Estimates of Ocean Macroturbulence: Structure Function and Spectral Slope from Argo Profiling Floats

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ABSTRACT

The Argo profiling float network has repeatedly sampled much of the World Ocean. This study uses Argo temperature and salinity data to form the tracer structure function of ocean variability at the macroscale (10–1000 km, mesoscale and above). Here, second-order temperature and salinity structure functions over horizontal separations are calculated along either pressure or potential density surfaces, which allows analysis of both active and passive tracer structure functions. Using Argo data, a map of global variance is created from the climatological average and each datum. When turbulence is homogeneous, the structure function slope from Argo can be related to the wavenumber spectrum slope in ocean temperature or salinity variability. This first application of structure function techniques to Argo data gives physically meaningful results based on bootstrapped confidence intervals, showing geographical dependence of the structure functions with slopes near $2/3$ on average, independent of depth.

1. Introduction

Understanding the nature of the turbulent processes in the atmosphere and ocean is crucial to determining large-scale circulation, and therefore climate prediction, but the relationship between large-scale circulation and small-scale turbulence is poorly understood. Atmospheric turbulence has been studied through spectral and structure function analyses for decades (Nastrom and Gage 1985; Lindborg 1999; Frehlich and Sharman 2010), and the results have been duplicated by high-resolution general circulation models (GCMs) and mesoscale numerical weather prediction (NWP) models as well (Koshyk and Hamilton 2001; Skamarock 2004; Frehlich and Sharman 2004; Takahashi et al. 2006; Hamilton et al. 2008). As realistic ocean climate models become increasingly turbulent, a similar dataset to the Nastrom and Gage (1985) spectrum would be a useful evaluation tool.

It is often assumed that constraining a horizontal power spectral density curve, or spectrum, requires a nearly continuous synoptic survey, such as by satellite (Scott and Wang 2005), tow-yo (Rudnick and Ferrari 1999), ship (Callies and Ferrari 2013), or glider (Cole and Rudnick 2012, hereinafter CR12). Near-surface spectra from tow-yo and satellite have been studied by the authors and collaborators, among many others (Fox-Kemper et al. 2011), but a similar comprehensive analysis has not been done deeper than 1000 m because of the limited availability of continuous observations.

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However, the recent atmospheric rawinsonde method of Frehlich and Sharman (2010) demonstrates that a collection of individual observations may be used to form the structure function, which is closely related to the power spectrum in stationary, isotropic, homogeneous turbulence. Bennett (1984) also used balloon soundings to determine the local versus nonlocal dynamics in the atmosphere. Furthermore, structure function analysis is quite common in the engineering literature on turbulence [She and Leveque (1994) is a well-known example]. With the increased density of Argo profiling floats sampling down to 2000 m over the past two decades, as well as the success of the rawinsonde method in the atmosphere, this method is attempted to quantify large-scale (>10 km) turbulence in the oceans. Roulet et al. (2014) recently used Argo to compute maps of eddy available potential energy to a similar end, but with a different method that does not specify the interactions of scales and turbulence cascades. The structure function statistic is a useful constraint on high-resolution models, as structure functions are easy to calculate in a model from even a single output snapshot. In this study, temperature and salinity data from Argo are used to characterize large-scale turbulence at depth by constructing structure functions and, when relevant, inferring the related temperature and salinity variance spectra.

2. Framework

Ocean surface observations suggest that the spectral behavior for scales larger than about 1 km differs from smaller-scale turbulence (e.g., Hosegood et al. 2006). Here, we call variability at scales between 10 and $10^4$ km "macroturbulence" to emphasize that, aside from being large scale (mesoscale and larger), little is known about which turbulent regime is being observed (see also Forget and Wunsch 2007). While dynamical frameworks for mesoscale, quasigeostrophic (QG) turbulence spanning this range of scales are heavily studied at subinertial frequencies, they may not fully describe the composite nature of observed variability seen in real ocean data. Macroturbulence as defined above includes mesoscale eddy activity, internal waves, and other signals such as responses to atmospheric forcing. A complementary approach is to distinguish among observed macroturbulence according to its spatial scale. To this end, structure functions provide an adequate tool that is here applied to in situ profiles of salinity collected by the global array of Argo floats.

a. Structure function–spectrum relationship

The tracer autocorrelation function $R_\theta$ is a statistical measure of the similarity (or difference) between a given location $x$ and another location separated from $x$ by the distance vector $s$ and is generally defined as

$$R_\theta(s, x) = \bar{\theta'(x)\theta'(x+s)}.$$  

where $\theta'$ is a generic tracer, usually conserved (e.g., potential temperature or salinity); the prime symbol denotes its deviation from an appropriate mean; and the overbar denotes averaging. The $n$th-order tracer structure function $D_{\theta,n}$ is accordingly defined as

$$D_{\theta,n}(s, x) = [\bar{\theta(x) - \theta(x+s)}]^n,$$  

and its $n = 2$ form is simply related to the autocorrelation function by

$$D_{\theta,2}(s, x) = 2[\bar{\theta''} - R_\theta(s, x)]$$  

for homogeneous turbulence. In the case of isotropic turbulence, $R_\theta$ and $D_{\theta,1}$ both are independent of direction [e.g., $D_{\theta,1}(s, x) = D_{\theta,1}(s, x)$], and for homogeneous and isotropic turbulence, they are further independent of $x$ [e.g., $D_{\theta,1}(s, x) = D_{\theta,1}(s)$]. Estimating $D_\theta(s) = D_{\theta,2}(s)$ and exploiting the relationship in Eq. (3) is of primary interest. Higher-order structure functions can be revealing of subtle aspects of intermittency, and the dissipation of energy and variance (Kraichnan 1994) and structure-function-like statistics formed from the combination of velocity and tracer correlations are potentially challenging tests for statistical theories of turbulence (Yaglom 1949). Unfortunately, the accuracy and data required for estimation of these statistics is beyond that of the second-order structure function, which as we will see is rather noisy in the ocean. Also, the assumptions of homogeneity and isotropy will not be commonly satisfied in the ocean, but the presentation of theory will begin following these assumptions. Later they will be relaxed as far as the data quantity and quality allow.

