Spontaneous Generation of Near-Inertial Waves by the Kuroshio Front

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(Manuscript received 25 April 2014, in final form 15 June 2015)

ABSTRACT

While near-inertial waves are known to be generated by atmospheric storms, recent observations in the Kuroshio Front find intense near-inertial internal-wave shear along sloping isopycnals, even during calm weather. Recent literature suggests that spontaneous generation of near-inertial waves by frontal instabilities could represent a major sink for the subinertial quasigeostrophic circulation. An unforced three-dimensional 1-km-resolution model, initialized with the observed cross-Kuroshio structure, is used to explore this mechanism. After several weeks, the model exhibits growth of 10–100-km-scale frontal meanders, accompanied by $O(10)$ mW m$^{-2}$ spontaneous generation of near-inertial waves associated with readjustment of submesoscale fronts forced out of balance by mesoscale confluent flows. These waves have properties resembling those in the observations. However, they are reabsorbed into the model Kuroshio Front with no more than 15% dissipating or radiating away. Thus, spontaneous generation of near-inertial waves represents a redistribution of quasigeostrophic energy rather than a significant sink.

1. Introduction

Most of the power into the ocean’s general circulation arises from stress exerted by the wind at the surface (Fofonoff 1981; Oort et al. 1994; Wunsch 1998). Because of the ocean’s boundaries, wind patterns, density distribution, and Earth’s rotation, this energy organizes into 100–1000-km gyres and currents and fields of 10–100-km mesoscale eddies. This large-scale quasigeostrophic dynamics arrests transfer of energy to smaller scales where it could be dissipated, instead cascading energy to larger scales. But energy in the general circulation must be dissipated at the same 1-TW rate that it is forced by wind.

Several dissipation mechanisms have been proposed involving interactions with bottom or lateral boundaries (Fig. 1) including 0.2 TW in internal lee-wave generation (Nikurashin and Ferrari 2011) and 0.1 TW of bottom drag (Wunsch and Ferrari 2004). However, these forces are negligible away from boundaries (Wunsch and Ferrari 2004) and fall short of the power input. Hence, there remains a gap in our understanding of how the interior ocean subinertial circulation dissipates.

Several studies have pointed out that fronts can spontaneously generate inertia–gravity waves. Using a two-dimensional numerical model, Snyder et al. (1993) showed that atmospheric fronts forced out of balance by frontogenetic confluence can spontaneously radiate inertia–gravity waves as they undergo geostrophic adjustment (Rossby 1938). Plougonven and Snyder (2007) found that near-inertial waves are radiated from a meandering atmospheric front and stall near the front. Ford (1994)
showed that barotropic shallow-water models with zonal bands of anomalous PV radiate inertia–gravity waves from evolving vortices. Similarly, Danioux et al. (2012) used a 2-km-resolution primitive equation ocean model to show that gravity waves are emitted from high-Rossby-number flow associated with density filaments near the surface. Shakespeare and Taylor (2014) showed analytically that a two-dimensional front undergoing frontogenesis can radiate inertia–gravity waves that remain trapped and amplified within the front by the confluent flow. In laboratory experiments, Williams et al. (2008) found that the amplitude of the emitted waves from balanced flow is linearly proportional to Rossby number, estimating that 1.5 TW could be transferred from the balanced flow to internal gravity waves in the ocean. Based on estimates of the mesoscale eddy/internal-wave momentum transfer coefficients in the Sargasso Sea (Polzin 2010), Ferrari and Wunsch (2009) inferred a global net transfer rate of 0.35 TW. From an analytic model for minimum-frequency near-inertial waves in a baroclinic front under the influence of confluent flow, Thomas (2012) suggested a 0.1-TW global loss of balanced energy to near-inertial waves.

Based on the order-of-magnitude spread in the above estimates, the energy that can be drained from the sub-inertial quasigeostrophic flows of the ocean general circulation by spontaneous generation of internal waves remains uncertain. Here, we explore this possible sink for quasigeostrophic energy numerically. We motivate this study with numerous recent measurements that show enhanced near-inertial shear in the Kuroshio Front and Gulf Stream (section 2). Unforced numerical simulations of the Kuroshio spontaneously generate near-inertial waves to reproduce the observed near-inertial shear and enhancement of turbulent dissipation under the
Global internal-wave power gain, through spontaneous radiation from fronts forced out of balance by confluent flows followed by reabsorption and dissipation, is estimated in section 4, and conclusions are summarized in section 5.

2. Observations of near-inertial waves in the Kuroshio

a. Kuroshio surveys

To motivate our study, we first describe four transect surveys. Three transects were sampled across the Kuroshio Front along 143°E during 8–10 August 2008 and 8–10 August 2011 and along 142°E during 9–11 August 2012. In these field programs, expendable bathythermographs (XBT T-7s) were deployed every 3.7 km in 2008 and 2011 and 2.8 km in 2012 to measure temperature in the upper 750 m from 36°36′N to 35°48′N during 2008, 36°30′N to 35°44′N during 2011, and 35°44′ to 34°19′N during 2012. A freefall towYo CTD (Underway CTD) was used to measure upper-500-m temperature and salinity every 14.8 km in the 2012 survey. During 2008, five Falmouth CTD profiles of temperature and salinity to 500-m and TurboMAP-II microstructure profiles to 300-m depth were collected every 28 km (Nagai et al. 2009). During 17–24 October 2009, five north–south transects were sampled across the Kuroshio to measure CTD and microstructure at 5–8 stations with 9-km resolution in each section (Nagai et al. 2012). During 2011 and 2012, five CTD and four TurboMAP-L (Doubell et al. 2009) profiles were collected every 9.3 km and expendable CTDs (XCTD) were dropped at the north and south ends of the transect.

To obtain salinity with the same resolution as XBT temperature, an optimal interpolation was performed for the CTD data with monthly salinity climatology from the World Ocean Atlas 2005 using 20 km and 40 m as horizontal and vertical decorrelation scales. Buoyancy is defined as

\[ b = \frac{1}{\rho_o} \left( \frac{\partial \rho}{\partial z} \right) \]

where \( \rho \) is the potential density calculated from temperature and salinity using the equation of state of seawater (EOS-80), the reference density \( \rho_o = 1025 \text{ kg m}^{-3} \), and \( g \) is the gravitational acceleration. Turbulent kinetic energy dissipation rates are computed by integrating microscale shear spectra over the wavenumber band where they agree with the Nasmyth (1970) model spectrum—that is, from approximately 1 m down to twice the Kolmogorov scale.

Currents were measured using a 38.4-kHz Teledyne RD ADCP (30° beam angle, 3.6° beamwidth) and 130-kHz Furuno ADCP. During 2008, the Teledyne measured flows to 1218-m depth in 16-m bins with 16-m transmitted pulse length while, during 2011, it measured to 1018-m depth in 10-m bins with 11.59-m transmitted pulse length. The vertical wavenumber spectra for zonal ADCP shear \( u_z(z) \) near the Kuroshio Front suggest signal attenuation around 30-m vertical wavelength (not shown) so that vertical shear on larger wavelengths is resolved.

