A Thickness-Weighted Average Perspective of Force Balance in an Idealized Circumpolar Current

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ABSTRACT

The exact, three-dimensional, thickness-weighted averaged (TWA) Boussinesq equations are used to diagnose eddy–mean flow interaction in an idealized circumpolar current (ICC). The force exerted by mesoscale eddies on the TWA velocity is expressed as the divergence of the Eliassen–Palm flux tensor. Consistent with previous findings, the analysis indicates that the dynamically relevant definition of the ocean surface layer is composed of the set of buoyancy coordinates that ever reside at the ocean surface at a given horizontal position. The surface layer is found to be a physically distinct object with a diabatic and force balance that is largely isolated from the underlying adiabatic region in the interior. Within the ICC surface layer, the TWA meridional velocity is southward/northward in the top/bottom half and has a value near zero at the bottom. In the top half of the surface layer, the zonal forces due to wind stress and meridional advection of potential vorticity act to accelerate the TWA zonal velocity; equilibrium is obtained by eddies decelerating the zonal flow via a downward flux of eastward momentum that increases with depth. In the bottom half of the surface layer, the accelerating force of the wind stress is balanced by the eddy force and meridional advection of potential vorticity. The bottom of the surface layer coincides with the location where the zonal eddy force, meridional advection of potential vorticity, and zonal wind stress force are all zero. The net meridional transport $S_f$ within the surface layer is a small residual of its southward and northward TWA meridional flows. The mean meridional gradient of the surface layer buoyancy is advected by $S_f$ to balance the surface buoyancy flux.

1. Introduction

The uniqueness of ocean dynamics within the Antarctic Circumpolar Current (ACC) has long been recognized (Munk and Palmén 1951; Rhines and Holland 1979; Johnson and Bryden 1989). The ACC balances wind stress forcing in a different manner than subtropical gyre systems that develop dissipative western boundary currents along meridionally oriented obstructions. Overall, the general paucity of the land–ocean boundaries in the vicinity of the ACC prevents lateral dissipation from playing a leading role. Rather, both scaling (Munk and Palmén 1951) and observational (Johnson and Bryden 1989; Phillips and Rintoul 2000) analyses suggest that the eddy-induced, vertical transport of momentum is of central importance in balancing wind stress forcing within the ACC.

The importance of ocean eddies in the vertical transmission of momentum required to balance wind stress forcing has been considered from the outset. Even with minimal observational data, Munk and Palmén (1951) concluded that the viscous transfer of momentum based on plausible estimates of viscosity and vertical shear is insufficiently small to transmit the wind stress from the ocean surface to the ocean bottom. Rather, Munk and Palmén (1951, p. 54) suggest that “each layer induces, by turbulent interchange of momentum, motion in the layer beneath, and in this manner the wind stress is transmitted to the sea bottom.” This physical phenomenon is now typically referred to as interfacial form stress.

Since mesoscale eddies are very well approximated by geostrophic balance, the vertical transport of zonal momentum is equivalent to a meridional transport of buoyancy (Johnson and Bryden 1989). As a result, the
importance of mesoscale eddies can be measured by either their vertical flux of eastward momentum or their meridional flux of buoyancy (McWilliams et al. 1978; Johnson and Bryden 1989; Phillips and Rintoul 2000; Marshall and Radko 2003; Howard et al. 2015). While both approaches offer insights into the role of eddies in the ACC, the two frameworks differ markedly in how we interpret and explain the role of mesoscale eddies in the ACC. The buoyancy flux framework leads to a perspective by which the Ekman layer–driven overturning is balanced by an eddy-induced overturning. This system is well described in Fig. 2 of Marshall and Radko (2003). But since the wind stress in and of itself is incapable of producing an overturning circulation, the buoyancy flux framework results in an explanation where two large overturning circulations exactly balance. So while geostrophic balance allows us to equate a wind stress forcing with an eddy-induced lateral flux of buoyancy, it is not clear that this leads to the most concise description of the underlying fluid dynamics.

Instead, we attempt below to further understand the influence of mesoscale eddies in circumpolar currents by diagnosing the balance of forces, that is, force equals mass times acceleration ($F = ma$), in the zonal direction of a circumpolar channel. Previously, such an analysis has been employed for idealized circumpolar currents (McWilliams et al. 1978) as well as the ACC (Mazloff et al. 2013), using both observational data (Johnson and Bryden 1989) and eddying ocean simulations (Gille 1997). Our goal is to contribute to this well-explored approach to understanding ocean dynamics in the ACC in two ways. Our first goal is to develop an analysis method that measures the role of eddies in the climate of circumpolar currents based on the exact mathematical framework developed in de Szoeke and Bennett (1993), Young (2012), and Maddison and Marshall (2013). Our second goal is to conduct the analysis with sufficient vertical resolution so that we can obtain an understanding of the force balance in different parts of the ocean water column.

In section 2, we review the three-dimensional, thickness-weighted averaged (TWA) equations that allow for the creation of exact ensemble mean-field equations. The idealized configuration used to study the force balance in an idealized circumpolar channel is described in section 3. The analysis of this system is presented in section 4, and conclusions are made in section 5.

2. Diagnosis of the TWA system

While the numerical model used to simulate the strongly eddying circumpolar current uses a standard Eulerian representation of the Boussinesq equations, the analysis of system dynamics is conducted entirely in buoyancy space. In particular, the goal is to diagnose the balance of forces in the buoyancy space representation of the TWA zonal momentum equation.

To begin, the TWA equations are obtained by first averaging over microstructural length scales (de Szoeke and Bennett 1993) in order to obtain a monotonic relationship between fluid depth and buoyancy. Such a relationship implies that the system is stably stratified. From Young (2012) and Maddison and Marshall (2013), the TWA Boussinesq equations expressed in buoyancy coordinates are written as

\[
\frac{D^t\tilde{u}}{Dt} + f\tilde{v} + \overline{m}_x + e_1 \cdot (\nabla \cdot \mathbf{E}) = \tilde{X}, \tag{1}
\]

\[
\frac{D^t\tilde{v}}{Dt} + f\tilde{u} + \overline{m}_y + e_2 \cdot (\nabla \cdot \mathbf{E}) = \tilde{Y}, \tag{2}
\]

\[
\tilde{\xi} + \overline{m}_b = 0, \tag{3}
\]

\[
\overline{\sigma}_t + (\overline{\sigma}\tilde{u})_x + (\overline{\sigma}\tilde{v})_y + (\overline{\sigma}\phi)_b = 0, \quad \text{and} \tag{4}
\]

\[
\frac{D^t\overline{b}_g}{Dt} = \tilde{S}. \tag{5}
\]

Equations (1)–(5) represent equations for zonal velocity $\tilde{u}$, meridional velocity $\tilde{v}$, hydrostatic balance, continuity, and buoyancy, respectively. The coordinates $\tilde{u}, \tilde{v}, \tilde{b}$ measure time, zonal distance, meridional distance, and buoyancy, where $e_1$ and $e_2$ are orthogonal zonal and meridional vectors in buoyancy space, respectively. Buoyancy is defined by $\rho = \rho_0(1 - g^{-1}b)$, where $\rho, \rho_0$, and $g$ are the potential density, reference density, and gravity, respectively. Derivatives are indicated with a coordinate label subscript. The $(\cdot)$ connected to coordinate variables $\tilde{i}, \tilde{x}, \tilde{y}, \tilde{b}$ indicates that measurements are made in buoyancy space. The $(\cdot)$ is the ensemble-averaging operator. The strict implementation of this operator requires averaging over an ensemble of realizations at a fixed $(\tilde{i}, \tilde{x}, \tilde{y}, \tilde{b})$. The simulations considered below are assumed to be in a statistically steady state, so the ensemble members can be obtained by sampling across the time coordinate. The material derivative is expressed as

\[
\frac{D^t\overline{b}_g}{Dt} = \partial_t + \overline{\omega} \partial_x + \overline{\omega} \partial_y + \overline{\omega} \partial_b, \tag{6}
\]

where $\overline{\omega}$ is the velocity across a buoyancy surface. The sharp superscript $(\cdot)^\dagger$ denotes variables that are not associated with averaged quantities but that are consistent with the dynamics of the averaged system.

