Semiempirical Dissipation Source Functions for Ocean Waves. Part I: Definition, Calibration, and Validation

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ABSTRACT

New parameterizations for the spectral dissipation of wind-generated waves are proposed. The rates of dissipation have no predetermined spectral shapes and are functions of the wave spectrum and wind speed and direction, in a way consistent with observations of wave breaking and swell dissipation properties. Namely, the swell dissipation is nonlinear and proportional to the swell steepness, and dissipation due to wave breaking is nonzero only when a nondimensional spectrum exceeds the threshold at which waves are observed to start breaking. An additional source of short-wave dissipation is introduced to represent the dissipation of short waves due to longer breaking waves. A reduction of the wind-wave generation of short waves is meant to account for the momentum flux absorbed by longer waves. These parameterizations are combined and calibrated with the discrete interaction approximation for the nonlinear interactions. Parameters are adjusted to reproduce observed shapes of directional wave spectra, and the variability of spectral moments with wind speed and wave height. The wave energy balance is verified in a wide range of conditions and scales, from the global ocean to coastal settings. Wave height, peak and mean periods, and spectral data are validated using in situ and remote sensing data. Some systematic defects are still present, but, overall, the parameterizations probably yield the most accurate estimates of wave parameters to date. Perspectives for further improvement are also given.

1. Introduction

a. On phase-averaged models

Spectral wave modeling has been performed for the last 50 years, using the wave energy balance equation (Gelci et al. 1957). This approach is based on a spectral decomposition of the surface elevation variance across wavenumbers \( k \) and directions \( \theta \). The spectra density \( F \) evolves in five dimensions that are the two spectral dimensions \( k \) and \( \theta \), the two physical dimensions of the ocean surface (usually longitude and latitude), and time \( t \):

\[
\frac{dF}{dt} = S_{\text{atm}} + S_{\text{nl}} + S_{\text{oc}} + S_{\text{bt}},
\]

where the Lagrangian derivative is the rate of change of the spectral density following a wave packet at its group speed in both physical and spectral spaces. In particular, this spectral advection includes changes in direction due to the earth’s sphericity, as well as refraction over varying topography (e.g., Munk and Traylor 1947; Magne et al. 2007) and currents, and changes in wavelength or period in similar conditions (Barber 1949).

The spectral source functions on the right-hand side of Eq. (1) are grouped into their atmospheric \( (S_{\text{atm}}) \), nonlinear scattering \( (S_{\text{nl}}) \), oceanic \( (S_{\text{oc}}) \), and bottom \( (S_{\text{bt}}) \)
sources. This grouping, like any other, is largely arbitrary. For example, waves that break are nonlinear. Thus, the effects of breaking waves, which are contained in $S_{oc}$, is intrinsically related to the nonlinear evolution term contained in $S_{nl}$. Yet, compared to the usual separation of deep-water evolution by wind input, nonlinear interactions, and dissipation (e.g., WAMDI Group 1988), this grouping has the benefit of identifying where the energy and momentum are going to or coming from, which is a necessary feature when ocean waves are used to drive or are coupled with atmospheric or oceanic circulation models (e.g., Janssen et al. 2004; Ardhuin et al. 2008b).

Here, $S_{atm}$, which gives the flux of energy from the atmospheric nonwave motion to the wave motion, is the sum of a wave generation term $S_{in}$ and a wind-generation term $S_{out}$ (often referred to as negative wind input, i.e., a wind output). The nonlinear scattering term $S_{nl}$ represents all processes that lead to an exchange of wave energy between the different spectral components. In deep and intermediate water depth, this is dominated by cubic interactions between quadruplets of wave trains, while quadratic nonlinearities play an important role in shallow water (e.g., WISE Group 2007). The ocean source $S_{oc}$ may accommodate wave–current interactions$^1$ and interactions of surface and internal waves. The oceanic source term $S_{oc}$ is restricted to wave breaking and wave–turbulence interactions.

The basic principle underlying Eq. (1) is that waves essentially propagate as a superposition of almost linear wave groups that evolve on longer time scales as a result of weak-in-the-mean processes (e.g., Komen et al. 1994). Recent reviews have questioned the possibility of further improving numerical wave models without changing this basic principle (Cavaleri 2006). Although this may be true in the long term, we demonstrate here that it is possible to improve our model results significantly by including more physical features in the source term parameterizations. The main advance that we propose is the adjustment of a dissipation function without any prescribed spectral shape, based on our empirical knowledge of the breaking of random waves (Banner et al. 2000; Babanin et al. 2001) and the dissipation of swells over long distances (Ardhuin et al. 2009a). The present formulations are not based on a detailed physical model of dissipation processes, but they demonstrate that progress is possible. This effort opens the way for physical parameterizations (e.g., Filipot et al. 2010) that will eventually provide new applications for wave models, such as the estimation of statistical parameters for breaking waves, including whitecap coverage and foam thickness. Other efforts, though less empirical in nature, are also under way to arrive at better parameterizations (e.g., Banner and Morison 2010; Babanin et al. 2007; Tsagareli 2008), but they have yet to produce a practical alternative for wave forecasting and hindcasting.

### b. Shortcomings of existing parameterizations

Until the work of van der Westhuysen et al. (2007), none of the wave dissipation parameterizations presented in the literature had a quantitative relationship with observed features of wave dissipation but rather they were adjusted to close the wave energy balance. Komen et al. (1984, hereinafter KHH) have produced a family of parameterizations loosely justified by the “random pulse” theory of Hasselmann (1974). In deep water, these parameterizations take a generic form:

$$S_{oc}(k, \theta) = C_{ds} \delta^{0.5} k_r^4 H_s^0 \left[ \frac{k_r}{k} \right]^\gamma \int \frac{F(k, \theta) dk d\theta}{k_r^2} ,$$

in which $C_{ds}$ is a negative constant, $F(k, \theta)$ is the spectral density, and $H_s = 4 \sqrt{\int \left| E(k, \theta) \right|^2 dk d\theta}$ is the significant wave height. The energy-weighted mean wavenumber $k_r$ is defined from the entire spectrum as follows:

$$k_r = \left[ \frac{16}{H_s^2} \int^\infty_0 \int k' E(k, \theta) dk d\theta \right]^{1/5} ,$$

where $r$ is a real constant. Usual choices are $r = -0.5$ (WAMDI Group 1988) or $r = 0.5$ (KHH; Bidlot et al. 2005).

These parameterizations are still widely used in spite of inconsistencies in the underlying theory. Indeed, if whitecaps do act as random pressure pulses, their average work on the underlying waves only occurs because of a phase correlation between the vertical orbital velocity field and the moving whitecap position, which travels with the breaking wave. In reality the horizontal shear is likely to be the dominant mechanism (Longuet-Higgins and Turner 1974), but the question of correlation remains the same. For any given whitecap, such a correlation cannot exist for all spectral wave components: a whitecap that travels with one wave leads to the dissipation of spectral wave components that propagate in similar directions, with comparable phase velocities. However, whitecaps moving in one direction will give (on average) a zero correlation for waves propagating in the opposite direction because the position of the crests of these opposing waves are completely random with

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$^1$ In the presence of variable current, the source of energy for the wave field is the work of the radiation stresses. It is not explicit when the energy balance is written as an action balance (e.g., Komen et al. 1994).
respect to the whitecap position. As a result, not all wave components are dissipated by a given whitecap (others should even be generated), and the dissipation function cannot take the spectral form (2).

A strict interpretation of the pressure pulse model gives a zero dissipation for swells in the open ocean because the swell wave phases are uncorrelated to those of the shorter breaking waves. There is only a negligible dissipation due to short-wave modulations by swells and preferential breaking on the swell crests (Phillips 1963; Hasselmann 1971; Ardhuin and Jenkins 2005). Still, the dissipation functions given by Eq. (2) are applied to the entire spectrum, including swells, without any physical justification.

