Dynamics of Cross-Isobath Dense Water Transport Induced by Slope Topography

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(Manuscript received 30 November 2010, in final form 24 July 2011)

ABSTRACT
Dynamics of cross-isobath downslope transport of a dense water mass induced by small-scale topographic variation is investigated based on a high-resolution numerical experiment with realistic settings, a simplified analytical model for water particle advection, and idealized sensitivity experiments. The existence of a submarine ridge induces two different processes for cross-isobath downslope transport of dense water: a strong but narrow and thin downslope current at the east side of the ridge and cyclonic eddies with dense water cores to the west of the ridge. The former downslope current is produced in response to the rapid increase of slope angle near the ridge. The latter eddies are formed by stretching of the dense water layer near the crest, where isobath curvature is so high that offshore centrifugal force overcomes the coastward Coriolis force. From a simple analysis on the equation of motion for a fluid particle placed on a slope with curved isobaths, a general criterion that describes whether a density current follows or crosses isobaths is derived, which is supported by idealized sensitivity experiments. The location where cross-isobath transport of dense water takes place is determined by relative magnitude between spatial derivatives of isobath curvature, planetary vorticity, and slope angle. Based on these arguments, a parameterization is proposed to represent the effect of unresolved small-scale topography in coarse-resolution models.

1. Introduction
Oceanic downslope gravity currents play very important roles in forming and maintaining the density structure and current system of the World Ocean (Killworth 1983; Price and Baringer 1994). At the Antarctic continental margin, for example, intense cooling and active sea ice formation create cold and saline dense water at surface. Such a dense water descends down to depth on the continental slope and provides a principal source of Antarctic Bottom Water (AABW) and hence drives the global thermohaline circulation (Orsi et al. 1999). Therefore, quantitative estimation is required with regard to where, how much, and with what water mass property such dense water mass descends to depth in understanding the global thermohaline circulation. Nevertheless, it is not easy because oceanic downslope gravity currents involve various processes with a wide range of spatial and temporal scales.

For nonrotating fluids, the behavior of dense gravity currents on a slope can be described by the source density anomaly, the background stratification, and the entrainment process induced by mixing between dense water and ambient less dense water. Downsloping dense water is gradually diluted by entrainment and descends to the depth where the density anomaly becomes zero, and then it departs from the slope and flows offshore at this level (Monaghan et al. 1999; Özgökmen et al. 2006).

In the presence of the earth’s rotation, however, the behavior of dense water on a slope is very different. The rotational constraint forces the geostrophic flow component to parallel isobaths and inhibits downslope flow. Some mechanisms that break this geostrophic balance are required for transporting the dense water to depth, and various processes have been suggested by many studies. For example, bottom friction induces Ekman transport in the downslope direction. Smith (1975) discusses the path and spatial density variation of a dense water plume at a steady state under the influence of bottom friction and entrainment by using a streamtub model. Baroclinic instability developing at the front between dense water and ambient less dense water is another major factor for

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DOI: 10.1175/JPO-D-10-05014.1

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breaking the geostrophic balance. Gawarkiewicz and Chapman (1995) estimate the cross-isobath eddy transport induced by the baroclinic waves developed at the interface of the two water masses using a three-dimensional numerical model. Jiang and Garwood (1996) perform similar experiments but with a wider region and show the large-scale structure of dense water plumes. In their results, although the mean flow is oriented to the along-isobath direction, cross-isobath eddy transport makes a plume wider as it travels away from the source of outflow. Gawarkiewicz (2000), Tanaka and Akitomo (2001), and Tanaka (2006) investigate sensitivity of this eddy transport to various parameters such as slope angle, background stratification, bottom friction, and the Coriolis parameter.

Although most of the studies, such as mentioned above, are based on an idealized slope of constant angle, some studies investigate the influence of topographic variation. Jiang and Garwood (1998) perform the same experiments as Jiang and Garwood (1996), but with topographic features such as a submarine ridge, a canyon, or a seamount on the slope. The results show that both submarine ridge and canyon steer some part of the dense water plume to the offshore direction and enhance cross-isobath transport. An offshore downslope current is trapped at the edge of canyon and the side of ridge. In the case with a seamount, however, no offshore downslope current is induced because an along-isobath flow can make a detour and bypass around the seamount. They also perform sensitivity experiments to the scale of the topographic features, but a discussion that relates the topographic features and the amount of offshore transport is not provided.

Darelius and Wåhlin (2007) discuss the along-ridge/canyon transport of dense water plume with an analytical 1.5-layer model and estimate the offshore velocity and the cross-isobath transport with several idealized topographic shapes. Darelius (2008) performs laboratory experiments that support this analytical model. However, their argument assumes the existence of Ekman transport at the side of the ridge/canyon a priori and depends on the bottom friction, which is hard to be correctly measured in the real ocean. Muench et al. (2009) performs similar idealized numerical experiments with not a single ridge/canyon but a periodic wavelike corrugation, and their results show that downslope transport of dense water increases as the corrugation wavelength decreases.