One reason to refer to this equation set as the thickness-weighted averaged equations is that velocities are obtained via the TWA operator as
\[
\dot{\vec{u}} = \frac{\nabla \vec{u}}{\sigma} \quad \text{and} \quad \dot{\vec{v}} = \frac{\nabla \vec{v}}{\sigma},
\]
where \(\sigma = \vec{z}\), and \(\vec{z}\) is the ensemble-averaged vertical position of the buoyancy coordinate. The thickness of a buoyancy layer \(\sigma\) measures the ensemble-mean distance between buoyancy coordinates. It has units of seconds squared and is equivalent to the square of the inverse Brunt–Väisälä frequency based on \(\vec{z}\). Nonconservative forces applied to the zonal and meridional velocity equations are given by \(\dot{X}\) and \(\dot{Y}\), \(\vec{m}\) is the Montgomery potential, and \(f\) is the Coriolis parameter.

Equation (5) is a statement for conservation of buoyancy \(b^f\), where \(\vec{S}\) contains all diabatic terms. But \(b_j^f\), \(b_j^g\), and \(b_j^h\) are identically zero in buoyancy space, so Eq. (5) reduces to the specification of the buoyancy velocity, \(\vec{u} = \vec{S}\), that is often referred to as the diabatic velocity.

The elegance of the TWA equations is evident after recognizing that the Reynolds-like correlation terms appear only in the momentum equations in the form \(\nabla \cdot \vec{E}\), where \(\vec{E}\) is the Eliassen–Palm flux tensor (Maddison and Marshall 2013). The TWA system is identical in structure to the unaveraged system, apart from the appearance of \(\nabla \cdot \vec{E}\). The Eliassen–Palm flux tensor is written as

\[
\vec{E} = \begin{pmatrix}
\frac{1}{2\sigma} \vec{z} \cdot \vec{2} & \vec{u} \cdot \vec{v}^f & 0 \\
\vec{u} \cdot \vec{v}^f & \vec{v} \cdot \vec{v}^f + \frac{1}{2\sigma} \vec{2} & 0 \\
\sigma \vec{u} \cdot \vec{v}^f & \vec{v} \cdot \vec{v}^f + \frac{1}{\sigma} \vec{2} & 0
\end{pmatrix},
\]
where

\[
u'' = u - \dot{u},
\]
and

\[
\zeta' = \zeta - \vec{z}.
\]

In the above equations, \((\cdot)''\) is a deviation from thickness-weighted mean values, and \((\cdot)'\) is a deviation from ensemble-mean values. The TWA system specifies the divergence of a vector \(\vec{F}^d(\vec{x}, \vec{y}, \vec{b}) = \vec{F}^d_1 + \vec{F}^d_2 + \vec{F}^d_3\) as

\[
\nabla \cdot \vec{F}^d = \sigma^{-1}(\sigma \vec{F}^d_1) + \sigma^{-1}(\sigma \vec{F}^d_2) + \sigma^{-1}(\sigma \vec{F}^d_3),
\]
where \(\vec{e}_1 = \vec{i} + \vec{z}\), \(\vec{e}_2 = \vec{j} + \vec{z}\), and \(\vec{e}_3 = \vec{k}\), with \(i, j, k\) denoting standard Cartesian unit vectors [see Eq. (53) from Young (2012)].

One subtlety in the analysis of the TWA system is the outcropping problem. As described by Young (2012), imagine an observer positioned at a fixed \((\vec{x}, \vec{y}, \vec{b})\) coordinate who is tasked with determining the ensemble-mean \(z\) position \(\vec{z}\) of this coordinate value based on a set of instantaneous measurement. The \(z\) position \(\vec{z}\) of this observer will naturally change as the observer’s buoyancy coordinate moves up and down in the water column. So long as the buoyancy value of the observer is contained in the water column for all time, the ensemble-mean \(z\) position of the coordinate is defined without ambiguity. However, the observer’s buoyancy value may not exist in the fluid at their \((\vec{x}, \vec{y})\) location at a given time, that is, the observer’s buoyancy value is outcropped. Assume, without loss of generality, that this observer is outcropped at the ocean surface. Based on the analysis of Andrews and McIntyre (1976), Young (2012) suggests following the Lorenz convention whereby the outcropped observer records the \(z\) position of the ocean surface when their buoyancy value is lighter than the surface buoyancy value. Therefore, we assume that all buoyancy coordinates lighter than those at the ocean surface are stacked on the surface. Data samples that record \(\vec{z}\) as residing at the ocean surface are just as valid in the estimate of the ensemble mean as data samples of \(\vec{z}\) from the ocean interior [for more detail, see appendix A, Eqs. (A1)–(A3)]. Our diagnosis of the TWA system follows the Lorenz convention for outcropped buoyancy coordinates and differs from that used by Mazloff et al. (2013), where outcropped buoyancy coordinates are excluded from the averaging procedure.

The model configuration to be studied below does not vary in the \(x\) direction; the system is zonally symmetric. We denote the zonal-averaging operator with \([\cdot]\). First, we change the advective form of Eqs. (1) and (2) into their vector-invariant form, then we apply the zonal-averaging operator and assume steady state to produce the following set of balance equations:

\[
\begin{align*}
\hat{\vec{u}}_b - [\eta^d \vec{v}] + [\vec{e}_1 \cdot (\nabla \cdot \vec{E})] &= \vec{X}, \\
\hat{\vec{v}}_b + [\eta^d \vec{u}] + [\vec{m} + \vec{K}]_b + [\vec{e}_2 \cdot (\nabla \cdot \vec{E})] &= \vec{Y}, \\
[\vec{z}] + [\vec{m}]_b &= 0, \\
[\vec{z}] + [\vec{m}]_b &= 0, \quad \text{and} \\
[\vec{m}]_b &= \vec{S},
\end{align*}
\]

where \(\eta^d\) is the absolute vorticity, with the \((\cdot)^d\) denoting an evaluation in buoyancy coordinates based on \(\vec{u}\) and \(\vec{v}\), and \(\vec{K}\) is the kinetic energy [see Young (2012), his Eqs. (111) and (112)]. The focus of the analysis is the zonal momentum shown in Eq. (12). Defining \([\vec{X}]\) and expanding \([\vec{e}_1 \cdot (\nabla \cdot \vec{E})]\) yields

\[
[\hat{\vec{u}}_b] - [\eta^d \vec{v}] + \frac{1}{\sigma} [\vec{m} \vec{v}^f]_b + \frac{1}{\sigma} [\vec{z} \vec{m}^f]_b = -\frac{1}{\sigma} \left[ \frac{1}{\rho_0} + \frac{1}{\sigma} \frac{D}{\rho_0} \right],
\]

where
where \( \tau \) and \( \overline{D} \) are the surface and bottom stress, respectively, measured in newtons per square meter. Both \( \tau \) and \( \overline{D} \) are boundary conditions that are imposed at the interface of the outermost buoyancy layers at the top and bottom of the fluid, respectively. The vertical derivative with respect to \( b \) of these boundary-imposed stresses results in a force applied to the zonal momentum equation. A numerical algorithm to estimate all terms in Eq. (17) is described in appendixes A and B.

The \( \vec{v} \tau \) represents the meridional transport of zonal momentum by eddies. The \( \tilde{\zeta} m^3_y \) represents the vertical transport of zonal momentum and is commonly referred to as interfacial form stress. We discuss all of the terms in Eq. (17) in section 4.

3. Modeling system and simulation configuration

a. Modeling system

The ocean component of the Model for Prediction Across Scales (MPAS-Ocean; Ringler et al. 2013) is used as the modeling system to produce the simulation described below. MPAS-Ocean is built on a mimetic, finite-volume discretization in the horizontal (Thuburn et al. 2009; Ringler et al. 2010) with an arbitrary Lagrangian–Eulerian (ALE) vertical coordinate (Petersen et al. 2015). The horizontal discretization possesses discrete analogs of Kelvin’s circulation theorem and conservation of mechanical energy, making the model well suited for the simulation of highly rotating flows. MPAS-Ocean solves the Boussinesq equations where scalars are expressed in flux form and are advected with a monotone transport algorithm (Skamarock and Gassmann 2011). Thus, the tracer advection algorithm locally conserves volume-weighted tracer concentration and is bounds preserving.