In spite of its successful use for the estimation of the significant wave height \( H_s \) and peak period \( T_p \), these fixed-shape parameterizations, from KHH up to Bidlot et al. (2007a), have built-in defects that limit the accuracy of the mode results. Most conspicuous is the spurious amplification of wind sea growth in the presence of swell (e.g., van Vledder and Hurdle 2002), which is contrary to all of the observations from Darbyshire (1958) to Ardhuin et al. (2009b). This effect is easily revealed by taking a sea state composed of a swell of energy \( E_1 \) and mean wavenumber \( k_1 \) and a windsea of energy \( E_2 \) and mean wavenumber \( k_2 \), with \( k_2 > k_1 \). The overall mean wavenumber is

\[
\kappa = \frac{(k_1^2 E_1 + k_2^2 E_2)/(E_1 + E_2))^{1/2}. \tag{4}
\]

At low frequency (i.e., small wavenumbers) the dissipation for a given value of \( \kappa \) is given by the first term, proportional to \( k_1^{-5/3} (E_1 + E_2) \). Now, if we keep \( k_1, k_2 \) an \( E_2 \) constant and only increase the swell energy \( E_1 \), the relative change in dissipation is, according to (2), proportional to \( x = 3.5[(k_1/k_2)^r - 1]/r + 2 \). For \( r = 0.5 \), as used by Bidlot et al. (2005, hereinafter BAJ), \( x \) is negative (i.e., the dissipation decreases with increasing swell energy) for \( k_1/k_2 < 0.51 \). For equal energy in sea and swell, this occurs when \( k_1/k_2 < 0.3 \), which is generally the case with sea and swell in the ocean. This erroneous decrease of swell dissipation with increasing swell steepness is reduced when the model frequency range is limited to a maximum frequency of 0.4 Hz, in which case the lowest winds (less than 5 m s\(^{-1}\)) are unable to produce a realistic wind sea level, hence limiting the value of \( k_\tau \) to relatively small values.

An alternative and widely used formulation has been proposed by Tolman and Chalikov (1996), and some of its features are worth noting. We refer to this parameterization as TC, including the later adjustments by Tolman (2002b). It combines two distinct dissipation formulations for high and low frequencies, with a transition at 2 times the wind sea peak frequency. Janssen et al. (1994) had already introduced the use of two terms, \( k \) and \( k^2 \) in Eq. (2), to represent different balances in the high- and low-frequency parts of the spectrum. However, both terms are still multiplied by the same factor, \( C_{\text{diss}} \ k_{\tau}^{0.5} k_{\tau}^{4.5} H_s^{4.5} \). In TC, these two dissipation terms are completely distinct, the low-frequency part being a linear function of the spectral density and proportional to wind friction velocity, \( u_w \); the high-frequency part is also linear and proportional to \( u_w^2 \). In TC the frequency dependence of the two terms is also prescribed. Tolman and Chalikov (1996) further included swell attenuation by the wind, here noted \( S_{\text{sw}} \), based on numerical simulations of the airflow above waves (Chalikov and Belevich 1993). At relatively short fetches, the \( S_{\text{sw}} \) and \( S_{\text{in}} \) terms in TC are typically a factor of 2–3 times smaller than those of Janssen et al. (1994). This small magnitude of the source terms was found to produce important biases in wave growth and wave directions at short fetch (Ardhuin et al. 2007). Another successful set of parameterizations, for high winds conditions, is the combination by Makin and Stam (2003), but it does not produce accurate results in moderate sea states (Lefèvre et al. 2004). Polnikov and Inocentini (2008) have also proposed new source term formulations, but their results appear to generally be less accurate than those with the model presented here, in particular for mean periods.

Considering the observed strong wave height gradients in rapidly varying currents, Phillips (1984) proposed a dissipation rate proportional to the nondimensional spectrum, also termed the saturation spectrum. Banner et al. (2002) indeed found a correlation of the direction-integrated saturation \( B \) with the breaking probability of dominant waves. In particular, they found that breaking occurs when \( B \) exceeds a threshold \( B_r \). Alves and Banner (2003) proposed to define the dissipation \( S_{\text{oc}} \) by \( B/B_r \) to some power, multiplied by a Komen-type dissipation term. Although this approach avoided the investigation of the dissipation of nonbreaking waves, it imported all of the above-mentioned defects of the KHH parameterizations. Further, their value for \( B_r \), which was much higher than suggested by the observations, tends to disconnect the parameterization from the observed effects (Babanin and van der Westhuysen 2008).

The use of a saturation parameter was taken up again by van der Westhuysen et al. (2007, hereafter WZB). They followed Banner et al. (2002) and integrated the saturation spectrum over directions:
$$B(k) = \int_{0}^{2\pi} k^2 F(k, \theta') \, d\theta'.$$

From this, the WZB source function was defined as

$$S_{oc,WZB}(k, \theta) = C_{db,WZB} \sqrt{gk} \left( \frac{B(k)}{B_r} \right)^{p/2} F(k, \theta),$$

where $C_{db,WZB}$ is a negative constant, $B_r$ is a constant saturation threshold, and $p$ is a coefficient that varies both with the wind friction velocity $u_*$ and the degree of saturation $B(k)/B_r$ with, in particular, $p \approx 0$ for $B(k) < 0.8B_r$. For nonbreaking waves, when $p \approx 0$, the dissipation is too large by at least one order of magnitude, making the parameterization unfit for oceanic-scale applications, with wave heights in the Atlantic underpredicted by about 50% (Ardhuin and Le Boyer 2006). Van der Westhuysen (2007) eventually replaced this dissipation of nonbreaking waves with the Komen-type form proposed by the WAMDI Group (1988).

Further, the increase in $p$ with the inverse wave age, $u_*/C$, tends to increase $S_{oc}$ at high frequency, and was needed to obtain a balance with the $S_{atm}$ term in Eq. (1). This indicates that, in addition to the value of the saturation $B_r$, other factors may be important, such as the directionality of the waves (Banner et al. 2002). Other observations clearly show that the breaking rate of high-frequency waves is much higher for a given value of $B$, probably due to the cumulative effects by which the longer waves are modifying the dissipation of shorter waves.

Banner et al. (1989) and Melville et al. (2002) have shown that breaking waves suppress the short waves on the surface. Young and Babanin (2006) arrived at the same conclusion from the examination of pre- and post-breaking wave spectra, and proposed a parameterization for $S_{oc}$ that included a new “cumulative term” to represent this effect. Yet, their estimate was derived for very strong wind-forcing conditions only. Their interpretation of the differences in parts of a wave record with breaking and nonbreaking waves implies an underestimation of the dissipation rates because the breaking waves have already lost some energy when they are observed and the nonbreaking waves are not going to break right after they have been observed. Also, since the spectra are different, nonlinear interactions must be different, even on this relatively small time scale (e.g., Young and van Vledder 1993, Fig. 5), and the differences in spectra may not be the result of dissipation alone.

Finally, the recent measurement of swell dissipation by Ardhuin et al. (2009a) has revealed that the dissipation of nonbreaking waves is essentially a function of the wave steepness, and a very important process for ocean basins larger than 1000 km. Because of the differences in coastal and larger-scale sea states (e.g., Long and Resio 2007), verifying the source function parameterizations at all scales is paramount, in order to provide a robust and comprehensive parameterization of the wave dissipation.

2. Parameterizations

In this section we present the results of the integration of the energy balance. Because numerical choices can have important effects (e.g., Tolman 1992; Hargreaves and Annan 2000), a few details should be given. All calculations are performed within the WAVEWATCH III modeling framework (Tolman 2008, 2009; hereinafter WWATCH), using the third-order spatial and spectral advection scheme, and including modifications of the source terms described here. The source terms are integrated with the fully implicit scheme of Hargreaves and Annan (2000), combined with the adaptation time step and limiter method of Tolman (2002a), in which a
minimum time step of 10 s is used, so that the limiter on wind-wave growth is almost never activated. The diagnostic tail, proportional to \(f^{-5}\), is only imposed at a cutoff frequency \(f_c\) set to

\[
f_c = f_{FM}f_m.
\]

Here, we take \(f_{FM} = 10\) and define the mean frequency as \(f_m = 1/\tau_{no1}\). Hence, \(f_c\) is generally above the maximum model frequency that we fixed at 0.72 Hz, and the high-frequency tail is left to evolve freely. Some comparison tests are also performed with other parameterizations using a lower value of \(f_c\), typically set at \(2.5f_m\) (Bidlot et al. 2007a). In such calculations, although the net source term may be nonzero at frequencies above \(f_c\), there is no spectral evolution due to the imposed tail.

a. Nonlinear wave–wave interactions

All the results discussed and presented in this section are obtained with the discrete interaction approximation (DIA) of Hasselmann et al. (1985). The coupling coefficient that gives the magnitude of the interactions is \(C_{nl}\). Based on comparisons with exact calculations, KHH adjusted the value of \(C_{nl}\) to \(2.78 \times 10^7\), which is the value used by Bidlot et al. (2005). Here, this constant will be modified slightly. The DIA parameterization is well known for its shortcomings (Banner and Young 1994), and the adjustment of other parameters probably compensates for some of these errors. This matter will be fully discussed in a future publication.

b. Swell dissipation

Observations of swell dissipation are consistent with the effects of friction at the air–sea interface (Ardhuin et al. 2009a), resulting in a flux of momentum from the wave field to the wind (Harris 1966). We thus write the swell dissipation as a negative contribution \(S_{\text{out}}\), which is added to \(S_{\text{in}}\) to make the wind-wave source term \(S_{\text{strm}}\).

