Wind Modifications to Density-Driven Flows in Semienclosed, Rotating Basins

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ABSTRACT

An analytical two-dimensional model is used to describe wind-induced modifications to density-driven flows in a semienclosed rotating basin. Wind stress variations produce enhancement, inversion, or damping of density-driven flows by altering the barotropic and baroclinic pressure gradients and by momentum transfer from wind drag. The vertical structure of wind-induced flows depends on $aH$, the nondimensional surface trapping layer, where $a$ is the inverse of the Ekman layer depth $d$ and $H$ is the maximum water depth. For $aH > 5$ wind-driven flow structures are similar to the Ekman spiral; for $aH < 2$ wind-driven flows are unidirectional with depth. The relative importance of density to wind forcing is evaluated with the Wedderburn number $W = \frac{t}{C_0^2H^2D}$, which depends on water density $\rho$, mean depth $H$, a proxy of the baroclinic pressure gradient $D$, and wind stress $t$. Because $D$ depends on $a$ and therefore on the eddy viscosity of water $A_z$, wind speed and $A_z$ both modify $W$. Moreover, wind direction alters $W$ by modifying the pressure gradient through the sea surface slope. The effect of $A_z$ is also evaluated with the Ekman number $E = \frac{A_z}{fH^2}$, where $f$ is the Coriolis parameter. The alterations of the density-driven flow by the wind-driven flow are explored in the $E$ and $W$ parameter space through examination of the lateral structure of the resulting exchange flows. Seaward winds and positive transverse winds (to the right facing up basin in the Northern Hemisphere) result in vertically sheared flow structures for most of the $E$ versus $W$ space. In contrast, landward winds and negative transverse winds (to the left facing up basin) result in unidirectional landward flows for most of the $E$ versus $W$ space. When compared to observed and numerically simulated flow structures, the results from the analytical model compare favorably in regard to the main features.

1. Introduction

The lateral structure of subtidal velocity in estuaries generally depends on bathymetry, tidal forcing, buoyancy input, and earth’s rotation (e.g., Fischer 1972; Nunez and Simpson 1985; Huzzey 1988; Friedrichs and Hamrick 1996; Li and Valle-Levinson 1999; Kasai et al. 2000; Valle-Levinson et al. 2003; Valle-Levinson 2008). On the other hand, wind is considered one of the main forcing mechanisms in estuaries controlling the salinity structure, mixing, stratification, and the longitudinal and transverse distribution of the exchange flow (e.g., Wang 1979a; Wong 1994; Valle-Levinson et al. 1998, 2001; Geyer 1997; Reyes-Hernandez 2001; Winant 2004; Sanay and Valle-Levinson 2005; Scully et al. 2005; Huijts et al. 2009; Guo and Valle-Levinson 2008). Understanding the effects of wind forcing on density-driven flows requires knowledge on the lateral structure of both wind-driven and density-driven flows. The lateral structure of density-driven flows may be depicted in terms of the Ekman number $[E = \frac{A_z}{fH^2}]$, where $f$ is the Coriolis acceleration ($s^{-1}$), $A_z$ ($m^2 s^{-1}$) is the vertical eddy viscosity, and $H$ is the maximum depth (m); Kasai et al. 2000]. For large $E$ the laterally sheared exchange flow consists of seaward flow over the flanks and inflow from bottom to surface in the middle of the channel. In contrast, for a low $E$, the flow is vertically sheared: the upper layer moves seaward and the bottom layer flows landward.

There is increased observational evidence that indicates that wind forcing can enhance the density-driven gravitational flow (Wang 1979a; Valle-Levinson et al. 1998, 2001; Geyer 1997; North et al. 2004; Scully et al. 2005), in addition to its propensity to mix the water...
column. The lateral structure of wind-driven flows has mainly been described theoretically (e.g., Wong 1994; Winant 2004; Sanay and Valle-Levinson 2005). These studies have described a pattern of downwind flow over shoals and upwind flow in the channel. Lateral flows develop in response to the earth’s rotation. Recent attempts to incorporate tidal and wind modifications to density-driven flows have included theoretical efforts as well. An analytical study (Huijts et al. 2009) has also pointed out the relevance of advective effects in driving residual flows. In this regard, numerical results by Cheng and Valle-Levinson (2009) showed that lateral advection is relevant in narrow basins but not necessarily in wide basins. A numerical study (Guo and Valle-Levinson 2008) indicated that the response to wind forcing (in mid Chesapeake Bay) depends on stratification conditions. If the basin is unstratified (high \( A_z \)), the Ekman layer can be as large as the maximum depth and the transverse structure of the flow is horizontally sheared. If the basin is stratified (low \( A_z \)), the Ekman layer is shallow compared to the depth of the channel and the exchange flow structure is vertically sheared. Furthermore, vertically sheared exchange flow develops when wind is downestuary, and laterally sheared flow occurs when wind is upestuary. The lateral structure of the longitudinal flow depends then on the relation between \( A_z \) and depth.

Even though the aforementioned studies have made considerable strides in understanding the influence of wind on gravitational circulation, most of them have been limited to examining longitudinal wind forcing or are based on few numerical solutions. Furthermore, some of the above studies treat the density-driven flow separately from the wind-driven flow, and some approaches have overlooked the effect of varying wind intensity and direction on the lateral velocity structures. In this study, an analytical solution (Valle-Levinson et al. 2003) is extended to include the effect of wind stress. In particular, this study examines the influence of \( A_z \) and wind velocity on the density-driven circulation. Solutions are synthesized through analysis of the exchange flow in the Ekman number versus Wedderburn number (defined in section 2) parameter space. Casting of numerical solutions in such parameter space would require thousands of numerical experiments; therefore, this constitutes a baseline framework for future studies.