A full listing of configuration parameters is provided in Table 1. The model is configured with 5-km resolution in the horizontal and 100 vertical layers. Since the horizontal domain is planar, we tessellate the 1000 km \( \times \) 2000 km region with 92 000 regular hexagons arranged on a 200 \( \times \) 460 grid. The ALE vertical coordinate is specified to mimic the traditional \( \gamma \) star coordinate (Adcroft and Campin 2004) with the layer thickness ranging from 0.63 m at the surface to 92.1 m at the bottom where the maximum ocean depth is 2500 m. Just over half of the 100 layers are contained in the top 250 m of the fluid.

The cascade of enstrophy to the grid scale is removed from the system with a variant of the biharmonic Laplacian operator where the hyperviscosity on the rotational and divergent parts of the flow are \( 7.8 \times 10^8 \) m\(^4\) s\(^{-1}\) and \( 7.8 \times 10^8 \) m\(^4\) s\(^{-1}\), respectively. This results in the simulation producing enstrophy and energy power spectra with slopes of approximately \(-1\) and \(-3\), respectively, extending down to wavelengths of 20 km or about four grid lengths.

The surface boundary layer is modeled using the K-profile parameterization (KPP) from Large et al. (1994). The background diffusivity and viscosity are set to \( 5.0 \times 10^{-6} \) m\(^2\) s\(^{-1}\) and \( 1.0 \times 10^{-4} \) m\(^2\) s\(^{-1}\), respectively. The bottom boundary layer stress is parameterized with quadratic bottom drag using a coefficient of \( 3.0 \times 10^{-3} \).

b. Simulation configuration

The configuration for the zonal idealized Southern Ocean (ZISO) follows closely that used by Abernathey et al. (2011), Stewart and Thompson (2013), and Saenz et al. (2015). The test case is zonally uniform with respect to all boundary conditions and forcing. The 1000 km spanned in the zonal direction \( L_x \) is periodic, and the 2000 km spanned in the meridional direction \( L_y \) is bounded by walls with a no-slip boundary condition.

The bathymetry \( h(y) \) is specified as

\[
h(y) = H_s + \frac{1}{2} (H - H_s) \left[ 1 + \tanh \left( \frac{y - Y_s}{W_s} \right) \right],
\]

where \( H = 2500 \) m is the maximum depth, \( H_s = 500 \) m is the shelf depth, \( Y_s = 250 \) km is the center \( y \) position of the shelf, and \( W_s = 500 \) km is the center \( y \) position of the shelf break.

The system is forced by a zonal wind stress and linear restoring of temperature to a specified meridional profile. Wind stress has the form

\[
\tau(y) = \begin{cases} 
\tau_0 \sin \left( \frac{\pi}{L_s} \frac{y - L_s}{L_y - L_s} \right)^2 & y \geq L_s \\
0 & L_s > y > L_s - W_sf \\
\tau_f \sin \left( \frac{L_s - y}{L_s} \right)^2 & y < L_s 
\end{cases}
\]

where \( \tau_0 = 0.2 \) N m\(^{-2}\), \( L_s = 800 \) km, \( W_sf = 600 \) km, and \( \tau_f = -0.05 \) N m\(^{-2}\). The flux of temperature \( T_f \) across the ocean surface is computed as \( T_f = -p(T - T_s) \), where \( p \) is the piston velocity with a value of \( 1.0 \times 10^{-5} \) m s\(^{-1}\) and \( T_s \) is the interior restoring time.
scale. Along the southern boundary on the shelf for $y = 0$ km interior restoring is used to mimic the production of deep bottom water, and interior restoring of temperature along the northern boundary is used to transform northward-flowing deep bottom water into southward-flowing middepth water for $y = 2000$ km, namely,

$$T_i(y, z) = T_r \exp \left( \frac{z}{z_e} \right), \quad (21)$$

with an interior restoring time scale of

$$\tau_i = \frac{1}{\tau_{T_r}} (e^{-y/L_c}), \quad (22)$$

$$y' = \begin{cases} y & \text{for } y = 0 \text{ km} \\ L_y - y & \text{for } y = 2000 \text{ km} \end{cases}, \quad (23)$$

where $z_e = 1000$ m, $\tau_{T_r} = 30$ days, $L_c = 80$ km, and $L_y = 2000$ km.

The simulation uses a linear equation of state with the form

$$\rho = \rho_{\text{ref}} - \alpha(T - T_{\text{ref}}),$$

where $\rho_{\text{ref}} = 1025.0$ kg m$^{-3}$, $\alpha = 0.255$ kg m$^{-3}$°C$^{-1}$, and $T_{\text{ref}} = 19.0$°C.

The simulation is started from rest with a uniform temperature of 3.0°C. The model is integrated for 100 yr at 20-km resolution and then interpolated to the 5-km resolution mesh and integrated for another 35 yr. The analysis is conducted on the last 20 yr of the simulation.

### 4. Results

Figure 1 shows a snapshot of relative vorticity at a level of $z = -100$ m with a color range spanning $-6.0 \times 10^{-5}$ to $+6.0 \times 10^{-5}$ s$^{-1}$. Relatively large eddies fill the portion of the domain forced by westerly winds for $1000 < y < 1800$ km. Within this region the Rossby radius of deformation (RRD) of the first baroclinic mode is approximately 20 km, so eddies in this portion of the domain are well resolved by 5-km grid spacing. Eddy activity on the shelf is also apparent but at significantly

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### Table 1. Parameters used in simulation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_y$</td>
<td>2000 km</td>
<td>Meridional extent of domain</td>
<td>Abernathey et al. (2011)</td>
</tr>
<tr>
<td>$L_x$</td>
<td>1000 km</td>
<td>Zonal width of periodic domain</td>
<td>Abernathey et al. (2011)</td>
</tr>
<tr>
<td>$H$</td>
<td>2500 m</td>
<td>Maximum channel depth</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$H_s$</td>
<td>500 m</td>
<td>Shelf depth</td>
<td>Stewart and Thompson (2013)</td>
</tr>
<tr>
<td>$W_s$</td>
<td>100 km</td>
<td>Slope half-width</td>
<td>Stewart and Thompson (2013)</td>
</tr>
<tr>
<td>$Y_s$</td>
<td>500 m</td>
<td>Slope center position</td>
<td>Stewart and Thompson (2013)</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>5 km</td>
<td>Grid resolution</td>
<td>Stewart and Thompson (2013)</td>
</tr>
<tr>
<td>$\Delta z$</td>
<td>0.63 to 92.1 m</td>
<td>Vertical resolution</td>
<td></td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>180 s</td>
<td>Model time step</td>
<td></td>
</tr>
<tr>
<td>$f_0$</td>
<td>$-1 \times 10^{-4}$ s$^{-1}$</td>
<td>Coriolis parameter</td>
<td>Abernathey et al. (2011)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$1 \times 10^{-11}$ m$^{-1}$ s$^{-1}$</td>
<td>Meridional gradient of Coriolis parameter</td>
<td>Abernathey et al. (2011)</td>
</tr>
<tr>
<td>$\tau_0$</td>
<td>0.2 N m$^{-2}$</td>
<td>Maximum wind stress</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$T_m$</td>
<td>3°C</td>
<td>Mean restoring temperature</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$T_a$</td>
<td>1°C</td>
<td>Hyperbolic tangent temperature deviation</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$T_b$</td>
<td>2°C</td>
<td>Linear temperature deviation</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$p$</td>
<td>$1.0 \times 10^{-3}$ m s$^{-1}$</td>
<td>Piston velocity for surface restoring</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$z_e$</td>
<td>1 km</td>
<td>Vertical sponge layer $e$-folding scale</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>6.18°C</td>
<td>Northern wall temperature magnitude</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$\tau_r$</td>
<td>30 days</td>
<td>Restoring time scale for sponge layer</td>
<td>Saenz et al. (2015)</td>
</tr>
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<td>$L_e$</td>
<td>80 km</td>
<td>Sponge layer decay length scale</td>
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<td>6°C</td>
<td>Initial temperature profile constant</td>
<td>Saenz et al. (2015)</td>
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<tr>
<td>$T_2$</td>
<td>3.6°C</td>
<td>Initial temperature profile constant</td>
<td>Saenz et al. (2015)</td>
</tr>
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<td>$h_1$</td>
<td>300 m</td>
<td>Initial temperature profile constant</td>
<td>Saenz et al. (2015)</td>
</tr>
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<td>$m_T$</td>
<td>$7.5 \times 10^{-5}$ C m$^{-1}$</td>
<td>Initial temperature profile constant</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.255 kg m$^{-3}$ C$^{-1}$</td>
<td>Linear thermal expansion coefficient</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1025 kg m$^{-3}$</td>
<td>Reference density</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$T_0$</td>
<td>19°C</td>
<td>Reference temperature</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$c_d$</td>
<td>$3.0 \times 10^{-5}$</td>
<td>Quadratic bottom drag coefficient</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$v_{tt}$</td>
<td>$7.8 \times 10^6$ m$^4$ s$^{-1}$</td>
<td>Biharmonic viscosity, rotational mode</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$v_r$</td>
<td>$7.8 \times 10^6$ m$^4$ s$^{-1}$</td>
<td>Biharmonic viscosity, divergent mode</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$v_o$</td>
<td>$1.0 \times 10^{-6}$ m$^2$ s$^{-1}$</td>
<td>Background vertical viscosity</td>
<td>Saenz et al. (2015)</td>
</tr>
<tr>
<td>$\kappa_o$</td>
<td>$5.0 \times 10^{-6}$ m$^2$ s$^{-1}$</td>
<td>Background vertical diffusivity</td>
<td>Saenz et al. (2015)</td>
</tr>
</tbody>
</table>
smaller spatial scales. On the shelf the RRD is approximately 5 km, so eddy activity is only marginally resolved in this region. These two regions of eddy activity are separated by a quiescent region between 400, y, 600 km that is associated with eastward- and westward-alternating surface front currents (SFCs) that are qualitatively similar to the Antarctic Slope Front.