During August 2011, the Kuroshio flowed along a relatively stable path with a strong eastward flow exceeding 2 m s\(^{-1}\) at the observational line (Fig. 2). The high-pressure warm-core eddy north of the Kuroshio (Fig. 2) provided confluent flow near the observation sites. During 2008, the Kuroshio was relatively unstable, exhibiting 100-km-scale meanders upstream of the transect (Fig. 1 of Nagai et al. 2009).

b. Near-inertial shear

To isolate the spatial structure of near-inertial shear across the Kuroshio Front, residual shear is extracted by subtracting alongfront geostrophic shear \( f u_z \) from ADCP-measured shear \( u_z \)--that is, \( u_z' = u_z - f u_z = u_z + b_y / f \), where \( y \) is the cross-front coordinate, \( b_y \) is based on our cross-front density measurements, and subscripts denote derivatives. Before subtraction, buoyancy and velocity data are smoothed and linearly interpolated of grid with a 3.7-km horizontal and 1-m vertical resolution. Because our transect is nearly normal to the Kuroshio, the ADCP-measured meridional shear \( u_z \) is assumed to have no geostrophic contribution.
(i.e., $v^\ast_z = v_z$), consistent with its alternating positive and negative signs (Fig. 3b).

Near-inertial shear rotates clockwise in time. To avoid temporal aliasing of the horizontal structure when observations span an interval (17 h in our case) that exceeds or is similar to the inertial time scale $f^{-1} = P_f/(2\pi) = 3.5$ h at our location where $P_f$ is the inertial period, back-rotated residual shear $u^\ast_z(t_0)$ referenced to time $t_0$ is estimated from the observations $Z(t) = u^\ast_z(t) + i\nu_z^\ast(t)$ at varying times $t$ as

$$Z(t_0) = \text{Re}[Z(t)e^{i(t-t_0)f}]. \quad (1)$$

Back-rotated residual vertical shear $Z(t_0)$ exhibits banding in the Kuroshio Front (Fig. 3a) of similar magnitudes as observed during 2008 (Nagai et al. 2009) and previous studies in the Kuroshio (Rainville and Pinkel 2004) and Gulf Stream (Winkel et al. 2002; Inoue et al. 2010). Horizontal and vertical wavelengths are $\lambda_h \sim O(10)$ km and $\lambda_z \sim O(100)$ m, respectively. The internal-wave dispersion relation then implies that the frequency of these waves $\omega = \sqrt{f^2 + N^2\lambda_z^2/\lambda_h^2} \approx f$, based on the estimated $N \sim O(10^{-3})$ s$^{-1}$ and local $f \sim O(10^{-4})$ s$^{-1}$. This is consistent with the assumption that the frequency of the shear bands is near inertial. Hodographs of the residual shear are rotary with depth and meridian as expected for propagating near-inertial internal waves, suggesting energy radiation northward and upward on the north side (Figs. 4b and 4d) and southward and downward on the south side of the Kuroshio Front (Figs. 4a and 4c).

During 2011, a typhoon passed 500 km south of the region two weeks prior to the observations, but the time elapsed since the storm far exceeds the 2–4-day time scale expected for propagation $|\Delta r|/C_g$, where $r$ is the distance and $C_g$ the group speed, or dissipation $E/\epsilon$, where $E$ is the inertial wave energy and $\epsilon$ is the dissipation rate of the waves. The wind during the 2011 cruise was weak with speeds of 5–10 m s$^{-1}$. Results from a slab model (Pollard and Millard 1970) for mixed-layer inertial motions forced with reanalyzed hourly winds from the week prior to the typhoon (GPV-MSM-S of the Japan Meteorological Agency; Saito et al. 2006)
show weaker cumulative wind power input to inertial motions (∼1000 J m$^{-2}$) compared to the near-inertial horizontal kinetic energy integrated between 100- and 750-m depth near the Kuroshio Front (∼5000 J m$^{-2}$; not shown).

In summary, observed bands of near-inertial shear (i) appear to be stronger near the front (Fig. 3a), (ii) recur from year to year as well as being found in other western boundary current fronts (Winkel et al. 2002; Rainville and Pinkel 2004; Inoue et al. 2010), and (iii) appear to be unrelated to surface forcing, though this cannot be established unequivocally with our limited data. Near-inertial waves can be trapped and amplified by frontal shears (Kunze 1985; Whitt and Thomas 2013) and wind-generated waves may stall in fronts long after forcing. While these alternative possibilities cannot be discounted based on available data, here these observations motivate numerical investigation into the hypothesis that the observed near-inertial shear arises from internal dynamics associated with frontogenesis (frontal strengthening) and frontolysis (frontal weakening) of the Kuroshio Front.

3. Model simulation of the Kuroshio Front

a. Model setup

We use the three-dimensional nonhydrostatic equation process study ocean model (PSOM) (Mahadevan et al. 1996a,b) to investigate generation of near-inertial waves by frontal instability. The model horizontal resolution is 1 km and domain dimensions 192 and 384 km in zonal ($x$) and meridional ($y$) directions, respectively. There are 64 vertical levels in the model with a flat bottom at 750-m depth. The vertical resolution is a few meters in the upper 150 m and telescopes from 5 to 30 m between 150- and 750-m depth. Zonal boundaries are periodic while meridional boundaries are rigid walls. Sponge layers of 35-km width along the north and south boundaries suppress reflection of propagating waves. The bottommost layer is also a sponge layer. Horizontal diffusivities are 500 m$^2$s$^{-1}$ with the decay rate $\alpha_o = 2$ days$^{-1}$.
and velocity vector $\mathbf{u}$. Based on vertical internal-wave energy fluxes, bottom generation of internal waves was found to be negligible in all of the simulations. Vertical diffusivities are $10^{-5} \text{m}^2 \text{s}^{-1}$ everywhere. No hyperviscosities or hyperdiffusivities are used.

To provide realistic initial conditions for the modeled Kuroshio, the 2008 grid-averaged meridional density section (Nagai et al. 2009) is used. To suppress initial unbalanced disturbances in the simulation, temperature and salinity at each vertical level across the front are fitted with high-order polynomial functions. Potential densities calculated from temperature and salinity are then sorted to be vertically monotonic so as to remove any gravitationally unstable density inversions; density inversions are present in 3% of the original unsorted 1-m data. The resulting density field is linearly interpolated onto the 1-km horizontal model grid.