If a given homogeneous, isotropic turbulence spectrum (of energy or tracer variance) has power-law behavior over a range of wavenumbers between the energy injection and dissipation scales, then a related scaling law for the structure function is expected (Webb 1964). Suppose the spectrum’s power law is given by $B(k) = \alpha_B k^\lambda$, with spectral slope $\lambda$. The structure function will also have a polynomial form: $D_\theta(s) = C_\theta s^\gamma + C_0$, with structure function slope $\gamma$ and a constant $C_0$ representing contributions from other portions of the spectrum not adhering to the $B(k) = \alpha_B k^\lambda$ law [shown to be negligible in Webb (1964)]. The relationship between the two slopes (derived in appendix A) is

$$\gamma = -\lambda - 1.$$  

(4)
However, structure functions calculated here often have a bend point with two slopes, so an analysis is needed to determine whether that was a sign of two separate power laws in the spectrum [a common example occurs in Nastrom and Gage (1985), where a spectral slope of $\lambda = -5/3$ is seen below 500 km and $\lambda = -3$ is seen above 500 km]. As shown in appendix A, if spectral slopes of the two power-law scalings are $\lambda_1$ and $\lambda_2$, then the structure function can be written as

$$D_\theta(s) = c_1 s^{\gamma_1} + c_2 s^{\gamma_2},$$

where the same relationship between structure function slope $\gamma$ and spectral slope $\lambda$, that is, Eq. (4), applies between the large-scale structure function slope versus small wavenumber spectral slope, and for the small-scale structure function slope versus large wavenumber spectral slope. As long as the inertial range over which each power law applies is large enough and $\gamma_1 < \gamma_2$, then the first term dominates the small scale and the second term dominates the large scale.

Observational estimates of structure functions often, and expectedly, show flat slopes at extreme separation distances. For large enough $s$, Eq. (3) indeed predicts that $D_\theta(s \to \infty) \to 2\theta^2$ [or $\theta^2_{(x_1)} + \theta^2_{(x_2)}$ in heterogeneous cases] in the limit where remote locations are fully uncorrelated [so that $R_\theta(s \to \infty) \to 0$]. For small enough $s$, a similar behavior can sometimes be seen, as the assumption of simultaneous observations can break down, the signal of interest goes to 0 and data noise becomes predominant. The scale of transition at which these limiting cases begin to dominate is difficult to predict and generally unknown. It may be that the scale of transition is meaningful (e.g., indicating the scale of the largest coherent structures), but conclusive evidence of a meaningful transition generally requires more information that just the structure function alone. However, the matter of interest here is $D_\theta(s)$ within the inertial range(s), away from these limiting cases.

A primary goal of this paper will be to estimate $\gamma$ from data over length scales where a single power law is suspected, or both $\gamma_1$ and $\gamma_2$ when a single linear fit is not apparent, and compare it to relevant theories, reviewed below, predicting $\gamma$ or a spectral equivalent.

### b. Relevant theories

**Kolmogorov (1941)** introduced the idea of an inertial range in isotropic, homogeneous turbulence through dimensional analysis, arriving at a kinetic energy spectrum of $E(k) \propto k^{-5/3}$. Using Kolmogorov-like dimensional arguments, Obukhov (1949) and Corrsin (1951) predict a temperature spectrum with slope $\lambda = -5/3$. Because of rotation, stratification, and limited total depth, large-scale [>$O(10)$ km] ocean flows are quasi-two-dimensional (dominantly horizontal) and are not expected to follow the simple scalings first derived by Kolmogorov, Obukhov, and Corrsin. Two-dimensional turbulence scalings by Kraichnan (1967) of the kinetic energy slope of $E(k) \propto k^{-3}$ in the enstrophy cascade range (plus a logarithmic correction neglected here) at small scales and $E(k) \propto k^{-5/3}$ in the inverse energy cascade at large scales could potentially describe barotropic motions. Batchelor (1959) and Vallis (2006) argue that in turbulence where each wavenumber is dominated by a single eddy-turnover time scale, a passive tracer spectrum should exhibit a slope of $\lambda = -1$ ($\gamma = 0$). The Obukhov and Batchelor passive tracer spectra are examined in a relevant limit by Pierrehumbert (1994), Charney (1971), Salmon (1982), and Blumen (1978) all describe kinetic energy spectra for the quasigeostrophic flows. For all cases, passive tracers should behave as Obukhov and Corrsin predict when $E(k) \propto k^{-5/3}$, or as the single eddy-turnover time-scale result of $\lambda = -1$ ($\gamma = 0$) when $E(k) \propto k^{-3}$. However, the wavenumber range where these spectral slopes should appear in quasigeostrophic flow is unclear as the effects of “surface” quasigeostrophy (SQG) and “interior” QG differ strongly in spectral slope and depth (Tulloch and Smith 2006; Callies and Ferrari 2013), with the former exhibiting $E(k) \propto k^{-5/3}$ at the surface and rapidly becoming much steeper below. Furthermore, Klein et al. (1998) predict a spectral slope of $\lambda = -2$ in locations of active frontogenesis in both active and passive tracer cascades. The predicted behavior below the surface is undefined for this case, in contrast to the SQG case, where the spectral slope is expected to get shallower because of a faster decay in variance at small scales.

Testing these competing theoretical predictions against global observations, and selecting the most adequate on a regional basis, is an important goal, and the present study attempts a step in that direction. Theories that predict slopes of $\gamma = 0$ could prove most difficult to invalidate, since any spectral slope of $k^{-1}$, uncorrelated geophysical variability (e.g., variability on scales larger than the largest eddies), or uncorrelated instrumental or other noise will translate into flat slopes. More generally, given the rather small range of slopes predicted by theory (Table 1), it is clear that highly accurate and precise estimates of $D_\theta(s)$ or $R_\theta(s)$ will be needed to eventually reach definitive conclusions, and it is also clear that having velocity data in addition to tracers would strengthen the selectivity of the structure function in constraining theory (Bühler et al. 2014). Whether available observations allow for sufficient accuracy and precision remains unclear. Our preliminary assessment sheds light on this matter, while deferring a more thorough assessment of methodological and observational requirements to further investigation.
c. Data analysis techniques

The data used in this analysis were obtained from Argo floats distributed over the World Ocean from 2000 to 2013. The extensive Argo float array introduced the first systematic, near-real-time sampling of temperature and salinity of the global ocean on a large spectrum of scales with accuracy of approximately 0.01°C and 0.01 psu, respectively (Argo Science Team 1998). The International Argo Program currently collects and provides profiles from an array of 3600 floats. Each Argo float takes a vertical profile of temperature and salinity as it ascends from 2000 m to the surface, where it transmits the data via satellite (using Argos or Iridium systems) before descending and drifting for, typically, 9 days. Calibration and quality control is done on all profiles at one of the national data centers, and though an incorrect or missing calibration could skew the statistics computed here, they are assumed to be correct (Carval et al. 2011).

In processing the data, we relied on the Argo delayed-mode procedures for checking sensor drifts and offsets in salinity and made use of the Argo quality flags. Density was computed for each Argo temperature/salinity profile, which was then interpolated to standard density levels, \(\sim 24.0 - 27.8 \text{ kg m}^{-3}\), in intervals of 0.1 kg m\(^{-3}\), and standard depth levels, 5 m at the surface, with increasing intervals down to 2000 m.