Using the method of Collard et al. (2009), a systematic analysis of swell observations by Ardhuin et al. (2009a) showed that the swell dissipation is nonlinear, which is possibly related to a laminar-to-turbulent transition of the oscillatory boundary layer over swells. Defining the boundary Reynolds number \(Re = 4u_{\text{orb}}a_{\text{orb}}/\nu_a\), where \(u_{\text{orb}}\) and \(a_{\text{orb}}\) are the significant surface orbital velocity and displacement amplitudes, and \(\nu_a\) is the air viscosity, we take, for \(Re\) less than a critical value \(Re_c\),

\[
S_{\text{out}}(k, \theta) = -C_{\text{dsv}}\frac{\rho_a}{\rho_w}(2k\sqrt{2\nu_a})F(k, \theta),
\]

where the constant \(C_{\text{dsv}}\) is equal to 1 in Dore (1978)’s laminar theory and the radian frequency \(\sigma\) is related to \(k\) via the dispersion relation.

When the boundary layer is expected to be turbulent, for \(Re \geq Re_c\), we take

\[
S_{\text{out}}(k, \theta) = -\frac{\rho_a}{\rho_w}(16f_c\sigma^2u_{\text{orb}}/g)F(k, \theta).
\]

A few tests have indicated that a threshold \(Re_c = (2 \times 10^5)/H_s\) provides reasonable results, although it may also be a function of the wind speed, and we have no explanation for the dependence on \(H_s\). A constant threshold close to \(2 \times 10^5\) provides similar—but less accurate—results. Here, we shall use \(C_{\text{dsv}} = 1.2\).

The parameterization of the turbulent boundary layer is a bit more problematic and, in the absence of direct measurements in the boundary layer, leaves room for speculations. Assuming a constant \(f_c\) in (9), Ardhuin et al. (2009b) found that swell observations are consistent with \(0.004 < f_c < 0.013\). From the analogy with an oscillatory boundary layer over a fixed bottom (Jensen et al. 1989), these values correspond to a surface with a very small roughness. Because we also expect the wind to influence \(f_c\), we include the adjustable effects of wind speed on the roughness, and an explicit correction of \(f_c\) in the form of a Taylor expansion to first order in \(u_a/u_{\text{orb}}^*\):

\[
f_c = f_{c,\text{GM}} + \left[3s_1 + s_2\cos(\theta - \theta_p)\right]\frac{u_a}{u_{\text{orb}}^*},
\]

where \(f_{c,\text{GM}}\) is the friction factor given by Grant and Madsen’s (1979) theory for rough oscillatory boundary layers without a mean flow. Adequate swell dissipation is obtained with constant values of \(f_c\) in the range 0.004–0.007, but these do not necessarily produce the best results when comparing wave heights to observations. Based on the simple idea that most of the air–sea momentum flux is supported by the pressure–slope correlations that give rise to the wave field (Donelan 1998; Peirson and Banner 2003), we have set the surface roughness seen by the oscillatory flow \(z_0^*\) to a small fraction of that seen by the mean flow \(z_0^*\):

\[
z_0^* = r_{\text{st}}z_0^*.
\]

Here, \(r_{\text{st}}\) is here set to 0.04. As a result, typical values of \(a_{\text{orb}}/z_0^*\) are as large as \(2 \times 10^3\), with \(f_{c,\text{GM}}\) of the order of 0.003.

The coefficients \(s_2\) and \(s_3\) of the \(O(u_a/u_{\text{orb}})\) correction have been adjusted to \(-0.018\) and 0.015, respectively, the former negative value giving a stronger dissipation for swells opposed to winds, when \(\cos(\theta - \theta_p) < 0\). This gives a range of values of \(f_c\) consistent with the observations, and reasonable hindcasts of swell decay (Fig. 1), with a small underestimation of dissipation for steep
swells. Because an increase of $s_1$ from 0.8 to 1.1 produces negative biases on $H_s$ of the order of 30% at all oceanic buoys, the magnitude of the swell dissipation cannot be much larger is than chosen here.

Further validation of the swell dissipation is provided by the global-scale hindcasts in section 4.

c. Wave breaking

Observations show that waves break when the orbital velocity at their crest, $U_c$, comes close to the phase speed, $C$, with a ratio $U_c/C > 0.8$ for random waves (Tulin and Landrini 2001; Stansell and MacFarlane 2002; Wu and Nepf 2002). It is nevertheless difficult to parameterize the breaking of random waves, since the only available wave information here are the spectral densities $F(k, \theta)$. These densities can be related to the orbital velocity variance in a narrow frequency band. This question is addressed in detail by Filipot et al. (2010). Yet, a proper threshold has to be defined for this quantity, and the spectral rate of energy loss associated with breaking has to be defined. Also, breaking is intricately related to the complex nonlinear evolution of the waves (e.g., Banner and Peirson 2007).

These difficulties will be ignored here. We shall parameterize the spectral dissipation rate directly from the wave spectrum, in a way similar to WZB. Essentially, we distinguish between spontaneous and induced breaking, the latter being caused by large-scale breakers overtaking shorter waves, resulting in their dissipation. For the spontaneous breaking we parameterize the dissipation rate directly from the spectrum, without the intermediate step of estimating a breaking probability.

We started from the simplest possible dissipation term formulated in terms of the direction-integrated spectral saturation $B(k)$ given by Eq. (5), with a realistic threshold $B_0 = 1.2 \times 10^{-3}$ corresponding to the onset of wave breaking (Babanin and Young 2005). This saturation parameter corresponds exactly to half of the $\alpha$ parameter defined by Phillips (1958). The value $B_0 = 4 \times 10^{-3}$, given by Phillips, corresponds to a self-similar sea state in which waves of all scales have the same shape, limited by the breaking limit.

This view of the sea state, however, ignores completely wave directionality. Early tests of parameterizations based on this definition of $B$ indicated that the spectra were too narrow (Ardhuin and Le Boyer 2006). This effect could be due to many errors. Because Banner et al. (2002) introduced a directional width in their saturation to explain some of the variability in the observed breaking probabilities, we similarly modify the definition of $B$. Expecting also to have different dissipation rates in different directions, we define a saturation $B'$ that would correspond, in deep water, to a normalized velocity variance projected in one direction (in the case $s_B = 2$), with a further restriction of the integration of directions controlled by $\Delta_\theta$:

$$B'(k, \theta) = \int_{\theta - \Delta_\theta}^{\theta + \Delta_\theta} k^2 \cos^2(\phi - \theta') F(k, \theta') \frac{C}{2\pi} d\theta'. \tag{12}$$

Here, we shall always use $\Delta_\theta = 80^\circ$. In our model, a sea state with two systems with the same energy but of opposite direction will thus produce less dissipation than a sea state with all the energy radiated in the same direction.