2. The analytical solution

The coordinate system is such that \( x \) is in the along-channel direction, positive seaward; \( y \) is the across-channel direction, positive to the right of the landward direction; and \( z \) is positive upward. Assuming linear and steady motion, the Coriolis acceleration, the pressure gradient, and the vertical friction (or vertical stress divergence) provide the momentum balance (e.g., Kasai et al. 2000):

\[
-fu = -\frac{1}{\rho_0} \frac{\partial P}{\partial x} + A_z \frac{\partial^2 u}{\partial z^2}
\]

and

\[
fv = -\frac{1}{\rho_0} \frac{\partial P}{\partial y} + A_z \frac{\partial^2 v}{\partial z^2},
\]

where \( u, v, A_z, g, P, \rho_0, \) and \( f \) are the along-channel velocity (m s\(^{-1}\)), the across-channel velocity (m s\(^{-1}\)), the vertical eddy viscosity coefficient (m\(^2\) s\(^{-1}\)), the acceleration due to gravity (m s\(^{-2}\)), the pressure (Pa), the mean density (kg m\(^{-3}\)), and the Coriolis parameter (s\(^{-1}\)), respectively. The system of equations can be reexpressed in terms of the complex velocity \( w = u + iv \) as

\[
A_z \frac{\partial^2 w}{\partial z^2} - ifw = g N - D_z,
\]

where \( i^2 = -1 \) is the imaginary number and \( N = V_H \eta \) and \( D = \rho_0^{-1} g N V_H \) are the complex sea level slope and the complex baroclinic horizontal pressure gradient contributions, respectively. The operator \( V_H \) indicates horizontal gradient. The solution of this equation is

\[
w(z, y) = g NF_1(z, y) + F_2(z, y)
\]

and is explained in detail by Kasai et al. (2000), where it is assumed that \( N, D, \) and \( A_z \) are independent of depth. Although rotational effects are considered in their solution, results are symmetric about the deepest part of the channel in contrast to observed flow structures in many basins. An exchange pattern closer to that expected for density-driven circulation is given by Valle-Levinson et al. (2003) for arbitrary bathymetries where, instead of a prescribed density gradient as in Kasai et al., a prescribed surface slope \( N(x, y) \) is used. By applying the no net transport condition,

\[
\int_B \int_{-H}^0 w dz dy = 0,
\]

it is possible to obtain a baroclinic pressure gradient \( D \) that is dynamically consistent with the surface slope. The baroclinic pressure gradient becomes

\[
D = \frac{-\alpha gI_1}{I_2},
\]

where
\[ I_2 = \int_0^B \frac{e^{-\alpha H} + \alpha H \tanh H - \frac{1}{2} e^{-\alpha H} + (e^{\alpha H})^2}{2} \, dy \]

and

\[ I_3 = \int_0^B N(\tanh H - \alpha H) \, dy. \]  

For simplicity, \( H = H(y) \), \( N = N(x, y) \) is given as in Valle-Levinson et al. [2003, Eq. (19)], and \( \alpha = (f/2A_z)^{1/2}(1 + i) \) represents the inverse of the Ekman layer depth (Kasai et al. 2000). Equation (6) indicates that \( D \) is proportional to the surface level slope, so for constant \( A_z \) the larger \( N \), the larger \( D \). In this work, the solution by Valle-Levinson et al. (2003) is modified to include also the effect of wind stress by changing the boundary condition at the surface \((z = 0)\):

\[ \rho g A_z \frac{\partial f}{\partial z} = \tau \]  

(9)

and

\[ \rho A_z \frac{\partial f}{\partial z} = \tau. \]  

(10)

where \( \tau = \tau_x + i \tau_y \) is the complex wind stress (in Pa), estimated as in Gill (1982). The solution (4), presented in the appendix, now can be reexpressed as

\[ w(z, y) = \frac{\partial f}{\partial z} + w_y(z, y); \]  

(11)

the linear superposition of the density-driven velocity \( w_g \) as in Valle-Levinson et al. (2003),

\[ w_g(z, y) = \frac{igN}{f} \left( 1 - \frac{\cosh \alpha z}{\cosh H} \right) + \frac{iD}{\alpha f} \]

\[ \times \left[ \frac{\cosh \alpha z - e^{-\alpha H} + (e^{\alpha H}) \cosh \alpha z}{\cosh H} \right]; \]

(12)

and the wind-driven velocity contribution \( w_z \) as

\[ w_z(z, y) = \left( \frac{\tau}{\rho A_z} \right) \left[ \frac{\tanh H \cosh \alpha z + \sinh \alpha z}{\cosh H} \right] - \left( e^{-\alpha H} \frac{\cosh \alpha z}{\cosh H} - e^{\alpha z} \right). \]

(13)

The vertical structure of (13) will be analyzed in the next section. It is reasonable to expect the surface slope \( N \), which appears in (12), to be modified by the wind stress \( \tau \).