The studies reviewed above estimate the influence of each process separately in very idealized situations, but those processes must take place at the same time and interact with each other in the real ocean. It is difficult to determine a priori which process is dominant. In the present study, we discuss the influence of small-scale topographic features based on the result of high-resolution numerical experiment with realistic settings performed by Matsumura and Hasumi (2010, hereafter MH10). The experiment by MH10 focuses on the overflow of cold dense water mass called ice shelf water (ISW) from the Filchner Depression in the southern Weddell Sea, where observations by Foldvik et al. (2004) suggest that a submarine ridge located near the exit of the depression plays a very important role in cross-isobath transport of ISW-origin dense water (see Fig. 1 for the location and topographic feature). The result of MH10 is consistent with the mooring observation near the ridge by Foldvik et al. (2004), and they estimate the formation rate of the bottom water whose origin is ISW overflowed from the Filchner Depression. In the present study, by using the result of MH10, we investigate the dynamics of the cross-isobath downslope transport induced by the existence of the submarine ridge. As shown in MH10, the existence of the ridge induces two different processes for cross-isobath downslope transport: a strong but narrow and thin downslope current at the east side of the ridge and cyclonic eddies with dense water cores formed when the dense water plume makes a detour around the ridge. We discuss the mechanisms for these ridge-induced processes and derive a general criterion where and when such processes take place through a simple analysis on the equation of motion of a dense water mass located on a slope. Then we validate the derived analytical model by numerical sensitivity experiments for various idealized topographies. Although the present study is based on the specific overflow phenomenon in the southern Weddell Sea, the discussion is rather general and the derived criterion for the effect of topography is also applicable to describing the path of downslope dense gravity currents in other regions.

2. Overview of the numerical experiment on the Filchner overflow

First, we briefly describe the settings and results of the experiment on the ISW overflow performed by MH10. The model domain is 75°S–72°S, 42°–30°W (indicated by the solid rectangle in Fig. 1), where topography is taken from the General Bathymetry Chart of the Oceans (GEBCO) one-minute bathymetry dataset. Note that a ridge of ~10-km width and ~300-m height exists at the immediate west of the exit of Filchner Depression. The initial stratification of the model is horizontally uniform, whose profile is based on observations (Foldvik et al. 2004), and ISW is provided in the model by restoring potential temperature θ and salinity S inside the Filchner Depression (from 35.5° to 30.5°W, south of 74.5°S, and below 400-m depth). A virtual tracer, which is initially zero and restored to unity inside the Filchner Depression as the same manner as θ and S, is introduced to trace the
ISW-origin dense water. The model resolution is $1/30 \times 1/120$ (approximately 900 m $\times$ 900 m) horizontally and 25 m vertically, which are sufficiently high for representing the small-scale downslope current and eddies induced by the ridge. The boundaries of domain are treated as rigid walls with sponge layers of 10-km width where $u$ and $S$ are restored to the initial stratification. The integration is performed for 180 days from the state of rest, by using a nonhydrostatic $z$-leveled model developed by Matsumura and Hasumi (2008).

Figure 2 shows the bottom concentration of the virtual tracer at day 120. A strong downslope dense water plume of less than 10-km width, which reaches as deep as 4000 m, is clearly seen at the east side of the ridge. Not all the ISW-origin dense water is trapped at the east side of the ridge and transported to the 4000-m depth, but some portion of it flows to the west by making a detour around the crest of the ridge at a wide range of depth. The closeup near the ridge (Fig. 3) depicts that the detouring branch accompanies cyclonic eddies. Such eddies contain the ISW-origin dense water inside and hence account for cross-isobath downslope transport of the dense water over a wide region to the west of the ridge. See MH10 for more detailed and quantitative results.

3. Eddy generation at the crest of the ridge

In this section, we discuss the eddy generation mechanism by treating a dense water mass located in a less dense ambient water as a fluid parcel. For simplicity, we neglect the influence of bottom friction and entrainment. On an ideal slope of constant angle $\theta$, a fluid parcel that represents a mass of dense water moves parallel to isobaths with geostrophically balanced velocity

$$ U = -\frac{g' \alpha}{f} $$

in a steady state (positive when seeing coast left), where $\alpha = \tan \theta$, $g'$ is the reduced gravity acceleration derived from the density anomaly, and $f$ is the Coriolis parameter.
(Fig. 4). It is not obvious whether a mass of dense water of finite size behaves as a fluid particle. Nof (1983) derives the condition for a mass of water to be advected parallel to isobaths with the geostrophic velocity without deformation based on the potential vorticity conservation. The condition is described by

\[ \frac{lh}{C^2 g} \leq \frac{g' H \alpha}{f^2} \]  

where \( l \) and \( h \) are horizontal and vertical scales of the mass of water, respectively, and \( H \) is the depth of the ambient fluid layer. A typical value of each parameter for the Filchner overflow in the model is \( g' = 5 \times 10^{-4} \text{ m s}^{-2} \), \( H = 2000\text{–}4000 \text{ m} \), \( \alpha = 0.05 \), \( f = 1.4 \times 10^{-4} \), and \( h = 100 \text{ m} \). Using these values, the condition yields \( l \ll 30 \text{ km} \). Because the width of downslope plume at the east side of the ridge is no greater than 10 km, the ISW-origin dense water that outflows from the Filchner Depression satisfies this condition and can be treated as a fluid particle, at least near the ridge.