The climatological state is produced by averaging over the last 20 yr of the simulation. In addition, all data are averaged zonally. Figure 2 shows the climatological surface heat flux $Q$, which results from the linear surface restoring. The amplitude of the surface heat flux ranges around approximately 10 W m$^{-2}$ in a sinusoidal structure with net cooling/warming southward/northward of $y = 1200$ km. For reference, Fig. 2 also shows the zonal wind stress along the second y axis.

a. Eulerian-averaged ZISO climate

Figure 3 shows the zonally averaged zonal velocity (colors) and zonally averaged temperature (contours) produced through Eulerian averaging (i.e., data averaged at fixed $x$, $y$, and $z$ coordinates). The climate is similar to that produced in Stewart and Thompson (2013). The simulation is dominated by what we will refer to as the idealized circumpolar current (ICC) that spans approximately one-half of the meridional extent of the domain. An eastward jet with a zonal velocity of approximately 0.35 m s$^{-1}$ is present at the surface and is located directly beneath the maximum in wind stress forcing near $y = 1400$ km. At the core of the ICC, the zonal velocity has vertical shear of approximately $8.0 \times 10^{-5}$ s$^{-1}$, resulting in a change in zonal velocity of 0.2 m s$^{-1}$ from top to bottom that is in thermal wind balance. The deep-water formation at the southern boundary of the shelf produces density currents that flow north along the ocean bottom. The northward density currents, along with geostrophy, produce westward zonal flow along the shelf break with a velocity of $-0.15$ m s$^{-1}$. Multiple SFCs across the shelf break are indicated by the eastward- and westward-alternating surface velocity. These fronts are qualitatively similar to the Antarctic Slope Front.

b. Thickness-weighted averaged ZISO climate

The remainder of the results are presented and discussed in buoyancy coordinates using the thickness-weighted averaging approach discussed in section 2. Since the simulations use a linear equation of state with constant salinity, surfaces of constant temperature are also surfaces of constant buoyancy. These surfaces are depicted as contours in Fig. 3. As described in appendix A, at intervals of 3 days during the course of the last 20 yr of the simulation, the model state is interpolated onto 100 predefined buoyancy surfaces that entirely span the buoyancy space of the simulation.

The TWA time- and zonal-mean zonal velocity is shown in Fig. 4 in colors. The range of buoyancy surfaces that exist in any given fluid column in the simulation varies in time. The black contours in Fig. 4 depict the fraction of time a given buoyancy surface exists, that is, the probability of existence $\tilde{f}$ as defined in Eq. (B1) of appendix B. While calculating averages in buoyancy space, the average $z$ position of a buoyancy coordinate $\tilde{z} = \tilde{z}(\tilde{x}, \tilde{y}, \tilde{b})$ can be computed; these heights are shown as white dashed contours in Fig. 4. The TWA zonal flow shown in Fig. 4 is qualitatively similar to its Eulerian-averaged counterpart shown in Fig. 3. The color bars of the two figures are different in order to accommodate the larger zonal velocity obtained with the thickness-weighted averaging of the lightest buoyancy classes in the vicinity of the ICC. In the region of the ICC, at approximately $y = 1400$ km, nearly a third of the buoyancy values ever occupied by
the fluid have values of $\bar{B} < 1.0$, indicating that a significant fraction of the buoyancy space in the ICC region is only occupied intermittently. In the region of the SFCs, contours of $\bar{B}$ collapse at the surface, indicating minimal variability in the range of buoyancy classes that visit the fluid surface in the region of the SFCs.

The TWA meridional flow is shown in Fig. 5 as colors and, in addition, the zero contour of the TWA meridional velocity is shown as a thick gray line. Also shown in Fig. 5 is the probability at which a given buoyancy coordinate is at the surface. The probability at surface value $\overline{v^2}$ is obtained by simply differencing the probability of existence values in adjacent buoyancy layers, that is, $\overline{v^2}(k) = \max[\bar{B}(k) - \bar{B}(k-1), 0]$. Across the entire meridional extent of the domain, the TWA meridional velocity is toward the south in the lightest buoyancy classes visited by the fluid at each $(\hat{x}, \hat{y})$. For all locations north of the SFCs, the explanation for the uniformly southward TWA flow in the lightest buoyancy classes is straightforward. The lightest buoyancy classes represent the warmest water. At any given $(\hat{x}, \hat{y})$ position, the warmest waters recorded at the surface will, on average, come from the north. Within the region of the SFCs where there is minimal variability in the surface conditions, the southward TWA flow is driven by the westward wind stress. Just as the lightest buoyancy classes are associated with southward TWA flow, the heaviest buoyancy classes that are ever in contact with the surface are associated with northward flow.

At the north boundary, an analog to Upper Circumpolar Deep Water (UCDW) is produced between the buoyancy coordinates of 0.002 and $-0.008 \text{ m s}^{-2}$, which corresponds to depths between $\tau = -500$ and $-2250$ m. The deep portion of the idealized UCDW reaches the surface just north of the SFCs and spreads southward, filling the shelf region. And finally, a wedge of northward TWA meridional flow starts at the surface just north of the shelf break at $y = 750 \text{ km}$ and expands to fill the top 500 m of fluid at the north boundary. This northward TWA meridional flow is a loose analog to the Antarctic surface water mass.

c. Zonal force balance of the ICC

With this summary of the zonal- and time-mean climate of the TWA system, we turn our attention to the TWA zonal momentum equation and the balance of forces within the ICC. Figure 6 shows the terms of the zonal- and time-mean TWA zonal momentum equation after averaging meridionally over the box shown in Fig. 5. This box is 200 km wide and is centered on the maximum wind stress forcing at $y = 1400 \text{ km}$. The five terms in the zonally averaged TWA zonal momentum equation (17) are shown in Fig. 6, along with the residual error. The mean depths of $\overline{\tau}$ at $-60, -230,$ and $-600 \text{ m}$ are indicated with arrows. The horizontal axis measures acceleration in meters per second per day with positive values acting to push the fluid toward the east. Buoyancy is on the vertical axis, where data are truncated at values below $\bar{B} = 0.01$ at the top and bottom. The residual error is shown as the dashed black line; the error is small relative to the dominant forcing terms, so we are confident that the force balance depicted in Fig. 6 is representative of the model simulation. Three buoyancy zones in the force balance are

---

1 As discussed in appendix B, $\overline{v^2}$ can only take on positive values. Also, the magnitude of $\overline{v^2}$ decreases as the number of buoyancy layers increases, but the integral of $\overline{v^2}$ at a given $(\hat{x}, \hat{y})$ across the entire buoyancy space will always be exactly 1.0.
emergent in Fig. 6: a surface layer, an upwelling zone, and a bottom boundary layer.

