Hysteresis and Dynamics of a Western Boundary Current Flowing by a Gap Forced by Impingement of Mesoscale Eddies

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(Manuscript received 20 April 2010, in final form 16 November 2010)

ABSTRACT

Hysteresis of a western boundary current (WBC) flowing by a wide gap of a western boundary and the dynamics of the WBC variations associated with the impingement of mesoscale eddies from the eastern side of the gap are studied using a 1.5-layer reduced-gravity quasigeostrophic ocean model. The study focuses on two issues not covered by existing studies: the effects of finite baroclinic deformation radii and time dependence perturbed by mesoscale eddies. The results of the study show that the hysteresis of the WBC of finite baroclinic deformation radii is not controlled by multiple steady-state balances of the quasigeostrophic vorticity equation. Instead, the hysteresis is controlled by the periodic penetrating and the leaping regimes of the vorticity balance. The regime of the vorticity balance inside the gap is dependent on the history of the WBC evolution, which gives rise to the hysteresis of the WBC path. Numerical experiments have shown that the parameter domain of the hysteresis is not sensitive to the baroclinic deformation radius. However, the domain of the periodic solution, which is determined by the lower Hopf bifurcation of the nonlinear system, is found to be sensitive to the magnitude of the baroclinic deformation radius. The lower Hopf bifurcation from steady penetration to periodic penetration is found to occur at lower Reynolds numbers for larger deformation radii. In general, the lower Hopf bifurcation stays outside the hysteresis domain of the Reynolds number. However, for very small deformation radii, the lower Hopf bifurcation falls inside the hysteresis domain, which results in the transition from the penetrating to the penetrating regimes of the WBC to skip the periodic regime and hence the disappearance of the upper Hopf bifurcation.

Mesoscale eddies approaching the gap from the eastern basin are found to have significant impact on the WBC path inside the gap when the WBC is at a critical state along the hysteresis loop. Cyclonic (anticyclonic) eddies play the role of reducing (enhancing) the inertial advection of vorticity in the vicinity of the gap so that transitions of the WBC path from the leaping (periodic penetrating) to the periodic penetrating (leaping) regimes are induced. In addition, cyclonic eddies are able to induce transitions of the WBC from the periodic penetrating to the leaping regimes through enhancing the meridional advection by its right fling. The transitions are irreversible because of the nonlinear hysteresis and are found to be sensitive to the strength, size, and approaching path of the eddy.

1. Introduction

The circulation induced by a western boundary current (WBC) flowing by a gap in the western boundary or in a mid-ocean ridge has a long history of study. The classical theory of Stommel and Arons (1960), which ignores stratification, topography, and nonlinearity, predicts existence of zonal jets extending westward from the gap.

Later studies (Pedlosky 1994a,b; Pedlosky and Spall 1999) found that baroclinicity and the positions of the gap within the gyre circulation is important for the structure of the gap circulation, but the zonal jets from the gap are always present in the linear models. Nof and Olson (1983) and Nof (1993) studied the nonlinear intrusion of a WBC into a gap on the order of one deformation radius wide. Their studies have shown that the zonal jets are absent in the nonlinear regime and that the intrusion is controlled by the beta effect and the pressure difference between the two basins. Nof’s solutions focus on the geostrophic circulation far away from the gap with the intruded current flowing along the eastern boundary of the western basin.
Recently, Sheremet (2001) presented a numerical analysis of a WBC flowing by a wide gap, showing that multiple steady states of the WBC and nonlinear hysteresis of the intrusion exist for certain ranges of nondimensional parameters. The solutions describe the paths of the WBC in the vicinity of the gap in detail, the steady-state regimes of which are dependent on the history of the WBC evolution. The existence of the multiple steady states has since been validated in laboratory experiments by Sheremet and Kuehl (2007) and Kuehl and Sheremet (2009). Despite all these studies, the vorticity balances that control the hysteresis evolution of the WBC path inside the gap remain to be disclosed.