In the presence of a submarine ridge, isobaths are not straight but have a curvature \( \kappa \) (defined positive when the isobath is convex). Because a fluid particle traveling along curved isobaths feels centrifugal force, which functions to make the fluid particle move in the cross-isobath direction, velocity \( U \) should satisfy the following balance for the particle to trace the curved isobaths,

\[ -fU - \kappa U^2 = g' \alpha. \]  

This quadratic equation for \( U \) has two solutions, but the one that converges to (1) at the limit of \( \kappa \to 0 \) is of physical interest; that is,

\[ U = \frac{-f + f \sqrt{1 - 4kg' \alpha f^{-2}}}{2\kappa}. \]

When the equilibrium velocity (4) is realized, cross-isobath transport does not occur even at curved isobaths. For such equilibrium velocity to exist, \( \kappa \) has an upper limit,

\[ \kappa < \kappa_c = \frac{f^2}{4g'\alpha}, \]  

or, using radius of curvature \( R = \kappa^{-1} \), this condition is expressed as

\[ \frac{g' \sin \theta}{u} \quad \text{deep} \]
\[ \frac{g' \sin \theta}{u} \quad \text{shallow} \]

(b)

Fig. 4. Schematic figures for the forces acting on a dense water mass located on a constant slope in the Southern Hemisphere in (a) plan view and (b) elevation view.
$R < 0$ or $R > R_C = 4 \frac{g' \alpha}{f^2}$.  

Therefore, when the isobaths are convex and the radius of curvature is less than $R_C$, the equilibrium velocity is never realized. In this case, the fluid particle cannot trace isobaths, and its trajectory separates offshore from isobaths. Note that the equilibrium velocity $U$ rapidly increases as the radius of curvature $R$ becomes close to $R_C$, and at the same time, the downslope motion of a fluid particle (2) and behaves like a fluid particle toward the gradient of ocean floor (positive when seen from the left). Hence, the intense downslope current at the east side of the ridge is located along the negative maximum of the isobath curvature. Because the centrifugal force pushes a fluid particle coastward and works to restrict the downslope motion where curvature of isobaths is negative, another force is required to maintain such a downslope current there. Darelius and Wahlin (2007) and Darelius (2008) argue a downslope plume trapped at the side of the ridge with assuming bottom Ekman transport, but we argue another possible cause.

4. General criterion for the location of cross-isobath transport

The argument in the previous section indicates that offshore separation of the trajectory of dense water occurs where the isobaths are convex and the curvature is sufficiently high. However, the intense downslope current at the east side of the ridge is located along the negative maximum of the isobath curvature. Because the centrifugal force pushes a fluid particle coastward and works to restrict the downslope motion where curvature of isobaths is negative, another force is required to maintain such a downslope current there. Darelius and Wahlin (2007) and Darelius (2008) argue a downslope plume trapped at the side of the ridge with assuming bottom Ekman transport, but we argue another possible cause.

We suppose that a dense water mass on slope satisfies the condition (2) and behaves like a fluid particle and argue the cause of the downslope current by considering the equation of motion of a fluid particle on the two-dimensional orthogonal curvilinear coordinate system $(s, n)$, where $s$ is directed parallel to isobaths (positive when seeing coast left) and $n$ is directed to the gradient of ocean floor (positive upslope). When the ocean bathymetry is given as $z = H(x, y)$ in the Cartesian coordinate system $(x, y)$, the base vectors of this orthogonal curvilinear coordinate system are defined as

$$s = n \times k, \quad n = \frac{\nabla H}{|\nabla H|},$$

where $k$ is a unit vector pointing upward. Let $u$ and $v$ be the velocity of a fluid particle toward $s$ and $n$ directions,
respectively; then the equations of motion for this fluid particle on the orthogonal curvilinear coordinate system are

$$\frac{du}{dt} = \frac{uv}{R} - \frac{v^2}{L} + fv + \frac{1}{\rho_0} \frac{\tau_s}{h}$$

and

$$\frac{dv}{dt} = \frac{uv}{L} - \frac{u^2}{R} - fu + \frac{1}{\rho_0} \frac{\tau_n}{h} - g' \alpha$$

where $\tau_s$ and $\tau_n$ are the stress due to bottom friction in each direction and $h$ is the thickness of the fluid layer.