The surface layer occupies the range $0 < \theta < 1$ in the lightest buoyancy classes. When a buoyancy class reaches the surface within the bounding box shown in Fig. 5, it is directly forced by the surface wind stress (Fig. 6, black line). As discussed in Eq. (B4) of appendix B, the method for evaluating the wind stress forcing in buoyancy space is to assume that at each instant in time the wind stress is entirely absorbed by the buoyancy layer that exists at the surface. This upper portion of the buoyancy space exists episodically when anomalously light (warm) waters enter the region from, on average, the north. As a result of treating the outcropped layers using the Lorenz convention, the layer thickness $s$ tends toward zero in the lightest buoyancy layers occupied by the fluid. Even though the probability at surface $y$ decreases monotonically above buoyancy values of $0.0 \text{ m s}^{-2}$, the acceleration due to wind stress continues to increase because $s$ decreases faster than...
For a fixed amount of applied stress $\tau$, this leads to a progressively larger wind stress acceleration as the buoyancy classes get lighter.

Within the upper portion of the surface layer, at mean depths between 0 and 60 m, meridional advection of thickness-weighted potential vorticity (MAPV; Fig. 6, blue line) acts to accelerate $\hat{u}$. As shown in Eq. (17), this force is the product of meridional velocity $\hat{v}$ and the absolute vorticity $\eta^\theta$. The relative vorticity based on the curl of the $(\hat{u}, \hat{v})$ velocity vector evaluated along buoyancy surfaces is small relative to the Coriolis parameter. As a result, the MAPV has a structure almost identical to $\hat{v}$. As shown in Fig. 5, $\hat{v}$ is southward in the lightest portion of the buoyancy space and, as a result, acts to accelerate $\hat{u}$. In the lower portion of the surface layer between $\phi = 0.5$ and 1.0, the meridional velocity is positive and northward, so MAPV opposes, and largely balances, the wind stress forcing. The second zero crossing of MAPV occurs at the bottom of the surface layer where the wind stress forcing also vanishes.
In the upper portion of the surface layer, the wind stress and MAPV are both acting to accelerate $\vec{u}$. Equilibrium is obtained by the eddies acting to decelerate $\vec{u}$. The zonal- and time-mean zonal component of the divergence of the Eliassen–Palm flux tensor, that is, the eddy terms in Eq. (17), is shown by the gray line in Fig. 6. The eddy force is the largest zonally integrated force in the surface layer reaching a magnitude of 0.4 m s$^{-2}$ day$^{-1}$. Only at the very bottom of the surface layer does the eddy-induced force act in concert with the wind stress to balance the MAPV. The eddy force also has a zero-value node at the bottom of the surface layer.

The upwelling zone is the portion of the buoyancy space with $f = 1.0$. This region extends from the bottom of the surface layer to the top of the bottom boundary layer. This zone is characterized by southward velocity throughout. The strongest southward velocities within this zone are found primarily in the $-0.004 < b < 0.0$ buoyancy range, which corresponds to average depths of $\tau = -850$ and $-230$ m, respectively. Within this zone the wind stress, surface-forced diabatic advection, and bottom drag are all zero by construction. Therefore, the MAPV and eddy-induced force must balance when a statistically steady state exists. The upwelling of UCDW produces a MAPV, dominated by the Coriolis force, that tends to accelerate $\vec{u}$. This force is exactly opposed by the action of mesoscale eddies.

The approximations we have made to diagnose the nonlinear bottom drag force based on mean quantities are not sufficient to close the force balance within the bottom boundary layer with the same level of precision as within the surface layer [see Eqs. (B5) and (B6)]. The zonal component of bottom stress is a nonlinear term based on correlations between kinetic energy and zonal velocity. We attempt to approximate this nonlinear term based on mean quantities. Furthermore, we assume that all of the bottom stress is deposited into the single, ensemble-mean buoyancy layer that rests on the bottom. As a result our approximation of the bottom drag force is only qualitatively correct. But we do have sufficient fidelity to conclude that the bottom drag force and northward-flowing geostrophic velocity are acting to decelerate the zonal flow while the eddies act in opposition to accelerate the zonal flow.

d. Eddy fluxes of zonal momentum

The eddy-induced force in the TWA zonal velocity equation is clearly playing a leading role in the force balance within the ICC. To better understand how eddies are contributing to the equilibrated $\vec{u}$, the meridional and vertical flux of zonal momentum is shown in Figs. 7 and 8, respectively. The meridional flux of zonal momentum is small, with characteristic values of $d_u^0 y^0 \approx 0.01$ m$^2$ s$^{-2}$ on the southward side of the ICC. The meridional flux of zonal momentum is negligible in the force balance of the ICC.

The vertical flux of zonal momentum is the primary mechanism by which eddies participate in the force balance of the ICC. The vertical flux of zonal momentum shown in Fig. 8 has been scaled by the reference density in order to convert to units of newtons per square meter. In
this way we can get a sense of the magnitude of the vertical flux of zonal momentum by comparing to the zonal wind stress, which has a characteristic value of 0.2 N m$^{-2}$. Throughout the broad region of the ICC, the vertical flux of zonal momentum is always negative. We interpret the negative values as the flux of positive (eastward) zonal momentum in the downward direction toward more negative buoyancy values. Except for a small pause at the bottom of the surface layer, the downward flux of eastward momentum grows continuously with increasing depth in the vicinity of the ICC. Starting at a value of zero in the light buoyancy classes that are never occupied, the downward flux of eastward momentum grows to a value of 0.20 N m$^{-2}$ at the bottom of the surface layer. Below the surface layer, the downward flux continues to increase while traversing the upwelling zone. Finally, the downward flux of eastward momentum converges in the bottom boundary layer and its value returns to zero in the heavy buoyancy classes that are never occupied by the fluid.

When shown in buoyancy coordinates, it is not possible to accurately sense the zonal force produced by the vertical divergence of the eastward stresses shown in Fig. 8. This is because, as shown in Eq. (17), the divergence of a vertical flux in buoyancy space is weighted by thickness, differentiated with respect to buoyancy, then deweighted by thickness. An alternative approach is to notice that $\sigma \varrho \bar{b} = \bar{\sigma} \varrho \bar{Z}$. By transforming from $\bar{\sigma} \varrho \bar{m} \{ \bar{y}, \bar{b} \}$ to $\bar{\sigma} \varrho \bar{m} \{ \bar{y}, \bar{Z}(\bar{y}, \bar{b}) \}$, the flux is presented such that its vertical divergence is linear with respect to the vertical coordinate. This transformation is presented in Fig. 9. When viewed in $z$ coordinates, the vertical flux of zonal momentum appears far more uniform in the vertical, especially within the core of the ICC. The vertical flux of zonal momentum increases rapidly from the surface down to $z = -500$ m. Then, from $z = -500$ to $-2250$ m, the vertical flux of zonal momentum continues to increase but at a much slower pace. Finally, the vertical flux of zonal momentum converges in the bottom 250 m of the domain.

e. Heat balance in the surface layer

We end our discussion of the results of this simulation by revisiting the rich structure of the surface layer. When viewed in buoyancy space from the thickness-weighted averaged perspective, the fluid motion within the surface layer shown in Fig. 5 is somewhat unintuitive, particularly within the region of the ICC. Across much of the meridional extent of the domain, the surface layer is characterized by southward meridional velocities in buoyancy classes lighter than average and northward velocities in buoyancy classes heavier than average. To obtain an integrated view of surface layer, Fig. 10 shows the meridional surface volume flux $S_f$ expressed as

$$S_f = \int_{h(0 < \bar{Z} < 1)} i \sigma \varrho \bar{b} \, db. \quad (25)$$

The meridional surface volume flux integrates the TWA meridional velocity across the surface layer weighted by $\sigma \varrho \bar{b}$. Note, again, that $\sigma \varrho \bar{b} = \bar{\sigma} \varrho \bar{Z}$, so $S_f$ has units of meters squared per second and represents the surface layer meridional volume flux per unit length in the zonal direction.
The second y axis in Fig. 10 shows the net surface heat flux $Q$ in watts per square meter. The y position of $Q = 0$ corresponds to the y position of $S_f = 0$. Given that the meridional gradient of surface temperature is positive definite and highly uniform across nearly the entire meridional extent of the domain (see Fig. 3), $S_f = 0$ must align with $Q = 0$ unless there is nonnegligible heat flux across the bottom of the surface layer. But Fig. 5 indicates that the TWA meridional velocity, and thus the heat transport, at the bottom of the surface layer is essentially zero across most of the ICC. So, on average, for $y$ positions greater than 1250 km, the surface layer carries colder waters northward to balance the heat flux into the ocean. And, in turn, for $y$ positions less than 1250 km, the surface layer carries warmer waters southward to balance the heat flux out of the ocean. We hypothesize that the dominant balance within the surface layer can be expressed as

$$\rho_0 c_p S_f T_y \sim Q, \quad (26)$$

implying that the surface heat flux is primarily balanced by the surface layer meridional advection of the meridional temperature gradient. Order of magnitude estimates indicate that such a balance is realized in the ICC. For example, between $y = 1250$ and 1600 km the simulation has a characteristic surface layer meridional flux of $0.25 \ m^2 \ s^{-1}$ and a characteristic meridional temperature gradient of $1^\circ C$ per 350 km. This results in an advective heat flux of

Fig. 9. Eddy-induced vertical flux of eastward momentum (m$^2$ s$^{-2}$, color and red contours) mapped from buoyancy coordinates to $z$ coordinates. Red contours are values of $-0.00025$, $-0.00020$, $-0.00015$, $-0.00010$, and $-0.00005$. Blue contours are probability of existence $\tilde{S}$ (nondimensional) values of $0.001$, $0.01$, $0.10$, $0.25$, $0.50$, $0.95$, and $0.999$.