Sheremet’s theory of the WBC hysteresis is based on the assumption that the deformation radius of a WBC is very small so that the transient evolution of the relative vorticity is neglected in the quasigeostrophic vorticity equation. In reality, the baroclinic deformation radius of a WBC is not always small so that the transient evolution of the relative vorticity is as important as the evolution of the streamfunction itself. As such, the direct application of Sheremet’s theory to midlatitude ocean problems, like the Luzon Strait circulation, is questionable. In addition, existing studies of the gap circulation have focused on the steady-state circulation. In the real ocean, however, various events like winds and mesoscale oceanic eddies can induce change of circulation near the gap. For example, the Kuroshio in the Luzon Strait between Taiwan of China and Luzon of the Philippines has been observed to exhibit multiple modes of intrusions (Nitani 1972), the paths of which are subject to strong intraseasonal variability associated with the impingement of mesoscale eddies from east of the Luzon Strait (Yuan et al. 2006). So far, the dynamics of the influence by these eddies are not known. The eddy–Kuroshio interactions are most likely related to the hysteresis of the Kuroshio system, the nature of which is not fully understood so far given the finite baroclinic deformation radius (~50 km) of the Kuroshio (Chelton et al. 1998). In this study, we use a simple 1.5-layer reduced-gravity quasigeostrophic ocean model to study the dynamics of the hysteresis WBC with finite deformation radii and its variability forced by the mesoscale eddies from the east.

The configuration of the simple model is described in the next section. The nonlinear bifurcation and hysteresis of the WBC system with finite baroclinic deformation radii are studied in section 3. The effects of mesoscale eddies on the WBC transitions are investigated in section 4. The application of the theory on the study of the Luzon Strait circulation is discussed in section 5. Section 6 contains conclusions of this study.

2. The models

The model used in this study is a full quasigeostrophic (QG) circulation model, with the transient evolution of the relative vorticity included in the model. In contrast to the Sheremet (2001) study, which focuses on the steady-state solutions and time-dependent solutions in the limit of a small baroclinic deformation radius, we use this model to study the vorticity balances of the WBC with finite deformation radii during its hysteresis transitions and under the influence of mesoscale eddies. So it is necessary to keep all the terms in the quasigeostrophic vorticity equation, written as

\[- \frac{1}{L_R^2} \psi_t + \zeta_i + J(\psi, \zeta) + \beta \psi_z = A_H \nabla^2 \zeta, \tag{1}\]

where $L_R$ is the baroclinic deformation radius, $\psi$ is the streamfunction of a depth-averaged flow, $\zeta = \nabla^2 \psi$ is the relative vorticity, and $A_H$ is the horizontal viscosity coefficient specified to be 300 m$^2$ s$^{-1}$ in this study.

The computational domain of the model is a rectangle of 3300 km in the zonal direction and 1800 km in the meridional direction, separated into a western and an eastern basin by a thin meridional wall with a gap in the middle connecting the two basins. The zonal lengths of the western and eastern basins are 2500 and 800 km, respectively. The meridional width of the gap in the middle of the barrier wall 2$\alpha$ is set to be 240 km. The model grid resolution is 10 km in both directions. The Munk WBC thickness $L_M = (A_H/\beta)^1/3$ in this model is 25 km with $\beta = 2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$, which means that the ratio of the half-width of the gap with the Munk WBC thickness, $\gamma = a/L_M$, is 4.87, larger than the critical ratio of $\gamma = 4.55$ for hysteresis to exist according to Sheremet (2001).

The northern, southern, and eastern boundaries of the eastern basin are open boundaries. The WBC in the model is driven by the streamfunction along the open boundaries as specified in the following: Along the northern and southern boundaries in the eastern basin, $\partial\psi/\partial y = \partial\zeta/\partial y = 0$. The streamfunction along the eastern boundary is $Q$ and the relative vorticity is zero. Along the barrier wall, the nonslip boundary condition is applied so that both the streamfunction and the velocity are zero. Based on the Taylor expansion, it is easy to verify that the relative vorticity at the barrier wall is $\zeta_i = 2 \psi_{i\pm 1}/\Delta z^2$, where $\psi_{i\pm 1}$ is the streamfunction one grid in the interior from the barrier (Roache 1972). Along the northern, southern, and western boundaries of the western basin, the streamfunction and the relative vorticity are set to zero for simplicity.