The first and second terms in the RHS of (10) and (11) are reflecting the curved coordinates, where $R$ is the radius of curvature of isobaths defined in the previous section and $L$ is the radius of curvature of ridge/trough lines perpendicular to isobaths defined as

$$\frac{1}{L} = \alpha \frac{1}{\partial s \alpha} = -\frac{1}{\alpha} \frac{\partial \alpha}{\partial s}.$$  

Because $L$ is the inverse of the spatial derivative of slope $\alpha$, it represents the length scale of convergence/divergence of isobaths.

Fig. 6. Six-day interval snapshots of the barotropic velocity (vector) and the thickness of a fluid layer whose tracer concentration is greater than 0.1 (color).
In the previous section, we neglect bottom friction. Here, we check the validity of this assumption. The bottom stress in the model is represented by the bulk formula

$$\tau = -C_D \rho_0 |\mathbf{u}| \mathbf{u},$$  \hspace{1cm} (13)$$

where $C_D$ is the drag coefficient, which is set to $5.0 \times 10^{-3}$. When the scale of velocity $u$ and $v$ is $\sim 10^{-3} \text{ m s}^{-1}$ and the thickness of dense water layer $h$ is $\sim 10^2 \text{ m}$, the scale of friction term in (10) and (11) is at most $O(10^{-6}) \text{ m s}^{-2}$. On the other hand, the scale of gravity force and Coriolis force are both $\sim O(10^{-5}) \text{ m s}^{-2}$ for $f \sim 10^{-4}$, $g' \sim 10^{-3}$, and $a \sim 10^{-2}$, so the frictional force is at least one order of magnitude smaller than these forces. The frictional force becomes significant only when the other terms cancel each other, but the following argument shows that such balance is not realized near the ridge.

When a fluid particle has the equilibrium velocity defined by (4), i.e., $u = U$ and $v = 0$, the RHS of (10) and (11) are zero and this fluid particle traces the isobaths without any acceleration. However, because $f$, $\alpha$, and $\kappa$ are not spatially uniform, the equilibrium velocity $U$ is a function of location, $U = U(s, n)$. Suppose that a fluid particle at a location $(s_0, n_0)$ has the local equilibrium velocity $u = U_0 = U(s_0, n_0)$ and $v = 0$. Although this fluid particle has no cross-isobath momentum and completely traces the isobath at $(s_0, n_0)$, it is accelerated perpendicular to isobaths after a travel of a small distance $\delta s$ along isobaths, because $U_0$ no longer satisfies the balance (3) at $(s_0 + \delta s, n_0)$. The acceleration in the $n$ direction $F_n$ that acts on a fluid particle with the velocity $u = U_0$ and $v = 0$ at the location $(s_0 + \delta s, n_0)$ is

$$F_n(s_0 + \delta s, n_0) = -U_0^2 \frac{\partial \kappa}{\partial s} \bigg|_{s_0, n_0} + U_0 \frac{\partial f}{\partial s} \bigg|_{s_0, n_0} + g \frac{\partial \alpha}{\partial s} \bigg|_{s_0, n_0} \delta s$$

$$+ U_0 \frac{\partial f}{\partial s} \bigg|_{s_0, n_0} + g \frac{\partial \alpha}{\partial s} \bigg|_{s_0, n_0} \delta s$$

$$= -U_0 \left[ \begin{array}{c} U_0^2 \frac{\partial \kappa}{\partial s} \bigg|_{s_0, n_0} + U_0 \frac{\partial f}{\partial s} \bigg|_{s_0, n_0} + g \frac{\partial \alpha}{\partial s} \bigg|_{s_0, n_0} \end{array} \right] \delta t, \hspace{1cm} (16)$$

where $\delta t = U_0^{-1} \delta s$ is the time taken for a fluid particle with velocity $u = U_0$ to travel the distance $\delta s$. The sign of $F_n$ determines the tendency of cross-isobath shift of the fluid particle trajectory; that is, it feels upslope (downslope) acceleration when $F_n$ is positive (negative). Although the discussion above is based on the case for Southern Hemisphere where $U_0$ is positive, the same formula as (16) is derived when we consider $F_n(s_0 - \delta s, n_0)$ for the Northern Hemisphere case where $U_0$ is negative.

Let $\Gamma(x, y)$ be a function of location $(x, y)$ defined as

$$\Gamma(x, y) = -U \left[ \begin{array}{c} U^2 \frac{\partial \kappa}{\partial s} + U \frac{\partial f}{\partial s} + g \frac{\partial \alpha}{\partial s} \end{array} \right] \cdot \mathbf{s}, \hspace{1cm} (17)$$

