Three-Dimensional Baroclinic Eddies in the Ocean: Evolution, Propagation, Overall Structures, and Angular Models

GUANGHONG LIAO
College of Oceanography, Hohai University, Nanjing, and Laboratory for Regional Oceanography and Numerical Modeling, Pilot National Laboratory for Marine Science and Technology, Qingdao, China

XIAOHUA XU
College of Oceanography, Hohai University, Nanjing, China

CHANGMING DONG
College of Marine Science, Nanjing University of Information Science and Technology, Nanjing, China, and Department of Atmospheric and Oceanic Science, University of California, Los Angeles, Los Angeles, California

HAIJIN CAO AND TAO WANG
College of Oceanography, Hohai University, Nanjing, China

(Manuscript received 19 November 2018, in final form 17 July 2019)

ABSTRACT

The evolution and dynamics of an initially Gaussian baroclinic vortex embedded in a resting stratification environment are investigated with a three-dimensional primitive equation model. The vortex evolution process strongly depends on multiple parameters. Particularly, the effects of the gradient of planetary vorticity, nonlinearity, and friction have been studied to evaluate their roles. Comparisons with previous results from a simplified model are made. We particularly focus on interactions between vortices at different levels to understand their evolution. Additionally, a set of numerical simulations has been performed to examine the role of parameter space (Froude–Rossby number) on the evolution of eddies. The evolution and propagation of the vortex is affected by the planetary vorticity gradient; nonlinearity accelerates the transfer of energy from low to high angular mode and speeds up the propagation of eddies. A new finding is that a “double dipole” structure is observed with the development of a vortex, which is located in the core and edge regions, respectively. Only eddies in a finite depth range can maintain synchronous motion, and the dispersive translation paths in all levels imply that initially aligned eddies finally develop into misaligned structures and lead the tilted axis. In contrast to the results in a previous study, eddies have a southward drift irrespective of the vortex polarity. The eddies in the upper level maintain strong stability; in the middle depths, eddies decay rapidly where they form mixed barotropic and baroclinic instabilities. The energy budget analysis demonstrates the complex energy conversion between eddy and angular modes. The Burger number is the most important factor affecting the pattern of eddy evolution.

1. Introduction

Eddies in the World Ocean are ubiquitous features that contain a large portion of the ocean’s mean kinetic and potential energy (Rhines 2001). These vortices, which are capable of important energy, heat, and salt transport, exert a vital impact in setting the ocean state and circulation, and therefore understanding their dynamics is essential. Examples of surface eddies include the Gulf Stream Rings near the eastern coast of Canada, the Kuroshio Warm Core Rings off the coast of Japan, the eddies in the Agulhas current system near South Africa, and the eddies in the Circumpolar region. Unlike surface eddies, the subsurface eddies are relatively difficult to find as they exist at various depths and are therefore not observed frequently (Richardson...
The most famous subsurface eddies are Mediterranean eddies (also called meddies), which are formed by the detachment of Mediterranean water from either the southern or western Portuguese coast (Bower et al. 1997). The Arctic region is another location where abundant subsurface eddies are usually formed by the intrusion of cold and fresh meltwater in warmer and stratified seawater (Chao and Shaw 1996). Vortices are also generated in the wake behind an island or seamount (Schär and Durran 1997; Dong et al. 2009; Chen et al. 2015; Perfect et al. 2018). Much of what we know about eddies comes from dynamic sea surface height signals measured by satellite altimetry (Chelton et al. 2011). However, it is clear that the surface of the oceans does not entirely control ocean dynamics at depth. It is now well documented that the ocean interior is populated by a large number of vortices (Castelao 2014; Xu et al. 2016; Zhang et al. 2014, 2016; Keppler et al. 2018). Vortices in a stratified ocean have a structure that is essentially 3D; they evolve and propagate in both time and space and are highly nonstationary and complex. Additionally, it has been extremely challenging to fully capture evolving oceanic vortices in terms of their detailed 3D structures from field observations (Zhang et al. 2016).