Therefore, a dynamically consistent \( N \) needs to be defined with respect to \( \tau \). For simplicity, the modification of \( N \) by \( \tau \) may be derived from a balance between surface slope and wind stress (e.g., Gutiérrez de Velasco and Winant 2004). According to the coordinate system chosen, the surface slope in each direction, \( x \) and \( y \), is scaled to the length \( L \) and width \( B \) of the basin, respectively. Therefore, the surface slope due to wind stress may be approximated as

\[ N_x = \frac{\text{Re}(\tau) x}{\rho g \overline{H} L}, \]  

and

\[ N_y = \frac{\text{Im}(\tau) y}{\rho g \overline{H} B}. \]  

where \( \overline{H} \) is the mean depth of the channel. Under this consideration, the surface slope should be expressed as

\[ N = N_{\text{prescribed}} + N_x. \]  

(14)

From the no net transport condition in the along-channel and across-channel directions, expression (6) for the baroclinic contribution must be rewritten as

\[ D = \left[ -\alpha g I_3 + \left( \frac{\tau}{\rho A_z} \right) (I_4 - I_3) \right] / I_2, \]  

(15)

where now \( I_3 \) is consistent with (14) and

\[ I_4 = \int_0^B \left( \tanh H \sinh H - \cosh H + 1 \right) \, dy \]  

(16)

and

\[ I_5 = \int_0^B \left[ e^{-\alpha H} (\tanh H + 1) - 1 \right] \, dy. \]  

(17)

Equation (15) indicates that the real and imaginary parts of the baroclinic contribution \( D \) are now a function of the sea level slope and the wind stress, but also the sea level slope is modified by wind forcing. It is noteworthy that, according to the model, wind stress modifies the surface slope or pressure gradient. Therefore, wind forcing can reduce or increase \( N \) and consequently modify the baroclinic part of the solution.

Customarily, the comparison between the horizontal density gradient and wind stress forcing in lakes has been performed through the dimensionless Wedderburn number (e.g., Imberger and Parker 1985), \( W = (g' h^2)/u^2 L \), where \( g' \), \( h \), \( u \), and \( L \) are the reduced gravity, the
depth of the pycnocline, the wind frictional velocity, and the characteristic length of the lake, respectively. For the subject dealt with herein, noticing that \( D \sim (\rho^{-1}g)\Delta \rho/L \) and \( u_\infty = (\rho^{-1}\tau)^{1/2} \), the Wederburn number can be rewritten as \( W = (\rho \tau^2 D)/\tau \), where \( \tau \) and \( L \) are now interpreted as the mean depth of a cross section and the length of the basin, respectively. Also, the dependence of \( W \) on \( \tau \) is specified in both terms for \( D \) in (15).

In the next section, the structure of velocity according to (11) is presented under the following assumptions: a channel of transverse form given by \( H(y) = 30 \exp[-(y/B - 0.15)^2/0.5^2]; \) three different values of the eddy viscosity coefficient \((10^{-3}, 10^{-2}, \text{and} \ 10^{-1} \text{m}^2 \text{s}^{-1})\) that imply three different Ekman numbers \((E = 0.01, 0.1, \text{and} \ 1.0); \) and three different values of wind speed and four different wind direction scenarios: 1) along the channel in the positive direction, 2) along the channel in the negative direction, 3) across the channel in the positive direction, and 4) across the channel in the negative direction. The magnitude of the longitudinal density gradient used by Wong (1994) and Kasai et al. (2000) and implied by Valle-Levinson et al. (2003, their Eq. 19) is \( O(10^{-4}) \). For the given bathymetry, different \( A_z \) and different wind speeds, the size of \( N \) and \( D \), may vary. However, the velocity structure ultimately depends on \( W \) and not on the absolute values of \( D \) or \( \tau \). As long as the magnitude of \( W \) is the same, the structure of the flow will also be the same regardless of the strength of the wind. Results then are presented in terms of \( W \), which is in the range \([100, 1]\). The value of \( W \geq 100 \) represents that density-driven exchange flow prevails, while \( W \leq 10 \) indicates that wind-induced exchange flow dominates. Also, the velocity contours and vectors representing the along-channel and the transverse velocity components are normalized with respect to the maximum of the two velocity components. Throughout this paper, the description of the resulting flow is for an observer looking into the basin.

3. Results

A baseline solution with no wind is compared to various wind forcing cases. In the baseline solution the wind velocity is null, \( E = 0.1 \), and solution (11) reduces to Eq. (12), that is, the solution given by Valle-Levinson et al. (2003). Here, an asymmetric longitudinal exchange of about the same magnitude consists of surface outflow over the shoals and inflow at the bottom of the channel (Fig. 1a). In the transverse direction, flows converge over the right side of the channel in response to the transverse pressure gradient contribution. Relatively high Ekman numbers cause the transverse flow to respond to the transverse pressure gradient, with rightward velocities near the surface and leftward velocities near the bottom. For low Ekman numbers, however, the structure of the transverse flow shows “the transverse flow adjusting to the along-estuary pressure gradient: flow to the left at surface and to the right at depth” (Valle-Levinson et al. 2003). It is worth noting that Fig. 1a is the sum of the barotropic and baroclinic components: the barotropic component yields asymmetric seaward velocities that are stronger over the left side of the channel and near the surface but weak over the shoulders of the channel as a result of the prescribed transverse sea level slope. On the other hand, the baroclinic component, \( D_0 = 1.1 \times 10^{-6} \text{ s}^{-2} \), produces symmetric inward velocities (as can be expected) that are stronger near the bottom but detached from it. The weaker baroclinic longitudinal velocities occur near the surface in the center of the channel and over its shoulders. In the transverse direction the barotropic component produces leftward transverse velocities.
that decay with depth, while the baroclinic component $D_x$ yields rightward velocities that also decay with depth.

The wind-driven longitudinal flow [Eq. (13), Fig. 1b] for a seaward wind and $E = 0.1$ depicts a two-layer exchange, in contrast to the solution given by Wong (1994). The surface flow is in the direction of wind and reverses direction at the central lower part of the channel; however, the magnitude of $D_x = 8.2 \times 10^{-7} \text{ s}^{-2}$ is smaller than for the baseline solution. In the transverse direction depth-decaying velocities are unidirectional toward the left and strongest at the central, deepest part of the channel as a result of Coriolis and viscosity, as will be seen in the next section.