Salinity is here analyzed along either isobars or isopycnals, taken as a representative of active and passive tracers, respectively. A reasonable alternative would be to analyze, for example, temperature on isobars (active) and “spice” on isopycnals (passive), but we choose to follow the simplest approach for a first assessment of Argo data. In interpolating salinity to standard density level, potential density is computed using the Thermodynamic Equation of Seawater (Millero et al. 2008). The presented

<table>
<thead>
<tr>
<th>Reference</th>
<th>Theory</th>
<th>(\lambda)</th>
<th>(\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obukhov (1949); Corrsin (1951)</td>
<td>Passive or active tracer cascade in energy cascade</td>
<td>(-5/3)</td>
<td>(2/3)</td>
</tr>
<tr>
<td>Batchelor (1959); Vallis (2006)</td>
<td>Passive tracer cascade in enstrophy cascade or other single dominant time scale</td>
<td>(-1)</td>
<td>(0)</td>
</tr>
<tr>
<td>Klein et al. (1998)</td>
<td>Surface frontogenesis active or passive tracer cascade</td>
<td>(-2)</td>
<td>(1)</td>
</tr>
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</table>

![Fig. 1. Potential density (kg m\(^{-3}\)) along (a) 23.5°W in the eastern Atlantic Ocean and (b) 180° in the Pacific, calculated from the OCCA climatology of temperature, salinity, and pressure with the Thermodynamic Equation of Seawater. The 25.7 and 27.3 kg m\(^{-3}\) isopycnals are highlighted for analysis in section 3d.](image-url)
results, however, are largely insensitive to a change in assumed equation of state compared to other approximations (e.g., neutral density, not shown). Figure 1 shows potential density from the Ocean Comprehensive Atlas (OCCA) climatology (Forget 2010) varying with depth in the western Atlantic (Fig. 1a) and central Pacific (Fig. 1b) Oceans and highlights the \( \sigma_0 = 25.7 \text{ kg m}^{-3} \) and \( \sigma_0 = 27.3 \text{ kg m}^{-3} \) isopycnals analyzed in this study.

The global data coverage by Argo is much denser and more homogeneous than that of ship-based measurements. This fact, and the continued growth of the profiles database, motivate our focus on Argo data. As compared with, for example, along-track altimetry, the distribution of Argo profiles is highly irregular, as a result of the complex drifting patterns of a multitude of individual floats. In this context, the use of structure functions is a rather obvious methodological choice. Fast Fourier transforms, for example, require data following a straight, regular, gap-free path (or statistical interpolation techniques to impute an equivalent).

Isotropic structure functions for salinity are computed according to

\[
D_S(s) = \left[ S'(x) - S'(x + s) \right]^2,
\]

where \( S'(x) - S'(x + s) \) denotes the difference in salinity anomalies for an Argo data pair separated by distance \( s \) and the double overbar denotes a weighted sample average. Computations are carried out in logarithmically spaced \( s \) bins, between 10 and 10,000 km. Other computational details (regarding salinity anomalies, weighted sample averages, weighting for unequal directional bins, and computational domains) are reported below. Isobaric structure function estimates are denoted as \( D_S(s)|_p \), while isopycnic structure function estimates are denoted as \( D_S(s)|_s \).

There are no pairs of Argo floats measuring at the exact same time, but the lack of strict simultaneity is not crucial. Indeed, observations that occur close enough together in time (\( \Delta t \)) and over sufficient spatial separation (\( s \)) form an effectively simultaneous pair, to the extent that oceanic signals cannot travel fast enough between paired observations. Thus, following Frehlich and Sharman (2010), data pairs such that \( s > c_{\max} \Delta t \) are considered “effectively simultaneous” and are included in the average. An example of the probability distribution of \( \Delta t \) and \( s \), for observations within a given Pacific

![Fig. 2. Log of the joint probability distribution of pairs (color) depending on separation distance (x axis) and separation time (y axis) for all observations in the heterogeneous region in the Pacific Ocean between 10° and 30°N and 140° and 160°W. Dotted lines show three different \( c_{\max} \) limits with increasing line thickness: 0.01, 1, and 10 m s\(^{-1}\).](image)

![Fig. 3. (a) Salinity structure function \( D_S(s)|_p \) along 158°W for 10°-40°N along isopycnals of 25.2–25.8, 25.8–26.4, 26.4–26.6, 26.6–26.8, 26.8–27.0, 27.0–27.2, and 27.2–27.3 kg m\(^{-3}\). The dashed line is the structure function model equivalent to the spectrum found by CR12. (b) Structure functions at 25.2–25.8 kg m\(^{-3}\) for the seasons specified in CR12. Dashed lines are the structure function model equivalent to a fit of CR12’s spectrum for April–May and November–December.](image)
The trade-off involved in choosing $c_{\text{max}}$ is as follows: a large $c_{\text{max}}$ (e.g., 10 m s$^{-1}$) reduces the number of qualifying pairs, inducing noise in structure functions, especially at short separations. A small $c_{\text{max}}$ (e.g., 0.01 m s$^{-1}$) leads to smoother results, but nonsynchronous pairs tend to distort structure functions, affecting slope in particular (i.e., flattening the structure function since the pairs are uncorrelated). The value of $c_{\text{max}} = 1$ m s$^{-1}$ is chosen as the approximate threshold where structure function slopes start to be majorly affected. This speed is also fast when compared to typical advective speeds, which would be primary dynamical adjustments to affect salinity anomalies at depth. It is not fast enough, however, to remove barotropic waves and some low-mode baroclinic gravity waves. Such waves would have a quite different effect when diagnosed by salinity anomalies on isopycnal and isobaric surfaces, so this additional step is examined here. The resulting structure functions are smooth enough to allow for physical interpretation (Fig. 3). Though it is possible that the slopes may not have reached their actual values before getting noisy, the agreement across locations and depths indicates that the behavior is realistic and not an artifact of noise, which would create unrelated slopes for each structure function. To solidify this empirical parameter choice, the value should be revisited in future studies and can certainly be increased (to reduce the time and distance lag between pairs) as more data, especially at small separation distances, become available.

In principle, an advantage of Eq. (2) over Eq. (1) is to alleviate the need to define a mean state explicitly. In practice, however, it is useful to subtract a time mean seasonal climatology before estimating Eq. (6), as the mean separation distances often span heterogeneous background salinities attributable to external forcing and the general circulation, not macroturbulence. Indeed, regional contrasts in the time mean hydrography,
as well as seasonal contrasts, can be as large as the macro-
roturbulence signal of interest and would contaminate
structure function estimates. Thus, the near-global mean
monthly OCCA climatology estimated for the 3-yr Argo
period from December 2003 through November 2006
(Forget 2010) is used to approximate the turbulence-free
mean for each location and each month and is subtracted
from Argo observations to obtain the salinity fluctua-
tions. The structure function average [the double bar in
Eq. (6)] is then computed for all simultaneous pairs, in-
dependent of season, though it is possible to analyze
those differences as in CR12 and Fig. 3.