The study of isolated vortices in rotating fluids—including their dynamics, instability properties, mutual interaction behavior, and effects due to bottom topography—is of fundamental interest for establishing refined models of geostrophic turbulence and general oceanic circulation. Abundant literature exists concerning the evolution of vortices (e.g., Flierl 1977; McWilliams and Flierl 1979; Flierl et al. 1980; Sutyrin et al. 1994; Orlandi and Carnevale 1999; Reznik et al. 2000; Early et al. 2011). However, much knowledge on the evolution of eddies comes from studies in which an eddy is confined to an active layer, such as the reduced-gravity shallow-water model (Early et al. 2011) or the simpler quasigeostrophic model, which can have either one layer (McWilliams and Flierl 1979; Smith and Reid 1982; Nycander and Sutyrin 1992; Benilov 1996; Stegner and Dritschel 2000) or two layers (Flierl et al. 1980; Smith and O’Brien 1983; Sutyrin and Dewar 1992; Nycander 1994; Pakyari and Nycander 1996). These models can be useful tools for understanding specific phenomena in designated limits of parameter space, but the evolution of eddies is constrained to remain within the asymptotic regime defined by some approximate assumptions. That is, such models lack many of the complexities associated with primitive equations, so 3D vortices in rotating, stratified fluids are subject to subtle physical processes that have not yet been investigated in detail and are thus not fully understood, although there has been a great deal of progress in interpreting and modeling the dynamics of monopolar vortices on the beta plane using simple models. To avoid this limitation, we try to study the evolution of 3D isolated eddies based on a primitive equation model.

Previous studies have demonstrated that steady vortices must propagate exactly westward at the maximum linear Rossby wave phase speed, irrespective of the vortex polarity (Chelton et al. 2011). A number of investigators have derived various expressions predicting the westward drift of isolated vortices in a single-layer model (Nof 1981; Killworth 1983; Cushman-Roisin et al. 1990). This ensures that these vortices do not radiate Rossby waves. However, the drift velocities of real oceanic vortices generally have significant components in the meridional direction (Chelton et al. 2007). Adem (1956) showed that a symmetric, isolated nondivergent cyclone on a beta plane would initially move westward. Then, as second-order circulations formed, the cyclone would turn and move northwestward under the combined poleward drift and vortex Rossby wave effects. This meridional drift can be a result of Rossby wave radiation. An initially symmetric vortex in the meridional planetary vorticity gradient will develop a pair of counterrotating gyres (beta gyres) due to Rossby wave dispersion and multiscale interactions (Rossby 1948). Specifically, in the absence of compensating pressure gradient forces, the stronger Coriolis force on the poleward versus the equatorward side produces a poleward force on a cyclone that is directly related to the cyclone rotation rate and its radial extent, as well as planetary vorticity gradient. In other words, quasigeostrophic monopolar vortices propagate westward and excite Rossby waves because of the meridional gradient of the Coriolis parameter. This wave radiation depletes the vortex energy, causing cyclones to drift northward and anticyclones to drift southward. These previous studies were mostly limited to one-layer systems, and the effect of baroclinicity on the migration of eddies was not closely examined. The principal reason for this is the complication caused by baroclinic processes arising when all layers are in motion. A formula predicting the speed of migration of eddies for a two-layer system was proposed by Cushman-Roisin et al. 1990, which contained a new term correlating the interfacial displacement and derivatives of bottom layer pressure; however, it is difficult to determine the signs and magnitudes of the term using generalities. The evolution of an isolated vortex in the 3D rotation and stratification environment is expected to be far more complicated, and has not been studied yet.

In this study, we attempt to address some basic issues, such as the dynamics of tall vortices in a stratified fluid; their evolution, propagation, and mutual interaction
behavior at different levels; and so on. These problems can be neither examined nor thoroughly understood unless three-dimensional model-solving primitive equations are used. More specifically, the role of nonlinear advection, planetary vorticity gradient, and viscosity on the evolution of the vortices is studied, and how different parameter space influences the evolution of vortices is investigated in detail. We mainly focus on the evolution of a 3D isolated vortex with the first baroclinic structure; this apparently very simple case is particularly interesting, and the investigation of such an isolated vortex can be a convenient starting point to gain a better understanding of 3D eddies in the real ocean. Furthermore, knowledge of the structure of the first baroclinic isolated vortex will be expanded in the future to describe the interaction process between vortex and current. Where possible, model results here will be compared with those of these previous studies using simplified models when input parameters are similar in value.