In the complete solution, the sum of Eqs. (12) and (13) (Fig. 1c), the wind stress modifies the structure of the baseline case (Fig. 1a) by direct surface drag and by changing the initially prescribed sea surface slope. The vertical shear owing to wind drag at the surface is able to reach deeper depths than at the baseline solution. On the other hand, the wind setup opposes the surface slope (associated with the density field) and reduces the barotropic pressure gradients; therefore, the baroclinic term $D_x$ is also reduced. Transverse velocities over the deep channel are toward the left side as a result of the predominance of wind-induced velocities and Coriolis. Over the right shoulder, velocities are also toward the left but here as result of both density gradient and wind stress forcing. Over the left shoulder, wind-driven flows are reduced by friction and density-driven flows dominate.

It is opportune here to analyze the behavior of the baroclinic pressure gradient, as given by (15), as a function of wind speed between 0 and 15 m s$^{-1}$ and different directions (Fig. 2). For no-wind conditions, the longitudinal and transverse components of $D$ are positive because of the prescribed negative sea surface slope components [Eq. (6)]. This can be interpreted as, given a higher sea level at the head of the estuary than at the mouth, the fresher/saltier water interface has an opposite slope so that $D_x > 0$ and the acceleration due to $D_x$ is negative (into the estuary). Sea level slopes downward from left to right, facing upestuary, and therefore $D_y$ is $> 0$ and the acceleration due to $D_y$ is negative (toward the left, facing upestuary). As the downestuary wind forcing increases, the response of $D_y$ decreases nonlinearly and changes sign for wind speed $\sim 14 \text{ m s}^{-1}$. This means that positive across-channel winds have a similar effect to downestuary winds because of the earth’s rotation. Here $D_y$ decreases drastically and changes its sign at wind speed $\sim 4 \text{ m s}^{-1}$. Wind in the negative across-channel direction creates the opposite effect and both components remain positive, increasing as wind speed increases. Finally and consistent with Eq. (15), wind speed different from zero introduces slight lateral asymmetries in the longitudinal baroclinic component of the flow. All of these effects on $D$ are localized at the estuarine cross section being portrayed. Future efforts on this topic will need to consider remote effects.

The vertical structure of the wind-driven velocity is a function of $\alpha H$, the degree of surface trapping (cf. Price and Sundermeyer 1999), modulated by the ratio of wind stress to $A_z$ (Fig. 3). For $\alpha H > 5$ (Fig. 3a), the solution approaches the expression

$$w(z) = \frac{2\tau e^{\alpha z}}{\rho A_z \alpha} = \frac{2\tau}{\rho A_z \alpha} e^{\alpha z} (\cos \alpha |z| + i \sin \alpha |z|)$$

and resembles the Ekman spiral for an infinite water depth, except that here the depth is finite. Velocity veers $\pi/4$ from the wind direction at the surface and decays exponentially to the depth $z = -1/\alpha$, the Ekman layer depth. For increased $A_z$ or decreased $H$, the veering decreases (e.g., Fig. 3b; $\alpha H = 2.1$) until the flow becomes
unidirectional and linearly decaying with depth (e.g., Fig. 3c, $\alpha H = 0.6$), where the solution approaches

$$w(z) = \frac{2\tau}{\rho A_z \alpha} \sinh \alpha z.$$  

For a constant $A_z$ and latitude, this means that, to observe a surface Ekman spiral, the depth of the water column $H$ has to be greater than five times the Ekman layer depth, keeping in mind that the Ekman layer is determined by $A_z$. It is worth noting that $\alpha H = 1/\sqrt{2E}$, so, for a given depth $H$ and latitude, the greater $A_z$ or $E$, the smaller the surface trapping because the Ekman layer depth approaches the value of $H$. A sufficiently low $A_z$ value results in two-layer wind-driven transports, and a sufficiently large $A_z$ value makes the flow unidirectional but weak. This result is consistent with observations that suggest that straining by wind is related to low $A_z$ values, whereas wind mixing is associated with high $A_z$ (Scully et al. 2005). In Fig. 1b, $\alpha H = 1.08$ and there is no opportunity for the Ekman spiral to form; therefore, the wind-driven longitudinal velocities are mainly downestuary and weak.

The effects of wind forcing on density-driven flows are discussed next. The direction of the wind will modify the velocity field by changing the pressure gradient, through direct drag on the water and through the respective component of the Coriolis acceleration. Wind in any direction will modify the respective pressure gradient as well as the downwind and transverse flows. In this analysis, the structure of the flow for each wind direction is presented in six panels: each column represents a different $A_z$ scenario and each row represents a different wind speed condition (e.g., Fig. 4). The most salient features in general are (i) the transformation from a mainly geostrophic, vertically sheared exchange ($E = 0.01$) to a mainly friction-controlled, laterally sheared exchange ($E \approx 1.0$) and (ii) the transformation from a pressure-gradient-controlled flow ($W > 100$) to a wind-driven flow ($W < 10$). It will be shown that, for wind-controlled flows, a two-layer structure may develop even under high $E$ values.