We finally define our dissipation term as the sum of the saturation-based term of Ardhuin et al. (2008a) and a cumulative breaking term $S_{bk, cu}$:

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\[
S_{oc}(k, \theta) = C_{oc}^2 \delta_d \left\{ \delta_d \max[B(k) - B_r, 0]^2 + (1 - \delta_d) \max[B'(k, \theta) - B_r, 0]^2 \right\} F(k, \theta) + S_{bk, cu}(k, \theta) + S_{turb}(k, \theta),
\]

where

\[
B(k) = \max[B'(k, \theta), \theta \in [0, 2\pi]].
\]

The combination of an isotropic part (the term that multiplies \( \delta_d \)) and a direction-dependent part (the term with \( 1 - \delta_d \)) was intended to allow some control of the directional spread in the resulting spectra. This aspect is illustrated in Fig. 2 with a hindcast of the 3 November 1999 case during the Shoaling Waves Experiment (SHOWEX; Ardhuin et al. 2007). Clearly, the isotropic saturation in the TEST442 dissipation (with the original threshold \( B_r = 0.0012 \)) produces very narrow spectra, even though it is known that the DIA parameterization for nonlinear interactions tends to broaden the spectra. The same pattern of behavior is obtained with the isotropic parameterization by van der Westhuysen et al. (2007), as demonstrated by Ardhuin and Le Boyer (2006). Further, using an isotropic dissipation at all frequencies yields an energy spectrum that decays faster toward high frequencies than the observed spectrum (Fig. 2a). On the contrary, a fully directional dissipation term (TEST443 with \( \delta_d = 0 \)) gives a better fit for all of the parameters. With \( s_B = 2 \), we reduce \( B_r \) to 0.0009, a threshold for the onset of breaking that is consistent with the observations of Banner et al. (2000) and Banner et al. (2002), as discussed by Babanin and van der Westhuysen (2008). The overall dissipation term in TEST443 is anisotropic due to the cumulative effect, but this does not significantly alter the underestimation of the directional spread.

The dissipation constant \( C_{oc}^2 \) was adjusted to \( 2.2 \times 10^{-4} \) in order to give acceptable time-limited wave growth and good directional fetch-limited growth as described by Ardhuin et al. (2007). As noted in this previous work, similar growth patterns in wave energy with fetch are possible with almost any magnitude of the wind input, but a reasonable mean direction in slanting fetch conditions selects the range of possible levels of input. Here, the mean directions at the observed peak frequency are still biased by about 25° toward the alongshore direction with the parameterizations proposed here (Fig. 2b), which is still less that the 50° obtained with the weaker Tolman and Chalikov (1996) source terms (Ardhuin et al. 2007, Fig. 11). A relatively better fit is obtained with the BAJ parameterization. This is likely due to either the stronger wind input or the weaker dissipation at the peak. Both features of the BAJ parameterization are more realistic than what we propose here.

The equilibrium sea state, achieved by most models for long durations with steady wind and infinite fetch, and often compared with the Pierson and Moskowitz (1964) spectrum, is largely controlled by the balance between the nonlinear flux of energy to low frequencies and the wind output term, as discussed below. Figure 3 shows the fetch-limited energy growth for the various parameterizations. We repeat here the sensitivity test to the presence of swell already displayed in Ardhuin et al. (2007). Whereas the 1-m swell causes an unrealistic doubling of the wind sea energy at short fetch in the BAJ parameterization, the new parameterizations, just like the one by van der Westhuysen et al. (2007), are by design insensitive to swell (not shown).

The dissipation \( S_{turb} \) due to wave–turbulence interactions is expected to be much weaker (Ardhuin and Jenkins 2006) and will be neglected here.

Finally, following the analysis by Filipot et al. (2010), the threshold \( B_r \) is corrected for shallow water, so that \( B'/B_r \) in different water depths corresponds to the same ratio of the root-mean-square orbital velocity and phase speed. For periodic and irrotational waves, the orbital velocity increases much more rapidly than the wave height as it approaches the breaking limit. Further, due to nonlinear distortions in the wave profile in shallow water, the height can be twice as large as the height of linear waves with the same energy. To express a relevant threshold from the elevation variance, we consider the slope \( kH_{lin}(kD) \) of an hypothetical linear wave that has the same energy as the wave of maximum height. In deep water, \( kH_{lin}(\infty) \approx 0.77 \), and for other water depths we thus correct \( B_r \) by a factor \( [kH_{lin}(kD)/kH_{lin}(\infty)]^2 \). Using streamfunction theory (Dalrymple 1974), a polynomial fit as a function of \( Y = \tanh(kD) \) gives

\[
B' = B_r Y(M_4 Y^3 + M_3 Y^2 + M_2 Y + M_1),
\]

such that \( B' = B_r \) in deep water. The fitted constants are \( M_4 = 1.3286, M_3 = -2.5709, M_2 = 1.9995, \) and \( M_1 = 0.2428 \). Although this behavior is consistent with the variation of the depth-limited breaking parameter \( \gamma \) derived empirically by Ruessink et al. (2003), the resulting dissipation rate is not yet expected to produce realistic results.

\footnote{This value of the maximum equivalent linear height, \( H_{lin} = 2\sqrt{2E} \), with \( E \) being the elevation variance, is smaller than the usual value \( KH = 0.88 \) due to the correction for the nonlinear wave profile for which \( H > \sqrt{2E} \).}
for surf zones because no effort was made to verify this aspect. This is the topic of ongoing work, which is outside of the scope of the present paper.

The cumulative breaking term $S_{bk, cu}$ represents the smoothing of the surface by big breakers with celerity $C'$ that wipe out smaller waves of phase speed $C$. Due to uncertainties in the estimation of this effect in the observations of Young and Babanin (2006), we use the theoretical model of Ardhuin et al. (2009b). Briefly, the relative velocity of the crests is the norm of the vector difference, $D_c = C - C'$, and the dissipation rate of short wave is simply the rate of passage of the large breaker over short waves, that is, the integral of $\Delta_c \Lambda(C) dC$, where $\Lambda(C) dC$ is the length of breaking crests per unit surface that have velocity components between $C_x$ and $C_x + dC_x$, and $C_y + dC_y$, (Phillips 1985).

However, because they used a zero-crossing analysis for a given wave scale, there are many times when waves are not counted since the record is dominated by another scale: in their analysis there is only one wave at any given time. This tends to overestimate the breaking probability by a factor of 1.5–2 (Manasseh et al. 2006), compared to the present approach in which we consider that several waves (of different scales) may be present at the same place and time. We shall thus correct for this effect, by simply dividing $P$ by 2.

With this approach we define the spectral density of the crest length (breaking or not) per unit surface $l(k)$ such that $\int |l(k)| d\mathbf{k}$ is the total length of all crests per unit surface, with a crest being defined as a local maximum of the elevation in one horizontal direction. In the wave-number vector spectral space we take

$$P = 56.8 \max \left( \sqrt{B''(k, \theta)} - \sqrt{B''_r, 0} \right)^2.$$  

However, because they used a zero-crossing analysis for a given wave scale, there are many times when waves are not counted since the record is dominated by another scale: in their analysis there is only one wave at any given time. This tends to overestimate the breaking probability by a factor of 1.5–2 (Manasseh et al. 2006), compared to the present approach in which we consider that several waves (of different scales) may be present at the same place and time. We shall thus correct for this effect, by simply dividing $P$ by 2.
\[ l(k) = \frac{1}{(2\pi^2 k),} \]  \hspace{1cm} (17)

which is equivalent to a constant in wavenumber-direction space \( l(k, \theta) = \frac{1}{(2\pi^2 k)} \). This number was obtained by considering an ocean surface full of unidirectional waves, with one crest for each wavelength \( 2\pi/k \) for each spectral interval \( \Delta k = k \); for example, one crest corresponding to spectral components in the range 0.5–1.5 \( k \). This potential number of crests is doubled by the directionality of the sea state. These two assumptions have not been verified and thus the resulting value of \( l(k) \) is merely an adjustable order of magnitude.

Thus, the spectral density of the breaking crest length per unit surface is \( \Lambda(k) = l(k)p(k) \). Assuming that any breaking wave instantly dissipates all the energy of all of the waves with frequencies higher by a factor \( r_{cu} \) or more, then the cumulative dissipation rate is simply given by the rate at which these shorter waves are taken over by larger breaking waves times the spectral density; namely,

\[ S_{bk, cu}(k, \theta) = -1.44C_{cu}F(k, \theta)\int_{0}^{r_{cu}}\int_{0}^{2\pi} \max(\sqrt{B(k', \theta') - \sqrt{B', 0^{2}}})\Delta \theta \, dk', \]  \hspace{1cm} (18)

where \( r_{cu} \) defines the maximum ratio of the frequencies of long waves that will wipe out the short waves.