The partial differential equation of the quasigeostrophic vorticity is solved using the finite difference method. For numerical stability, the WBC transport per
unit depth $Q$ is set to be roughly $1/10$ of that of the Kuroshio. So, for the same nondimensional time, the real-world events evolve about 10 times faster than in the model.

The eddy is inserted into the model initial condition by specifying a streamfunction of Gaussian distribution 300 km east of the barrier wall in the eastern basin, which then migrates westward freely on a $\beta$ plane. With the above configuration, the QG model is able to simulate the nonlinear hysteresis of the WBC near the gap and its variability forced by impingement of mesoscale eddies from the east.

3. Hysteresis of the WBC

We first use the QG model to study the hysteresis of the WBC with a finite baroclinic deformation radius. This kind of investigations involves a large amount of computation, and the use of the QG model is helpful because of its simplicity and fast time stepping.

The hysteresis maps of the WBC in the vicinity of the gap have been calculated for deformation radii of 60, 50, 40, and 25 km as shown in Fig. 1. For each baroclinic deformation radius, the hysteresis loop of the WBC evolution is computed in the following way: For a low WBC transport $Q$ at the open boundaries, the QG model is integrated into a steady state and the westernmost coordinate (called $X_p$ in this paper following the Sheremet’s study) of the $\psi = Q/2$ streamline in the gap is determined, which corresponds to the point in the bottom-left corner of the hysteresis map in Fig. 1. The WBC transport $Q$ at the open boundaries is then increased and the QG model is integrated further to yet another steady state so that another point is determined in Fig. 1. The change of the $Q$ corresponds to the Reynolds number defined as $Re = Q/A_H$ to increase from 20 to 48 and then decrease back to 20 (Fig. 1). The history of the WBC path evolution is thus remembered by this kind of integration.

In the steady-state regimes, our QG model integration shows similar nonlinear hysteresis of the WBC in the vicinity of the gap as investigated by Sheremet (2001). At low Reynolds numbers, the WBC intrudes into the western basin through the gap in an anticyclonic loop so that $X_p$ is negative and distant into the western basin from the gap. As the Reynolds number increases, the WBC intrusion into the western basin gradually decreases until, at a critical Reynolds number $R_c$, the WBC becomes periodic (expressed by the double lines). This is the lower Hopf bifurcation experienced by the WBC during its transition from the penetrating to the leaping regimes as indicated by Sheremet (2001). Numerical experiments show that, for $L_R = 60, 50, 40$, and 25 km, $R_c = 26, 28, 31$, and 36, respectively, suggesting that the Hopf bifurcation occurs at larger Reynolds numbers for smaller deformation radii. For deformation radii less than 25 km, $R_c$ is even larger, which is in agreement with the Sheremet (2001) calculation with the approximation $L_R \to 0$.

The WBC sheds eddies periodically to the western basin for Reynolds numbers larger than $R_c$ and smaller than $R_L = 43$. For Reynolds numbers larger than $R_L$, the WBC becomes steady with the main stream of the WBC leaping across the gap as if the gap is nonexistent. For the four different values of baroclinic deformation radii, our computations have shown nearly the same $R_L$ value, suggesting that $R_L$ is not sensitive to the magnitude of the deformation radius of the WBC (Fig. 1).

Along the upper branch of the hysteresis loop as the Reynolds number decreases from larger than $R_L$, the WBC keeps its leaping path until at a critical Reynolds number $R_P = 37$, where the WBC path shifts back into the periodic eddy shedding state quickly. Notice that $R_c < R_P$, which suggests that periodic and steady states of the WBC path coexist in the vicinity of the gap for the same Reynolds numbers between $R_P$ and $R_L$, the final solutions of which depend on the history of the WBC evolution. If the initial path of the WBC is in the leaping regime, the WBC will stay in the leaping regime. On the other hand, if the WBC is intruding into the western basin initially, the final WBC path will be in the periodic penetrating regime. This is the nonlinear hysteresis of the gap flow investigated by Sheremet (2001). Figure 1 indicates that $R_P$ is almost independent of the deformation radius of the WBC (Fig. 1).