**a. Wind in the seaward direction**

The modifications of a seaward wind to density-driven flows depend on the wind speed. Weak wind speeds ($W > 100$, Figs. 4a–c) produce minor modifications to the purely density-driven case (cf. Fig. 1a to 4a). The barotropic pressure gradient and the wind-induced Coriolis acceleration dominate near the surface, but the baroclinic pressure gradient becomes dominant underneath. This is the case for all other wind directions under weak wind forcing. It is noteworthy that the core of maximum landward velocity in Figs. 4a–c lifts off from the bottom as $E$ (bottom friction) increases (cf. Valle-Levinson et al. 2003, their Fig. 5). For $E = 0.01$, the flow is nearly geostrophic: asymmetric seaward at the surface and landward underneath. The transverse flow is also in two layers and shows two maxima with depth in response to Coriolis and barotropic pressure gradient at the surface and Coriolis and baroclinic pressure gradient at depth.
As friction increases, $E = 0.1$ and $E = 1.0$, the velocity is reduced and the structure of the longitudinal flow gradually shifts from vertically sheared to horizontally sheared (Figs. 4b,c). As $E$ increases, the asymmetries reduce and the exchange flow becomes symmetric about the thalweg. For $E = 0.1$, both components of velocity resemble the solution given by the balance between pressure gradient and friction (cf. Fig. 1a). The same balance operates for $E = 1.0$, but now the solution resembles those by Wong (1994) and Kasai et al. (2000), although velocity at the surface in the center of the channel is slightly different from zero. The structure of the transverse velocity, on the other hand, shows convergence at the core of maximum longitudinal velocity over the deepest part of the channel. For $E = 0.1$, the main transverse balance at the surface is between Coriolis and the barotropic pressure gradient, but at depth it changes to pressure gradient, Coriolis, and friction. For $E = 1.0$, the transverse balance is given by the pressure gradient balanced by friction.

As wind velocity increases (100 > $W$ > 10), there is a seaward setup that opposes the initial sea level slope, but the wind stress enhances the seaward flow throughout the section (Figs. 4d–f). The wind setup also results in the decrease of the baroclinic pressure gradient (Fig. 2) that weakens the inflow underneath, almost cancelling it at $E = 1.0$ (Fig. 4f). The structure of the transverse velocities shows the higher leftward velocities near the surface, consistent with the higher-longitudinal velocities and Coriolis acceleration. For $E = 0.01$ (Fig. 4d), the surface trapping can develop over the deepest channel and rightward velocities are observed at depth. As friction increases, $E = 0.1$ and $E = 1.0$ (Figs. 4d and 4f), transverse velocities become unidirectional, leftward over the right side of the channel and rightward over the left side of the channel. Despite the progress toward unidirectional velocity components with increased friction, the no net transport condition is satisfied by the form of the no-net-transport condition [Eq. (5)]. Positive, seaward velocities are compensated by negative leftward velocities.

For increased wind speed (10 > $W$ > 1) and $E = 0.01$ (Fig. 4g), asymmetrical wind-driven seaward longitudinal flow occupies the surface trapping layer across the section. Very weak upestuary flow underneath and detached from the bottom occupies most of the right section of the channel. In the transverse direction, the flow is mainly leftward. Along-channel and transverse flows satisfy the no-net-transport condition. For $E = 0.1$ and $E = 1.0$ (Figs. 4h,i), the longitudinal flow is seaward throughout the whole channel section, while transverse flow is unidirectional toward the left, satisfying the no-net-transport condition. These patterns indicate that wind setup overcomes the prescribed surface slope in such a way...
that the wind and the baroclinic forcing act in the same direction. For $E = 0.1$ the longitudinal velocity structure shows a clearer vertical structure than for $E = 1.0$.

b. Wind in the landward direction

For weak wind ($W > 100$) and different $E$ numbers, wind stress in the landward direction produces slightly different flow structures (Figs. 5a–c) than those for seaward winds (Figs. 4a–c). For $E = 0.01$ (Fig. 5a), the longitudinal flow is still seaward at the surface but relatively weaker than for downestuary wind (Fig. 4a). Also, although the landward wind is comparatively weak, it increases the prescribed longitudinal surface slope and also the magnitude of $D_x$ (Fig. 2). This results in slightly stronger landward velocity near the bottom compared to Fig. 4a. For $E = 0.1$ and $E = 1.0$ (Figs. 5b,c), the structure of the longitudinal flow is fairly similar to that for a seaward wind (Figs. 4b,c). The longitudinal balance remains about the same except that seaward velocities are weaker and landward velocities stronger. For $E = 1.0$, the flow is landward throughout the whole water column in the center of the channel and seaward over the shoals. The transverse direction no major changes are noticeable for $E = 0.01$, but for $E = 0.1$ and $E = 1.0$ transverse weak velocities are rightward in response to increased Coriolis acceleration and increased bottom friction.

As landward wind increases, $100 > W > 10$ (Figs. 5d–f), the structure of the longitudinal flow shifts from a three-layer system for low $E$ to a laterally sheared exchange for higher $E$. For $E = 0.01$ (Fig. 5d), the landward wind stress drives water into the estuary and increases the longitudinal barotropic and baroclinic pressure gradients. Over the shoals where $\alpha H$ is small, the flow is in the direction of the wind. At the surface on the right side of the channel, the landward flow expands horizontally to the center of the channel as a consequence of the asymmetric structure of the density-driven flow (cf. Fig. 1a). In the middle of the channel, the wind-induced flow opposes the density-driven upper layer flow; near the bottom, however, the flow is relatively strong, consistent with $D_x$. Within the surface trapping layer, leftward transverse velocities are weak because of the competition between the wind-induced velocity and Coriolis-induced velocity. For $E = 0.1$ and $E = 1.0$ (Figs. 5e,f), landward wind produces unidirectional flow over the shallow sides of the channel and over the right portion of the channel where the asymmetric density-driven outflow is the weakest (cf. Fig. 1a). The asymmetry and the vertical structure fade as viscosity increases. In the transverse direction, the leftward flow increases toward the right side of the channel, and transverse divergence and convergence is observed from left to right. Divergences and convergences occur at the maxima longitudinal seaward and landward flows at each depth, respectively. Because of the asymmetry for $E = 0.1$, the line of convergence leans to the right from bottom to surface, while for $E = 1.0$ it is vertical, in the middle of the channel where the core of maximum longitudinal velocity is located.