To induce a frontal meander, a sinusoidal fluctuation is introduced in the initial condition by shifting the initial vertical section meridionally by $\delta y = A \sin(x\pi/L_x)$, where $\delta y$ is the meridional displacement, $A = 3$ km is the displacement amplitude, $x$ is the zonal coordinate, and $L_x = 96$ km is half the zonal wavelength. The velocity is initialized to be everywhere in geostrophic balance. The initial quasigeostrophic imbalance is quantified by computing the magnitude of the frontogenetic $\mathbf{Q}$ vector ($|u_x b_y - u_y b_x;|\); see also section 4a) on the initial state which is found to be only 1% of that after frontal meanders develop fully on day 11.25. Large-amplitude inertial oscillations are not generated at the outset but only arise after the meanders develop fully as described below. The Ertel potential vorticity of the initial condition is confirmed to be everywhere positive. Accordingly, the initial flow field is gravitationally and symmetrically stable.

**b. Evolution of the model flow field**

As the model solution evolves, baroclinic instability generates a $\sim 100$-km-scale meander and eddy on a time scale of weeks. At this $O$(weeks) time in the simulation, model mesoscale fields (Fig. 5) are similar to those often observed in the Kuroshio Front (e.g., Fig. 1a in Nagai et al. 2009; Kouketsu et al. 2007). The magnitude of the model 30-h average confluence $\sqrt{(u_x - u_y)^2 + (u_y + u_z)^2}$ is comparable to the magnitude of cross-front confluence $|v_z| \sim O(10^{-6} - 10^{-5}) \text{s}^{-1} \sim O(0.01 \text{f} - 0.1 \text{f})$ in the upper 300 m near the Kuroshio estimated from a 45-km meridionally smoothed ADCP transect. Within this meandering mesoscale field, vertical shear, vorticity $(u_z - u_y)/f$, and horizontal divergence $(u_z + u_y)/f$ exhibit submesoscale banding (Figs. 5 and 6) within fronts with similar horizontal wavelengths of $O(10)$ km to those observed—although the particular model front snapshot displayed is shallower and less steep than the observed front (Fig. 3). The banded structures emerge, not in the initial stages of the simulation, but within an inertial period after the development of meanders and mesoscale eddy interaction with the main stream. As established below, these banded structures are near-inertial waves.

To filter out the subinertial flow, Eulerian high-pass quantities are obtained by subtracting 30-h averages from instantaneous data, where 30 h was chosen because the lowest internal-wave frequencies are modulated by geostrophic shears (Kunze 1985; Whitt and Thomas 2013) and Doppler shifting in the Eulerian frame. The high-pass fields include both internal waves and Doppler-shifted subinertial structures; Eulerian 30-h high-pass velocity is strongly correlated with Eulerian high-pass pressure gradient (not shown), suggesting a significant contribution from thermal wind.

Frequencies in an Eulerian frame can be difficult to interpret dynamically owing to Doppler shifting by subinertial front and eddy flows. To characterize the frequencies of the banded structure in a Lagrangian frame, we introduce 10,000 virtual massless particles (drifters) near the center of the model front at 50–150-m depth. These particles are advected passively by the three-dimensional flow without influencing the dynamics. Four Lagrangian particles are selected as examples to record high-pass flow fields for 20 days as they pass through the region of strong banded signals that mostly lie along the main stream (Fig. 7a). The resulting Lagrangian rotary spectra are clockwise in time for frequencies between $f$ and $2f$ (Figs. 8a and 8b), confirming that the banding in the Eulerian high-pass quantities is near-inertial waves. The relatively broad peaks in the spectra are likely caused by modulation of the lowest allowed internal-wave frequency by geostrophic vorticity (Kunze 1985) and baroclinicity (Mooers 1975; Whitt and Thomas 2013).

Hodographs of residual shear $(u_x'' = u_x + b_y/f$, $v_z'' = v_z - b_x/f)$ exhibit signatures of vertically propagating near-inertial waves (Fig. 9), rotating clockwise in time (Figs. 9c and 9e), clockwise with depth over 200–400-m depth on day 10.5 (Fig. 9d), and counter-clockwise with depth over 100–200-m depth on day 16 (Fig. 9f). Lack of perfect circularity (Figs. 9c–f) may be a signature of the superinertial nature of the waves, for which the major-to-minor axis ratio should be $\omega/f$, or due to subinertial baroclinicity that rectifies cross-front waves with phase lines parallel to steeply sloping
isopycnals (Whitt and Thomas 2013) as observed (Fig. 3).

Unlike geostrophic motions, internal waves are associated with horizontal divergence $u_x + v_y$ (Fig. 6; Müller and Siedler 1976; Lien and Müller 1992; Lelong et al. 1999; Plougonven and Snyder 2007; appendix B). High-pass horizontal divergences are strongly intensified after passage of a meander or eddy during days 10–11 (Figs. 6c,d and 10a). Such wave-intensifying events recur as the front evolves and subinertial confluent regions pass a fixed location (diamonds in Figs. 5c,d; Figs. 9 and 10). Divergence recorded by the particles provides several Lagrangian time series that also exhibit spectral peaks between $f$ and $2f$ (Fig. 7b). Eulerian

FIG. 5. Plan views of normalized 30-h running average surface vorticity $\zeta/f$ for days (a) 6.5 and (b) 10.5 and normalized Eulerian high-pass surface vorticity $\zeta'/f = (\zeta - \zeta)f = (u_x' - u_y')/f$ for days (c) 6.5 and (d) 10.5. Closed black contours indicate the regions where 30-h running averaged surface horizontal velocity magnitude exceeds $1.2 \text{ m s}^{-1}$. Diamonds in (c) and (d) are Eulerian stations for the time series shown in Figs. 9–10.
divergence spectra (dashed curve in Fig. 7b) show a greater dominance of subinertial variance, indicating that the near-inertial waves’ Eulerian frequency is subinertial, like internal lee waves, so these waves might be expected to be confined within the frontal jet and would not be identified as near inertial in fixed mooring measurements.