The structure function is a statistic that is adequate, in
its own right, to describe ocean macroturbulence. The
slope’s relation Eq. (4) makes it interchangeable with
power spectra, but only under assumptions of homoge-
neity and isotropy. When these assumptions are violated
(and in practice they are never perfectly valid), the in-
terpretation of either statistic, and of their mutual re-
lation, becomes much more difficult. Hence, caution in
analyzing either statistic is recommended, and it is
crucial to assess and possibly mitigate departures from
homogeneity and isotropy. We note that it is possible,
for statistically stationary turbulence, to reduce or re-
move spatial and directional averaging in Eqs. (1)–(3),
retaining only a temporal or ensemble average, pro-
ducing structure functions suitable for heterogeneous
and anisotropic conditions. Nonetheless, the continued
growth of Argo is bound to allow for refined analyses in
the future. The heterogeneity seen in Argo data is dis-
cussed in section 3.

To mitigate the impact of anisotropy, structure func-
tions are first computed in directional bins, and a
weighted average of directional bins is then performed
(see appendix B). Since structure function slope estimates
are of particular interest, and to gain insight into their
statistical significance, they are presented with bootstrap
confidence intervals (see appendixes B and C). A detail of
importance is that slope calculations should omit large
separations, where \( D_{5} \) expectedly asymptotes to \( 2\theta^{2} \). To
this end, a bend point is determined in \( D_{5} \) and slopes are
computed below the bend point (appendix D).

Fig. 6. (a) Isobaric salinity structure functions \( D_{5}(s) \) between 10° and 30°N and 140° and 160°W in the Pacific
Ocean (a heterogeneous region with high and low salinity variance) on pressure surfaces (5–1900 m, represented by
color). (b) Small-scale slopes of the structure function at each latitude band, including 90% bootstrap confidence interval.
(c) Amplitude of the large-scale structure function at each latitude band, including 90% bootstrap confidence interval.
Reference slopes of \( \gamma = 0 \) (dashed) and \( \frac{2}{3} \) (solid) are shown as thick lines in (a) and as dashed-dotted lines in (b).
3. Structure function results

a. Preliminary assessment

It is of immediate importance to note that it is possible to use the Argo data to retrieve the structure function over macroscale separation distances. Because of a lack of simultaneous nearby observations at scales smaller than \(O(10)\) km, the structure function is noisy and slope is not discernible for submesoscales yet, but at scales larger than \(O(10)\) km, a clear slope can be seen (Fig. 3; computed within \(10^\circ\)–40\(^\circ\)N, 156\(^\circ\)–160\(^\circ\)W). This first example allows for direct comparison with the salinity spectra calculated in the same region by CR12 on seven isopycnal bands along 158\(^\circ\)W from 22.75\(^\circ\) to 29\(^\circ\)N, based on density surfaces (25.1–27.6 kg m\(^{-3}\), represented by color).
on 2 years of glider repeat transects. The structure function expressions of the CR12 estimates are shown as the dashed lines in Fig. 3: the average spectrum over the whole range 25.2–25.8 kg m$^{-3}$ in Fig. 3a, and for both seasons in colors corresponding to the structure functions in Fig. 3b. CR12 observed a spectral slope of $\lambda = -2$ (see their Fig. 9), consistent with the structure function slope near $\gamma = 1$ seen in Fig. 3. It is noteworthy that the range of scales represented in CR12 (the length of the dashed thick line) is surpassed at large scales by the use of Argo data. The difference in magnitude can be attributed to the inclusion of several years of data (and therefore interannual variability), while CR12 only have 2 years. Seasonal Argo estimates (Fig. 3b) are also in qualitative agreement with CR12, with higher correlations in spring over winter, and slopes near $\gamma = 1$ for both seasons. This first assessment shows that it is possible to use the Argo data to retrieve the structure function over macroscale separation distances and to obtain physically meaningful results.

The method is next applied to a highly anisotropic and heterogeneous region of the tropical Pacific. Thus, Fig. 4 shows the isobaric salinity structure function for the region between 10°S and 10°N and 180°W and 150°W (thick curve) and for four subregions (thin curves). The decisive result in Fig. 4 is the agreement in slope (Fig. 4b) between the various estimates, indicating that the four subregions are not governed by fundamentally different dynamics despite heterogeneity in simpler statistics (e.g., salinity variance, as plotted in Fig. 5). The 90% confidence interval, shown for the full region, is indicative of the statistical significance of the differences between the average and the subregion estimates. Bootstrap confidence intervals are expectedly wider for the data subgroups (since the sample size is smaller; see Table B4 in appendix B). Despite the overall agreement, it is still possible that such differences could be an artifact resulting from heterogeneity and uneven sampling. The first-order conclusion from Fig. 4, however, is that robust structure function patterns (with confidence intervals) can emerge, even in the presence of anisotropy and heterogeneity, when considering regions of similar dynamics.

Taken all together, the relative success of the two presented tests (Figs. 3, 4) warrants further investigation of Argo structure function estimates. The rest of this section thus proceeds to assess the dependence of Argo

![Fig. 9. (a) Isobaric salinity structure function $D_s(s)$ in the Kuroshio uniform variance region at depths from 5 to 2000 m, represented in color. (b) Small-scale slopes of the structure function at each latitude band, including 90% bootstrap confidence interval. (c) Amplitude of the large-scale structure function at each latitude band, including 90% bootstrap confidence interval. Reference slopes of $\gamma = 0$ (dashed) and $\gamma = \frac{2}{3}$ (solid) are shown as thick lines in (a) and as dashed-dotted lines in (b).](image)
structure function estimates on depth, level of eddy energy, and latitude.

b. Depth dependence

One tantalizing aspect of the Argo data is that estimates of structure function can be made at depths exceeding the depth where continuous data are presently available. Glider and submarine data remain rare, and tow-yos at substantial depth are not feasible. Thus, the first analysis here concerns how the structure function estimates depend on depth.

Beginning the assessment of ocean turbulence with the structure function as its own statistic, both isobaric and isopycnal structure functions are calculated at different depths, first in a relatively quiet region of the midlatitude Pacific (Figs. 6, 7). This region shows some degree of heterogeneity in salinity variance (see Fig. 5) but is far removed from the most energetic ocean jets (the Kuroshio and Equatorial Undercurrent in particular). Variance in the upper 250 m is near constant in Fig. 6, which is consistent with a low level of eddy energy (a point further discussed in the next section).