FIG. 5. As in Fig. 4 but modified by landward wind stress.
For wind-driven flow dominating over density-driven flow \((W < 10)\) and \(E = 0.01\) (Fig. 5g) an asymmetrical three-layer structure is well developed. At the surface and bottom, the flow is upestuary in response to wind drag and baroclinic forcing, respectively. In the transverse direction, velocities are to the right of the channel due to the wind forcing at the surface and the Coriolis-induced forcing below. For \(E = 0.1\) and \(E = 1.0\) (Figs. 5h,i), the longitudinal and transverse flows are completely upestuary and rightward, showing dominance by wind forcing. Similar to the seaward wind case, the net-transport condition \([\text{Eq. (5)}]\) is satisfied by negative landward velocities and positive rightward velocities.

c. Wind in the positive across-channel direction

Wind stress in the positive \(y\) direction (Fig. 6) decreases the prescribed transverse pressure gradient, but, at the same time, Coriolis acceleration contributes to enhance the longitudinal density-driven exchange. Consequently, the resulting longitudinal and transverse flow structures are similar to those related to seaward wind stress (Fig. 4), especially for \(W > 100\).

For high \(W\) and mid to low \(E\), the structure of the flow (Figs. 6a,b) remains without considerable changes compared to seaward wind (Figs. 4a,b). For \(E = 1.0\) (Fig. 6c), the structure of the longitudinal flow is closer to the structure given by Wong (1994) and Kasai et al. (2000), possibly because of a less effective surface seaward wind drag.

For \(10 < W < 100\), the transverse wind produces stronger surface seaward flow throughout the section and modifies the longitudinal pressure gradient. This modification, however, is not as effective as for the longitudinal seaward wind (Fig. 2a); consequently, the landward flow near the bottom owing to the longitudinal baroclinic pressure gradient is weaker and constrained to a deeper depth for relatively low bottom friction, \(E = 0.01\) and \(E = 1.0\) (Figs. 6d,e). As bottom friction increases, \(E = 1.0\), the wind-induced flow is not effective in constraining the bottom landward flow. In the transverse direction and \(E = 0.01\), the flow at the very surface responds to the direct wind forcing but below, although weak, responds mainly to the longitudinal pressure gradient (Fig. 6d). For \(E = 0.1\) and \(E = 1.0\) (Figs. 6e,f), the transverse flow becomes unidirectional toward the right.

d. Wind in the negative across-channel direction

Wind stress in the negative direction increases the prescribed transverse pressure gradient but also weakens the
surface longitudinal density-driven flow through Coriolis accelerations (Fig. 7). Therefore, the structure of the long-channel flow is similar to that for longitudinal wind in the landward direction (see Fig. 5), especially for $W > 100$ (Figs. 7a–c). Consequently, these results are not discussed further. For $100 > W > 10$ and $E = 0.01$ (Fig. 7d), the long-channel flow is laterally partitioned in outflow over the left and right sides and inflow in the center-right side. The long-channel component of the wind-driven flow at the surface overcomes the weak seaward density-driven flow at the right side of the channel (see Fig. 1a) and coalesces with the bottom landward flow. Over the left and right sides where the cores of maximum density-driven velocity occur (Fig. 1a) seaward flow persists. In the transverse direction the flow is made of two layers: Coriolis and wind-forced leftward flow at the surface and Coriolis-induced rightward flow underneath. For $E = 0.1$ and $E = 1.0$ (Figs. 7h,i), the structure of the longitudinal flow is lateral and symmetrical with seaward flow by the sides and landward flow by the center. In the transverse direction the flow is unidirectional toward the left. For $E = 0.1$ velocity maxima are observed over the slope of the channel. For $E = 1.0$ the velocity maximum occupies the center of the channel.

e. Flow over $H_{\text{max}}$ as a function of the Ekman and Wedderburn numbers

The results presented above are cast in the parameter space of $E$ and $W$ to generalize the nature of the competition between wind-driven flow and density-driven flow. The property analyzed here is the difference between the maximum of the absolute value of the normalized seaward velocity and the maximum of the absolute value of the normalized landward velocity at the deepest part of the section $H_{\text{max}}$; that is,

$$\Delta u_{H_{\text{max}}} = \left( \max \left| u_{\text{seaward}} \right| - \max \left| u_{\text{landward}} \right| \right)_{H_{\text{max}}}. $$

This quantity represents the strength of the vertical shear at the deepest part of the section and is portrayed as a function of $E$ and $W$ (Fig. 8). Values of $\Delta u < 0$ indicate

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**Fig. 7.** As in Fig. 4 but modified by negative across-channel wind stress.
that landward flow dominates in \( H_{\text{max}} \). For \( \Delta u = -1 \), the flow is landward throughout the entire water column. Values of \( \Delta u > 0 \) indicate that seaward flow dominates in \( H_{\text{max}} \). For \( \Delta u = 1 \), the flow in the entire water column is seaward. Furthermore, the value of \( \Delta u = 0 \) indicates that both inward and landward flows are equally important. Accordingly, isolines in Fig. 8 close to unity (positive or negative) indicate net seaward or landward flow at \( H_{\text{max}} \), suggesting laterally sheared exchange through the section. Isolines close to zero indicate vertically sheared exchange flow at \( H_{\text{max}} \). To facilitate interpretation on the structure of the flow at \( H_{\text{max}} \), areas of \( \Delta u > 0.3 \) and areas of \( \Delta u < -0.3 \) were arbitrarily shadowed with dark and light gray respectively, leaving the range \( 0.03 > \Delta u > -0.3 \) in white. The labels inserted in each of the panels in Fig. 8 correspond to the cases portrayed in Figs. 4–7.