In contrast to the finding of Sheremet (2001), which suggests direct transition of the WBC from the leaping to the penetrating regimes at $R_P$, our numerical experiments using the full nonlinear reduced-gravity QG model suggest that no such transition is possible for a WBC with a finite baroclinic deformation radius ($L_R \geq 25$ km in the
above experiments). Instead, the transition is from the leaping to the periodic penetrating regimes at $R_p$. This is because, for finite-amplitude deformation radii, the transient evolution of the relative vorticity becomes important at low Reynolds numbers, which induces eddy shedding at the gap (Fig. 1). For very small baroclinic deformation radii, $R_c$ is larger than $R_p$, which suggests no Hopf bifurcation during the leap-to-penetrataion transition.

The dynamics of the WBC path are determined by the vorticity balance among the inertial term, the horizontal viscosity term, and the beta term in the steady-state regimes. According to the conceptual model of Sheremet (2001), if the WBC is strong so that the meridional advection dominates and is balanced by the beta term, the WBC takes the leaping path. On the other hand, if the WBC is weak so that the beta term dominates and is balanced by the zonal advection in the inertia jets to and from the tips of the barrier, the WBC intrudes into the western basin in an anticyclonic path. In this paper, we use the vorticity balance at the point $X_p$, which is the westernmost position of the $Q/2$ streamline that experiences hysteresisitic evolution, to understand the regime dynamics. These regimes of vorticity balances are shown in the next section in the discussions of the eddy–WBC interactions. In addition, the vorticity analyses in the next section disclose the vorticity balances during the transitional stage and during the periodic eddy shedding stage of the WBC, which are complicated and important for the understanding of the hysteresistic dynamics of the WBC at the gap.

4. Effects of mesoscale eddies on WBC path

In this section, the impact of mesoscale eddies on the nonlinear evolution of the WBC path in the vicinity of the gap is investigated. To isolate the eddy effects, we insert an eddy 300 km east of the gap in the initial condition and let the eddy propagate to the gap naturally.

The hysteresis map in Fig. 1 suggests two critical states of the WBC at the Reynolds numbers of $R_L$ and $R_p$, where the WBC is likely to shift regimes at small perturbations. At $R_L = 42$, the WBC is at a critical state of transition from the periodic penetrating to the leaping regimes, whereas, at $R_p = 37$, it is critical to shifting from leap to penetration. Our numerical experiments suggest that mesoscale eddies can induce such transitions. For simplicity, all the numerical experiments in this section are conducted with a baroclinic deformation radius of $L_R = 50$ km.

a. From penetration to leap

The effects of an anticyclonic eddy on a WBC critical from the periodic penetrating to the leaping transition are illustrated by the following experiment: At $Re = 42$, where a WBC is in a periodic state at the gap (Fig. 2), an anticyclonic eddy of Gaussian streamfunction distribution, with the maximum streamfunction value of $\psi_0 = 5000$ m$^2$ s$^{-1}$ at the center and the maximum azimuthal velocity at about 80 km from the center, is specified 300 km due east of the center of the gap in the initial condition. As the eddy approaches the gap on the beta plane, the WBC is seen to stop shedding eddies into the western basin and eventually take the leaping path across the gap. After the anticyclonic eddy is advected northward away from the gap and eventually out of the model domain through the northern boundary, the WBC stays in the leaping regime permanently. Evidently, the anticyclonic eddy has induced an irreversible transition of the WBC from the periodic penetrating regime to the leaping regime.