The paper is organized as follows. In the next section, we briefly describe the quasigeostrophic model, with particular attention given to the balanced initialization procedure in the 3D primitive equation numerical model. Several numerical experiments were designed for investigating the effect of dynamical factors on the evolution of an eddy; a broad parameter experiment has also been considered. In section 3, we describe the evolution, propagation, and overall structures of 3D eddies in some detail; dynamic factors effecting evolution of eddy are explained; and we also will analyze evolution of the vortex as a function of the parameters. The azimuthal mean and angular modes decomposition and kinetic energy budget are quantified. The last section summarizes the major findings of this work.

2. Numerical model and experiments

In this study, the Stanford Unstructured Nonhydrostatic Terrain-Following Adaptive Navier–Stokes Simulator (SUNTANS) ocean model (Fringer et al. 2006) is used to simulate the evolution of an initial 3D eddy in an ocean exhibiting continuous stratification. The SUNTANS model solves the Navier–Stokes equation under the Boussinesq approximation on a finite-volume grid. Some technical details can be found in Fringer et al. (2006).

a. Initialization

To initialize the three-dimensional structures of vortices, we first consider the nonlinear quasigeostrophic model in a continuously stratified ocean on the β plane:

$$\frac{\partial q}{\partial t} + J(\psi, q) = 0,$$

where \(q\) is the quasigeostrophic potential vorticity (PV), which is defined as follows:

$$q(x, y, z, t) = \nabla^2 \psi + \frac{f^2}{N^2(Z)} \frac{\partial \psi}{\partial Z} + \beta y,$$

where \(\psi(x, y, z, t)\) is the streamfunction, \(J(a, b) = (a, b_y - a_y, b_x)\) is the Jacobi operator, and \(N(z) = \left[ -(g/\rho_0)(\partial \rho/\partial z) \right]^{1/2}\) is the buoyancy frequency (and \(\rho_0\) is a constant density). Parameter \(\beta\) is the meridional derivative of the Coriolis parameter \(f = f_0 + \beta y\), and \(f_0\) is the local Coriolis parameter. The solutions for velocities include geostrophic and ageostrophic components to emphasize that quasigeostrophy is a correction to geostrophy. For a general situation, that is, buoyancy frequency variability with depth, then the streamfunction is recovered from the PV by first diagonalizing the operator \(\partial/\partial z [f^2/N^2(Z)]^{1/2} \partial \psi/\partial z\) (Carton 2001):

$$\frac{d}{dz} \left[ \frac{f^2}{N^2(N)} \frac{dF_m}{dz} \right] = -\lambda_m^2 F_m,$$

which is subject to the following boundary condition:

$$\frac{dF_m}{dz} = 0 \quad z = 0, -H,$$

where \(F_m(z)\) is the vertical modes. There are an infinite number of solutions (modes) for the eigenvalue equation in (5). For each mode \(m\), the eigenvalue \(\lambda_m\) is the inverse of the radius of the baroclinic Rossby deformation \(R_d\). The deformation radius physically expresses the scale of the horizontal deformation of an isopycnal due to the motion with a given vertical scale. Mode 0 is the barotropic mode, and higher modes are baroclinic modes. The vertical structures of ocean eddies are often expressed in terms of “baroclinic modes.” Observations have shown that the first baroclinic mode dominates the vertical structure oceanic eddies (Krauss et al. 1990).
Then, PV and streamfunction are projected on the $F_m$ basis:

$$q(x, y, z) = \sum_m q_m(x, y) F_m(z),$$

(7)

$$\psi(x, y, z) = \sum_m \psi_m(x, y) F_m(z),$$

(8)

where $q_m(x, y)$, $\psi_m(x, y)$ is the horizontal structure of PV and streamfunction, respectively.