Both isobaric and isopycnal structure functions generally show positive slopes, with 90% confidence based on bootstrap distributions, and bootstrap mean slopes that are often near 2/3. Differences between pressure and density surfaces can be instructive about the effects of internal waves and eddies on the structure function. Furthermore, since the region analyzed in Figs. 6 and 7 is eddy-poor, isobaric structure functions may rather characterize internal wave activity, while structure functions computed along isopycnals are expected to filter out some of the internal wave signals. However, the current level of uncertainty indicated by bootstrap intervals is too high to draw conclusions with a high confidence on those grounds (Figs. 6b, 7b). Additional data will be needed to reduce uncertainties and challenge the general behavior seen in Figs. 6 and 7—that is, the fact that slopes are positive with 90% confidence, with a mean slope near 2/3 throughout the upper 1900 m, on isopycnals as well as on isobaric surfaces.

The isobaric salinity structure function estimates in this region are relatively constant in amplitude (quantified as the average of the large-scale fit line) near the surface mixed layer (i.e., within 250 m of the surface) and then decay roughly exponentially with depth. The isopycnal structure function is nearly constant until a much greater depth (near 26.8 kg m$^{-3}$, near 500-m depth) and then decays. It is tempting to compare this result to SQG theory, where an exponential decay with depth is

![Figure 10](image-url)

**Fig. 10.** (a) Isopycnal salinity structure function $D_s(s)|_s$ in the Kuroshio region, chosen as uniform on the 5-m isobar, from 25.4 to 27.6 kg m$^{-3}$, represented by color. (b) Small-scale slopes of the structure function at each latitude band, including 90% bootstrap confidence interval. (c) Amplitude of the large-scale structure function at each latitude band, including 90% bootstrap confidence interval. Reference slopes of $\gamma = 0$ (dashed) and $2/3$ (solid) are shown as thick lines in (a) and as dashed–dotted lines in (b).
predicted and has nearly constant amplitude within the mixed layer. However, the slope of the structure function is consistent across all depths, where SQG predicts strong steepening with depth as short-separation scales become decorrelated. As will be shown below, however, this pattern of slope and amplitude is not universal.

c. Eddy-rich versus eddy-poor regions

A similarly contrasting yet conclusive picture emerges when comparing regions of high versus low mesoscale eddy energy. To assess the effect of eddies on structure functions, the Kuroshio region, where eddy activity is very high, is compared to the eddy-poor region discussed above. Figure 8, based on interpolated data distributed by AVISO (http://www.aviso.oceanobs.com/), confirms the clear contrast in eddy energy between the two selected regions. To mitigate the impact of heterogeneity associated with the quieter surroundings of energetic jets, the region of analysis was further guided by the map of salinity variance at 5 m (Fig. 5).

Figure 9 shows much increased isobaric variance near the surface, again consistent with the exponential decay expected from SQG theory (see Callies and Ferrari 2013). Furthermore, variance on isopycnals is near constant above \( \sim 26.5 \text{ kg m}^{-3} \), both in eddy-rich and eddy-poor regimes (Figs. 7, 9), and it decreases gradually below \( \sim 26.5 \text{ kg m}^{-3} \). If one takes the isobaric structure functions as indicating primarily (or dominated by) internal waves, and the isopycnal structure function as indicating (or dominated by) geostrophic variability, then this result is the opposite of what is expected from popular theories: SQG (strong decay in isopycnal structure function) and bottom-generated internal waves (increasing variability as the bottom is approached). The increased surface variability in isobaric structure functions may be an indicator of strong near-inertial internal waves generated by winds at the surface (Kunze 1985). Again, a slope of \( \gamma = 2/3 \) is seen in the Kuroshio region, on both isobars and isopycnals, with no obvious dependence on depth (Figs. 7b, 9b), corresponding (in homogeneous turbulence) to the spectral slope of \( \lambda = -5/3 \). Figure 10 also adheres to this slope at all depths, but again the isopycnal structure function stays nearly constant until a much greater depth (26.4 kg m\(^{-3}\)), beyond which exponential weakening with depth occurs.

The \( \gamma = 2/3 \) behavior may also be described by the theory of passive tracer variance of Obukhov (1949) and Corrsin (1951), with structure function slopes equivalent to a spectral slope of \( \lambda = -5/3 \). The persistence of the \( \gamma = 2/3 \) slope deep in the water column is an indicator of the energy (and therefore tracer) cascade to larger scales as a function of depth on isopycnals. At larger scales, the slope of \( \gamma = 0 \) may indicate that the structure function slope is uncorrelated, random motions or that it is equivalent to a spectral slope of \( \lambda = 0 \), which coincides with the theory of Vallis (2006) of the passive tracer. This theory would suggest that the largest eddies are the size of the onset of the \( \gamma = 0 \) regime, and the bend point is found around 200 km.

The fact that isobaric structure function slopes only weakly depend on depth may reflect the presence of internal waves throughout the water column. Differences between isobaric and isopycnal structure functions could be attributed to internal wave signals that should be partially omitted by construction of isopycnal structure functions. In further investigation, such hypotheses could be investigated by calculating predictions from, for example, the Garrett–Munk spectrum (Garrett and Munk 1972) of internal waves. Such theories are still evolving but would not change the diagnosis of this dataset. For more recent discussion and observations of the internal wave spectrum, the reader is referred to Klymak and Moum (2007) and Callies and Ferrari (2013).
Figures 6, 7, 9, and 10 do not exclude the possibility that subtle differences may be found between eddy-rich versus eddy-poor, isopycnal versus isobaric structure functions. However, additional data are necessary to increase the degree of confidence and draw more definitive conclusions. At this stage, the null hypothesis being tested, based on the robust behavior seen in Figs. 6, 7, 9, and 10, is that structure function slopes are positive with high confidence and near $2/3$ on average. Structure function amplitude tends to decay with depth, and more slowly in the isopycnal structure function than in the isobaric, but without changing the slope. How universal this behavior may be and the implications for theoretical work remains to be established.

d. Latitude dependence

Latitude is anticipated to be a determining factor in the structure function slope and amplitude, because the Rossby deformation radius rapidly decreases with increasing latitude (see, e.g., Chelton et al. 1998). In extending the analysis of structure functions to latitudinal contrasts, however, it is clear that particular attention should be paid to heterogeneity. In particular, the great contrasts in salinity variance between oceanic basins seen in Fig. 11 lead us to focus our analysis on an individual basin (the Atlantic is chosen below). Slopes near $2/3$ are consistently found, yet again, in each basin (Fig. 11). But variances differ by an order of magnitude near the surface, and even more at depth, between the Atlantic and Pacific. It is particularly striking that the deep North Atlantic shows as much salinity variance as the near-surface Pacific—a point further discussed below. Based on Fig. 11, we make no attempt at estimating global mean or even global zonal mean structure functions. Here we focus on North Atlantic zonal means (Figs. 12–15), which could be more meaningful.