The dynamics of the transition, which is governed by the QG vorticity equation, are analyzed in Fig. 3. Here, the QG vorticity balances are extracted at a grid point nearest to the point $X_p$, which is the westernmost position of the $Q/2$ streamline. The grid point moves with the point $X_p$ during the hysteresis evolution. Because of the definition of $X_p$, the zonal advection term of the extracted vorticity balance is very small, with the inertia term dominated by the meridional advection. All terms are moved to the left side of the equation. At the beginning of the numerical experiment, the WBC sheds eddies periodically and the vorticity balance in the vicinity of the first $X_p$ point west of the gap is between the beta term and all the other terms of the QG vorticity equation (Fig. 3, top). As the anticyclonic eddy approaches the gap, the magnitude of the meridional advection increases dramatically to balance an increased beta term (Fig. 3, bottom). The evolution of the QG vorticity balance in the vicinity of the point $X_p$ shows that the magnitude of the meridional advection has increased by more than an order of magnitude, whereas the beta term and the friction term both increase to balance the meridional advection during the eddy–WBC interactions between days 2000 and 3000. The QG model obviously has two regimes of balance—one is periodic and one is steady—for the same forcing of the open boundary conditions. The WBC finally takes the leaping path across the gap after the eddy passage. The transitions are thus induced by the anticyclonic eddy through enhanced northward advection in the gap, which pushes the WBC vorticity balance at the gap from the periodic regime into the leaping regime. The hysteresis map in Fig. 1 suggests that the WBC stay in the leaping regime after the eddy moves away from the gap. Thus, an irreversible transition of the WBC path is induced after the passage of the anticyclonic eddy.
Not all anticyclonic eddies induce the transition of the WBC path from periodic penetration to leap. For a fixed transport of the WBC at $Re < R_L$, the transition of the WBC path requires that the anticyclonic eddies to overcome the threshold of $\delta = R_L - Re$ temporally in the vicinity of the gap for the nonlinear hysteresis to take effect. To determine the sensitivity of the WBC regime shift to the strength and size of the anticyclonic eddies, numerical runs have been conducted with anticyclonic eddies released at a point due east of the middle point of the gap in the initial condition (experiment B). The WBC initially sheds eddies periodically at the gap at $Re = 42$ in all of the runs. The minimum strength and size of the eddies that are capable of inducing the regime transition are marked in Fig. 4. The results suggest that the WBC is sensitive to both the strength and size of small eddies, whereas it is only sensitive to the strength of large eddies.

An explanation of the relation between the critical strength and size of the eddies that are able of inducing the regime transition in Fig. 4 is given in the following: The total circular momentum of the eddy is proportional to

\[
\int_0^{2\pi} \int_0^\infty \psi \frac{\partial \psi}{\partial r} r \, d\theta \, dr = -\frac{\pi^2}{2} \psi_0^2 R.
\]

Therefore, the depth-averaged circular momentum is proportional to $\psi_0 R$, which results in the relation in Fig. 4.

The WBC regime transition is also sensitive to the approaching latitude of the anticyclonic eddies. This is evident by the results of experiments A, C, and D in Fig. 4, in which the eddies are released 50 km north, 50 km south, and 120 km south, respectively, of the initial position of the eddy release in experiment B. The releasing position in experiment D corresponds to the latitude of the southern edge of the gap. Figure 4 suggests that the regime shift is more sensitive to anticyclonic eddies approaching the gap from the upstream of the WBC.

Not only anticyclonic eddies but also cyclonic eddies can induce WBC path transition from penetration to leap. This kind of behavior is associated with the right fling of the eddy that enhances the meridional advection of the WBC as the eddy migrates westward through the gap. Figure 5 shows the experiment of a cyclonic eddy acting on a periodic eddy-shedding WBC at $Re = 42$, the
critical Reynolds number of transition from penetration to leap. As the cyclonic eddy moves into the center of the gap, it is squeezed and distorted between the WBC and its shed eddy so that the feeding to the shed eddy by the WBC is terminated (Fig. 5, top right). After the eddy passage, the WBC is unable to return to the periodic penetrating path. In contrast to the experiment in Fig. 2, it is the right fling, instead of the left fling, of the eddy that induces the WBC regime shift in this case, because the initial penetrating path of the WBC facilitates the westward propagation of the eddy through the gap. The right fling of the eddy, therefore, is able to act on the WBC in the gap to induce the regime shift.