We now obtain \( \Lambda \) by extrapolating Eq. (16) to higher frequencies. With 56.8/(4\( \pi^2 \)) = 1.44, it gives the source term

\[ S_{bk, cu}(k, \theta) = C_{cu}F(k, \theta)\int_{f_{cu}}^{\infty} \Delta \Lambda(k') \, dk', \]  \hspace{1cm} (19)

This feature helps to produce realistic spectral shapes near the peak, with a steeper low-frequency side and a more gentle slope on the high-frequency side, contrary to the backward-facing spectra produced by BAJ and TC. However, this localized strong relative dissipation, \( S_{\infty}(k, \theta)/F(k, \theta) \), is hard to reconcile with the time and spatial scales of breaking events, and is thus probably exaggerated. Indeed, there should be no significant difference in the relative dissipation among the spectral components that contribute to a breaking wave crest, provided that they do not disperse significantly over the breaker lifetime, which is less than a wave period. There is no physical reason why a breaking event would take much more energy, relatively speaking, from the spectral band (1.1–1.2)\( f_p \) than from (1.2–1.3)\( f_p \). The factor of 2 difference found here in the relative dissipation rates thus appears to be unrealistic. This strong relative dissipation at the peak (50% higher than in BAJ) is one important factor that leads to slower growth of the wave spectrum in TEST441 compared to BAJ. It is possible that the localization of \( S_{\infty} \) at the peak compensates for the broader spectrum produced by the DIA compared to results with an exact nonlinear interaction calculation.

We now consider “fully developed” conditions, as illustrated by Fig. 5. At low frequency, the nonlinear swell damping term \( S_{out} \) (the negative part of \( S_{in} \)) cancels about 30%–50% of the nonlinear energy flux, so that the sea state grows only very slowly. As a result, “full development” does not exist, but the resulting energy is still compatible with the observations of mature sea states (Alves et al. 2003). In contrast, the linear swell damping adjusted by Tolman (2002b) is designed to produce reasonable swell heights in the tropics, but it is much smaller.
than the nonlinear energy flux to low frequencies, even with the reduced interaction coefficient proposed by Tolman and Chalikov (1996). A nonlinear swell dissipation appears to be necessary to obtain both a realistic damping of observed swells and a satisfactory agreement with mature wind waves. Nonlinearity also brings within the same order of magnitude the decay scales estimated for short (Högström et al. 2009) and very long swells (Ardhuin et al. 2009b).

Both parameterizations are physically very different from the parameterizations of the KHH family, including Bidlot et al. (2005). In these cases, the swell energy is lost to the ocean via whitecapping. Here, we propose...
that this energy is lost to the atmosphere, with an associated momentum flux that drives the wave-driven wind observed in wave flumes (Harris 1966) and for very weak winds at sea (Smedman et al. 2009).

In the inertial range, a reasonable balance of all the source terms is obtained for $C_{cu} = -0.4$ (Fig. 5). In this case, the spectrum approaches an $f^{-4}$ shape up to 0.4 Hz. The behavior of the high-frequency tail is best seen when displayed in nondimensional form, as is done in Fig. 6. Figure 6 shows the unrealistically high level of the tail without cumulative effect nor modification of the wind input, for fully developed waves. With the TEST441 parameterization ($C_{cu} = 0.4$ and $s_u = 1$), the spectral level at 0.7 Hz (3 m wavelength) may be 30% too high compared to other analyses by Long and Resio (2007), but they are in the range 0.0045–0.008 given by Babanin and Soloviev (1998). Further, for average oceanic conditions, the resulting spectral moments $m_3$ is still slightly underpredicted (Ardhuin et al. 2009b). Spectral levels for shorter waves (20 cm–2 m) were estimated at $\alpha = 1.8 \times 10^{-3}$ by Banner et al. (1989). There is thus a strong need for more spectral measurements in the range of wavelengths from 0.5 to 5 m, outside of the range of buoy measurements.

Adding the cumulative effect can be a means of controlling the tail level, but this degree of freedom is not enough. Indeed, in strongly forced conditions the dominant waves break frequently, and a high cumulative effect, $C_{cu} = -1$, reduces the energy level in the tail below observed levels. This effect can be verified with mean square slopes estimated from satellite altimeter measurements (Fig. 8), or high moments of the frequency spectrum derived from buoy data (not shown but similar).

A more accurate shape of the spectrum tail may be obtained with a lower $C_{cu}$ or a higher $r_{cu}$, so that dominant breaking waves will only wipe out much smaller waves. Instead, and because the wind to wave momentum flux was too high in high winds, we chose to introduce one more degree of freedom, allowing a reduction of the wind input at high frequency.

d. Wind input

The wind input parameterization is thus adapted from Janssen (1991; see also Chalikov 1993) and the following adjustments performed by Bidlot et al. (2005, 2007a). The full wind input source term reads

$$S_m(k, \theta) = \frac{\rho_u \beta_{max}}{\rho_a \kappa^2} \frac{c^2}{C} Z_{0}(u_{*}/C)$$

$$\times \max[\cos(\theta - \theta_u), 0]^2 \sigma_F(k, \theta),$$

where $\beta_{max}$ is a nondimensional growth parameter (constant), $u_{*}$ is the wind friction velocity, and $\kappa$ is von Kármán’s constant. In the present implementation the air–water density ratio is constant. The power of the cosine is taken constant with $p = 2$. We define the effective wave age $Z = \log(\mu)$, where $\mu$ is given by Janssen and corrected for intermediate water depths, so that

$$Z = \log(kz_1) + \kappa/[\cos(\theta - \theta_u)(u_{*}/C + z_a)].$$

where $z_1$ is a roughness length modified by the wave-supported stress $\tau_w$ and $z_a$ is a wave age tuning parameter. We define $z_1$ implicitly by

$$U_{10} = \frac{u_{*}}{\kappa} \log(\frac{z_1}{z_1})$$

$$z_0 = \min \left(\alpha_{0}, \frac{\tau_w}{g}, z_{0,\text{max}}\right),$$

$$z_1 = \frac{z_0}{\sqrt{1 - \tau_w/\tau}},$$

where $z_0$ is the height at which the wind speed is specified, usually 10 m. The maximum value of $z_0$ was added to reduce possible unrealistic wind stresses at high winds that are otherwise given by the standard parameterization. For example, $z_{0,\text{max}} = 0.0015$ is equivalent to setting a maximum wind drag coefficient of $2.5 \times 10^{-3}$. For the TEST441 parameterization, we have adjusted $z_a = 0.006$ and $\beta_{max} = 1.52$ (Fig. 7).

An important part of the parameterization is the calculation of the wave-supported stress $\tau_w$, which includes the resolved part of the spectrum, as well as the growth of an assumed $f^{-3}$ diagnostic tail beyond the highest frequency. This parameterization is highly sensitive to the high-frequency part of the spectrum since a high energy level there will lead to a larger value of $z_1$ and $u_{*}$, which gives a positive feedback and reinforces the energy levels.
To allow a balance with the saturation-based dissipation, the wind input at high frequency is reduced by modifying the friction velocity $u_w$. This correction also reduces the drag coefficient at high winds. In particular, the wind input is reduced for high frequencies and high winds, loosely following Chen and Belcher (2000). This is performed by replacing $u_w$ in Eq. (19) with a wave-number-dependent $u_w(k)$ defined by

$$(u_w^*)^2 = u_w^2 (\cos^2 \theta, \sin^2 \theta) - \left| s_u \right| \int \int \frac{2\pi}{C} \frac{S_m(k', \theta)}{(\cos \theta, \sin \theta) dk' d\theta},$$

where the sheltering coefficient $\left| s_u \right| \sim 1$ can be used to tune the wind stresses for high winds, which would be largely overestimated for $s_u = 0$. For $s_u > 0$, this sheltering is also applied within the diagnostic tail, which requires the estimation of a three-dimensional lookup table for the high-frequency stress. The shape of the new wind input is illustrated in Fig. 7 for fully developed seas. Clearly, for relatively young waves the energy levels at the spectral peak are lower with $s_u = 1$ (TEST41) than in other runs; this is largely due to a reduced feedback of the wind age on the wind stress via the $\tau_w/\tau$ term in Eq. (23). The reduction of $s_u$ from 0.011 to 0.006 strongly reduces the input for frequencies in the range 0.15–0.2 Hz, which is probably overestimated in BAJ when average (20%) levels of gustiness are considered: now the wind input goes to zero for $f = 0.13$, which corresponds to $C/U_{10} = 0.83$, whereas it is still significant at that wave age in the BAJ parameterization. As a result the much lower input level needs a readjustment, performed here by increasing $\beta_{max}$ to 1.52. Yet, this high value of $\beta_{max}$ produces very high wind stress values and thus a very strong high-frequency input. Adding the sheltering term $s_u = 1$ allows a decent balance at high frequency. Finally, the addition of the air–sea friction term that gives swell dissipation produces a significant reduction of the input to the wind sea at $f = 0.25$ Hz. It is questionable whether this mechanism also applies in the presence of the critical layer for those waves. This matter clearly requires more theoretical and experimental investigation.