For a single propagating near-inertial wave, vertical vorticity $\zeta = (\nabla \times \mathbf{u})_z = (u_y - u_x)/f$ and horizontal divergence $(u_x' + v_y')/f$ should have similar magnitudes but be 90°
out of phase, forming a vector that rotates with time, depth, and horizontal propagation direction. Rotation of the vector composed of high-pass vorticity and divergence is seen in the simulations (Fig. 10), although the relatively larger magnitudes in high-pass vorticity than divergence implies nonnegligible contributions from quasigeostrophic flow in the Eulerian 30-h high-pass velocity. Energy propagation inferred from the rotary properties in the high-pass quantities is downward during days 10–11 to the north of the front (not shown) and upward at 300–400-m depth during days 7–8 to the south of the front (Fig. 10) as was also inferred from the residual vertical shear (Figs. 9c–f). Lagrangian rotary spectra of a vector composed of high-pass horizontal divergence \( (u_x + u_y)/f \) and relative vorticity \( (v_x - u_y) \), and ratios of horizontal divergence to relative vorticity \( (u_x + u_y)(v_x - u_y) \) are also both consistent with the banding being dominated by near-inertial waves (Figs. 8b and 8c); dominant clockwise rotation of the vector composed of high-pass vorticity and divergence with time in the frequency band \( f \) and \( 2f \) is also found along the Lagrangian particle trajectories (Fig. 8b). Variances of horizontal high-pass divergence \( (u_x + u_y)^2 \) and relative vorticity \( (v_x - u_y)^2 \) are of the same order of magnitude (Fig. 8c), consistent with near-inertial waves but not geostrophic flow (Kunze 1993).

c. Internal-wave energy fluxes and flux divergences

Near-inertial waves in our unforced model draw their energy from instabilities of the geostrophic flow and may provide a route for transferring energy from low-frequency geostrophic flow to breaking and turbulent dissipation. Using our numerical results, we quantify internal-wave energy \( E_{IW} \) conservation following Müller (1976) and Polzin (2010):
Fig. 8. (a) Lagrangian spectral ratios of clockwise $\Phi_{CW}^{ACW}$ rotation in time of high-pass velocities $u'$ and $v'$, obtained by four sample passive floats out of 10 000 released in the numerical simulation. (b) Lagrangian spectral ratios of clockwise $\Phi_{CW}^{ACW}$ rotation for the vector composed of high-pass vorticity $(u' - u')$ and divergence $(u' + u')$. (c) Lagrangian spectral ratios of high-pass divergence $\Phi_{v,\omega}$ to vorticity $\Phi_{\omega}$. Each estimate of auto- and co-spectra have $-26$ equivalent degrees of freedom with a cosine window, 400 points (20 days every 72 min), and overlapping. High-pass fields are relative to 30-h Eulerian averages. The dashed horizontal lines correspond to ratios of one which are what is expected for continuous ($\omega \gg f$) internal waves.

$$
\frac{DE_{IW}/Dt + \nabla \cdot \langle p' u' \rangle}{-\rho_o \left( \frac{\partial U}{\partial x} + \langle u' \rangle \frac{\partial V}{\partial y} \right)} - \rho_o \left( \frac{\partial U}{\partial x} + \langle u' \rangle \frac{\partial V}{\partial y} \right)
- \rho_o \left( \frac{\partial W}{\partial z} + \langle w' \rangle \frac{\partial W}{\partial z} \right)
- \rho_o \left( \frac{\partial U}{\partial x} + \langle u' \rangle \frac{\partial V}{\partial z} \right)
- \rho_o \left( \frac{\partial N^{-2} (b' b')}{\partial z} \right) - \varepsilon_{X, IP}^{IW},
\tag{2}
$$

where $DE_{IW}/Dt$ is the semi-Lagrangian time derivative of internal-wave energy (semi-Lagrangian quantities are obtained with a second set of passive particles that are isobaric so that they only follow the horizontal flow in order to avoid averaging data from different depths with large fluctuations in pressure). In (2), $\nabla \cdot \langle p' u' \rangle$ is the energy-flux divergence, where $u'$ and $p'$ are semi-Lagrangian high-pass velocity and pressure relative to low-pass 30-h fifth-order Butterworth fields and $\langle \rangle$ denotes 30-h semi-Lagrangian running averages of the covariance. Uppercase variables are the 30-h semi-Lagrangian running averages for zonal $U$, meridional $V$, and vertical $W$ velocities and buoyancy $B$. The dissipation rate of internal-wave kinetic and potential energy is represented by $\varepsilon_{X, IP}^{IW}$.

Internal-wave energy consists of high-pass kinetic energy $E_{K, IW}^{IW} = \rho_o (\langle u' u' \rangle + \langle v' v' \rangle + \langle w' w' \rangle)/2$ and available potential energy $E_{P, IW}^{IW} = -g (\rho' P' / [2 \rho_o (\zeta)]$, where $\zeta$ is averaged vertical density gradient as a function of depth alone. Energy transfers into and out of the internal-wave field are evaluated using the left-hand side of (2), with positive values indicating sources (defined as $\int (DE_{IW}/Dt) \, dz + \nabla \cdot \langle p' u' \rangle > 0$) and negative values sinks (defined as $\int (DE_{IW}/Dt) \, dz + \nabla \cdot \langle p' u' \rangle < 0$). To obtain the semi-Lagrangian high-pass quantities, 8 million isobaric (fixed depth) passive particles for each simulation case were released in the entire model domain at the outset to record pressure and velocity with reduced Doppler smearing. Because (2)
includes spatial derivatives, data recorded along particle trajectories are interpolated back onto the model grid after filtering for sub- and superinertial signals. Although particles following only horizontal flows could undergo flow divergence which induces inhomogeneity, the 8 million particles did not exhibit this problem. Also, because Doppler shifting by subinertial vertical flow of $O(10^{2}–10^{3}) \text{ms}^{-1}$, $0.01–0.1f$ is smaller than that of horizontal flow $0.1–1f$, most Doppler shifting is mitigated by the passive isobaric particles.

The internal-wave energy flux is a 30-h semi-Lagrangian temporal running average of the high-pass pressure and velocity covariance $p' u'$ that is grid averaged on each vertical level with 5-km horizontal resolution. It is readily shown from the geostrophic balance that geostrophic motions do not contribute to energy-flux divergence:

$$\frac{\partial u_x p_s}{\partial x} + \frac{\partial v_z p_g}{\partial y} = \frac{1}{f} \left[ \frac{\partial}{\partial x} \left( \frac{\partial p_g}{\partial x} p_s \right) + \frac{\partial}{\partial y} \left( \frac{\partial p_g}{\partial y} p_s \right) \right]$$

$$= \frac{1}{2f} \left( -\frac{\partial^2 p_g}{\partial x^2} + \frac{\partial^2 p_g}{\partial x \partial y} \right) = 0,$$  

where $p_g$ and $u_x$ are geostrophic reduced pressure and velocity that may be present in the high-pass fields. Furthermore, semi-Lagrangian high-pass velocities are not correlated with high-pass pressure gradients (Figs. 11c and 11d) as was the case for the Eulerian high-pass signals.

To average internal-wave energy flux over a typical 20-km wavelength (Figs. 5 and 6), a 20-km two-dimensional Gaussian horizontal average is applied. The resulting internal-wave energy-flux divergence is not sensitive to small changes in spatial and temporal filtering scales.