where $U$ is the local equilibrium velocity defined by (4). If an appropriate value of $g'$ is provided, $\Gamma$ can be derived a priori only from the depth of ocean floor. Figure 8a shows the map of $\Gamma$ calculated by using $g' = 5 \times 10^{-4}$ m s$^{-2}$, which is the typical value for the Filchner overflow, and the time-averaged bottom velocity of the numerical
experiment result performed by MH10. It is clear that the location of strong downslope current at the east side of the ridge coincides with the place where $\Gamma$ takes a large negative value. On the other hand, the direction of mean flow is parallel to isobaths at the northwestern region of the domain, where $\Gamma$ is relatively small. Therefore, the function $\Gamma$ is an index of stability for an along-isobath geostrophic current. The RHS of (17) consists of three terms, and the location where the cross-isobath transport of dense water takes place is determined by the relative magnitude between these three terms. The first, second, and third terms represent the spatial derivatives of isobath curvature, planetary vorticity, and slope angle, respectively, in the along-isobath direction. Figures 8b,c show the first and third terms in the RHS of (17), respectively. In the domain of the present study, the maximum absolute value of the second term is three orders of magnitude smaller than the first and third terms, so the influence of the planetary vorticity gradient is negligible. The contribution of the third term is dominant where the strong downslope current takes place, so the cause of such a strong downslope current at the east side...
of the ridge is concluded to be a response to the rapid increase of the slope angle along isobaths. Figure 9 presents a schematic view of this process.

5. Numerical validation

In this section, we validate the derived criterion for the location of cross-isobath dense water transport, $\Gamma < 0$, by numerical sensitivity experiments. The numerical model used is the same one as MH10, but the settings are very idealized. The domain is 160- and 120-km widths in $x$ and $y$ directions, respectively, and 3000 m deep in the Cartesian coordinate system, and the $f$-plane approximation is used with $f = -1.4 \times 10^{-4}$ s$^{-1}$. In this section, the terms east and north mean the $x$ and $y$ directions, respectively, and the coordinate origin is at the southwest corner of the domain. The bathymetry is set as

$$h(x, y) = -0.025y + H \exp \left[ -\frac{(x - x_0)^2}{L^2} \right]; \quad (18)$$

that is, the floor deepens in the $y$ direction with a slope angle of 2.5%, and a ridge/canyon of Gaussian shape is located at $x_0 = 40$ km. The terms $H$ and $L$ are the height and the horizontal scale, respectively, of the ridge (when $H > 0$) or canyon (when $H < 0$); see the bathymetry contours of Fig. 10 for the $H = 300$ m, $L = 10$ km case. The boundaries are periodic in the $x$ direction and rigid walls exist at the southern and the northern boundaries. The model resolution is 1 km horizontally and 25 m vertically. The potential temperature and salinity are initially $\theta = 0.5^\circ$C and $S = 34.7$ psu over the domain and are restored to $\theta = -2.0^\circ$C and $S = 34.6$ psu at 120 km $< x < 140$ km, $y < 5$ km with a time scale of 6 h. Like MH10, a virtual tracer, which is initially zero over the domain and restored to unity in the same manner as $\theta$ and $S$, is also introduced. We perform sensitivity experiments with $H = \pm 150$, $\pm 300$, and $\pm 600$ m and $L = 5$, 10, and 20 km (total of 18 cases; see Table 1 for reference name of each case). Each case is integrated for 90 days from the state of rest.

![Fig. 9. Schematic figures for a downslope current formation at the side of the ridge where isobaths are locally convergent in (a) plan view and (b) elevation view.](image1)

![Fig. 10. Result of case R300–15. (a) Snapshot of bottom velocity (vector) and bottom tracer concentration (color) at day 45. (b) Time-averaged bottom velocity of model result (vector), $\Gamma$ (color), and the estimated path of dense water by $\Gamma$-based particle-tracking model. Contours are bathymetry with 200-m intervals.](image2)
First, we investigate the result of R300–10, which is the closest case to the situation of the Filchner overflow modeled by MH10. Although this experiment is very idealized, Fig. 10a shows the similar processes as MH10. The dense water provided at the southeast margin flows westward and intermittently arrives at the ridge (discussion on the cause of this intermittency is in MH10). A strong downslope flow is found at the east side of the ridge, but some part of the dense water makes a detour around the ridge and flows to the west of the ridge. We expect that such a pathway of dense water can be predicted by $\Gamma$ (indicated by color of Fig. 10b), which is calculated only from bathymetry and typical density anomaly.

Recall that the quantity $\Gamma \delta t$ is the apparent cross-isobath force acting on a dense water particle, which tries to trace isobaths, where $\delta t$ is the time scale in which the particle travels with keeping the local equilibrium along-isobath velocity $U$. The cross-isobath force at the location of large $|\Gamma|$ induces the cross-isobath motion $u$, which results in the along-isobath Coriolis force $fu$ in the RHS of (10), and hence the state is adjusted to a new equilibrium where the gravity, the Coriolis, and the centrifugal forces are balanced in a time scale of $|f|^{-1}$. Thus, $|f|^{-1}$ is the appropriate time scale to substitute for $\delta t$, so the cross-isobath force induced by curved isobath is estimated as $\Gamma|f|^{-1}$. Now we can predict the trajectory of dense water particle $\mathbf{r}(t)$ by a very simple Lagrangian particle-tracking model using $\Gamma$. We assume that the along-isobath velocity $u$ always coincides with the local equilibrium velocity $U(\mathbf{r})$ defined by (4) as long as the curvature of isobaths is less than the critical value (5), and we substitute $\kappa_C$ for $\kappa$ in (4) where the curvature exceeds the critical value $\kappa_C$. The cross-isobath velocity $v$ is initially zero but is accelerated by the apparent force $\Gamma|f|^{-1}$. Here, the $\Gamma$-based particle-tracking model that predicts the dense water pathway is as follows:

$$u(t) = \begin{cases} \frac{-f + f \sqrt{1 - 4g' \alpha f^{-2}}}{2k} & \text{when } \kappa < \frac{f^2}{4g' \alpha} \\ \frac{2g' \alpha}{f} & \text{when } \kappa \geq \frac{f^2}{4g' \alpha} \end{cases}$$

$$v(0) = 0, \quad \frac{\partial v}{\partial t} = \frac{\Gamma(\mathbf{r})}{|f|}, \quad \text{and}$$

$$\mathbf{r}(t) = \mathbf{r}(0) + \int_0^t \mathbf{u}(t') \, dt'.$$

The three green lines in Fig. 10b are the trajectories of dense water particles predicted by this $\Gamma$-based particle-tracking model with using the typical value for the reduced gravity $g' = 3 \times 10^{-4}$. This value of $g'$ is approximately equivalent to that of the mixture of source dense water ($\theta = -2.0^\circ\text{C}$ and $S = 34.6$ psu) and ambient less dense water ($\theta = 0.5^\circ\text{C}$ and $S = 34.7$ psu) with a mixing ratio of 1:1. The $\Gamma$-based trajectories initially trace isobaths in the region of $\Gamma = 0$ and then separate offshore at the east side of the ridge, where $\Gamma$ takes a large negative value. After detouring around the crest, the lines trace the isobaths again to the west, where $\Gamma = 0$, but the levels of trajectories descend down by about 300 m while passing the ridge. Note that the trajectory does not run in the upslope direction, even in the region where the sign of $\Gamma$ is positive, because it represents cross-isobath acceleration, not velocity. The cross-isobath velocity, which has a negative value after passing the region of negative $\Gamma$, returns to almost zero while passing the region of positive $\Gamma$, and hence the trajectory traces isobaths again to the west of the ridge. During this process, the dense water is predicted to descend about 300 m in R300–10 case. Such $\Gamma$-based trajectories well coincide with the time-averaged bottom velocity of the numerical model result (denoted by vectors in Fig. 10b).

Comparison between the $\Gamma$-based dense water trajectories and the results of numerical experiments for other cases is in Fig. 11 (not all cases are shown because of space limitations.) Roughly speaking, the former (green lines) well predicts the latter (vectors) for all cases, so we can conclude that the $\Gamma$ is a good index for the location of cross-isobath transport of dense water. It is notable that the $\Gamma$-based trajectories are completely trapped at the ridge (or the canyon) and cannot make a detour around

| Table 1. Topographic parameters and the results for cases of sensitivity experiments. |
|---|---|---|---|---|
| Case | $H$ (m) | $L$ (km) | $|\Gamma| \times 10^{-5}$ | Is trajectory trapped? | Ratio of tracer below 2500 m |
| R150–05 | 150 | 5 | 0.357 | No | 8.4% |
| R150–10 | 150 | 10 | 0.074 | No | 4.4% |
| R150–20 | 150 | 20 | 0.011 | No | 2.8% |
| R300–05 | 300 | 5 | 1.031 | Yes | 24.2% |
| R300–10 | 300 | 10 | 0.241 | No | 8.9% |
| R300–20 | 300 | 20 | 0.045 | No | 6.7% |
| R600–05 | 600 | 5 | 2.629 | Yes | 42.6% |
| R600–10 | 600 | 10 | 0.667 | Yes | 30.8% |
| R600–20 | 600 | 20 | 0.166 | No | 15.1% |
| C150–05 | -150 | 5 | 0.519 | No | 1.9% |
| C150–10 | -150 | 10 | 0.117 | No | 3.9% |
| C150–20 | -150 | 20 | 0.018 | No | 3.8% |
| C300–05 | -300 | 5 | 1.746 | Yes | 17.3% |
| C300–10 | -300 | 10 | 0.387 | No | 6.1% |
| C300–20 | -300 | 20 | 0.062 | No | 9.2% |
| C600–05 | -600 | 5 | 7.149 | Yes | 19.6% |
| C600–10 | -600 | 10 | 1.192 | Yes | 25.1% |
| C600–20 | -600 | 20 | 0.208 | No | 16.6% |
FIG. 11. As in Fig. 10b, but for other cases.
the ridge (or cross over the canyon) for the cases with steep topography (R300–05, R600–05, R600–10, C300–05, C600–05, and C600–10) where the maximum |Γ| is $O(10^{-10})$ m s$^{-3}$. For such cases, the numerical model results also exhibit strong downslope plumes, which descend down to as deep as 3000 m and reach the northern boundary of the domain. Figure 12 shows the vertical distribution of the horizontally integrated virtual tracer content for cases with $H = 300$ m. It is clear that R300–05 where the Γ-based trajectories are trapped at the ridge exhibits a much larger amount of tracer content below 2500-m depth compared to the others. This characteristic is also clear for other cases; see Table 1, where the fifth column indicates whether the Γ-based trajectory is trapped at the ridge/canyon and the sixth column is the ratio of virtual tracer distributed below 2500-m depth to that below 1000 m at day 90 for each case. These results suggest that the downslope dense water transport is highly enhanced where the small-scale topography is so steep that |Γ| takes as large as $O(10^{-10})$ m s$^{-3}$.