The initial density is specified in terms of a reference density profile with shifted isopycnals induced by eddies. Thus, the density can be expressed as follows:

$$\rho(x, y, z) = \rho(z) + \rho',$$

(9)

where $\rho(z) = \rho_0(1 - N^2 z/g)$; $\rho_0 = 1020$ kg m$^{-3}$ is a constant density; $g$ is the gravitational acceleration constant; and $\rho'$ is the perturbation density induced by an eddy, which is determined by the streamfunction in (4). Here, we determine the background buoyancy frequency $N(z)$ structure according to the Lorentzian formula:

$$N^2(z) = \frac{(N_p \Delta H/2)^2}{(z + H_p)^2 + (\Delta H/2)^2},$$

(10)

where $H_p$ is the depth of maximum buoyancy frequency $N_p$, and $\Delta H_p$ is the range of depth where the buoyancy frequency is higher than $N_p/2$. The background density field is obtained using the specified $N^2(z)$. Then, the vertical structure function $F_m(z)$ is obtained by numerically solving the eigenvalue equations (5) and (6).

The eddy field in the 3D primitive equation model is analytically initialized according to the steps as follows: the sea surface height is specified first using a Gaussian-shaped eddy perturbation over the entire domain, that is

$$\eta(x, y, 0) = \eta_0 \exp\left[-(L/L_0)^\alpha\right],$$

(11)

where $L = \sqrt{(x - x_0)^2 + (y - y_0)^2}$, $(x_0, y_0)$ is the initial location of the eddy center, $L_0$ is the eddy $e$-folding radius, and $\eta_0$ is the maximum sea surface height. The constant $\alpha$ is a steepness parameter, specifying the radial vorticity gradient of the eddy. Once the sea surface height field is determined, the streamfunction in the horizontal direction can be recovered using the simple relationship between $\psi(x, y) = g\eta(x, y)/f$. Using (8), the 3D streamfunction for a Gaussian-shaped eddy is constructed, where the vertical structure is solved when the background density profile is given. Finally, based on the quasigeostrophic theory described above [(3) and (4)], the matched velocity and perturbation density field induced by an eddy can be recovered; it should be noted that the $\beta$ and time derivative terms are removed in (3) during the initialization process. Thus, the 3D eddy field is initialized using consistent sea surface height, density, and velocity. Furthermore, the Jacobi term is relatively small in comparison to the other terms, and the initialization field primarily satisfies geostrophic balance.

b. Experiments setup

Given a velocity scale $U$, a length scale $L$, and a height scale $H$, one constructs the Rossby and Froude numbers in the conventional way: $R_o = U/f_0L$ and $F_r = U/NH$, where $N$ is defined as $1/H \int_{-H}^H N(z) \, dz$. These two numbers define the two-dimensional parameter space of the problem. Another dimensionless number is the Burger number, $B_u = (NH/f_0L)^2 = (R_o/L^2)$, the square of the ratio of the Rossby deformation radius $R_d = NH/f_0$ and the typical length scale of motion, the Burger number measure the relative importance of rotation and stratification. As we will show, the Burger number plays a major role in the evolution of the vortex. So we first design a reference simulation (named Exp0 hereafter), in which the radius of eddy is set to the first baroclinic Rossby deformation radius, thus leading to a corresponding Burger number equal to 1, and it implies that the rotation and stratification are equally important. An idealistic pycnocline-type profile is employed as the background stratification in Exp0; the background density and buoyancy frequency profiles are shown in Figs. 1a and 1b. The maximum buoyancy frequency is located at a depth of 900 m with a value of $4 \times 10^{-3}$ s$^{-1}$. At a latitude of 24°, $f_0 = 6.0 \times 10^{-5}$ s$^{-1}$, and the Rossby deformation radius $R_d$ values determined from the background field are $1.2 \times 10^7$ and 90 km for the barotropic and first baroclinic modes, respectively. The corresponding first baroclinic Rossby wave phase speed is $c = \beta R_d^2 = 13.8$ cm s$^{-1}$, and the $\beta$ effect over this time scale is $\beta(1/R_d) = 7.5$ days. Based on statistical study from 16 years of sea surface height data observed by the satellites (Chelton et al. 2011), more than 90% have radius values between 50 and 150 km; the radius scales of ocean eddies are comparable with local Rossby radius of deformation. According to Chelton et al.’s (2011) statistical results, the eddy amplitudes are broadly distributed, with 40% of eddies having amplitudes of less than 5 cm and 25% having amplitudes of greater than 10 cm. Therefore, a typical minimum (maximum) sea surface height of 15 cm is selected for the initial cyclonic (anticyclonic) eddies, with a typical tangential velocity of 0.15 m s$^{-1}$.