Within the Atlantic itself, there is a marked asymmetry in isobaric salinity variance between the northern and southern midlatitudes (Fig. 11). This meridional asymmetry is quite clear in deep isopycnal variance (Fig. 12) and in deep isobaric variance (Fig. 13). It may reflect deep convection and deep water formation injecting salinity variability to depth, as proposed by Yeager and Large (2007). In their theory, seasonal injections of spice (i.e., density compensated variability in both temperature and salinity) in the North Atlantic increase the salinity variability on outcropping density surfaces, and this added variability is then subducted and transported southward by the meridional overturning circulation. This theory is supported by
the continuous southward decrease in observed salinity variance shown in Figs. 12 and 13. Another noteworthy, statistically significant result is that low latitudes show a maximum in isobaric salinity variance near the surface (Fig. 15), possibly because of fast planetary waves propagating through a highly stratified upper ocean, and minima at midlatitudes, which may characterize the quieter interior of subtropical gyres. This behavior is qualitatively, and significantly, different from the case of 1000 m and the two isopycnal cases (Figs. 12–14). Comparisons between these surface data (Figs. 14, 15) and tow-yos or the Prediction and Research Moored Array in the Atlantic (PIRATA) array would be interesting, but there are no comparable data to compare to the deep Argo data (Figs. 12, 13).

At most latitudes, structure functions in Figs. 12–15 show positive slopes, with 90% confidence and bootstrap-estimated means that are often near 2/3. Thus, the proposition that this behavior is near universal is further supported by Figs. 12–15. It is unclear whether the one counterexample seen in Fig. 12 is of physical origin or an artifact of the still-limited data collection. Bootstrap mean slopes show signs of meridional asymmetry—hints of slightly steeper slopes in the Northern Hemisphere, and maybe of tropical slope minima. However, these slope differences are small and far from being statistically significant, which again leads to the conclusion that further accumulation of Argo data is needed to challenge the proposed null hypothesis.

4. Conclusions

This first application of structure function techniques to Argo data gives physically meaningful results. The 90% confidence intervals estimated by bootstrapping show that there is both regional and depth dependence of the structure functions. The majority of the estimates discussed here have a slope near 2/3 on average (the equivalent of a $k^{-5/3}$ tracer spectrum) in an inertial range between 10 and 100 km that varies with location, and slope shows little dependence on depth. Many aspects of the method should be reevaluated (homogeneity, isotropy, simultaneity, noise handling, potential biases, mean handling, etc.), but a map of slopes from Argo, as is done for sea surface height spectra in Xu and Fu (2012), will be possible in the near future. Unlike Xu and Fu (2012), the Argo-based map will vary with depth as the estimates do here (Figs. 12–15). This work provides a first step in that direction. The scale of the bend point—if it indeed signifies the largest scale of coherent variability—is also a potentially useful measure. Estimates of eddy scales (e.g., Tulloch et al. 2011) rarely

![Fig. 13. (a) Isobaric salinity structure function $D_s(s)$ in the deep Atlantic, at 1000 m, with the colors representing structure function value for each $s$ and latitude bin. (b) Small-scale slopes of the structure function at each latitude band, including 90% bootstrap confidence interval. (c) Amplitude of the large-scale structure function at each latitude band, including 90% bootstrap confidence interval.](image-url)
use in situ data, as the data volume required is enormous. These scales can be read off of Figs. 12a, 13a, 14a, and 15a as the $s$ value for each latitude where all larger values are of similar magnitude. Roughly, this scale is 100 km, but latitudinal and depth variations are indicated (although noisily). The structure function of Argo offers a potentially inexpensive estimate of these scales.

Finally, it is possible, or even likely, that sampling biases are inherent in the style of sampling based on Lagrangian float technology. That is, floats will be unlikely to drift into or out of coherent structures and are likely to be ejected from regions of high eddy activity toward lower energy regions (e.g., Davis 1991). Without a substantially higher density of observations, such biases due to sampling heterogeneity are not easily detected generically and so are neglected here. However, all structure functions result from a large number of observational pairs (see appendix B), and instrument error analysis and bootstrapping confidence intervals are used to verify these assumptions (see appendixes B and C). A comparison between the Argo structure functions and those from stationary Eulerian moorings, for example, Tropical Atmosphere Ocean/Triangle Trans-Ocean Buoy Network (TAO/TRITON), Research Moored Array for African–Asian–Australian Monsoon Analysis and Prediction (RAMA), and PIRATA, would help quantify such biases.

Neither structure function nor spectral slope is a conclusive proof of any particular behavior, but they are very useful in eliminating theories or models that are erroneous. Even with discontinuous and spotty temperature or salinity measurements, an appreciation of the turbulence statistics at greater depths and over broader geographic regions than previously observed is now possible and will only improve with the growth of the Argo dataset. The ability to infer a spectrum from a structure function, even in a case where two distinct structure function slopes are present and data are filled with gaps is suited to Argo data analyses. The primary limitation is data density, as spatial refinement reduces the amount of observation pairs that can be used. As more Argo data become available, the noisiness in the structure functions can be smoothed, the limiting velocity $c_{\text{max}}$ can be increased to include more pairs with smaller separation times, and bootstrap intervals can be narrowed.

This work has opened many possibilities for future studies beyond the results already presented. Alongside the increasing number of Argo floats measuring at depth, it would be beneficial to include other sources of data (e.g., mooring data) to fill in the spatial gaps in Argo’s network that would allow the structure function to be calculated further into the inertial range of the

![Fig. 14. As in Fig. 12, but for the shallow Atlantic, at 25.7 kg m$^{-3}$.](image)
oceans at smaller scales. Adding a method for estimating the velocity and velocity-tracer covariances would greatly enhance the dynamical detail possible from structure function analysis. This method can also be extended to scattered velocity observations in order to directly measure the kinetic energy structure function.

Acknowledgments. This paper was inspired by conversations with Rod Frehlich. We wish that there had been more time with Rod, so that we could learn more from him. The Argo Program is part of the Global Ocean Observing System. K.M. was supported by the CIRES/NOAA-ESRL Graduate Research Fellowship. B.F.-K. was supported by NSF 0855010 and 1245944, and G.F. was supported in part through NASA project “Estimating the Circulation and Climate of the Ocean (ECCO) for CLIVAR” and NSF 1023499.