The vorticity balance of the above experiment in the vicinity of the point $X_p$ shows that the effects of the cyclonic eddy are to promote the meridional advection as it is squeezed and distorted between the WBC and its shed eddy (Fig. 6). The transition of the regimes occurs slightly later than in Fig. 3 because the right wing of the eddy takes longer time to reach the gap than the left wing. As the cyclonic eddy moves away from the gap, the meridional advection dominates and is balanced by the beta term and the friction term, which are both enhanced, suggesting the leaping regime. Because of the nonlinear hysteresis of the WBC path, the circulation in the gap is thus shifted permanently from the periodic penetrating regime to the leaping regime.

The critical strength and size of the cyclonic eddies that are capable of inducing WBC path transition from penetration to leap are shown in Fig. 7, which suggests that the transition is more sensitive to cyclonic perturbations in the southern part of the gap. The experiments in Fig. 7 are configured similarly as those in Fig. 4. It is, however, interesting to notice that the WBC path is less sensitive to cyclonic perturbations shooting at the up-stream of the gap, as indicated by the sensitivity of curve D in Fig. 7. For these cyclonic eddies, the effect of the interactions is to reduce the WBC transport temporally by the left wing of the eddy, which is to enhance the intrusion into the western basin. Meanwhile, the cyclonic eddy is distorted by the WBC enough to lose its circular structure. In short, the cyclonic eddy needs to be inside the gap for the right fling of the eddy to be effective on a penetrating WBC.

b. From leap to penetration

The cyclonic eddies can also force the WBC to penetrate into the western basin through the gap if the WBC is at the critical state of transition from leaping to penetrating regimes. This is done through reducing the meridional advection in the vicinity of the gap. Figure 8 shows a sequence of the streamfunction field in an experiment with a cyclonic eddy impinging on a WBC leaping critically across the gap at $Re = 37.5$. The cyclonic eddy, with an initial streamfunction of $-16$ 000 m$^2$ s$^{-1}$ at the center and the maximum azimuthal velocity at about 80-km radius from the center, approaches the gap from 300 km due east of the southern edge of the gap. Under the influence of the eddy, the WBC path is changed permanently from the leaping state into the periodic eddy shedding state,
evidently because the cyclonic eddy reduces the WBC speed in the vicinity of the gap (Fig. 9). The reduced speed causes the meridional advection term in the vorticity equation to decrease, which results in the growth of anticyclonic vorticity in the western basin. Because of the nonlinear hysteresis, the WBC stays in the periodic penetrating regime in the lower branch of the hysteresis loop after the passage of the cyclonic eddy. Thus, an irreversible change of the WBC path from leap to penetration is induced by the passage of the cyclonic eddy.

Numerical experiments have also indicated that only cyclonic eddies approaching the gap from the south can induce the WBC regime transition. Eddies approaching the gap from the east and north are unable to induce the WBC transition from leap to penetration. These are in contrast to the experiments in Fig. 7. Figure 10 shows the critical strength and size of the cyclonic eddies that are capable of inducing the transition of the WBC from leap to penetration at \( Re = 37.5 \). Experiments D and E are different only in the latitude of the initial eddy releasing positions. The cyclonic eddies in experiment D are released 300 km due east of the southern edge of the gap, whereas the eddies in experiment E are released 180 km south of the initial position of experiment D. The entire eddy in experiment E is south of the gap. Both experiments show that, for eddies with a radius larger than 100 km or so, the WBC is sensitive only to the strength of the eddy. However, when the eddy radius is less than 70 km or so, no eddies in experiment D can actually induce the WBC transition from leap to penetration.

Why is the WBC transition from leap to penetration only sensitive to the cyclonic perturbations from the south? Examination of the current speed at a point 50 km east of the southern edge of the gap shows that the northward current speed is reduced if the cyclonic eddy is released south of the gap. However, for eddies released initially east of the middle point of the gap, the current speed east of the southern edge of the gap has actually increased (Fig. 11).
this section east of the southern edge of the gap is the entrance of the WBC into the gap circulation, the variations of the currents here explain the failure of the cyclonic eddies from the east to induce the WBC regime shift. Analysis of the anomalous flow fields induced by the eddy indicates that, because of the absence of the mirror effects of the eddies from the east, the cyclonic eddy induces a companion anticyclonic eddy in the south (Fig. 12, top), which plays the role of enhancing the WBC speed east of the southern edge of the gap. In contrast, the cyclonic eddy is unable to induce the companion anticyclonic eddy if approaching the gap from the south because of the mirror effects of the southern barrier (Fig. 12, middle and bottom).