The parameters used for the initialization field are listed in Table 1. Figure used on the initialization field is shown in Fig. 2a, which is
visualized by the isosurface of the pressure anomaly ($\pm 50 \text{N m}^{-2}$) and shading by the PV anomaly. The initialized eddy is a tall column vortex penetrating the full water depth in our model. The Gaussian vortices defined in the model described above are shielded, that is, in a cyclonic eddy, the central core of the vortex is a contiguous region where the vertical component of its vorticity has the same sign as the Coriolis parameter, and the shield area is an annular ring with the opposite vorticity around its core. This circulation quickly decays outside the shield. The radius–vertical cross sections of pressure, density, and relative vorticity are shown in Figs. 2b–d.
In the vertical direction, we select the first baroclinic mode structure to initialize the eddy. For the barotropic mode with a very large deformation radius \((1.2 \times 10^7 \text{ km})\), the corresponding motion scale is far greater than the scale of a mesoscale eddy, and such a barotropic eddy is thus unstable. For the first baroclinic mode, the vertical structure function changes its sign at a depth of 1500 m (figure omitted). The cyclonic eddies with low pressure anomalies are initialized in the upper layers above a depth of 1500 m; however, anticyclonic eddies exist in lower layers. In such eddy structures, a positive density anomaly appears in the midwater column (Fig. 2c). A positive (negative) relative vorticity anomaly is surrounded by the negative (positive) relative vorticity in the upper (lower) layers, exhibiting an annulus structure. In the middle layer, the relative vorticity is weak and consistent with the prescribed first baroclinic structure.

Based on the parameters listed in Table 1, the conventional Rossby number \(R_o = 0.03\) is fairly small for \(f\)-plane dynamics, indicating that the role of nonlinearities is fairly unimportant in the momentum balance, especially in the region away from the eddy on the shelf. For a \(\beta\)-plane eddy, a different measure is the quasigeostrophic nonlinear parameter, defined to be the ratio of the relative vorticity advection to the planetary vorticity advection (McWilliams and Flierl 1979): \(Q = U/\beta L^2 = 1.1\), which indicates the weak nonlinearity of isolated eddies on the \(\beta\) plane. The Froude number \(F_r = 0.03\) is also small, thus, the Burger number \(B_u = 1\), which is indicative of equivalent important for stratification and rotation effects in the reference simulation. To investigate dynamic factors and the parameter space that probably affect the evolution of an eddy, another two types of experiments are designed in this study: a dynamical factor experiment and a parameter dependence experiment. The descriptions of the experiments are given in Table 2, and details will be given in section 3. This study includes a wider range of values than the previous numerical study (Flierl 1977; McWilliams and Flierl 1979; Sutyrin et al. 1994; Pakyari and Nycander 1996; Reznik et al. 2000; Early et al. 2011).