Large depth-integrated fluxes $\int (\rho' u') dz \sim O(1) \text{KW m}^{-1}$ and flux divergences $\nabla \cdot \int (\rho' u') dz \sim O(10) \text{mW m}^{-2}$ occur in the meander trough and crest (Fig. 12). Both zonally averaged { } depth-integrated internal-wave energy sources ($\int (DE_{IW}/Dt) dz + \int (\rho' v') dz > 0$; Fig. 13a) and sinks ($\int (DE_{IW}/Dt) dz + \int (\rho' v') dz < 0$; Fig. 13b), each averaged separately by sign, intensify near the meandering model front as it becomes unstable, becoming statistically stationary near day 6. The strongest generation sites are associated with frontogenesis and

FIG. 9. Depth-time series of WKB-normalized residual (a) zonal shear $u'_x = u_x - b_x/f$ and (b) meridional shear $v'_x = v_x - b_y/f$ north of the front (northern diamond in Figs. 5c,d and 6c,d). Depth is WKB-normalized $z^* = [N(z)/N_0] dz$. Temporal hodographs of residual vertical shears $u'_z$ and $v'_z$ (c) along the black–white–black horizontal line at the lower left in (a), and (e) along the white–black–white horizontal line at the upper right in (a). Depth hodographs along the (d) black–white–black vertical line and (f) white–black–white vertical line in (a). Green bars along the top axis represent the periods when surface 30-h running averaged horizontal velocity exceeds 0.7 m s$^{-1}$, showing when fronts and intensified near-inertial waves pass this Eulerian location.
frontolysis, consistent with geostrophic adjustment being the source of the waves, as discussed in section 4a.

Local recurrent events of internal-wave generation, or internal-wave power gain $\mathcal{E}_{\text{IW}}$ from the submesoscale subinertial flow, are associated with hotspots of 0.02–0.05 W m$^{-2}$ (Fig. 13a). Zonally and temporally averaged depth-integrated meridional energy-flux $\int_0^h \rho f \frac{\partial \phi}{\partial y} dz$ do not radiate far. Maximum fluxes of $O(0.1)$ kW m$^{-2}$ within 50 km from the front axis (Fig. 13d) fall to near-zero 100 km from the front. Zonally and temporally averaged depth-integrated internal-wave energy sources reach 0.02 W m$^{-2}$, which are largely balanced by sinks within $\pm 100$ km north and south of the front with sources and sinks appearing to be nearly coincident within the front (Figs. 13a,b,e). This is consistent with Shakespeare and Taylor’s (2014) finding that spontaneously generated internal waves might be associated with dissipation (Kunze et al. 1995) or reabsorption into the mean since wave energy is not conserved in steady shear flow (Booker and Bretherton 1967; Polzin 2008).

Zonally averaged depth-integrated positive values of the wave-mean flow interaction terms on the right-hand side of (2) show similar time variability and magnitudes to the internal-wave energy source (Figs. 14a and 13a). This is confirmed by the positive correlation coefficient of 0.89 with a near-zero $p$ value between internal-wave energy source and positive interaction terms (Fig. 14b), indicating dynamical consistency. Normal Reynolds-stress deformation terms $-\rho_d \langle \sigma' \rho' \rangle \frac{\partial U}{\partial x} + \langle \sigma' \rho' \rangle \frac{\partial V}{\partial y}$ dominate over the off-diagonal horizontal shear-stress terms $-\rho_d \langle \sigma' \rho' \rangle \frac{\partial V}{\partial x} + \langle \sigma' \rho' \rangle \frac{\partial U}{\partial y}$ and by an order of magnitude over other interaction terms. Geostrophic horizontal nondivergence forces $U_x \sim V_y$ so it is the partial rectilinearity of the near-inertial fluctuations, as discussed in section 3b, that is responsible for the normal Reynolds-stress term being nonzero; model near-inertial rotary signals are not perfectly circular such that $\langle \rho' \sigma' \rangle$ is $\sim O(10^{-3})$ m$^2$s$^{-2}$. This is consistent with internal waves extracting energy from subinertial confluent flows (Müller 1976; Bühler and McIntyre 2005; Polzin 2010; Thomas 2012).

d. Model internal-wave energy sinks and dissipation rates

Similarly, negative values of the right-hand side of (2) agree with the internal-wave energy sinks
(Figs. 15a and 13b) with the normal-stress terms once more dominating over the off-diagonal horizontal shear-stress terms and by an order of magnitude over other interaction terms. Thus, subinertial (QG) confluence both emits and absorbs near-inertial waves.

The explicit model kinetic $\varepsilon^\text{KE}_{xp}$ and potential $\varepsilon^\text{PE}_{xp}$ energy dissipation rates can be written

![Fig. 11.](image-url)
These results suggest that, in our numerical simulations, spontaneous internal-wave energy is generated then reabsorbed by the submesoscale subinertial flow (Booker and Bretherton 1967; Polzin 2008, 2010) with only 5–10% dissipating. Both wave energy generation and loss are dominated by normal-stress interaction terms $-\rho_u \langle u' \delta U \rangle \partial U / \partial x + \langle u' \delta V \rangle \partial V / \partial y$.

e. Summary of modeling results

Motivated by in situ measurements of elevated near-inertial shear and turbulent kinetic energy dissipation in the Kuroshio, numerical experiments were conducted that spontaneously emit internal waves (Fig. 12) from an unforced model Kuroshio Front with properties consistent with near-inertial waves. The simulations find similar banded shear along frontal isopycnals (Figs. 3c and 3d) as the observations (Figs. 3a and 3b). Lagrangian frequencies of these banded features are (1–2)$f$. The estimated average internal-wave power gain within 200 km of the Kuroshio Front is $O(0.01)$ W m$^{-2}$ (Fig. 13e) with little energy escaping the front (Fig. 13d). The bulk of the internal-wave energy appears to be reabsorbed within 50–100 km north and south of the source regions (Fig. 13e) with little lost to explicit model dissipation. This implies that most of the subinertial (QG) energy lost to internal waves is redistributed rather than dissipating. Evidence that the model internal-wave energy, energy fluxes, and their divergences and convergences based on...
the high-pass fields are dominated by internal waves comes from (i) the Lagrangian spectra, (ii) the spatial and temporal rotary properties of the residual shears, (iii) absence of correlation between Lagrangian high-pass velocity and pressure gradient, and (iv) that geostrophically balanced flows have no energy-flux convergence associated with them (3).

4. Discussion

The previous laboratory study by Williams et al. (2008) estimated that 1.5 TW could be lost from the wind-driven circulation to spontaneous generation of near-inertial waves globally, while Ferrari and Wunsch (2009) suggested a more modest 0.35 TW based on Polzin’s (2010) momentum transfer coefficients, and Thomas (2012) extrapolated 0.1 TW from analytic theory. To quantify the global contribution of gross inertial-wave generation and reabsorption from our simulations, we now estimate the global power transferred to the internal-wave field from the subinertial flow and back again using a two-step process.