5. Discussion

We have studied the hysteresis of WBCs with finite baroclinic deformation radii flowing by a wide gap of the western boundary and investigated the effects of mesoscale eddies on the regime transitions of the WBCs near critical states. The results can be applied to the Kuroshio study in the vicinity of the Luzon Strait. Analysis of satellite altimeter, ocean color, and sea surface temperature data have shown that the Kuroshio path in the Luzon Strait is subject to intraseasonal transitions between the leaping and the penetrating regimes (Yuan et al. 2006). Historically, these transitions have been attributed to monsoonal wind forcing (Farris and Wimbush 1996; Wang et al. 2010) and instability of the WBC (Metzger and Hurlburt 2001; Jia and Liu 2004). None of these existing theories, however, is able to explain the observed fact that the Kuroshio leaps across the Luzon Strait during about twice as much the time as it penetrates into the South China Sea in an anticyclonic loop.

However, if the Kuroshio path transitions are understood as the nonlinear hysteresis of the WBC under the
perturbations of mesoscale eddies, the statistics of the Kuroshio path shifts can be explained. The mesoscale eddies in the Philippine Seas are generated by baroclinic instabilities of oceanic currents, which can be approximated to the leading order as randomly distributed in both cyclones and anticyclones. These eddies are frequently observed to approach the Luzon Strait from the Philippine Sea side and perturb the Kuroshio path (Yuan et al. 2006). Because both cyclonic and anticyclonic eddies can induce transitions of the Kuroshio path from the penetrating to the leaping regimes, whereas only the cyclonic eddies can induce the transition from the leaping to the penetrating regimes, the chances for the Kuroshio to leap across the Luzon Strait should be approximately 3 times as much as to penetrate into the northern South China Sea in an anticyclonic loop. In an numerical experiment, where $g = 4.66$, $Re = 34$, and random cyclonic and anticyclonic eddies of $\Psi_0 = 10,000 m^2 s^{-1}$ and of the 80-km radius are specified to approach the gap from the east, the $X_p$ indeed stays $\frac{1}{4}$ of the time in the leaping regime and $\frac{3}{4}$ of the time in the periodic penetrating regime (not shown here). In addition to the eddy perturbations, the seasonal reversing monsoonal winds can also induce the regime transitions of the WBC (Wang et al. 2010). Because the annual average of the monsoon is northerly, which tends to push the WBC path into the penetrating regime through the surface Ekman transport, the chances of the penetration is higher than $\frac{1}{4}$. We are currently investigating the combined effects of monsoon, annual variations of the WBC, and the impingement of mesoscale eddies on the regime statistics of the WBC; the results of this will be summarized in a separate paper.

6. Conclusions

The dynamics of a WBC flowing by a wide gap of the western boundary is studied using a 1.5-layer reduced-gravity quasigeostrophic model. Numerical experiments of the QG model show that the steady-state WBC path in the gap is subject to nonlinear hysteresis, which is determined by the multiple vorticity balances existed in certain ranges of the nondimensional parameters. The multiple balances suggest coexistence of the steady state and periodic penetrating state of the WBC for the same (figures omitted). In this case, the westward propagation of the eddies is blocked by the leaping Kuroshio so that very little mass and energy from the eddies are leaked into the western basin. The westward propagation of the meso-eddies through the gap into the western basin under the influence of different states of the WBC is the subject of an ongoing research; the results of this research will be summarized in a separate paper.

![Figure 9](image1.png)

**Fig. 9.** As in Fig. 6, but for Re = 37.5.

![Figure 10](image2.png)

**Fig. 10.** As in Fig. 7, but for Re = 37.5. Curve E represents eddies released 180 km south of the initial position of the curve D eddy.