APPENDIX A

Structure Function–Spectrum Relationship in Detail

The spectral and structure function theory will be addressed starting from the isotropic temperature variance spectrum \( B(k) \), found similarly to the approach used in Webb (1964):

\[
\theta'^2 = \int_0^\infty B(k) \, dk \tag{A1}
\]

for wavenumbers \( k \), where \( \theta' \) is temperature variance, defined in section 2c. The salinity variance spectrum is the same as Eq. (A1), with \( S' \) instead of \( \theta' \), and from here on, temperature variance and salinity variance will be discussed interchangeably. The temperature variance autocorrelation function \( R(s) \) and \( n \)th-order structure function \( D_n(s) \) for spatial separation \( s \) are defined by

\[
R(s) = \theta'(x)\theta'(x+s), \quad \text{and} \tag{A2}
\]

\[
D_n(s) = [\theta'(x) - \theta'(x+s)]^n. \tag{A3}
\]

The second-order \((n = 2)\) structure has the unique relationship to \( R(s) \) by

\[
D_2(s) = 2[\theta'^2 - R(s)]. \tag{A4}
\]

The autocorrelation may be represented spectrally for isotropic, homogeneous turbulence by

\[
R(s) = \int_0^\infty B(k) \cos(ks) \, dk. \tag{A5}
\]

Using the relationship between the autocorrelation and structure function from Eq. (A4), and the spectral
definition of autocorrelation in Eq. (A5), the structure function can be written spectrally by

\[ D_\theta(s) = 2 \int_0^\infty B(k)[1 - \cos(ks)] dk. \quad (A6) \]

As mentioned in section 2a, for a given spectrum \( B(k) = \alpha_B k^\lambda \) with a single spectral slope \( \lambda \) over a range from \( k_{\text{min}} < k < k_{\text{max}} \) and a given structure function \( D_\theta(s) = \alpha_D s^\gamma \) with a single structure function slope \( \gamma \), a change of variables \(( ks \to \xi \) yields

\[ D_\theta(s) = 2 \int_0^\infty \alpha_B k^\lambda [1 - \cos(ks)] dk \\
= 2\alpha_B s^{-\lambda - 1} \int_0^\infty \xi^\lambda (1 - \cos \xi) d\xi \\
= s^\gamma \left[ 2\alpha_B \int_0^\infty \xi^\lambda (1 - \cos \xi) d\xi \right]. \quad (A7) \]

This shows that \( \gamma = -\lambda - 1 \), relating the slope of the structure function \( \gamma \) to the spectral slope \( \lambda \). Webb (1964) shows that outside of the inertial range \([k_{\text{min}}, k_{\text{max}}] \), the contribution to the spectrum is small, so Eq. (A7) can be truncated and written as

\[ D_\theta(s) = s^\gamma \left[ 2\alpha_B \int_{k_{\text{min}}}^{k_{\text{max}}} \xi^\lambda (1 - \cos \xi) d\xi \right]. \quad (A8) \]

One could make the same argument for the kinetic energy spectrum \( E(k) \) and velocity structure function \( D_U(s) \). Thus,

\[ \overline{U^2} = \int_0^\infty E(k) dk \quad (A9) \]

and

\[ D_U(s) = (u(x) - u(x + s))^2. \quad (A10) \]

Following the same method, \( D_U(s) \propto s^{\beta_U} \) and \( E(k) \propto k^{\beta_E} \) produce the same relationship: \( \beta_D = -\beta_E - 1 \).

In the case of a tracer variance spectrum with a direct and indirect cascade producing two power-law scalings [as is the case in Nastrom and Gage (1985)], Eq. (A6) can be split into four pieces spanning intervals in \( k \):

\[ D_\theta(s) = 2 \int_0^\infty B(k)[1 - \cos(ks)] dk \\
= 2 \left\{ \int_{k_{\text{min}}}^{k_1} B(k)[1 - \cos(ks)] dk + \int_{k_1}^{k_{\text{max}}} \alpha_1 k^{\lambda_1}[1 - \cos(ks)] dk \right\} \\
+ 2 \left\{ \int_{k_{\text{min}}}^{k_1} \alpha_2 k^{\lambda_1}[1 - \cos(ks)] dk + \int_{k_{\text{max}}}^\infty B(k)[1 - \cos(ks)] dk \right\}. \quad (A11) \]

Since the first and the last integrals are definite and negligible (Webb 1964), inserting the continuity of \( B(k) \) \((\alpha_1 k_{\text{min}}^{\lambda_1} = \alpha_2 k_1^{\lambda_1}) \) produces

\[ D_\theta(s) = 2\alpha_1 \left\{ \int_{k_{\text{min}}}^{k_1} k^{\lambda_1}[1 - \cos(ks)] dk \\
+ k_1^{1 - \lambda_2} \int_{k_1}^{k_{\text{max}}} k^{\lambda_2}[1 - \cos(ks)] dk \right\}. \quad (A12) \]

Assuming each of the two inertial ranges is large \((k_{\text{min}} \ll k_1 \ll k_{\text{max}}) \) the structure function is dominated by only one of the two integrals in Eq. (A12), depending on scale of \( s \) when compared with the wavenumber \((k_{\text{min}} < 1/s < k_1 \) or \( k_1 < 1/s < k_{\text{max}} \)). Performing the change of variables as done above for the single power law case, gives the structure function in terms of \( s \):

\[ D_\theta(s) = 2\alpha_1 s^{-\lambda_1 - 1} \int_{k_{\text{min}}}^{k_1} \xi^{\lambda_1}[1 - \cos(\xi)] d\xi \\
+ \alpha_2 s^{-\lambda_2 - 1} \int_{k_1}^{k_{\text{max}}} \xi^{\lambda_2}[1 - \cos(\xi)] d\xi. \quad (A13) \]

Thus, when the inertial ranges are deep, the structure function is closely approximated by a polynomial with two terms:

\[ D_\theta(s) = c_1 s^{\gamma_1} + c_2 s^{\gamma_2}, \quad (A14) \]

with \( \gamma_1 = -\lambda_2 - 1 \) and \( \gamma_2 = -\lambda_1 - 1 \), and an internal dependence on \( s \) that determines which term dominates the spectrum. The analysis of Nastrom and Gage (1985) confirmed that the bend point where the structure function switches from being dominated by the second to the first term happens near \( s \approx 1/k_1 \), although this
result was much clearer when the inertial ranges were made wider than those in the actual observations of Nastrom and Gage (1985). Other prototypical dual cascade spectra were also tested, yielding similar results [e.g., the direct and indirect cascades of 2D turbulence from Kraichnan (1967)].