3. Consequences of the source term shape

We have already illustrated the effects of various parameters on spectral shapes in academic time-limited and more realistic fetch-limited conditions. We now look at real sea states observed in the World Ocean. Although wave spectra are difficult to compare to the few available observations, we have investigated the systematic variation of spectral moments:

$$m_n(f_c) = \int_0^{2\pi} \int_0^{2\pi} F(f, \theta) d\theta df,$$

with $n = 2, 3, 4$, and cutoff frequencies in the range 0.2–0.4 Hz. The spectral density $F(f, \theta)$ is estimated as $2\pi F(k, \theta)/C_g$, using linear wave theory. Such moments are relevant to a variety of applications. Ardhuin et al. (2009b) investigated the third moment, which is proportional to the surface Stokes drift in deep water, and found that buoy data are very well represented by a simple function of $U_{10}$, $H_s$, and $f_c$, which typically explains 95% of the variance of $m_3$.

This relationship is well reproduced in hindcasts using $C_{\text{cu}} = -0.4$ and $s_u = 1$, while the BAJ source terms give almost a constant value of $m_3$ when $H_s$ varies and $U_{10}$ is fixed (Ardhuin et al. 2009b).

Here, we consider the fourth moment $m_4$, which, for linear waves, is proportional to a surface mean square slope filtered at the frequency $f_c$. For modeled values a constant 0.011 is added to account for the short waves that contribute to the satellite signal and that are not resolved in the model. This saturated high-frequency tail, independent of wind speed, is consistent with the observations of Banner et al. (1989) and Vandemark et al. (2004).

Figure 8 shows that for any given wind speed mss, increases with the wave height (Gourrion et al. 2002), whereas this is not the case for $m_4$ in the BAJ parameterization, or, for very high winds, when $C_{\text{cu}}$ is too strong. In the case of BAJ, this is due to the $(k/k_s)^2$ part in the
The dissipation term [Eq. (2)], which plays a role similar to the cumulative term in our formulation. For $C_{cu} = -1$ and $s_u = 0$, the cumulative effect gets too strong for wind speeds over $10 \text{ m s}^{-1}$, in which case $m_4$ starts to decrease with increasing wave height, whereas for high winds and low (i.e., young) waves, the high-frequency tail is too high and the mean square slope gets to be as large as 6%, which is unrealistic. For $s_u = 0$ the high-frequency tail responds too much to the wind, hence our use of $s_u = 1$ in the TEST441 combination. The presence of a cumulative dissipation term allows for a different balance in the spectral regions above the peak, where an equilibrium range with a spectrum proportional to $f^{-4}$ develops (Long and Resio 2007), and in the high-frequency tail where the spectrum decays like $f^{-3}$ or possibly a little faster. The spectral level in the range 0.2–0.4 Hz was carefully compared against buoy data and was found to be realistic.

These interpretations rely on the assumption that the model results are really wavenumber spectrum $E(k, \theta)$ converted to frequency densities using linear wave
theory, which is expected to be closer to the frequency spectra obtained from Lagrangian buoy measurements, and the wavenumber spectrum of nonlinear waves (Janssen 2009). This matter is left for further studies, together with a detailed interpretation of the altimeter radar cross sections. Although it covers much less data, the analysis of $m_4$ obtained from buoy heave spectra produces results similar to Fig. 8.

4. Verification

To provide simplified measures of the difference between the model time series $X_{\text{mod}}$ and the observations $X_{\text{obs}}$, we use the following definitions for the normalized root-mean-square error (NRMSE),

$$\text{NRMSE}(X) = \sqrt{\frac{\sum (X_{\text{obs}} - X_{\text{mod}})^2}{\sum X_{\text{obs}}^2}}; \quad (26)$$

the normalized bias,

$$\text{NB}(X) = \frac{\sum X_{\text{obs}} - X_{\text{mod}}}{\sum X_{\text{obs}}}; \quad \text{and} \quad (27)$$

Pearson’s linear correlation coefficient,

$$r(X) = \frac{\sum (X_{\text{obs}} - \bar{X}_{\text{obs}})(X_{\text{mod}} - \bar{X}_{\text{mod}})}{\sqrt{\sum (X_{\text{obs}} - \bar{X}_{\text{obs}})^2 (X_{\text{mod}} - \bar{X}_{\text{mod}})^2}}; \quad (28)$$

where the overbar denotes the arithmetic average.

The normalization of the errors allows a quantitative comparison between widely different sea state regimes. Because previous studies have often used (nonnormalized) RMSEs, we also provide RMSE values. In addition to the coastal fetch-limited case of SHOWEX, presented above, the parameterizations are calibrated on at the global scale and validated in two other cases.

a. Global-scale results

We present here results for the entire year 2007, using a stand-alone 0.5° resolution grid, covering the globe from 80° south to 80° north. The model has actually been adjusted to perform well over this dataset, but the very large number of observations (more than 2 million altimeter collocation points) makes the model robust and an independent validation on 2008–10 gives identical results. The interested reader may also look at the monthly reports for the Service Hydrographique et Océanographique de la Marine (SHOM) model (e.g., Bidlot 2008), generated as part of the model verification project of the Intergovernmental Oceanographic Commission–World Meteorological Organization (IOC–WMO) Joint Commission on Oceanography and marine Meteorology (JCOMM), in which the TEST441 parameterization ($C_{\text{cu}} = -0.4$ and $v_0 = 1$) is used, except for the Mediterranean where, the TEST405 has been preferred for its superior performance for younger seas. These SHOM models are run in a combination of two-way nested grids (Tolman 2007). The monthly JCOMM reports include both analyses and forecasts, and many SHOM forecasts from December 2008 to June 2009 have been affected by wind file transfer problems.

Comparing the model results for $H_s$ to well-calibrated altimeter-derived measurements (Quefféroulou and Croizé-Fillon 2008) provides a good method of verification for the model performance in a number of different wave climates. Figure 9 shows that, as expected, the important positive bias in the swell-dominated regions when using the BAJ parameterization has been largely removed. This is essentially the signature of the specific swell dissipation that is parameterized in $S_{\text{out}}$. The largest bias pattern now appears in the Southern Ocean, reaching 30 cm in the southern Atlantic in 2007 and more than 80 cm in 2004. Although this bias is small compared to the local averaged wave height, it is rather odd when the model errors are plotted as a function of wave height in Fig. 10. Why would the model overestimate the Southern Ocean waves but underestimate the very large waves?

The structure of the large bias, also seen in model results of BAJ, is reminiscent of the observed pattern in iceberg distribution produced by Tournadre et al. (2008) for the year 2004. These observed iceberg distributions are enough to give a cross section for incoming waves of the order of 1%–10% for a 250-km propagation length. Assuming a full blocking of wave energy by icebergs larger than 1 km in diameter is enough to cancel most of the bias.

Also noticeable is a significant negative bias in the equatorial South Pacific, amplified from the same bias obtained with the BAJ parameterization. It is possible that the masking of subgrid islands (Tolman 2003) introduces a bias by neglecting shoreline reflections. This model defect could be exacerbated in this region by the very large ratio of shoreline length to sea area. This will also require further investigation. Finally, the negative biases for $H_s$ along midlatitude east coasts are reduced but still persist. It is well known that these areas are also characterized by strong boundary currents (such as the Gulf Stream, Kuroshio, and Agulhas current) with warm waters that are generally conducive to wave amplification and faster wind-wave growth (e.g., Vandemark et al.)
Neither effect is included in the present calculation because the accuracy of both modeled surface currents and air–sea stability parameterizations are likely to be insufficient (Collard et al. 2008; Ardhuin et al. 2007).