c. Model setup

Using the initialization described above, the symmetric vortices in the first baroclinic structure are embedded in a resting environment with a constant planetary vorticity gradient. After it is configured for the present application, the numerical model is tested to assess the evolution of an isolated eddy with flat bottom topography. The model adjustment is performed by initializing the SUNTANS model (Fringer et al. 2006) with one of the above-described experiments and then integrating the model forward for 160 days, considering that the typical lifetime of an open-ocean eddy is approximately 130 days. The data are output in 2-day intervals. Simulations are performed in the basin-scale rectangular domain with a length of \(L_x = 3000 \text{ km}\) and a width of \(L_y = 1600 \text{ km}\), and the eddy with the first baroclinic structure is seeded at eastern side in the basin. The lateral grid has a resolution of 5 km. In the vertical direction, 80 layers are used, with a minimum resolution of 15.1 m near the surface expanding to a resolution of 74.0 m at the bottom cell. The initial location of the eddy center is located at a distance of 5 times the radius of the eddy away from the eastern boundary and in the center of the meridional direction (i.e., \(x_0 = L_x - 5L_0\), \(y_0 = L_y/2\)). To prevent transient oscillations with the impulsive starting of vortices, the initial vortices are spun up over 1 day, that is, the sea surface height, density perturbation, and velocities slowly increase during the initial transient period (1 day). The boundary condition chosen for this study, which is the simplest but by no means the best, is a sponge layer in the model. Within a given distance next to the boundary, a damping term is added to the right-hand side of the horizontal momentum equation:
FIG. 2. (a) 3D structure of the initialized first baroclinic eddy, visualized by the isosurface of the pressure anomaly ($\pm 50 \text{ N m}^{-2}$) shading by potential anomaly, density slice, and velocities. (b) Azimuthal mean pressure anomaly profile, (c) azimuthal mean density anomaly profile, and (d) azimuthal mean relative vorticity profile.
3. Results

a. Evolution of vortices

In the reference and dynamic factor experiments, rotation rate and background stratification are kept constant, fixing the background deformation radius. Most of our discussion is concerned with our reference simulation (Exp0), unless stated otherwise. In Fig. 3 we show the pressure anomaly (contoured lines) at various stages of evolution of eddy at two different depths for Exp0, which represents cyclonic and anticyclonic eddies located in the upper and lower layer, respectively. The physical variable that best describes the rotation and the density anomaly associated with vortices is potential vorticity, which is defined as \( q = (\omega + f)/f \bar{\nabla} \rho \); therefore, the normalized PV anomalies (in color) are overlaid in these figures, and the cyclone (anticyclone) can be treated as a negative (positive) PV anomaly. In our model, although the initial field of motion in the above-described eddy is geostrophically balanced, the geostrophic adjustment of the model is performed due to nonlinear effect. The initial conditions excite some transient small-amplitude inertial gravity waves due to the stratification of the background ocean. As a result, the sea surface height is adjusted, and the eddy radiates some of its energy outward as inertial gravity waves (Wang and Özgökmen 2016). Thus, only the results output after 10 days of the spinup model are adopted to analyze the evolution of the eddy, which is greater than the \( \beta \)-effect time scale of 7.5 days. The initial vortex is axially symmetric, with a core of positive vorticity surrounded by an annulus of weak negative vorticity (Fig. 2d). For such a shielded vortex, the Rayleigh inflection point criterion is satisfied; thus, it will undergo the shear instability that produces satellite eddies in the periphery of the main eddy core as report by Beckers et al. (2003), which is similar to the Kelvin–Helmholtz instability for planar shear flow. However, for most eddies of upper layer in our simulation, this instability is weak and does not lead to the breakdown of the initial circular vortex structure into smaller vortices as observed from Fig. 3. The main eddy core can maintain its shape for a long time and can be identified by tracing pressure or PV anomaly field. Over about 20 days, an anticyclonic (cyclonic) vorticity grows to the northeast of the initial cyclonic (anticyclonic) vortex; thus, a vortex pair consisting of counter-rotating eddies has been formed. Associated with these PV anomalies are two eddies with opposite signs, which form an asymmetrical flow over the vortex core that affects the motion of the vortex. Such a PV structure satisfies the condition necessary for barotropic instability and may result in the development of some small-scale eddies. We found that barotropic instability may be one of the causes of stirring and mixing. It will be clarified later that these opposite polarity vortices emerge due to the \( \beta \)-effect according to the conservation of potential

\[
s_j(x, y, z, t) = \frac{u_j(x, y, z, t)}{\tau} s(r). \tag{12}
\]

Here, \( j = 1, 2 \) is the component of horizontal velocity; \( s(r) = \exp(-4r/L_s) \), which causes the sponge layer to decay exponentially over the distance of \( L_s = 80 \text{ km} \) from the nearest boundary, which is located at a distance \( r \) from the point \( x, y \); and \( \tau = 720 \text{ s} \) is the damping time scale. A constant drag coefficient \( (C_d = 0.0025) \) is given at the bottom and sidewalls. The turbulent horizontal coefficient is constant and given by \( \nu_H = 50 \text{ m}^2 \text{ s}^{-1} \), thus the Reynolds number \( R_e = UL/\nu_H = 1440 \), and the vertical eddy viscosity coefficient is constant and given by \( \nu_V = 10^{-4} \text{ m}^2 \text{ s}^{-1} \). These viscosity coefficients are set by stability requirements of central differencing for momentum advection. We have carried out several simulations with varying \( \nu_H \) in order to examine the parameter dependencies of the features of the eddy evolution (see Exp4).