### APPENDIX B

#### Structure Function Details

The calculation of the structure function from Argo data was completed as follows. The data were sorted in time and the flagged profiles and individual values were thrown out, according to the quality-control scheme introduced above. The dataset was then limited to the depth level for calculation and for geographical region. At this point, the OCCA climatological value, which is available at the same location as each Argo observation, was subtracted from the Argo observation to obtain the perturbation $S$. Bins of separation distance were defined as $10^3$ to $10^4$ in intervals of $10^2$. The value for $c_{\text{max}}$ was defined, and separation time bins were defined as the distance bins divided by $c_{\text{max}}$. After the time between each pair of observations was calculated, the dataset was narrowed down to the pairs with separation times between 0 and the maximum time separation defined by separation distance and $c_{\text{max}}$. After the distance between all pairs of points was calculated, the dataset was then limited again to only the points whose separation distance divided by separation time were greater than $c_{\text{max}}$. The direction between each pair of observations was calculated, and the direction is saved for later use in structure function averaging. The difference between every pair in the limited dataset is then squared and is the content of the averaging in the structure function.

For the averaging procedure for the structure function, a limit was set to determine if a directional weight was applied. If more than 10% of pairs were in the same directional bin, then a weight was used. The average in each 18° directional bin was computed, and then the average of the averages was used as the final value. If there was no need for directional weights to be applied, then all points were averaged together. The average was calculated of all pairs that fall within the range between midpoints of the separation distance bins. The values that contributed to each separation distance’s bin were saved for calculation of the confidence intervals, which will be discussed below.

The tables included here show the details of the structure function calculations; Tables B1 and B2 list the numbers of float profiles in each calculation and the number of “simultaneous” pairs used, and Tables B3 provide the structure function calculations; Tables B1 and B2 list the number of profiles and pairs used to compute the structure function in the heterogeneous region analysis in Fig. 4.

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<td>34 228</td>
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<td>60 546</td>
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and B4 list the 95% bootstrap confidence intervals for the structure functions. Tables B5, B6, and B7 include the total number of profiles and the number of simultaneously-measured pairs included in each structure function average in Figs. 6, 7, and 9.

The 95% bootstrap confidence interval was calculated because the population of pairs that contribute to the average in the structure function is not normally distributed, so the standard deviation of the observations is not sufficient. Using the Central Limit Theorem (Devore 2009), which states that the means \( \bar{x}_n \) from \( n \) samples of a population (here, the pairs of simultaneous observations) are normally distributed, and therefore, the population mean (\( \mu \), the true quantity of the structure function) is the mean of the sample means (\( \mu = \bar{x}_n \)). Therefore, the confidence interval is the area with a 95% probability that it contains the true structure function value. This theorem is only true when \( n \) is sufficiently large (usually larger than \( n = 30 \), though some populations may require more), so \( n = 200 \) was used here.

### Table B5. Number of profiles and profile pairs used to compute the isobaric structure function for each depth in the heterogeneous region of the Pacific, shown in Fig. 6.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>No. of profiles</th>
<th>No. of pairs</th>
<th>Depth (m)</th>
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The Kuroshio homogeneous region, which is “uniform” at 5 m, is bounded by 33°8 N, 141°8 E to the southwest; 43°8 N, 142°8 E to the northwest; 42°8 N, 155°8 E to the southeast; and 42°8 N, 157°8 E to the northeast. The heterogeneous region, which is “nonuniform” at 25.9 kg m\(^{-3}\) and 5 m, is bounded by 10°8 N, 160°8 W to the southwest; 30°8 N, 160°8 W to the northwest; 10°8 N, 140°8 W to the southeast; and 30°8 N, 140°8 W to the northeast.

**APPENDIX C**

**Error Analysis**

An important aspect of structure function analysis that must be included is an understanding of random noise. Lester (1970) showed that the structure function of Gaussian white noise has a slope of \( \gamma = 0 \), so that

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<th>No. of pairs</th>
<th>Density (kg m(^{-3}))</th>
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**TABLE B6. Number of profiles and profile pairs used to compute the isopycnal structure function for each density level in the heterogeneous region of the Pacific, shown in Fig. 7.**

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<th>Depth (m)</th>
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**APPENDIX C**

**Error Analysis**

An important aspect of structure function analysis that must be included is an understanding of random noise. Lester (1970) showed that the structure function of Gaussian white noise has a slope of \( \gamma = 0 \), so those

**TABLE B7. Number of profiles and profile pairs used to compute the isobaric structure function for each depth in the “uniform” region of the Kuroshio, shown in Fig. 9.**
results were replicated with a randomly generated dataset of temperatures and salinities with changing standard deviations. The same calculation was particularly important to determine the noise level generated by measurement error. Using the square of the known standard error of the Argo measurements of temperature and salinity (0.01°C and 0.01 psu, respectively) as the standard deviation, and a typical temperature and salinity value for the mean, a Gaussian dataset was created, and the structure function was calculated. A noise floor for the structure function including the error from the standard deviation, $\sigma_{\text{tot}} = \sqrt{\sigma_{\text{Argo}}^2 + \sigma_{\text{clim}}^2}$. This more realistic noise floor of $O(10^{-4})$ is still below the majority of the structure functions calculated, allowing this analysis of turbulence from Argo data to continue without fear of data measurement errors interfering.

**APPENDIX D**

**Line-Fitting Algorithm**

To quantify the differences among structure functions, a line-fitting algorithm was created to extract the slopes of the structure functions with one (or two) linear fits. A test was first performed to decide whether more than one linear fit was needed. On the structure functions with only one slope, a least squares method of linear regression was performed, using the bootstrap method of sampling to obtain a confidence interval. The advantage of using the bootstrap method for the confidence interval is that the assumption of normality for the individual observations is not necessary.

Since the relationship to the spectral slope no longer holds in heterogeneous regions, there could be two separate linear slope regimes with no relation to the spectrum. In this case, the same linear regression was performed, but in steps so as to find the amplitude and approximate bend point where a change in slope occurs. A bootstrap analysis was completed for this process. First, randomly chosen data points were fitted by two lines with the bend point at each separation distance bin. A least squares error was calculated for the lines fit for each bend point, and the bend point with the smallest error was chosen ($s_{\text{bend}}$). Using that bend point, all data points were then considered for the best-fit line. Another bootstrap regime was then run, choosing random data points and calculating the resulting slopes of the best-fit lines for the subsets of the original data. A bootstrap interval using 200 subsamples was calculated from these results, providing a confidence interval for the chosen best fit from all points. The amplitudes discussed are determined as the average of the points above the bend point.

The resulting bend points were not presented here because the changes in bend point between structure functions were small compared to the confidence intervals. With the addition of more data, and subsequently less noisy structure functions, this metric can also be used to quantify the bend point, which is the largest eddy scale measured.

In the homogeneous regions where two structure function slopes are discerned, the same linear fitting regime was used, and the relation to the spectral slope was applied to the results. The bend point for the spectral slope could then computed to be $k_1 = 1/s_{\text{bend}}$.

**REFERENCES**


Fox-Kemper, B., and Coauthors, 2011: Parameterization of mixed layer eddies. III: Implementation and impact in global ocean