In general, \( \Delta u \) isoline slopes show three regions: 1) where \( \Delta u \) depends mainly on \( E \) and weakly on \( W \), 2) where \( \Delta u \) depends on \( W \) and weakly on \( E \), and 3) where \( \Delta u \) depends on both \( E \) and \( W \). For seaward winds (Fig. 8a) \( \Delta u \) isolines slope upward toward high \( E \) values; therefore, \( \Delta u \) contours are mainly dependent on \( W \) and the flow at \( H_{\text{max}} \) is vertically sheared (white area). For relatively low \( W \) (wind forcing dominant) and high \( E \), the flow becomes seaward through the water column (dark gray). It can be said that a two-layer structure over \( H_{\text{max}} \) is maintained by moderate and strong seaward wind forcing except for high bottom friction conditions. The labels 4a–4h, corresponding to the seaward wind cases dealt in the previous section, fall in the region where the flow is vertically sheared. However, labels 4a–4c correspond to slightly landward-dominated flow and labels 4d–4h correspond to slightly seaward-dominated flow. Label 4i corresponds to the region of essentially seaward flow. This is not the case for landward wind forcing (Fig. 8b). Here the \( \Delta u \) slopes indicate a high dependence on \( E \), except for \( W > 10 \). Additionally, the flow at \( H_{\text{max}} \) is mainly landward although vertically sheared, except for \( E > 0.1 \) when it becomes unidirectional.

Wind in the positive across-channel direction (Fig. 8c) produces \( \Delta u \) to depend mainly on \( W \), in a similar fashion to seaward wind forcing, except that now \( \Delta u \) is mainly controlled by \( E \) for \( E > 0.5 \) and \( W > 10 \) (relatively weak wind). The flow at \( H_{\text{max}} \) is mainly vertically sheared; however, for \( W < 6 \) it is seaward through the water column or unidirectional landward for \( W > 70 \) and \( E > 1 \). For wind in the negative cross-channel direction (Fig. 8d), \( \Delta u \) contours are similar to those for landward wind forcing. A net landward flow at \( H_{\text{max}} \) dominates in the channel for most of the parameter space. Only for \( W > 70 \) and \( E \sim 5 \) the flow shows some vertical structure.

4. Discussion

The analytical solution presented here includes several assumptions that are subject to improvement, perhaps with future numerical approaches. The solution neglects
advective effects; assumes constant eddy viscosity, no net transport (local but no remote effects), and a linear superposition of density-driven and wind-driven flows; and considers a straightforward dynamical consistency between wind-forcing and pressure gradient. The first assumption may be limiting as recent studies have shown that the lateral advection term in the longitudinal momentum balance is one of the main mechanisms driving the residual flow (Lerczak and Geyer 2004; Huijts et al. 2009; Scully et al. 2009). However, lateral advection is relevant only in narrow or deep estuaries (Cheng and Valle-Levinson 2009) where the “estuarine Reynolds number” Re is large (>20). According to Cheng and Valle-Levinson (2009),

\[
Re = \frac{UH^2}{A_z B} = \frac{R_o}{E},
\]

where \( U \) is the magnitude of the density-driven exchange flow, \( A_z \) is the vertical eddy viscosity, \( B \) and \( H \) are the channel width and depth, and \( R_o \) and \( E \) are the Rossby and Ekman numbers. Narrow or deep estuaries are characterized by an aspect ratio \( H/B \) (contained in Re) that is sufficiently high, combined with sufficiently low friction, to yield \( Re > 20 \). In wide and shallow estuaries (\( H/B \) small) where \( Re < 0.1 \), Coriolis accelerations are more important to the dynamics than lateral advection. For example, taking for the Chesapeake Bay entrance (CBE) an average depth of 12 m, an average speed of 0.15 m s\(^{-1} \), and a width of 15 600 m, \( Re \) ranges between 0.013 and 1.3 for \( A_z \) between \( 1 \times 10^{-3} \) and \( 1 \times 10^{-1} \). Therefore, the linear analytical solution portrayed here is fairly appropriate for wide and shallow systems.

The no-net-transport condition implies that the sectionally integrated horizontal velocity components are zero. This is accomplished in three main forms: 1) through vertically sheared velocities with opposing signs in both velocity components; 2) through convergence of the transverse component at the core of maximum longitudinal velocity and two-layer exchange flow in the longitudinal direction; or 3) through unidirectional transverse component with opposite sign in the longitudinal component, or vice versa. The last form is particularly observed in Figs. 4h,i and 5h,i under high frictional conditions and strong seaward and landward wind scenarios. The condition is limiting in the sense that it may not represent properly the details of observed responses to wind forcing. However, it is consistent with observations related to depth-independent seaward transport for intense downestuary wind (e.g., Goodrich 1987; Valle-Levinson et al. 1998) or observations that describe the existence of an upestuary underflow in response to moderate downestuary winds (e.g., Wang 1979b; Valle-Levinson et al. 1998; North et al. 2004). Any consideration in regard to the remote wind effect is beyond the applicability of this 2D solution. However, local winds, as prescribed here, can produce the slope that drives the subtidal flow in most of the estuary (Guo and Valle-Levinson 2008).

The effects of spatially and temporally variable \( A_z \) may be relevant in modifying the details of the flow patterns described here. Nonetheless, as also described by Guo and Valle-Levinson (2008), the essence of the exchange flow patterns remains unaltered by the assumption of constant \( A_z \). Future studies will have to concentrate on this issue by examining detailed modifications induced by varying \( A_z \).

The independence of the surface level slope in the transverse direction to wind forcing is a limitation of the model that cannot be disregarded. It is expected that in real estuaries the transverse barotropic and baroclinic pressure gradients will be modified by wind. An appropriate formulation of this modification is quite problematic but will undoubtedly improve results. Such formulation is beyond the scope of this work.

