Why Do LES of Langmuir Supercells Not Include Rotation?

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(Manuscript received 11 April 2016, in final form 20 October 2016)

ABSTRACT

Existing large-eddy simulations (LES) of Langmuir supercells (LS) do not include rotational terms. Despite the fact that the actual coastal ocean is certainly affected by rotation, such simulations are found to provide excellent agreement with a wide range of features of LS observed in the shallow coastal ocean. This note explains why it is indeed acceptable to compare results of a nonrotational LES of LS in a laterally unbounded domain with observations of LS made in a rotating fluid with a lateral boundary.

1. Introduction

Langmuir supercells, wind-/wave-driven full-depth Langmuir circulation (LC) in homogeneous water columns, were first discovered in observations from the LEO15 cabled coastal observatory (Gargett et al. 2004; Gargett and Wells 2007) off New Jersey, where they occur under storm conditions with winds from the northeast. As long as the wind/wave forcing remained quasi-steady in magnitude and direction, LS structures in this shallow shelf environment were found to be steady (when averaged, in time and/or downwind spatial dimension, over minor fluctuations associated with the turbulent nature of flow). In associated work, Tejada-Martínez and Grosch (2007, henceforth TMG) created a specialized LES model using wave-averaged momentum equations containing the Craik–Leibovich (C–L) vortex force as derived by Craik and Leibovich (1976) and partially resolving surface and bottom boundary layers. With forcing parameters derived from observational conditions, their LES results agreed with the large-eddy structure of observed LS in many details: structure of 3D velocity fields, normal and shear stress profiles, Lumley invariants, and the steady-state nature of the turbulent structures. Unremarked at the time (and since) was the omission of rotational terms from the LES of TMG and subsequent shallow-water simulations (Kukulka et al. 2011; Martinat et al. 2011; Akan et al. 2013). In contrast, deep-ocean LES of LC normally include rotational terms; onset of a steady wind in this case produces "mean" (inertial) currents that rotate off the wind/wave direction with time, breaking up originally structured LC into much less organized "Langmuir turbulence" (McWilliams et al. 1997) that cannot be considered "steady" unless averaged over many inertial periods (Tejada-Martínez et al. 2009). The purpose of this note is to demonstrate that the results of TMG are nonetheless valid for the shallow-water case because of the difference in the effects of rotation in the presence of a lateral boundary. Specifically, it is shown that to first-order, identical steady mean flows result from the action of a constant surface force (wind stress) on a shallow homogeneous water column in two cases: case A, with rotation and with a lateral boundary to the right of the direction of the surface force, and case B, without rotation and without a lateral boundary, the former being the case of the observational study and the latter that of the LES of TMG. The (first order) equivalence of the mean flows in both cases implies that LES run without the C–L vortex force will exhibit the same turbulence in both flows (weak Couette turbulence) and that addition of the C–L force will produce the same changes to the turbulence of both flows.

2. Formulation of the model problem

Figure 1 is a sketch of a long straight coast at $x = 0$, with the x axis normal to the coast and y axis parallel to the coast. The water off the coast has constant depth H and constant density ρ. A surface wind stress $\boldsymbol{\tau}$ acts

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DOI: 10.1175/JPO-D-16-0092.1

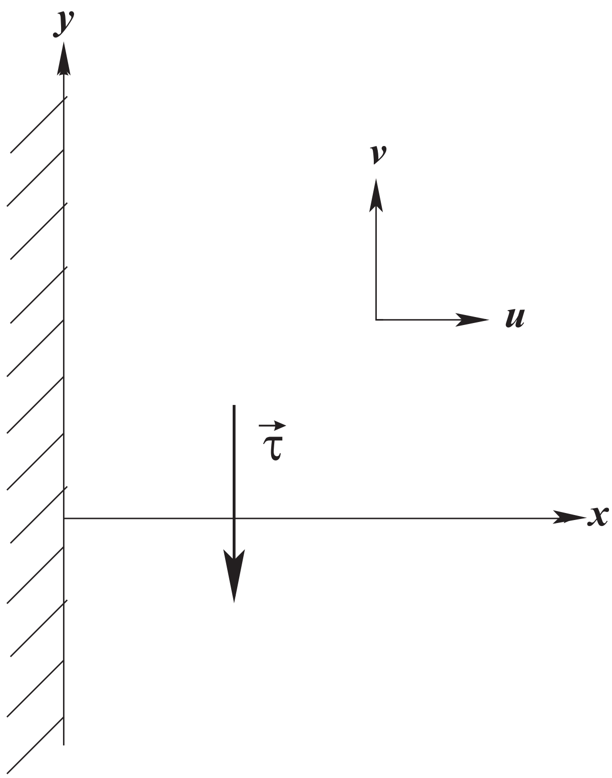

FIG. 1. Sketch of a long, straight coast at $x = 0$. The x axis is normal to the coast, and the y axis is parallel to the coast. The water off the coast has constant depth and constant density. Alongshore surface wind stress $\boldsymbol{\tau}$ acts in the direction shown.

parallel to the coast in the direction shown. It is assumed that the governing equations are the depth-averaged mass conservation and momentum equations, including a Coriolis term. The velocity components are u and v in the x and y directions, respectively, and η is the deviation of the surface from H. The resulting (linearized) shallow-water equations are