Fig. 10. The surface layer–integrated meridional volume flux per unit length in the zonal direction (m$^2$ s$^{-1}$, left axis, black contour). Net surface heat flux (W m$^{-2}$, right axis, blue contour).
3 W m$^{-2}$, which is very close to the surface heat flux averaged between $y = 1250$ and 1600 km (see Fig. 10).

5. Conclusions

Interpreting eddy–mean flow interactions through the lens of the thickness-weighted averaging approach results in a simple, yet complete, explanation of the equilibrium in zonally symmetric circumpolar currents. The powerful perspective provided by the TWA analysis stems from the concise and exact decomposition of the full three-dimensional Boussinesq equations into mean and eddy components and by conducting the analysis in buoyancy space.

Supporting previous thinking from Tréguier et al. (1997) and Marshall and Radko (2003), a key finding from the TWA analysis is the identification of the surface layer based on ventilation as opposed to traditional boundary layer dynamics. The surface layer is defined as the set of buoyancy coordinates that ever reside at the ocean’s surface. This is the set of buoyancy values that directly feel the nonconservative forcing due to wind stress and surface heat flux. Tréguier et al. (1997) suggest a similar definition where the boundary between the adiabatic interior and diabatic surface zones should be placed between sporadic and uninterrupted isopycnal layers. Marshall and Radko (2003, p. 2344) suggest that the mixed layer depth should likely be defined as “the $z$-position of the deepest isopycnal that occasionally grazes the surface.” By construction the surface layer isolates the diabatic ocean layer from the adiabatic region of the ocean. While such a definition is naturally accommodated in buoyancy coordinates by finding the lightest, uninterrupted buoyancy value at each $(x, y)$ position, making a similarly unambiguous definition in Eulerian coordinates does not seem straightforward. It is essential to realize that this definition of the surface layer is distinct, both physically and conceptually, from the ocean boundary layer. Figures 4 and 5 clearly indicate that the bottom of the surface layer (bottommost black contour in both figures) is not a material surface. Yet, we find that the TWA meridional velocity is essentially zero at the bottom of the surface layer (thick gray contour in Fig. 5). In addition, by definition the diabatic vertical velocity is also zero at the bottom of the surface layer. This idealized simulation of a circumpolar current suggests that the surface layer is a physically distinct object with dynamics that are largely isolated from the underlying adiabatic region of the ocean.

Figure 10 lends additional support to the notion of a dynamically isolated surface layer. The linear restoring to a prescribed surface temperature results in net heat flux in/out of the ocean over the northern/southern half of the domain. By integrating the TWA meridional velocity across the surface layer we were able to elucidate the primary heat (buoyancy) balance. While the TWA meridional velocity is both southward and northward within the surface layer, the integrated surface layer meridional flux $S_y$, shown in Fig. 10, is strongly related to the surface heat flux $Q$, shown in the same figure. The zero value of $Q$ corresponds to the zero value of $S_y$. North/south of this nodal point, positive/negative values of $S_y$ carry cold/warm water to balance the positive/negative surface heat flux. Our conjecture is that the primary balance equation has the form of $\rho_0 c_p S_y T_y = Q$, where $T_y$ is the meridional temperature gradient. This result suggests that the surface layer–integrated meridional flux is not directly related to the surface stress $\tau$ but is instead determined by the surface flux of buoyancy. Surface stress can certainly indirectly modify the surface-integrated buoyancy balance through modification of the buoyancy coordinate outcropping positions. This relationship is similar to that derived by Marshall and Radko (2003) in their Eq. (11). While Marshall and Radko (2003) conduct their analysis in $z$ coordinates, which precludes defining the surface layer based on ventilation, the analysis conducted above in buoyancy coordinates supports their hypothesis developed using $z$ coordinates.

While the configuration is idealized, the large-scale structure of the TWA meridional velocity is broadly similar to that observed in the Southern Ocean. Namely, the production of bottom water on the shelf flows northward along the ocean floor with southward, upwelling flow residing above and extending across the majority of the meridional domain. In addition, across the ICC portion of the domain we find that the TWA meridional velocity is northward in the bottom half of the ventilation-defined surface layer. So if we move in the positive meridional direction along, for example, the $b = -0.002$ m s$^{-2}$ surface in Fig. 5, we know that there has to be a zero crossing of the TWA meridional velocity. But the following question remains: why does the zero crossing of the TWA meridional velocity coincide almost exactly with the very bottom of the ventilation-defined surface layer? The zero crossing cannot occur inside the ventilation-defined surface layer because this would imply that the coldest waters to ever reach the surface would, on average, come from the north. Given the zonal symmetry of the system and the linear restoring of surface buoyancy, such a scenario is not dynamically tenable. Alternatively, the zero crossing cannot occur outside of the ventilation-defined surface layer due to continuity. Just outside of the ventilation-defined surface layer the system is adiabatic, and conservation of mass equation (15) implies $[\sigma \nu]_z = 0$, which,
in turn, forces $\bar{\mathbf{v}}$ to be a constant along a buoyancy surface. The meridional velocity in the bottom portion of the ventilation-defined surface layer is a factor of 10 larger in magnitude than the upwelling velocity. If the zero crossing occurred in the adiabatic region, the stratification, which goes as $1/\sigma$, would have to change by a factor of 10 in order to keep $\bar{\mathbf{v}}$ a constant along a buoyancy surface. But the stratification of the upwelling waters is set by the restoring along the northern boundary, thus making a factor of 10 change in $\sigma$ along a constant $b$ surface to be implausible within the adiabatic region of the fluid. So it might be that the idealized nature of this system constrains the zero crossing of the TWA meridional velocity to coincide with the bottom of the ventilation-defined surface layer. But the dynamical simplicity of the crossing to be coincident with the interface of the diabatic and adiabatic regions of the fluid suggests that the finding might be applicable to more realistic ocean conditions.

The analysis clearly highlights the leading role that mesoscale eddies play in the force balance of the circumpolar current. The eddies participate in the force balance through the differential vertical flux of eastward momentum. While the importance of downward transport of vertical momentum by eddies in the ACC is well accepted (Munk and Palmén 1951; Johnson and Bryden 1989; Gille 1997; Phillips and Rintoul 2000), by diagnosing the Eliassen–Palm flux tensor $\nabla \cdot \mathbf{E}$ in buoyancy coordinates, we are able to diagnose the vertical structure of the force balance with a high level of fidelity and precision, particularly within the surface layer (see Fig. 6). Integrated across the surface layer, the net force produced by MAPV is small, and the primary balance is between the accelerating force produced by wind stress and the decelerating force produced by mesoscale eddies. At the bottom of the ventilation-defined surface layer, the downward transport of eastward momentum by eddies almost exactly balances the eastward momentum injected into the fluid by the wind stress. Essentially, the eddies are required to move the momentum associated with the nonconservative wind stress forcing across and then out of the surface layer into the adiabatic ocean interior. It is important to note that without the decelerating force produced by the mesoscale eddies, the surface layer zonal flow would continue to accelerate, the isopycnals would continue to steepen due to the thermal wind balance, and the resulting equilibrium would be a barotropic, unstratified circumpolar current (Samelson 1999).