First, a scaling relation between the internal-wave energy source \( \left\{ \int \int (DE_{IW}/Dt) \, dz > 0 \right\} \), which we equate with the internal-wave power gain \( \mathcal{E}_{IW} \), and quasigeostrophic vertical kinetic energy \( VKE_{QG} \) is derived from the 1-km regional PSOM model. Similarly, a second scaling relationship between internal-wave energy reabsorption rate \( \mathcal{E}_{Absp} \) and \( VKE_{QG} \) is obtained from the regional PSOM model. These relationships are then applied to the global 0.1°-resolution OGCM for the Earth Simulator (OFES) (Masumoto et al. 2004), which allows estimation of \( VKE_{QG} \) but not spontaneous generation of near-inertial waves, to obtain global estimates of the internal-wave power gain, reabsorption rates, and the net loss to radiation and dissipation.

a. Scaling for the quasigeostrophic vertical kinetic energy (\( VKE_{QG} \))

Horizontal confluence and frontogenesis that arise along meandering fronts create up- and downwelling motions that can be diagnosed using the Eliassen–Sawyer, or quasigeostrophic (QG) \( \omega \) equation

\[
b \cdot \left[ \frac{\partial^2 \psi}{\partial y^2} + f^2 \frac{\partial^2 \psi}{\partial z^2} \right] - 2 \frac{\partial \nu^x}{\partial y} \frac{\partial b}{\partial y} - \alpha |\nabla_b b| \quad (6)
\]
Hoskins et al. (1978), where \( \psi \) is the cross-front streamfunction \( (\partial\psi/\partial y + \partial\psi/\partial z = -v^a) \) with \( w \) and \( u^a \) as the quasigeostrophic vertical and across-front velocity, respectively), \( y^* \) is the across-front direction, \( b \) is the buoyancy frequency squared \( N^2 \), \( f \) is the Coriolis frequency, \( y_g \) is the across-frontal geostrophic flow, and \( \alpha = \sqrt{(u_x - u_y)^2 + (v_x + u_y)^2} \) is the subinertial horizontal confluence. The first equality on the right-hand side of (6) [i.e., \(-2(\partial v^a/\partial y^*)(\partial b/\partial y^*)\)] is the across-front component of the frontogenetic \( Q \) vector (Gill 1982), defined as a product of horizontal buoyancy gradient and confluence, so scales more generally as \( \alpha |V_h b| \) as in the second \((\sim)\) equality on the right-hand side of (6). The \( \omega \) equation [6] illustrates that quasigeostrophic up- and downwelling \( \partial\psi/\partial y^* \) arise when nondivergent subinertial flow sharpens or weakens isopycnal slopes in fronts. The frontogenetic \( Q \) vector on the right-hand side of (6) can be interpreted as the forcing due to this mechanism acting to break geostrophy, while the cross-front secondary circulation \( \psi \) arising from this forcing acts to restore geostrophy. Geostrophic adjustment is a well-known mechanism for generation of internal waves (Rossby 1938). Frontogenesis and frontolysis by confluent flows provide the mechanism to force fronts out of balance so they will undergo geostrophic adjustment. Although the vertical velocity, \( w = \psi_y \), diagnosed from (6) is formally in quasigeostrophic balance, it deforms the buoyancy field and can become unstable, radiating internal waves (Snyder et al. 1993).

Scaling \( y^* \) by the front width \( L \), \( z \) by the front depth \( H \), and streamfunction \( \psi \) by \( wL \), the QG-\( \omega \) equation [6] implies

\[
 w \sim \frac{H}{L} \frac{\alpha |V_h b| H}{N^2 \left( \frac{H}{L} \right)^2 + f^2}.
\]

FIG. 14. (a) Time evolution of positive zonally averaged depth-integrated interaction terms on the right-hand side of (2). Gray contours demark surface density. (b) Scatterplot of internal-wave energy source and positive zonally averaged depth-integrated interaction terms. Data exceeding 1.5 standard deviations are excluded from these. The black line represents bin-averaged interaction terms as a function of internal-wave energy source. Gray shading indicates one standard deviation. The bottom sponge layer is excluded in these calculations.

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Replacing $H$ in the numerator by the vertical integral, $H/L$ by isopycnal slope $s = |\nabla b|/N^2$, vertical kinetic energy scales as

$$VKE_{OG} = \frac{1}{2}w^2 \sim \left( \frac{\langle \int_{-H_b}^0 \alpha|\nabla b|dz \rangle}{\langle N^2 \rangle^{\frac{1}{2}} + f^2} \right)^2,$$

where $H_b$ is the bottom depth and $\int_{-H_b}^0 \alpha|\nabla b|dz$ is the frontogenetic integral. The overbar represents a depth average over $H_f = \int_{-H_b}^0 |\nabla b|dz/\max(\alpha|\nabla b|)$ and $\langle \cdot \rangle$ is a temporal average (over a week). The first term in the denominator of (8) is typically smaller than $f^2$ for large Richardson number $R_i \gg 1$, implying that $VKE_{OG}$ is larger for narrower, stronger fronts driven out of geostrophic balance by stronger confluence.

b. Relation between $VKE_{OG}$ and $E_{IW}$

Using the 1-km-resolution regional PSOM model, we demonstrate that the energy transfer rate to internal waves $E_{IW}$ is related to the vertical kinetic energy $VKE_{OG}$ in the cross-frontal circulation that restores balance (Hoskins et al. 1978). By running the regional PSOM model for a range of frontal strengths, a relation between quasigeostrophic vertical kinetic energy $VKE_{OG}$ and internal-wave power gain $E_{IW}$ is established. To produce a range of isopycnal slopes and frontogenetic integrals, the initial and day-7 buoyancy fields in the original run of PSOM for 750- and 2000-m-deep runs, respectively, are decomposed into mean and deviations as

$$b = \overline{b}(x,z) + cb'(x,y,z),$$

with $c = 1$ for the original Kuroshio cross section. Here, $\overline{b}(x,z)$ is the meridional mean that is a function of alongfront coordinate $x$ and depth $z$, and $b'(x,y,z)$ the deviation from $\overline{b}$ due to across-front variability in the Kuroshio Front structure. Frontal strength $c$ is varied ($c = 0.25, 0.5,$ and $0.75$) to produce different initial conditions in thermal-wind balance. The impact of varying frontal strength $c$ and Coriolis frequency $f$ on the dependent variables—isopycnal slope $s = |\nabla b|/N^2$ and frontogenetic integral $\int_{-H_b}^0 \alpha|\nabla b|dz$—in these PSOM simulations is listed in Table 1; vertical stratification $N^2$ is nearly constant ($3.0 - 3.3 \times 10^{-4} \text{s}^{-2}$ not shown), while isopycnal slope and frontogenetic integral vary by an order of magnitude. $VKE_{OG}$ is not determined directly but inferred from the frontogenetic forcing of the QG $\omega$ equation [(6)] using (8). Each term in (8) is first computed by 30-h lowpass-filtered subinertial flow along the semi-Lagrangian isobaric floats, which is then averaged onto an Eulerian grid before the temporal $\langle \cdot \rangle$ and vertical averaging in (8). After development of a 100-km-amplitude meander of a few 100-km wavelength around day 6, the rms internal-wave flux divergence is relatively invariant with time. To reduce the spatial gap between peak locations of
VKE_QG and internal-wave energy-flux divergence evident in Fig. 12, the VKE_QG, internal-wave power gain $\mathcal{E}_{IW}$, and internal-wave energy reabsorption rate $\mathcal{E}_{Absp}$ are further smoothed over 40 km $\times$ 40 km. As the reabsorption rate estimated with all the terms on the right-hand side of (2) has large scatter because of the gradients (Fig. 15), the reabsorption rate is inferred from negative terms on the left-hand side of (2) minus dissipation, $\left(\int (DE_{IW}/Dt)\,dz + \mathbf{V} \cdot \int (\mathbf{p}'\mathbf{u}')\,dz < 0 \right) - \int (\mathcal{E}_{VKE})\,dz$. Averaging more than a week is sufficient to produce an empirical power law for the top 70 percentile of each internal-wave power gain $\mathcal{E}_{IW}$ and VKE_QG:

$$\mathcal{E}_{IW} = \beta_1 \mathcal{VKE}_{QG}^\beta,$$

where $\beta_1 = (1.217 \pm 0.006, p = 0.025)$ (Fig. 16a). The internal-wave energy reabsorption rate $\mathcal{E}_{Absp}$ is also related to VKE_QG as

$$\mathcal{E}_{Absp} = \xi_1 \mathcal{VKE}_{QG}^\xi,$$

where $\xi_1 = (5.837 \pm 0.002)$ and $\xi_2 = (1.217 \pm 0.006, p = 0.025)$ (Fig. 16b). Results are not significantly altered when model horizontal resolution is refined to 500 m (appendix A), the model depth is extended to 2000 m (appendix B), or when a hydrostatic model is used (appendix C), suggesting that the phenomenon is hydrostatic and well resolved in the 1-km-resolution 750-m-deep PSOM model.

c. Global estimation of internal-wave power gain

We next use the 0.1°-resolution global OFES simulation to estimate VKE_QG [(8)] and hence $\mathcal{E}_{IW}$ from

![Graphs showing internal-wave energy sources and reabsorption rates](image-url)
scaling (10) for the World Ocean. VKEQG in the global model is based on arithmetic averages of each term in (8) over a week. To be consistent with the 40-km spatial resolution of VKEQG computed in PSOM, OFES is further smoothed over $0.4^\circ \times 0.4^\circ$. Quasigeostrophic VKEQG [(8)] is comparable in PSOM and OFES models when solutions are averaged over a week. In the global model, major ocean frontal systems, including western boundary currents and Antarctic Circumpolar fronts, stand out for their contributions to VKEQG (Fig. 17a). The Indian Ocean also stands out, likely because of the monsoon wind forcing to the north and the Agulhas

FIG. 17. Global distribution of (a) the gross internal-wave power gain $\mathcal{E}_{IW}$ and (b) the maximum net loss $\mathcal{E}_{IW} - \mathcal{E}_{Abs}$ as inferred from the empirical relations to VKEQG in (10) and (11) evaluated from daily output of the 0.1°-resolution OFES general circulation model and smoothed over 2 weeks and over $0.4^\circ \times 0.4^\circ$. The maximum net loss is computed from the confidence intervals in (10) and (11). The globally integrated internal-wave power gain $\mathcal{E}_{IW}$ is $\sim-0.36$ TW (Table 2) while the net loss is $\sim0.001$–$0.047$ TW. As with wind power (Wunsch 1998), the equatorial band between $\pm3^\circ$ is omitted.
retroflection to the south. The globally integrated inferred internal-wave power gain $E_{\text{IW, global}}^\text{global}$, excluding a zonal band within $\pm 3^\circ$ of the equator, is 0.36 TW (see Table 2) from the quasigeostrophic circulation and eddy field to the internal-wave field. Reabsorption rates $E_{\text{Absp}}$ have similar magnitudes to the internal-wave power gain $E_{\text{IW}}$. With the combined uncertainty in the fits of (10) and (11), it is hard to provide reliable estimates for the

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<th>Upper bound</th>
<th>$\beta_2 = 1.253 \pm 0.003$</th>
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Fig. 18. (a) Upper-300-m-depth-averaged $\sigma_\theta$ as a function of meridional distance $y$ relative to the Kuroshio Front from 2008, 2009, 2011, and 2012 density measurements (solid) and model density (dashed), and (b) 80–250-m-depth-averaged dissipation rates $\epsilon$ as a function of meridional distance $y$ relative to the Kuroshio Front from 2008, 2009, 2011, and 2012 fine- and microstructure measurements (solid) and model inferences of total kinetic energy dissipation rates, $E_{K, exp}$ (dashed). Shadings for the solid curves are 95% confidence intervals estimated with 300 bootstrap resamplings and for the dashed curves 95% confidence intervals based on normal distribution fits.
net losses, $\mathcal{E}_{IW} - \mathcal{E}_{Absp}$, but the net loss is 0.001–0.047 TW, no more than $O(15\%)$ of the gross internal-wave power gain $\mathcal{E}_{IW}$ (Fig. 17b), indicating that most of the inertial-wave generation is reabsorbed rather than being dissipated. Thus, these interactions represent a redistribution of subinertial quasigeostrophic energy rather than a significant net loss.

5. Conclusions

Banded shear signals are observed in the Kuroshio Front under calm summer conditions (Fig. 3a). The residual shear has rotary properties in depth and time (Fig. 4), consistent with propagating near-inertial waves. Previous work (Williams et al. 2008; Polzin 2010;
Thomas (2012) has suggested that frontal instabilities could transfer energy out of the quasigeostrophic flow field into near-inertial waves at rates of 0.1–1.5 TW, suggesting that this mechanism could represent a major sink for the large-scale balanced circulation.

These results motivated numerical simulations of an unforced meandering Kuroshio Front which spontaneously generates large-amplitude near-inertial internal waves associated with frontogenesis/frontolysis. Internal-wave energy is generated at a rate of $O(10)$ mW m$^{-2}$, about 10 times higher than estimated in the North Pacific Subtropical Front (Alford et al. 2013). Global estimates of the spontaneous internal-wave generation are 0.36 TW based on a scaling [(10)] between internal-wave power gain $E_{IW}$ and quasigeostrophic vertical kinetic energy $VKEQG$ (Fig. 16), similar to the global magnitudes inferred by Ferrari and Wunsch (2009) following Sargasso Sea transfer rate estimates by Polzin (2010) and analytically by Thomas (2012). However, our simulations find that most of the internal-wave power gain is reabsorbed into the subinertial flow so that the net loss from quasigeostrophic energy to model dissipation is no more than 15% of the internal-wave power gain. Thus, compared to other mechanisms such as (i) drag in bottom boundary layers (0.1 TW; Wunsch and Ferrari 2004) and (ii) topographic lee-wave generation (0.2 TW; Nikurashin and Ferrari 2011), spontaneous generation of near-inertial waves (Fig. 1) does not represent a major sink for the large-scale circulation as previously suggested, but only a local redistribution. The gross generation rates are also comparable to low end estimates of 0.2–1.5-TW wind forcing of near-inertial waves (Alford 2001; Furuichi et al. 2008; Plueddemann and Farrar 2006; Rimac et al. 2013), making it a potentially significant, if frontally confined (Fig. 17), source of ocean internal waves.