<table>
<thead>
<tr>
<th>Experiment case</th>
<th>Experiment name</th>
<th>Experiment description</th>
<th>Dimensionless parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference simulation</td>
<td>Exp0</td>
<td>Nonlinear advection; beta plane</td>
<td>( R_e = 0.03; F_r = 0.03; B_a = 1; R_c = 1440 )</td>
</tr>
<tr>
<td>Dynamical factor experiment</td>
<td>Exp1</td>
<td>No nonlinear advection; beta plane</td>
<td>( R_e = 0.03; F_r = 0.03; B_a = 1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp2</td>
<td>Nonlinear advection; ( f ) plane</td>
<td>( R_e = 0.03; F_r = 0.03; B_a = 1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp3</td>
<td>No nonlinear advection; ( f ) plane</td>
<td>( R_e = 0.03; F_r = 0.03; B_a = 1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp4</td>
<td>No nonlinear advection; beta plane; large Reynolds number</td>
<td>( R_e = 0.03; F_r = 0.03; B_a = 1; R_c = 72000 )</td>
</tr>
<tr>
<td>Parameter dependence experiment</td>
<td>Exp5</td>
<td>Small Rossby number case; nonlinear</td>
<td>( R_e = 0.03; F_r = 0.09; B_a = 0.1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp6</td>
<td>advection; beta plane</td>
<td>( R_e = 0.03; F_r = 0.003; B_a = 10; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp7</td>
<td>Large Rossby number case; nonlinear</td>
<td>( R_e = 0.3; F_r = 0.3; B_a = 1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp8</td>
<td>advection; beta plane</td>
<td>( R_e = 0.3; F_r = 0.9; B_a = 0.1; R_c = 1440 )</td>
</tr>
<tr>
<td></td>
<td>Exp9</td>
<td></td>
<td>( R_e = 0.3; F_r = 0.03; B_a = 10; R_c = 1440 )</td>
</tr>
</tbody>
</table>
vorticity. The main core vortex elongates in a direction and then flattens; the flow is no longer axisymmetric and meanders develop on the original eddy boundary. Evolution of eddies in the upper and lower layers is independent, with complicated evolution and three-dimensional structure in the lower layer, for example, at the 1500-m level (bottom row in Fig. 3), whereas small-scale plumes dominate over the vortex region with small-scale meanders developing around the main eddy. At a much later stage, several opposite polarity eddies appear with the original eddy. The eddies migrate away from the starting point, destroying the original preconditioning eddy, and the initialized cyclonic vortices in the upper layer move toward the southwest. This is different than a previous report that stated that cyclonic vortices in the ocean propagate northwestward (Nof 1981; Cushman-Roisin et al. 1990; Reznik et al. 2000). The trajectories of the eddy center shown in the next section (Fig. 6) further illustrate the various stages of eddy evolution, where propagation of vortices will be addressed in detail.

To examine the evolution of the whole eddy column, Fig. 4 shows the vertical slice structure of a vortex at three different stages. The impression of an eddy is shown in pressure, density, and PV anomaly fields. In Fig. 4a, the isolated eddies still are aligned in the vertical direction, the eddy interior has sloping isopycnals, and the negative pressure anomaly regions are associated with strong cyclonic eddies, accompanied by negative potential vorticity anomaly. In Fig. 4b, the beta-induced opposite polarity vorticity (called $\beta$ eddy hereafter) is seen to be formed, but it is weak and confined in the narrow depth range (Fig. 4b). At day 160, these $\beta$ eddies become stronger (Fig. 4c). The development of a $\beta$ eddy in the lower layer is faster than that in the upper layer. The sloping isopycnals gradually become flat, which means the available energy is released. From wave viewpoint, the negative PV (pressure) anomaly propagates downward with an angle with the vertical direction, and the positive PV (pressure) anomaly propagates upward, as evident from Fig. 4. At the depths around the node of the first baroclinic mode, the pattern of flow is complicated and changeable, and eddies at these depths form mixed barotropic and baroclinic instabilities. Eddies in the bottom layer evolve to form multipolar structures. Along the depth direction, the initialized first baroclinic model spawns into a secondary baroclinic model (Fig. 4c).