In this idealized configuration, the bottom of the ventilation-defined surface layer coincides with the location where all of the forces in the zonal momentum balance equation [Eq. (17)] are very nearly equal to zero. By definition, the wind stress force is zero at the bottom of the surface layer. As explained above, the bottom of the surface layer also coincides with a node in the meridional velocity and thus a node in the force produced by MAPV. This then constrains the eddy force to also be zero in order for equilibrium in the balance of forces to be obtained.

While we have not used the theoretical constructs associated with Ertel potential vorticity (EPV) to interpret the eddy–mean flow interaction within the ICC, the eddy force $\nabla \cdot \mathbf{E}$ is equivalent to an eddy flux of EPV in the TWA framework [see Young (2012), his Eq. (126)]. We note that this flux is both down and up the meridional EPV gradient within the surface layer. The meridional EPV gradient is single signed and northward (not shown). In the lightest surface layer buoyancy classes, the eddy EPV flux is strong and downgradient. In the densest surface layer buoyancy classes, the eddy EPV flux is weak, but it is up the EPV gradient. Based on the analysis of Eq. (23) of Rhines and Holland (1979), this upgradient zone is not only permitted but is required to exist somewhere within the equilibrated forced–dissipated system. We note this upgradient flux of EPV because it represents a potential challenge to mesoscale eddy parameterizations based on downgradient flux of mean EPV (e.g., Eden and Greatbatch 2008; Marshall and Adcroft 2010; Marshall et al. 2012).

We have demonstrated that the TWA framework developed by de Szoeke and Bennett (1993), Young (2012), and Maddison and Marshall (2013) is exceptionally powerful in its ability to exactly and concisely express the role of eddies within the mean, climatological ocean state. Development of variations and extensions of the broad class of thickness-weighted approaches to the eddy–mean flow decomposition suggests that even further advances are possible (e.g., Aiki and Greatbatch 2014; Aoki 2014). In reference to eddy–mean flow decomposition, Rhines and Holland (1979) suggest that our goal in developing equation sets that capture eddy–mean flow interaction is to have a mean-field equation in which eddy effects are as transparent as possible. We find that the TWA framework is possibly optimal in this regard.

Still, it is important to note that our in situ implementation of the TWA analysis framework is incomplete as compared to the possibilities offered by the underlying theory. First, because of the amount of data required to conduct the analysis, the only viable path to analysis of global eddying simulations is through in situ implementation where the diagnostics are computed during the simulation (see appendix A). In terms of analysis, we are only diagnosing
the zonal-mean TWA system [see Eq. (17)], whereas the TWA framework can be applied to understand eddy–mean flow interaction in the full 3D system. We have made assumptions about how and where the nonconservative surface forcing and bottom drag are modeled in the force balance [see Eqs. (B4) and (B6)]. We have also not addressed more complicated buoyancy forcings, such as penetrative solar heating, because of the idealized nature of this study. Finally, the TWA analysis framework can be used to measure the spurious contributions to the diabatic vertical velocity in the ocean interior due to imperfect numerics. We have not accounted for spurious numerical buoyancy fluxes in our analysis. While incomplete, the analysis framework is sufficient to explain eddy–mean flow interaction in an idealized circumpolar channel. We see no impediment to a complete in situ TWA analysis framework that could be applied to realistic global eddy ocean configurations. All of the results obtained above will have to be reconsidered in more complex ocean configurations. In particular, it is unclear if the ventilation-defined surface layer will remain dynamically isolated in more complex geometries with more complex forcing. Also, evidence clearly shows that the bottom form stress force plays a leading role in the force balance at the bottom of the Southern Ocean (Johnson and Bryden 1989; Gille 1997; Phillips and Rintoul 2000). Since this configuration employs bottom bathymetry without variations in the zonal direction, bottom form stress cannot contribute to the zonal force balance. Our next step is to add ridges and plateaus to this idealized configuration in order to quantify how the force balance is impacted by bottom form stress. While demonstrating the robustness of our results within increasingly complex physical settings is required, we suspect that our findings with respect to the diabatic and force balance in the ventilation-defined surface layer will prove useful in describing the role of mesoscale eddies in the climate of the Southern Ocean.

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APPENDIX A

Calculation of the Eliassen–Palm Flux Tensor

This appendix describes how the terms in the thickness-weighted averaged Boussinesq equations are constructed based on an ensemble of ocean simulation states measured with Eulerian coordinates.

The TWA discrete coordinate system is \( (l_h, x_i, y_j, b_k) \). Increasing \( i, x, \) and \( y \) is with increasing \( h, i, \) and \( j \) subscripts, respectively, as would be expected. But increasing \( k \) moves downward in the flow and thus toward more negative buoyancy values. Assume that the ensemble is composed of \( N \) members. The ensemble members sample across simulations at fixed \( (l_h, x_i, y_j, b_k) \). If the system is considered to be in a state of statistical equilibrium, then the members can be obtained by sampling a single simulation across time, preferably at sampling intervals greater than the autocorrelation time scale. Here, we sample the instantaneous model state with \( b_k \) increments of 3 days. In the description below we assume a statistical steady state, so averaging across ensembles results in a collapse and disappearance of the \( i \) coordinate dimension. We require the range of \( b_k \) to include all values of buoyancy realized in the simulation.

Now assume that the states are provided in a discrete Eulerian coordinate system composed of \( (l_h, x_i, y_j, z_k) \). The first step in the procedure is to use Eulerian state variables and an equation of state formula to produce \( b(l_h, x_i, y_j, z_k) \). In each vertical column for each ensemble member, that is, for each \( (l_h, x_i, y_j) \), all Eulerian model state variables \( A \) are interpolated to buoyancy coordinates, that is, \( A(b_k) = I(A[b(z_k)]) \), where \( I \) is the vertical interpolation operator. Buoyancy coordinate values \( b_k \) that are lighter and heavier than those buoyancies that exist in the water column are assumed to be positioned at the top and bottom of the fluid column, respectively. Outcropped buoyancy coordinate values are treated in exactly the same as any other buoyancy coordinate. This is the Lorenz convention (Andrews 1983). Hereafter, all calculations are carried out in buoyancy coordinates. In addition, the notation drops an explicit listing of variable dependency on the \( (x_i, y_j) \) coordinates.

Once in buoyancy coordinates, the ensemble average \( \mathcal{A}(b_k) \) of a state variable \( A(l_h, b_k) \) is defined as

\[
\mathcal{A} = \frac{1}{N} \sum_{i=1}^{N} A_i, \tag{A1}
\]

The thickness-weighted average operator \( \hat{A}(b_k) \) is constructed based on ensemble-mean values as
\[
\hat{\sigma} = \frac{\sigma A}{\sigma}, \quad (A2)
\]

where

\[
\sigma = \frac{\bar{\zeta}(\hat{b}_k) - \bar{\zeta}(\hat{b}_{k+1})}{b_k - b_{k+1}} = \left[\frac{1}{N} \sum_{i=1}^{N} \bar{\zeta}(\hat{b}_{k,i}) - \frac{1}{N} \sum_{i=1}^{N} \bar{\zeta}(\hat{b}_{k+1,i})\right]/(b_k - b_{k+1}). \quad (A3)
\]

The quantities \(\bar{\zeta}(\hat{b}_k)\) and \(\bar{\zeta}(\hat{b}_{k+1})\) measure the ensemble-mean \(z\) position of the \(\hat{b}_k\) and \(\hat{b}_{k+1}\) buoyancy surfaces, respectively, produced via Eq. (A1). So \(\sigma\) measures the rate of change of \(\bar{\zeta}\) with respect to \(\hat{b}\).

At a given \((\hat{x}_i, \hat{y}_i)\) position the buoyancy surface \(\hat{b}_k\), on which \(A\) is recorded, might be outcropped some of the time, none of the time, or all of the time. If \(\hat{b}_k\) is outcropped all of the time, then Eq. (A2) results in a 0/0 and this \((A3)\) position the buoyancy surface to be the same as the sea surface height. The Lorenz convention requires that these outcropped \(z\) positions be included as valid data used to produce the ensemble average defined in Eq. (A1) that is used, subsequently, in Eq. (A3).