### 6. Discussion

**a. Suggested parameterization for coarse-resolution models**

Although descent of dense water and consequent bottom water production have a great impact on the global thermohaline circulation and the earth’s climate system, ocean general circulation models (OGCMs) cannot reproduce such bottom water formation processes explicitly even in recent high-resolution experiments. Present OGCMs rely on bottom boundary layer (BBL) parameterizations in representing the amount and location of the bottom water formation (Nakano and Suginohara 2002), but such parameterizations are not very successful so far. In particular, many OGCMs have been suffering from the problem that most of the dense water formed at the continental margin of Antarctica tends to flow westward at the shelf break instead of descending down to depth. One possible cause of insufficiency of present BBL parameterizations is a lack of consideration for the effect of unresolved small-scale topographic features and their spatial variation. Various studies (e.g., Foldvik et al. 2004; Darelius and Wåhlin 2007; Darelius 2008; Muench et al. 2009; Wilchinsky and Feltham 2009; MH10) have pointed out that small-scale ridges and canyons of $O(1)$ km, which are never resolved in OGCMs, play an essential role in transporting dense water to deep ocean and the formation of bottom water. In the present study, we quantified that the effect of topography on the cross-isobath dense water transport is dominant where topography is so steep that |Γ| > $O(10^{-10})$ m s$^{-3}$. Such a value of |Γ| is found in the real ocean bathymetry not only at the site of the Filchner overflow (Fig. 8) but also in other regions of dense water overflow. Because Γ can be calculated only from the bathymetry data and typical density anomaly, it is possible to parameterize the effect of unresolved topographic features in coarse-resolution models. As presented in the section 5, the apparent cross-isobath force due to curved isobaths acting on the dense water mass is estimated as $Γ/|Γ|^{-1}$, so we can parameterize this effect of unresolved topography by adding this term to the equation of motion of coarse-resolution models. It should be noted that this cross-isobath force is an apparent one because of inertia induced by curved isobaths, so it must not be an energy source/sink. To satisfy this condition, the additional force should act in the direction perpendicular to velocity $u$, not to the isobaths. Therefore, it should be expressed as

$$\pm \frac{Γ}{|Γ|} \frac{u}{|u|} \times k,$$

(22)

where the sign is decided below. It is not inconsistent with the fact that the force is originally defined for the cross-isobath direction, because dense water tends to flow parallel to isobaths when small-scale topography is not resolved. Because this additional force parameterizing the effect of unresolved topography is perpendicular to velocity $u$, it can be merged with the Coriolis force term,

$$f^u \times k = \left( f + C\frac{Γ}{|Γ|} \right) u \times k,$$

(23)

where $f^*$ is an effective Coriolis parameter including the effect by unresolved curved isobaths and $C$ is a non-dimensional parameter which controls the influence of this parameterization. The sign of additional force is chosen so that it acts to reduce the Coriolis force when Γ is negative. Replacing $f$ with $f^*$ in the bottom boundary...
layer of coarse-resolution models will introduce the effect of unresolved topography on dense gravity currents. How to decide the value of $\Gamma$ in each coarse-resolution grid is a problem, but it might be good to set as the maximum negative value of $\Gamma$ calculated from high-resolution bathymetry of corresponding area. The appropriate value for $C$ may depend on the ability to represent dense bottom water on continental slopes of the coarse-resolution model to which this parameterization applies. These issues should be discussed elsewhere.

b. Relationship with the separation problem of large-scale boundary currents

In the present study, we assert that whether a dense gravity current follows isobaths is determined by relative importance among spatial derivatives of the curvature of isobaths, the planetary vorticity, and the slope angle. The influence of spatial variation of topographic features on a path of along-isobath flow has previously been discussed in the context of offshore separation of large-scale coastal boundary currents. Stern (1998) shows that convergence of isobaths is one of the reasons for separation of the Gulf Stream. The slope angle becomes very steep locally near 35°N, 76°W, where the axis of the Gulf Stream separates from the shelf break. Such a mechanism is qualitatively equivalent to the mechanism of cross-isobath downslope currents discussed in the present study. He discusses the separation problem of coastal boundary currents from the viewpoint of relation between isobath convergence [corresponding to the third term in the RHS of (17)] and topographic $\beta$ effect induced by stretching [corresponding to the second term in the RHS of (17); see discussion below]. Marshall and Tansley (2001) assert that the condition for the boundary current separation from a coastline is