![Figure 11](image3.png)

**Fig. 11.** Current speed at 50 km east of the southern edge of the gap in the WBC under the influence of a cyclonic eddy. The eddy is released initially at 300 km due east of the center of the gap in experiment B. In experiment D, the eddy is released 300 km due east of the southern edge of the gap. The eddy in experiment E is released 180 km farther south.
set of external forcing. For these nondimensional parameters, if the beta effect dominates and is balanced by the other terms, the WBC takes the anticyclonic periodic penetrating path. On the other hand, if the meridional advection dominates and is balanced by the beta effect and the friction term, the WBC takes the leaping path. The final states of the WBC depend on the history of the WBC evolution in the gap, which is the hysteresis.

In between the leaping and the penetrating regimes of the WBC path, there usually exist periodic eddy shedding states of the WBC, which are the Hopf bifurcations of the nonlinear system. During the periodic solution, the effect of the transient variations of the relative vorticity, which is important in the QG vorticity balance, is very much influenced by the magnitude of the baroclinic deformation radius. Results of the numerical experiments have shown that the Hopf bifurcation from penetration to leap occurs at lower Reynolds numbers for larger baroclinic deformation radii. However, the domain of the WBC hysteresis (i.e., the range of the Reynolds number for the coexistence of the periodic and steady states) is not sensitive to the magnitude of the baroclinic deformation radius. Thus, the range of the Reynolds number for eddy-shedding states of the WBC gets larger for larger deformation radii. In contrast to the finding of Shemeret (2001), which suggests direct transition of the WBC from the leaping to the penetrating regimes in the limit of a zero baroclinic deformation radius, our study indicates that no such transition is possible for a WBC with a finite deformation radius. Instead, the transition is from the leaping to the periodic eddy shedding states.

Mesoscale eddies approaching the gap from the east are found to modify the WBC path in the vicinity of the gap significantly. For a WBC near the critical states of hysteresis, both cyclonic and anticyclonic eddies can induce transitions of the WBC path from the periodic penetrating to the leaping regimes, whereas only the cyclonic eddies are able to induce transitions from the leaping to the periodic penetrating regimes. The dynamics are that the eddies modify the vorticity balances in the vicinity of the gap so that a new steady or periodic state of the WBC is established irreversibly because of the hysteresis after the passage of the eddies. The cyclonic (anticyclonic) eddies play the role of reducing (enhancing) the WBC speed in the gap so that the WBC path shifts from leap (periodic penetration) to periodic penetration (leap). In addition, cyclonic eddies can induce transitions from the penetrating to the leaping regimes by enhancing the meridional advection through its right fling. The results suggest that intraseasonal variations of the WBC can be induced by impingement of mesoscale eddies coming from the eastern basin, which explains the intraseasonal variability of the Kuroshio in

FIG. 12. Anomaly circulation induced by the cyclonic eddies released at different initial positions east of the gap at day 832 in experiments (top to bottom) B, D, and E. The contour interval is 1000 m$^2$ s$^{-1}$. 
the vicinity of the Luzon Strait. The theory of eddy-induced variability is also consistent with the observed fact that the Kuroshio stays more likely in the leaping regime than in the penetrating regime, assuming random cyclonic and anticyclonic eddy impingement.

The numerical experiments have suggested that the WBC path transition is generally more sensitive to eddy perturbations in the upstream of the WBC. The smaller the eddy, the stronger the eddy strength that is needed to induce the path transition. Cyclonic eddies approaching the gap from the east are not able to induce transitions of the WBC from the leaping to the penetrating regimes because of a companion anticyclonic eddy generated to the south due to the absence of the mirror effects. The anticyclonic eddy enhances the WBC speed at the southern edge of the gap, which prevents the transition from leap to penetration. When a WBC is far away from the critical states of the hysteresis, the regime transitions are difficult to induce.

Acknowledgments. This work is support by the NSFC National Outstanding Youth Grant 40888001, by the Shandong Provincial Outstanding Youth Grant, and by the “Hundreds-Talent Program” of CAS.

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