Two decompositions are used to define eddy fields. The Reynolds decomposition measures fluctuations from the ensemble mean, for example,

\[
\zeta'(\hat{b}_k) = \zeta(\hat{b}_k) - \bar{\zeta}(\hat{b}_k), \quad \text{and} \quad (A4)
\]

\[
m'(\hat{b}_k) = m(\hat{b}_k) - \bar{m}(\hat{b}_k), \quad (A5)
\]

measure the eddy height and eddy Montgomery potential defined as perturbations from their respective ensemble means. The Montgomery potential is computed after interpolating into buoyancy space based on Eq. (2.1b) from Higdon (1999), assuming that the density is constant within each buoyancy layer. The thickness-weighted decomposition measures fluctuations from the TWA, for example,

\[
u''(\hat{b}_k) = u(\hat{b}_k) - \hat{u}(\hat{b}_k), \quad \text{and} \quad (A6)
\]

\[
u''(\hat{b}_k) = v(\hat{b}_k) - \hat{v}(\hat{b}_k). \quad (A7)
\]

The adiabatic terms in the Eliassen–Palm flux tensor are then expressed as

\[
E = \left(\begin{array}{ccc}
\bar{u}u'' + \frac{1}{2\sigma} \bar{\zeta} & \bar{u}v'' & 0 \\
\bar{v}u'' & \bar{v}v'' + \frac{1}{2\sigma} \bar{\zeta} & 0 \\
\frac{1}{\sigma} \bar{\zeta}m_x & \frac{1}{\sigma} \bar{\zeta}m_y & 0
\end{array}\right). \quad (A8)
\]

For the simulation presented above, approximately \(N = 2000\) ensemble members are included in the computation of \(E\). Writing these ensemble members to disk and computing \(E\) as a postprocessed diagnostic would require approximately 6 TB of storage. Instead, \(E\) is built during the simulation at the cost of adding approximately 1% to the total compute time.\(^2\) This in situ computation of \(E\) does require additional variables to be computed and stored in order to keep track of the eddy correlations as the simulation progresses. In particular, for the adiabatic system we are required to compute and store these 11 fields: \(\bar{\sigma}, \bar{\zeta}, \bar{\zeta}^2, \bar{m}, \bar{\zeta}m_z, \bar{\zeta}m_x, \bar{\sigma}u, \bar{\sigma}w, \bar{\sigma}v, \bar{\sigma}v, \bar{\sigma}w, \bar{\sigma}m_x, \bar{\sigma}m_y, \bar{\sigma}m_z\), and \(\bar{\sigma}m\). At any point during the simulation we can then back out all of the eddy correlations needed to compute \(E\), for example,

\[
\bar{\zeta}^2 = \bar{\zeta}^2 - \bar{\sigma}^2. \quad (A9)
\]

If the current ensemble contains \(N\) members and we wish to add an additional member, we simply use

\[
\bar{A} = (N \times \bar{A} + A)/(N + 1), \quad \text{and} \quad (A10)
\]

\[
N = N + 1. \quad (A11)
\]

APPENDIX B

Calculation of Momentum and Diabatic Forcing in Buoyancy Coordinates

The idealized configuration allows for a relatively straightforward computation of the forcing terms in the momentum and buoyancy equations. These computations are closely connected to the probability of existence and probability at surface values. For each ensemble member measured in Eulerian space, define \(b_{\text{min}}(t_n, x_n, y_n)\) and \(b_{\text{max}}(t_n, x_n, y_n)\) to be the minimum and

\(^2\) Note that the in situ approach to computing \(E\) provides a natural path to perform this same computation in a realistic, strongly eddy global ocean simulation. For global eddying simulations, writing the required data to disk for postprocessing \(E\) would require a few tens of petabytes of storage.
maximum buoyancy values in each water column. Now define a mask \( \phi(\hat{t}, \hat{x}, \hat{y}, \hat{b}) \) in buoyancy coordinates with

\[
\phi = \begin{cases} 
0, & \text{for } \hat{b} < \hat{b}_{\text{min}} \\
1, & \text{for } \hat{b}_{\text{min}} \leq \hat{b} \leq \hat{b}_{\text{max}} \\
0, & \text{for } \hat{b} > \hat{b}_{\text{max}}
\end{cases}
\] (B1)

where \( \phi \) is valued 1 within the buoyancy space occupied by the fluid and 0 otherwise. Using Eq. (A1), \( \overline{\phi} \) is the ensemble mean of \( \phi \). In our simulation, \( \overline{\phi} \) represents the fraction of time a given position in buoyancy coordinates contains fluid, that is, \( \overline{\phi} \) is the probability of existence with range \([0, 1]\). Zonal-mean values of \( \overline{\phi} \) are shown in Fig. 4.

The probability that a given layer is at a top or bottom boundary and has nonzero thickness is obtained by simply differencing \( \overline{\phi} \) across adjacent buoyancy surfaces, that is,

\[
\nu(\hat{b}_k) = \overline{\phi}(\hat{b}_{k+1}) - \overline{\phi}(\hat{b}_k). \] (B2)

Positive and negative values of \( \nu \) indicate the presence of layer \( \hat{b}_k \) at the ocean surface and bottom, respectively. Let \( \overline{\nu} \) represent the probability at surface values and \( \overline{\nu}^b \) represent the probability at bottom values. If a given \( \hat{b}_k \) visits both the ocean surface and ocean bottom, the algorithm described below fails. Zonal-mean values of \( \overline{\nu} \) are shown in Fig. 5.

Given the linear equation of state with constant salinity, a buoyancy layer is equivalent to a constant temperature layer. So at any point we can exchange \( \hat{b}_k \) with its temperature equivalent, that is, \( \hat{T}_k \). The temperature flux \( T_f \) at the ocean surface is \( T_f = -p_T(T_k - T_s) \), where \( p_T \) is the piston velocity, \( T_s \) is the restoring temperature, and \( T_k \) is the ocean temperature surface. The surface buoyancy flux is linearly related to the surface temperature flux as \( b_f = T_f \Delta g / \rho_0 \). When \( \hat{b}_k \) is at the surface, \( T_s = \hat{T}_k \), so \( b_f(\hat{b}_k) = -p_T(\hat{T}_k - T_s) \Delta g / \rho_0 \). If we assume that the buoyancy flux is deposited entirely into the layer in contact with the surface, then the diabatic velocity is \( \overline{m}(\hat{b}_k) = b_f / (\sigma \Delta \hat{b}) \).

The \( \sigma \) appearing in the TWA equation is thickness weighted, that is, \( \overline{\sigma} = \overline{\sigma_f} \hat{b} = \overline{\sigma_f} / (\sigma \Delta \hat{b}) \), which is invariant across ensemble members. The ensemble mean of the surface buoyancy flux \( \overline{b_f} \) is

\[
\overline{b_f} = \overline{\nu} \overline{b_f}. \] (B3)

The ensemble mean is simply the buoyancy flux assuming the buoyancy layer is at the surface multiplied by the fraction of time that the layer is actually at the surface. Note that the diabatic velocity computed here represents the diabatic tendency due to explicit surface forcing. Additional diabatic velocity due to nonexact numerical methods is ignored.

The analysis above also applies to the wind stress \( \tau \), but it is even more straightforward since the wind stress does not vary with \( \hat{b}_k \). The ensemble-mean wind stress applied to any \( \hat{b}_k \) surface is just \( \tau_k = \overline{\nu} \tau \). As we did with the diabatic forcing, if we assume that all of the wind stress is deposited in the buoyancy layer that is in contact with the surface, then the resulting thickness-weighted averaged force due to wind stress forcing appears in the zonal momentum equation as

\[
\hat{X}_s(\hat{b}_k) = -\overline{\nu} \tau / \rho_0 \sigma \Delta \hat{b}. \] (B4)

This force is shown as the black line in Fig. 6.

At this point in the analysis, no approximation has been made other than assuming 1) discrete buoyancy coordinates and 2) that surface forcing is deposited entirely in the buoyancy layer residing at the surface. We approximate the bottom drag stress in the TWA zonal velocity equation as

\[
\hat{D}(\hat{b}_k) = \rho_0 c_d \sqrt{\hat{u}^2 + \hat{v}^2} \hat{u}. \] (B5)

The resulting thickness-weighted averaged force due to bottom drag in the zonal momentum equation is then

\[
\hat{X}_b(\hat{b}_k) = \overline{\nu} \hat{b} \sqrt{\hat{u}^2 + \hat{v}^2} \hat{u} / \sigma \Delta \hat{b}. \] (B6)

This force is shown as the green line in Fig. 6. Embedding all of these force calculations into the in situ TWA diagnostic will be required for broader application of this technique.

REFERENCES