At present time, it is challenging to demonstrate with measurements the variety of patterns illustrated by the analytical model results. However, data on the lateral structure of the wind-driven flow at the CBE are used to compare to model results from the real bathymetry of the CBE. On 20 and 21 February 1997 and 7 and 8 May 1999, velocity data were collected at the Chesapeake Bay entrance during 25-h periods centered at spring and neap tides, respectively (Reyes-Hernandez 2001). The sampling consisted of towing a 614.4-kHz broadband RD Instruments acoustic Doppler current profiler (ADCP) across the bay entrance. High input buoyancy conditions provided by river discharge (~3800 and ~2377 m\(^3\) s\(^{-1} \)) and southeast (1.0 m s\(^{-1} \)) and southwest (3.8 m s\(^{-1} \)) were the average conditions during each respective survey. On 20 and 21 February (Fig. 9a), longitudinal two-layer exchange occupied the first half of the section from the Chesapeake Channel to Middle Ground Shoal. Over Six-Meters Shoal the flow was landward; over the North Channel a rather complicated three-layer structure composed of asymmetrical seaward flow on the left surface and middle layer, and landward flow on the right surface and bottom was observed. Assuming \( W = 21.2, E = 0.06 \), and an oblique landward wind, the analytical model was able to satisfactorily represent the main features of the longitudinal and transverse velocities across the section, except those at the North Channel. It is interesting, however, to realize that Figs. 5d,e (landward wind) resemble the observed structure of velocity at the North Channel. On 7 and 8 May (Fig. 9b), a two-layer exchange flow was observed over most of the section (Fig. 9a), suggesting a relatively low \( A_z \). By assuming \( W = 20, E = 0.01, \)
and wind in the transverse positive direction, the analytical solution reproduced the essential structure of the longitudinal and transverse velocity.

The limited number of observations available for comparisons with the analytical model may be compensated with comparisons to numerical model results. The lateral structure of the longitudinal flow from the analytical model (Fig. 10) is comparable to the flow structures at mid Chesapeake Bay and the Chesapeake Bay entrance obtained from numerical experiments of wind-driven circulation \((W - 7)\) (Guo and Valle-Levinson 2008, their Figs. 7h and 8h). At mid Chesapeake Bay, an upestuary southeast wind \((W = 13.5, E = 0.18)\) produces an asymmetrical longitudinal flow structure, whereas downwind velocities develop over the sides of the section and an upwind velocity occupies the entire column in the center of the channel. At the bay entrance, an upestuary southeast wind \((W = 89, E = 0.08)\) produces the main features of the longitudinal flow: downestuary velocities across the bay entrance section and subsurface upestuary velocities in the Chesapeake Channel.

Despite potential drawbacks represented by the assumptions used in the analytical model, its solution shows promise in understanding patterns of density-driven flow modified by wind forcing in relatively wide estuaries. The solution is consistent with the observed data at the Chesapeake Bay entrance and also with the vertically sheared and horizontally sheared flow patterns as a function of the Ekman number, pointed out by Guo and Valle-Levinson (2008). The solution allows elucidation of the roles of rotation, wind, and internal friction in modifying the density-driven flows. To gain the insights provided in Fig. 8, for example, several hundred or even thousands of numerical model runs will have to be executed. Therefore, the approach presented here is only a step toward a more thorough understanding of this topic.

5. Conclusions

The modifications of wind forcing to density-driven flows depend on the wind velocity and may be cast in the parameter space of the Wedderburn \(W\) and Ekman \(E\) numbers. At low \(E\) numbers the structure of the flow is mainly controlled by the pressure gradient and Coriolis. At High \(E\) the structure is controlled by pressure gradient and friction. For high \(W\), wind effects are small and for low \(W\) wind effects are substantial. Wind forcing can invert the pressure gradient sign and therefore the velocity structure. The dependence of the flow structure on \(E\) and \(W\) was analyzed through the difference between the normalized inflows and outflows at the maximum depth of the channel (Fig. 8). Seaward or positive transverse wind show vertical structure for most of the \(E\) and \(W\) space. In contrast, landward or negative transverse winds show unidirectional landward flows for most of
the $E$ and $W$ space. When compared with observed or numerical flow structures, the results from the model compare favorably in regard to the essential features of vertical versus laterally sheared exchange flows.

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APPENDIX

Analytical Wind-Driven Solution

The general solutions for $F_1$ and $F_2$ are

$$F_1 = C_1 \cosh \alpha z + C_2 \sinh \alpha z - \frac{1}{if} \text{ } \text{ } (A1)$$

and

$$F_2 = C_3 e^{\alpha z} + C_4 e^{-\alpha z} + \frac{Dz}{if}. \text{ } \text{ } (A2)$$

Making use of the boundary conditions at the surface and at the bottom allows

$$C_1 = C_2 \tanh \alpha H + \frac{1}{if \cosh \alpha H}.$$  

$$C_2 = \frac{\tau}{\rho A_z \alpha gN},$$

$$C_3 = \frac{\tau}{\rho A_z \alpha} - \frac{D}{if \alpha} + C_4,$$

and

$$C_4 = \left[ \frac{D}{if \alpha} \left( e^{-\alpha H} + \alpha H \right) - \frac{\tau}{\rho A_z \alpha} \left( e^{-\alpha H} + \frac{1}{e^{\alpha H}} \right) \right].$$

Replacing the respective constants in (A1) and (A2) and these expressions in (4) results in (11).

REFERENCES


Reyes-Hernandez, A. C., 2001: Tidal and subtidal lateral structures of the density and velocity in the Chesapeake Bay entrance. Ph.D. dissertation, Old Dominion University, 131 pp.