$$R < \left( \frac{U}{\beta} \right)^{1/2},$$

where $U$ is the velocity of the boundary current, $R$ is the curvature radius of the coastline, and

$$\beta = \frac{\partial \gamma}{\partial \lambda}$$

is the gradient of planetary vorticity along the coastline. This condition is the same as the separation condition $\Gamma < 0$ of this study when the third term in (17) is neglected (beware of the sign of the Coriolis parameter and the direction of currents). Therefore, the function $\Gamma$ defined by (17) is a generalized criterion for separation of an along-isobath flow that covers both downslope dense gravity currents of less than $O(10)$ km scale discussed in the present study and larger-scale coastal boundary currents of $O(100)$ km scale discussed in Stern (1998) and Marshall and Tansley (2001). The planetary vorticity gradient term is not important for small-scale gravity currents, but it becomes significant when we deal with a larger-scale separation problem. Note that, although the velocity $U$ in the previously discussed separation problem of large-scale boundary currents is determined by external factors such as basin-scale wind-driven circulation, the equilibrium velocity $U$ in a dense gravity current on a slope discussed in the present study is determined by local topographic features and density anomaly.

We mentioned in section 3 that the cause of generation of cyclonic eddies at the crest of the ridge is the gain of negative relative vorticity due to stretching of a fluid layer. In section 4, on the other hand, we discussed the mechanism of the formation of a strong downslope current at the east side of the ridge by treating the ISW-origin dense water as a fluid particle, and stretching of the layer is not considered. It might seem arbitrary that stretching of a dense water layer and consequent gain of relative vorticity are taken into account in the former case but not in the latter case. Such a different treatment is based on the fact that the cyclonic eddies existing to the west of the ridge are associated with a thick layer of the dense water, whereas the downslope plume at the east side of the ridge is a thin dense layer, as presented in Fig. 3. This difference is qualitatively explained as follows: In the former case, the principal force that induces the cross-isobath motion of a dense water mass is a lateral centrifugal (or an inertial) force. When a dense water mass moves laterally in the offshore direction, it will be located above a less-dense layer as the sea bed deepens along its trajectory. Thus, the dense water layer stretches to avoid the density inversion and gains relative vorticity (see Fig. 7b). In the latter case, however, the principal force that induces the cross-isobath motion is the along-slope component of the downward gravity force. Therefore, the dense water mass is accelerated in the downslope direction, not horizontal, so the dense water layer remains thin (see Fig. 9b). It should not be the case that stretching of a fluid layer does not take place at all along the downslope plume. In fact, there is an upslope counter current at the immediate east of the downslope plume (Fig. 3a), which suggests gain of negative relative vorticity due to stretching there. The magnitude of stretching for the downslope current at the east side of the ridge is roughly estimated below.

Marshall and Tansley (2001) discuss that whether stretching of a fluid layer promotes or restricts the separation of a boundary current depends on the sign of relative vorticity obtained due to stretching. For example, in the separation of a western boundary current in the Northern Hemisphere, gain of positive relative vorticity
due to stretching acts to shift the axis of flow offshore and promote the separation. Although (17) does not explicitly include such an effect of stretching, it can be included by adding the topographic $\beta$ term, which represents the gain of relative vorticity due to stretching, to the planetary $\beta$ (spatial derivative of the planetary vorticity) term in the RHS in (17). Because a dense gravity current on a slope in the Southern Hemisphere flows with seeing a coast left, it can be used to represent the effect of unresolved small-scale topography on dense gravity currents in coarse-resolution OGCMs.

Note that, although the map of $\Gamma$ at the site of the Filchner overflow well coincides with the modeled cross-isobath dense water transport by MH10 and the simple $\Gamma$-based particle-tracking model well predicts the dense water pathway of idealized numerical experiments, it does not mean that the route of dense overflow is always controlled by $\Gamma$ in the real ocean, because the numerical experiments ignore or do not resolve some important processes other than the topographic variation. For example, dense gravity currents at the northwestern Ross Sea are significantly affected by strong tides (Gordon et al. 2009; Padman et al. 2009). The present study quantifies the effect of topographic variation, and other processes such as bottom Ekman transport, tidal flows, and entrainment induced by shear-driven instability should also be quantified in other studies to determine the complete behavior of dense gravity currents in the real ocean. Finally, it should be noted that high-resolution bathymetry of about $O(1)$ km has not been robustly surveyed yet and there is uncertainty for small-scale topographic features particularly in the polar oceans. Although we use the GEBCO one-minute dataset, there are some nonnegligible differences from other datasets such as 2-minute gridded elevations/bathymetry for the world (ETOPO2). More quantitatively robust analysis can be possible when higher-resolution bathymetry data are available.

Acknowledgments. This study is supported by JST/CREST.

REFERENCES