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} - \gamma u + \frac{\tau_x}{\rho H}, \quad \text{and} \tag{2}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} - \gamma v + \frac{\tau_y}{\rho H}, \tag{3}$$

with f as the Coriolis parameter, g as the acceleration of gravity, and $\tau_x/\rho H$ and $\tau_y/\rho H$ as the x and y components of the acceleration generated by the wind stress $\boldsymbol{\tau}$. Linear Rayleigh damping with damping coefficient γ is used as the simplest possible friction model. In the most basic problem, $\tau_x = 0$ and $\tau_y = -\tau_o$, where $\tau_o > 0$, with boundary conditions $u(0, y, t) = 0$ and (u, v, η) bounded as $x \to \infty$ and as $y \to \pm\infty$. In general there will be an initial condition so the problem is an initial-boundary value problem; however, only the resulting steady mean flow is of interest.

a. Case A: With rotation and a lateral boundary

Imposition at $t = 0$ of the constant stress shown in Fig. 1 generates initial flow in the negative y direction. In a rotating system, the subsequent sequence of events is well known; the Coriolis "force" turns the initial flow to the right, causing water to pile up at the coast and generating a pressure gradient in the positive x direction. This offshore pressure gradient drives a flow in the positive x direction, which is then also turned to the right by the Coriolis force. The eventual result is a steady current in the negative y direction.

For steady flow, $\partial(\cdot)/\partial t = 0$. With $\tau_x = 0$ and τ_y constant in time and in y, the simplest way to satisfy the boundary conditions is to set $u \equiv 0$ and assume that v and η are independent of y, that is, functions only of x. Under these assumptions, the steady-state equations are

$$\frac{\partial v}{\partial y} = 0, \tag{4}$$

$$-fv = -g\frac{\partial \eta}{\partial x}, \quad \text{and} \tag{5}$$

$$0 = -\gamma v - \frac{\tau_o}{\rho H}, \tag{6}$$

$$\text{with solution } u \equiv 0, v = -\frac{\tau_o}{\gamma \rho H}. \tag{7}$$

With v known, $\eta(x)$ can be found by integrating Eq. (5), the geostrophic balance in the offshore direction.

b. Case B: No rotation and no lateral boundary

With $f = 0$, the steady-state Eqs. (1)–(3) reduce to

$$\frac{\partial v}{\partial y} = 0, \tag{8}$$

$$0 = -g\frac{\partial \eta}{\partial x}, \quad \text{and} \tag{9}$$

$$0 = -\gamma v - \frac{\tau_o}{\rho H}, \tag{10}$$

$$\text{with solution } u \equiv 0, v = -\frac{\tau_o}{\gamma \rho H}. \tag{11}$$

In this case, η is a constant (which could without loss of generality be taken as zero). Although the free-surface slope differs between the two cases, the time-mean velocities [Eqs. (7) and (11)] are identical.

3. Discussion and conclusions

In case A, an offshore pressure gradient is balanced by the Coriolis term associated with alongshore flow $v(x)$. The acceleration of this alongshore current is balanced by a frictional force, here modeled by linear Rayleigh friction. In this case, while the steady current $v(x)$ is parallel to the applied stress, it is not the direct result of this stress. It is instead a geostrophically balanced current caused by interaction of the stress, the Coriolis force, the boundary, and frictional drag. For case B, the mean velocity components are identical to those of case A, but in the absence of geostrophic balance ($f = 0$), alongshore flow is driven directly by the imposed stress. The only difference between the two cases lies in the free-surface slope; the velocity components are identical in both cases. We conclude that, to first order, identical steady mean flows result from the action of a surface wind stress on a shallow homogeneous water column in two apparently quite different cases: case A, with rotation and with a lateral boundary to the right of the direction of the surface stress, and case B, without rotation and without a lateral boundary, the former being the case of the observational study and the latter that of the LES of TMG.

Although the above interpretation of the actual mean flow as geostrophically balanced is highly simplified, its validity is supported by observations at two separate locations: LEO15 in 15 m of water ~7 km off the New Jersey shore and R2, a naval tower in 26m of water ~70 km off the coast of Georgia. At both sites, time-mean, depth-averaged flow observed during conditions of strong constant winds and homogeneous water columns is not accurately downwind but instead approximately shore parallel [~13° to the left of the downwind direction at LEO15 (Gargett et al. 2004) and ~23° to the left of downwind at R2 (D. Savidge 2016, personal communication)]. In addition, immediately after the onset of a strong wind event at LEO15, Gargett et al. (2014) document the shoreward surface layer flow expected during setup of the offshore pressure gradient that results in the steady geostrophic flow of case A.

With the mean flow of case B, LES without wave forcing produces characteristic (Couette) turbulence; since (to first order) the mean flow of case A is identical, the turbulence associated with case B will be the same. Addition to either case of the C–L vortex force parameterizing wave forcing will produce identical modifications to this basic turbulence. Prima facie evidence for this equivalence is found in the close agreement of results produced by the LES of case B with all available features of the observed turbulence structures. These features include the trajectory of invariants of the Reynolds stress anisotropy tensor in the Lumley triangle, a particularly sensitive diagnostic of turbulent structures.

In conclusion, to first order it is indeed acceptable to use the results of a nonrotational LES of LS in a laterally unbounded domain (case B) for comparison with existing observations of LS in a rotating fluid with a lateral boundary to the right of the surface force direction (case A). However, despite the formal equivalence of the mean flows demonstrated above, the actual difference in the physical nature of this flow in each case should be acknowledged; in the LES, mean flow is stress driven, and in the observational setting, it is geostrophic.

Acknowledgments. The authors gratefully acknowledge support from the National Science Foundation (OCE-0136403 and OCE-0927724) and NOAA (NA06RU0139).

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