The reduction of systematic biases clearly contributes to the reduction of RMSEs, as is evident in the equatorial eastern Pacific (Fig. 11). However, the new parameterization also brings a considerable reduction to the scatter, with reduced errors even where biases are minimal, such as the trade winds area south of Hawaii, where the NRMSE for $H_s$ can be as low as 5%. When areas within 400 km from continents are excluded, because the global model resolution may be inadequate, significant errors (>12.5%, in yellow to red) remain in the northern Indian Ocean, on the North American and Asian east coasts, and in the southern Atlantic. The parameterizations TEST405, TEST437, and TEST441 produce smaller errors on average than BAJ. It is likely that the model benefits from the absence of swell influences on wind seas: swell in BAJ typically leads to a reduced dissipation and stronger wind wave growth. As models are adjusted to average sea state conditions, this adjustment leads to reduced wind sea growth along east coasts, where there is generally less swell.

For the highest waves, the model scatter is smallest, with some important biases for some parameterizations. Although the altimeter estimates are not expected to be valid for $H_s$ larger than about 12 m, due to the low backscatter and a waveform that is too short to properly estimate $H_s$, this good model performance contradicts some claims (e.g., Cavaleri 2009) that models may be less accurate for severe weather.

Although much more sparse than the altimeter data, the in situ measurements collected and exchanged as part of the JCOMM wave model verification are very useful for constraining other aspects of the sea state. This is illustrated here with mean periods $T_{m02}$ for data provided by the U.K. and French meteorological services, and peak periods $T_p$ for all other sources. It is worth noting that the errors in $H_s$ for in situ platforms are comparable to the errors found for altimeter data.

With the BAJ parameterizations, the largest errors in the model results are the 1.2–1.8-s biases in peak periods along the U.S. west coast (Fig. 12), and the underestimation of peak periods along the U.S. east coast. However, peak and mean periods off the European coasts are generally very well predicted. When using the TEST441 parameterization, the explicit swell dissipation reduces the bias over periods along the U.S. west coast, but the problem is not completely solved, with residual biases of 0.2–0.4 s. This is consistent with the validation using satellite synthetic aperture radar (SAR)
data (Fig. 1), which showed a tendency to underpredict steep swells near the storms and overpredict them in the far field. A simple increase in the swell dissipation was tested but it tended to deteriorate the results of other parameters; this trend is possibly associated with an underestimation of wave heights in the center of severe storms using TEST441. Along European coasts, despite a stronger bias, the errors in $T_{m02}$ are particularly reduced. Again, this reduction of the model scatter can be largely attributed to the decoupling of swell from windsea growth.

The general performance of the parameterizations is synthesized in Table 1. It is interesting to note that the parameterization TEST405, which uses a diagnostic tail for 2.5 times the mean frequency, gives good results in terms of the scatter and bias even for parameters related to short waves ($m_3, m_4$). This use of a diagnostic tail is thus a reasonable pragmatic alternative to the more costly explicit resolution of shorter waves, which requires a smaller adaptive time step, and more complex parameterizations. The diagnostic tail generally mimics the effects of both the cumulative and sheltering effects. Yet, the parameterization TEST441 demonstrates that it is possible to obtain slightly better results with a free tail. The normalized biases indicated for the mean square slopes are only relative because of the approximate calibration of the radar cross section. They show that the BAJ parameterization (Bidlot et al. 2005), and to a lesser extent the use of a $f^{-5}$ tail, produce energy levels that are relatively lower at high frequency.

b. Lake Michigan

At the global scale, the sea state is never very young, and it is desirable to also verify the robustness of the parameterization in conditions that are more representative of the coastal ocean. We thus follow the analysis of the performance of wave models in Rogers and Wang (2007), hereinafter RW2007, and give results for Lake Michigan, which is representative of relatively young waves. The model was applied with the BAJ, TC, TEST437, and TEST441 parameterizations over the same time frame as was investigated by RW2007: 1 September–14 November 2002. The model is run with 4-km resolution and with other settings and forcing fields similar to those defined by RW2007 (e.g., $10^8$ directional resolution and wind field derived from in situ observations).

Using the directional validation method proposed by these authors, the TC parameterization underestimates the directional spread $\sigma_d$ by $1.2^\circ$–$1.6^\circ$ in the range $(0.8–2.0)f_p$, and more at higher frequencies. The underestimation with BAJ is about half, and the TEST441 and TEST437 simulations overpredict the directional spread by about $2.3^\circ$–$5.9^\circ$ in the range $(0.8–2.0)f_p$, and less so for...
Higher frequencies. It thus appears that the broadening introduced to fit the SHOWEX 1999 observations may not be optimal for other situations.

Further results are presented in Table 2 and Fig. 13. The top panel in Fig. 13 shows a comparison between the summed values of collocated model and observed spectral densities for the duration of the simulation. This presentation provides a frequency distribution of the bias of the various models, while also indicating the relative contribution of each frequency to the wave climate for this region and time period. The bottom panel in Fig. 13 shows the correlation coefficient $r$ for the equivalent significant wave heights computed for multiple frequency bands. This is presented in terms of $f/f_p$ (bin width = 0.1), with $f_p$ being calculated as the stabilized “synthetic peak frequency” of the corresponding buoy spectrum, as defined in RW2007.

The most noticeable outcome of these comparisons is the relatively poor performance of the TC parameterizations. Taken in context with other TC results presented herein and a prior undocumented application of the model in the Great Lakes with model wind fields, this suggests that these parameterizations have some undesirable dependence on scale, with the parameters adjusted by Tolman (2002b) being most optimal for ocean-scale applications and nonoptimal for small-scale applications.

The KHH dissipation scheme performs well. This is consistent with prior published applications with the Simulating Waves Nearshore (SWAN) model, (Rogers et al. 2003) and RW2007. However, as noted by those authors, the KHH physics have an advantage in this simple wave climate, and should not be expected to perform well in mixed sea swell conditions.

The BAJ, TEST437 and TEST441 models also perform well here. Taken together with the global comparisons above, we observe no apparent dependency of model skill on the scale of the application with these three physics. In the bias comparison (Fig. 13, top), the BAJ model is...
nearly identical to the KHH model. Similarly, the two new models are nearly identical. In terms of $H_s$ (Table 2), the TEST441 and TEST437 yield minor underestimations, but give slightly more accurate results compared to observations than BAJ or KHH.

It thus appears that for such young seas, the directional spreading of the parameterization could be improved, but the energy content of various frequency bands, and as a result the mean period, are reproduced with less scatter than with previous parameterizations.

c. Hurricane Ivan

Although the global hindcast does contain quite a few extreme events, with significant wave heights up to 17 m, these were obtained with a relatively coarse wave model grid and wind forcing (0.5° resolution and 6-h time step).
that are insufficient to resolve small storms such as tropical cyclones (Tolman and Alves 2005). Hurricane waves do share many similarities with more usual sea states (Young 2006), but the high winds and their rapid rotation are particularly challenging for numerical wave models. It is thus necessary to verify that the new source functions perform adequately under extreme wind conditions. A simulation of Hurricane Ivan (Gulf of Mexico, September 2004) is chosen for this purpose because it was extensively measured (e.g., Wang et al. 2005) and hindcasted.

Winds for this simulation are based on 3-hourly gridded surface wind analyses created by National Oceanic and Atmospheric Administration’s (NOAA) Hurricane Research Division (HRD). As an intermediate step, fields are reprocessed at 30-min intervals, with the storm position updated at each interval (thus, semi-Lagrangian interpolation). The wind speeds are reduced by factor of 1/1.11 to convert from the maximum sustained gust to an hourly mean. The HRD winds do not cover the entire computational domain. For areas falling outside the domain, the nearest National Data Buoy Center (NDBC) wind observation is used. This produces some non-physical spatial discontinuities in the wind field, but these

![Figure 13](image-url)
are smoothed in the wave model integration and, in any event, only affect weaker wind seas far from the storm center.

Bathymetry is taken from the Naval Research Laboratory’s 2′-resolution database (DBDB2) coarsened to the computational grid resolution (0.1′). The directional resolution is 10°, and the frequency range is limited to 0.0418–0.4117 Hz. The model was applied from 13 to 16 September 2004. Model results are illustrated in Fig. 14. The models were validated at all of the buoys in the Gulf of Mexico. Results from buoy 42040, where the waves where largest, are shown here.

