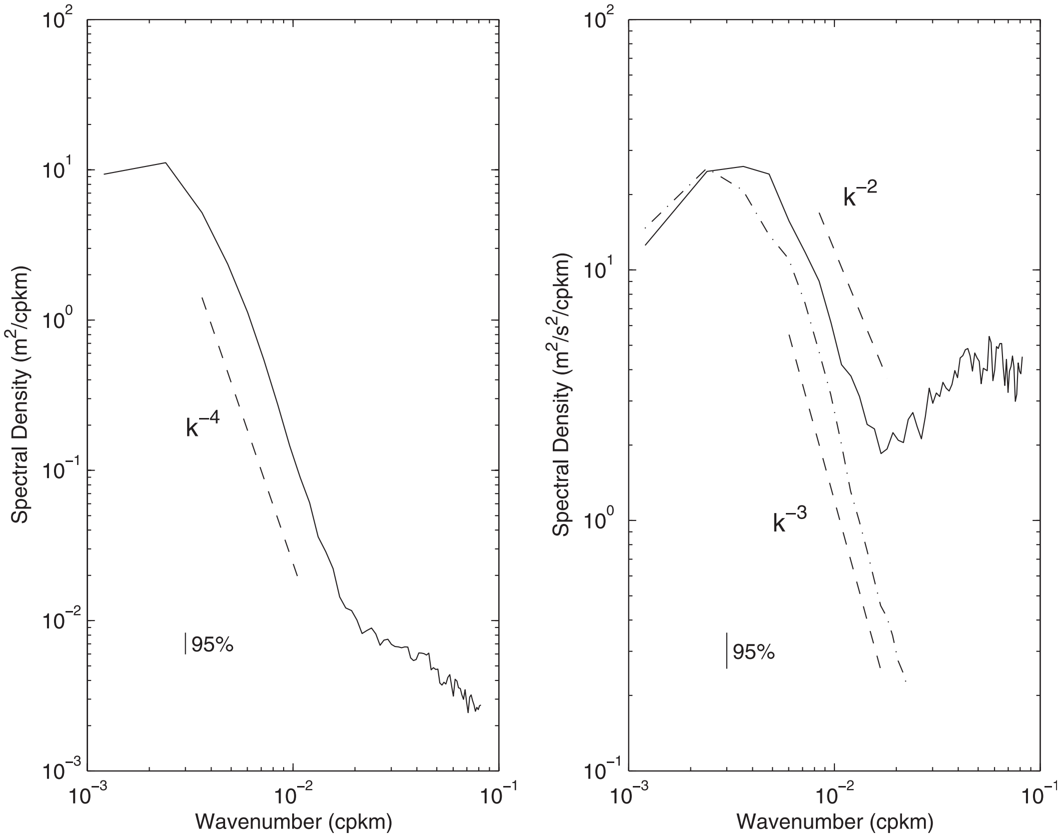

FIG. 3. (left) SSH and (right) geostrophic velocity spectra from altimetry measurements, superimposed with kinetic energy spectrum (dashed–dotted) from *Oleander* observations. Dashed lines indicate slopes of −4, −3, and −2, respectively. The 95% confidence interval is marked.

spectrum, isotropy of horizontal velocity, and an equipartition between kinetic and potential energy. This is in good agreement with the two-dimensional QG turbulence theory.

The k^{-3} slope obtained from ADCP observations does not agree with the k^{-2} slope derived from altimetry measurements cited earlier. To make a careful comparison relevant for the Oleander dataset, we looked at the two closest T/P tracks (cycles 50 and 126) for the 10-yr T/P repeat-orbit period (1992–2002) over the same latitudinal extent between 32° and 39°N (Fig. 1). Because the surface slope estimate is sensitive to spatial interpolation (filtering), only tracks with no missing data are included in the analysis, which results in a total of 162 tracks (out of ~700 tracks). Counting only the T/P tracks (~140) not taken consecutively (~10 day apart), results in about 280 DOF. The SSH and geostrophic velocity spectra are calculated following the same procedure as for the Oleander data. Also, because the altimetry along-track resolution is coarser, ~6.2 km, three adjacent *Oleander* velocity samples are averaged to produce an equivalent 6-km resolution. The spectrum from the resampled 6-km Oleander data though is essentially the same as from the original 2-km data.

Figure 3 shows SSH and satellite-derived geostrophic velocity spectra; the latter is superimposed over the Oleander kinetic energy spectrum. The SSH spectrum shows a k^{-4} slope, and the corresponding geostrophic velocity spectrum shows a k^{-2} slope. These agree with Stammer (1997) and Le Traon et al. (2008). In the high-wavenumber (<70 km) tail, the altimetric velocity spectrum is badly impacted by measurement noise in the SSH spectrum. However, even in the ''signal'' range between 70 and 200 km, the energy level of the altimetric velocity spectrum is considerably higher—by about 50%—than that of the Oleander. This suggests that the altimetric velocity spectrum is probably contaminated over the entire mesoscale range and that the resultant k^{-2} slope cannot be trusted. The total eddy kinetic energies summed over all wavelengths >70 km are 0.136 and 0.178 $m^2 s^{-2}$ for the Oleander and altimetry, respectively.

The Oleander velocity spectrum is calculated from a small subset of 213 transects. Relaxing the data screening to 80% good, the total number of useful ADCP transects is doubled (461). The velocity and temperature spectra determined from this much larger dataset are indistinguishable from the original spectra. We also note that the Oleander and altimetry spectra are not calculated for the same time intervals. To test if this may introduce bias, we recompute the spectrum using only the *Oleander* transects taken within 2 days of the satellite pass. To have adequate sample size, we use the larger (80% good) Oleander dataset and the filtered altimetry time series. This results in a total of 121 overlapping records. The spatial filter impacts the altimetry velocity spectrum for shorter wavelengths. However, for wavelength >70 km, the spectral shapes are essentially the same as in the original spectra.

3. Discussion

The apparent good agreement between the Oleander spectra and QG turbulence theory is surprising. In QG theory, temperature variations are assumed to be located at the thermocline (the first baroclinic mode). In the Gulf Stream, large temperature variances are concentrated in the upper ocean, associated with the movement of fronts, filaments, and eddies. The SQG theory, which predicts equivalence between surface buoyancy and kinetic energy, in fact, seems to be more consistent with the observation. However, the $k^{-5/3}$ spectrum predicted by the SQG theory differs significantly from the observed k^{-3} spectrum. How a k^{-3} law remains valid despite large surface temperature variance requires further theoretical investigation. We note, for example, the semigeostrophic theory, which perhaps is more appropriate to describe Gulf Stream frontogenesis, predicts a $k^{-8/3}$ spectrum, very close to k^{-3} (Andrews and Hoskins 1978). It also is not clear if the k^{-3} kinetic energy spectrum in the Gulf Stream region is universal. It would be very useful if the present results can be verified for other major current systems, such as the Kuroshio and the Antarctic Circumpolar Current.

Neither the magnitude nor the shape of altimetric velocity spectrum agrees with the direct velocity observations. The assumption that the altimetric velocity spectrum is not affected by the SSH noise in wavelengths >70 km is perhaps too optimistic. How the SSH noise should be filtered, which may vary from region to region, is a challenging question. Interpretation of altimetric wavenumber spectrum in the global context should also be treated with caution.

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