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**Fig. A2.** (a) Scatterplot of normalized horizontal divergence of the full velocity $(u_i + v_i)/f$ at 100-m depth for 1000- and 750-m-resolution model runs using days 7.5, 9.5, and 11.5. The data from coarser-resolution horizontal grids are interpolated onto the 500-m grid. (b) As in (a), but for 1000- and 500-m model resolutions. (c) Histogram of horizontal divergence for (blue) 1000-, (green) 750-, and (red) 500-m resolution. (d) Quantile–quantile plots for horizontal divergence between (blue) 1000- and 750- and (red) 1000- and 500-m resolution.
Directly measured TKE dissipation rates during August 2008, October 2009, August 2011, and August 2012, averaged over 80–250-m depth and within 30 km of the Kuroshio Front, are over $10^{-2} \text{Wkg}^{-1}$, which is an order of magnitude larger than the similarly averaged model subgridscale kinetic energy dissipation rates (Fig. 18). We emphasize that our model is unforced and contains no internal waves at the outset so that all waves originate from the initially geostrophically balanced flow through frontal instability. In the ocean, superposition of frontally generated waves with the background wave field from winds and tides may allow more of the spontaneously generated near-inertial wave energy to be dissipated before it can be reabsorbed as in the numerical model. But wind-forced near-inertial waves may also be trapped and dissipated in fronts so we cannot be certain whether dissipation rates are much larger in the ocean than in the model because of wind-forced or spontaneously generated near-inertial waves.

Scalings from QG vertical kinetic energy $\text{VKE}_{\text{QG}} \quad ([10]–[11])$ provide an empirical parameterization that might be used for global models with sufficient resolution. However, more direct observations and modeling are needed to identify and understand spontaneous generation of the near-inertial internal waves, elucidate how much spontaneously generated wave energy dissipates through wave breaking versus being reabsorbed, and verify that (10) and (11) parameterize these mechanisms in climate models. The question of how the wind-induced large-scale circulation dissipates remains.

**Acknowledgments.** We thank Captain Noda and crews of R.T.V Umitaka-maru, Captain Ukekura and crews of R/V *Natsushima*, Dr. Sasaki at JAMSTEC for OFES outputs, JSPS (KAKENHI 24684036), “The Study of Kuroshio Ecosystem Dynamics for Sustainable Fisheries (SKED)” supported by MEXT, MIT-Hayashi Seed Fund, ONR (Awards N000140910196 and N000141210101), NSF (Award OCE 0928617, 0928138) for support, and two anonymous reviewers for their insights.
APPENDIX A

Sensitivity to Model Resolution

Previous work has shown that model resolution impacts the amplitude and frequency of internal-wave generation in atmospheric fronts (Plougonven and Snyder 2007). Divergences \( u_x + v_y \) at 100-m depth differ little between 1000-, 750-, and 500-m-resolution PSOM runs. Near-inertial convergence and divergence is most frequently observed in highly confluent regions of order \( \lesssim 10 \text{ km} \) wide, regardless of resolution (Fig. A1).

A histogram shows that divergence approaches \( \pm 0.8f \) at 1000-m resolution and \( \pm f \) at 750- and 500-m resolution (Fig. A2). The extreme values occupy 0.02% and 0.04% of the model domain for 750- and 500-m-resolution runs, respectively. A quantile–quantile plot shows that the difference in the distribution of divergence between 500- and 1000-m-resolution runs only becomes noticeable for divergence magnitudes \( |u_x + v_y| > 0.5f \). The Kolmogorov–Smirnov test shows that the null hypothesis of equal distribution between 1000- and 500-m-resolution cases is rejected with 95% confidence intervals (\( p \) value: \( 4.7 \times 10^{-100} \)) because of large extreme values at 500-m resolution (Fig. A2).

The vertical velocity at 100-m depth shows more similarity among the three resolutions (Fig. A3), although the same null hypothesis is rejected (\( p \) value: \( 9.3 \times 10^{-31} \)) because \( w \) is skewed toward stronger downwelling (Mahadevan and Tandon 2006) in the 500-m-resolution run (Fig. A3). Stronger downwelling is associated with superinertial \( w' \) mostly by internal waves in highly stratified simulations initialized with summer observation data. Here, 8.2% (1.6%) of the data points differ in flow divergence \( u_x + v_y \) (vertical velocity \( w \) ) between 1000- and 500-m resolution by more than one standard deviation. Compared to the 1000-m run, the magnitude of the power gain increases by as much as 6% and 11% at 750- and 500-m runs. However, the timing and locations of the sources and sinks are indistinguishable among different resolutions. These results suggest that our 1-km model resolution is sufficient to simulate near-inertial wave generation by the unstable Kuroshio Front, though it may underestimate the power transfer from balanced flow to internal waves by up to 10%.

APPENDIX B

Sensitivity to Depth of the Model Domain

Our PSOM model domain is limited to 750 m vertically. The real Kuroshio region has a depth of about 4000 m. The depth of the domain may alter baroclinic instability and associated confluence fields, which are important for wave generation. Although the mesoscale meander takes several weeks longer to develop in a 2000-m-deep domain than the 750-m domain, the 2000-m simulations exhibit similar banded structures in horizontal divergence, signifying generation of internal waves (Fig. B1). Downward propagation of the waves below 1000-m depth and their reflection from the bottom are seen, because the bottom sponge layer has been removed.
results from the 2000-m-depth model scale identically to the 750-m domain for the internal-wave power gain \([6]\), suggesting that the scaling law can be applied regardless of domain depth (Fig. 16) and results from the global OFES model are still valid.

**APPENDIX C**

**Hydrostatic versus Nonhydrostatic**

We also ran the hydrostatic primitive equation ROMS model (Shchepetkin and McWilliams 2005) to examine internal-wave generation by meandering fronts. This model was initialized with the same August 2008 density section (section 3a). The model’s horizontal resolution is \(\sim 1000\) m and uses 100 vertical levels. No explicit diffusion is included. Banding in horizontal divergence emerges at the meander trough and crest with similar magnitude and wavelengths as the PSOM simulations, suggesting that inertial-wave generation is hydrostatic.

**REFERENCES**