Model runs with parameterizations BAJ, TEST437, and TEST441 give very similar results: close to the observations, except for the highest waves ($H_s > 13$ m at buoy 42040), where TEST437 and TEST441 give slightly smaller values. Results with the TC parameterization are generally lower in terms of $H_s$ than are all of the parameterizations that share a Janssen type of input. It appears that the new source term perform similarly to BAJ and is able to reproduce such young waves and severe sea states.

Because the wind forcing enters the wave model through a wind stress parameterized in a way that may not apply to such conditions, it is worthwhile to reexamine some choices made above. In particular the surface roughness was allowed to exceed 0.002 in TEST441b, which resulted in better estimates of $H_s$. Lifting this constraint shows that, for these very high winds, the wind-sheltering effect plays a similar role in limiting the roughness to $z_{0,max}$, with the difference that it tends to narrow the wind input spectrum (Fig. 7). This narrower wind input has a limited effect of the wind stress and $H_s$, but is has a noticeable effect on the spectral shape. This is illustrated by the low-frequency energy that appears to be strongly underestimated before the peak of the storm for TEST441b. In general the new parameterizations provide results that are as reasonable as those of previous parameterizations, given the uncertainty of the wind forcing.

5. Conclusions

A set of parameterizations for the dissipation source terms of the wave energy balance equation have been proposed, based on known properties of swell dissipation and wave breaking statistics. This dissipation includes an explicit nonlinear swell dissipation and a wave breaking parameterization that contains a cumulative term, representing the dissipation of short waves by longer breakers, and different dissipation rates for different directions. These dissipation parameterizations have been combined with a modified form of the wind input proposed by Janssen (1991), in which the questionable gustiness parameter $z_a$ has been reduced, and the general shape of the wind input has been significantly modified. The resulting source term balance is thus markedly different from the previous proposed forms, with a near balance for very old seas between the air–sea friction term that dissipates swell and the nonlinear energy flux to low frequencies. Also, the wind input is concentrated in a narrower range of frequencies.

For younger seas the wind input is weaker than that given by Janssen (1991) but stronger than that given by Tolman and Chalikov (1996). However, the dissipation at the peak is generally stronger because it is essentially based on a local steepness and these dominant waves are the steepest in the sea state. As a result, the short fetch growth is relatively weaker than that with the source term combination proposed by Bidlot et al. (2007a) (BAJ). The choice of parameters tested here tends to produce broader directional spectra than those observed in Lake Michigan and global hindcasts, and slanting fetch directions that are too oblique relative to the wind (Fig. 2). In this respect the new source terms are intermediate between BAJ and TC.

Another defect comes from the definition of the saturation level used to define the breaking-induced dissipation. Here, as in the work by van der Westhuysen et al. (2007), the saturation is local in frequency space, whereas wave breaking is naturally expected to have a relatively broad impact due to its localization in space and time (Hasselmann 1974). This is expected to produce an overestimation of the energy just below the peak, and an underestimation at the peak of the saturation spectrum.
These effects likely contribute to the persistent overestimation of low-frequency energy in the model.

In spite of these defects, the new parameterization produces robust results and clearly outperforms the Bidlot et al. (2007a) parameterization in global hindcasts, whether one considers dominant wave parameters, $H_s$, $T_{m02}$, and $T_p$ or parameters sensitive to the high-frequency content, such as the surface Stokes $U_{ss}$ drift or the mean square slope. At global scales, errors in $H_s$, $T_p$, and $U_{ss}$ are—on average—reduced by 15%, 25%, and 50% relative to those obtained with the parameterization by Bidlot et al. (2007a). Global and regional hindcasts (North and Irish Seas, English Channel, Tuamotu, Lesser Antilles, and more), from 2002 to 2010 at least, are available online for further analysis (at http://tinyurl.com/yetsofy).

Another important aspect of this study was the validation at regional to global scales. Tolman and Chalikov (1996) included verification with steady-state, fetch-limited growth curves. Though such a verification is a useful step, the outcome of the Lake Michigan hindcast suggests that such verification gives no indication of the model’s skill in real subregional-scale applications. One of the parameterizations proposed here (TEST441) also gives slightly poorer performance for young seas, which is not obvious in the case of Lake Michigan, but was revealed by hindcasts of Mediterranean waves (not shown).

Because our intention was only to demonstrate the capability of new dissipation parameterizations and the resulting source term balances, we have not fully adjusted the 18 parameters that define the deep water parameterizations, compared to the roughly 9 used by Bidlot et al. (2007a). The results presented here are thus preliminary in terms of model performance, which is why the parameterizations are still given temporary names like TEST441. As illustrated by the Hurricane Ivan hindcast, some parameters, such as $z_{0,\text{max}}$, are probably unnecessary; in that particular case, the removal of $z_{0,\text{max}}$ improved the results, but for global-scale results it had no impact at all (not shown).

Because five of the extra parameters define the air–sea friction term that produces swell dissipation, and two define the cumulative breaking term, it is feasible to define a systematic adjustment procedure that should increase the model accuracy by separately adjusting the swell, wind sea peak, and high-frequency properties. In particular, the directional distribution may be improved by making the dissipation term more isotropic (i.e., taking $d_{\text{dsV}} \approx 0.3$) or modifying the definition of the saturation parameter $B'$ in Eq. (12). In a forthcoming paper we shall further investigate the responses of the wave field to varying currents, from global scales to regional tidal currents. From that research, we expect that wave steepening will produce much more dissipation due to breaking, as envisaged by Phillips (1984).

Obviously, it is well known that the discrete interaction approximation used here to compute the nonlinear interactions is the source of large errors, and further calculations will be performed using a more accurate estimation of these interactions in the future.

**Acknowledgments.** This research would not have been possible without the dedication of Hendrik Tolman, Henrique Alves, and Arun Chawla in putting together the core of the WAVEWATCH-III code. Florent Birrien performed the integration of Aaron Roland’s routines into the WWATCH framework. Wind and wave data were kindly provided by ECMWF, Météo-France, and the French Centre d’Études Techniques Maritimes Et Fluviales (CETMEF). The SHOM buoy deployments

<table>
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<th>Parameter</th>
<th>Eq.</th>
<th>Variable in WWATCH</th>
<th>WAM cycle 4</th>
<th>BAJ</th>
<th>TEST405</th>
<th>TEST437</th>
<th>TEST441</th>
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TABLE A2. Dissipation parameters as implemented in version 3.14 of SHOM in the WWATCH code, and values used in the tests presented here. In WWATCH, all parameters are accessible via the namelist “SDS3.” The only parameters not defined in the present paper are $C_{atm}$ and $C_{sat}$, which act like switches to activate the BAJ parameterization for the part of the spectrum with saturation below and above the spectrum, respectively. Most of these parameters are also included in version 3.14, except for $C_{cu}$, which is needed for TEST437 and TEST441 with results described here. The TEST405 can be run with version 3.14. The parameter $M_0$ is a switch for the correction or not of $B_1$ into $B_c$, when $M_0 = 1$, as is the case here, the correction is not applied.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Eq.</th>
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<th>WAM4</th>
<th>BAJ</th>
<th>TEST405</th>
<th>TEST437</th>
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<td>9.9</td>
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were managed by David Corman with invaluable help from Guy Amis. Wave data were kindly provided by the U.S. Navy, ESA, and CNES, and the many in situ contributors to the JCOMM (WMO-IoC) exchange program coordinated by Jean-Raymond Bidlot.

APPENDIX

Parameter Settings

The numerical values of all parameters that define the source functions are listed in Table A1 for the wind–wave interaction term $S_{atm}$ and Table A2 for the wave–ocean interaction term $S_{oc}$. We also recall that the nonlinear coupling coefficient (variable NLPROP in WWATCH) is set to $2.78 \times 10^2$ in all cases, except for the two parameterizations mostly frequently used here, with $C_{atm} = 2.5 \times 10^2$ in TEST437 and TEST441. Although the best performance for most parameters is obtained with the TEST441 settings, its underestimation of extreme sea states may be a problem in some applications for which TEST437 may be preferred. A full tuning of the model has not been attempted yet and it is possible that simple adjustments to $\beta_{max}$, $C_{cu}$, $r_{ca}$, and $s_n$ may produce even better results. Finally, these parameters have been mostly adjusted for deep water conditions using European Centre for Medium-Range Weather Forecasts (ECMWF) winds. Using other sources of winds for large-scale applications may require a retuning of the wind source function, which can be performed by a readjustment of $\beta_{max}$.

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