We investigate the probable dynamic factors that affect the evolutions of eddies through several numerical experiments (Exp1–4), which examine the influence of nonlinear advection term, gradient of planetary
vorticity, and viscosity separately. We turn off the advection term in the primitive equations in the linear case (Exp1). In Exp2, the Coriolis parameter is taken as constant, thus the beta effect is not taken into account; both advection term and beta effect are excluded in Exp3. In Exp4, we only examine the viscosity effect by changing horizontal viscosity coefficient. The experiment descriptions and dimensionless parameters are given in Table 2. Figure 5 shows the final status of eddies at 250-m depth in the different dynamical experiments. In all experiments, the vortices undergo varying degrees of deformation and cannot sustain their initial perfect circulation. The result from the linear case (Fig. 5a) is highly similar to that in the reference simulation (Fig. 3), and it is easy to understand that the nonlinearity is weak in these simulations because of a small Rossby number ($R_o \approx 0.03$). Flierl’s (1977) work showed that in the absence of friction, a linear vortex decays rapidly due to dispersive effects on the beta plane. For stronger, larger-amplitude vortices, McWilliams and Flierl (1979) and Mied and Lindemann (1979) showed that nonlinear advective processes stabilize eddies against beta dispersion effects and allow them to propagate as stable units for longer periods of time than their linear counterparts. However, comparing Figs. 5a and 5b, the periphery boundary of the eddy is torn into four spiral arms, as observed from the PV field in Exp2 simulation (Fig. 5b). We find that nonlinearity speeds up the instability of the annulus of the eddy and leads to the breakage of the outside annulus, and the $\beta$ effect is the only dynamical factor for formatting a $\beta$ eddy. In the case of the $f$ plane and linear dynamics (Exp3), the eddy also suffers slight deformation, and it is especially evident from the PV anomaly field. The PV maintains its negative anomaly during the entire evolution, but tends to develop into four lobes.

The planetary vorticity gradient is the primary factor that controls the development of a $\beta$ eddy, and the $\beta$ eddy signal cannot be detected in Exp2 and 3 (Figs. 5b,c), where the planetary vorticity gradient is not included. An interesting point is that the $\beta$ effect destroys the rotational symmetry of the eddy, and the flow field exhibits anisotropy during development of eddies; the flow velocity on the south side of the vortex strengthens and the flow velocity on the north side weakens. The difference of Coriolis forces along the latitude direction can be used to explain such phenomena under the geostrophic balance theory frame. Another primary result is that the $\beta$ effect drives the translation of an eddy. In Exp4, the horizontal viscosity coefficient $\nu_H = 1 \text{ m}^2 \text{s}^{-1}$, less than 20 times than that in the reference experiment, thus the Reynolds number is 72,000, which characterizes the influences of dissipation to motion. A higher turbulent environment makes the evolution of eddies more complicated, and small-scale eddies can be seen in Fig. 5d.
We also examine the slice structures for four dynamical factor experiments (figures omitted). Comparing with the reference experiment, we find that $\beta$ eddy mainly appears in the middle layers in the linear case (Exp1). Without $\beta$ effect, the vortex still keeps the first baroclinic structure throughout the evolution (Exp2 and 3). It is shown that the eddies in the middle layers gradually disappear in Exp3, where the nonlinearity is not taken into count. It is demonstrated that the nonlinear effect plays a role in stabilizing the eddies. In Exp4, the higher turbulence characteristics make the initial isolated eddy split into some small-scale eddies in the middle layers.

**b. Propagation of vortices**

In the section, we will discuss the propagation velocity and trajectory of eddies. First, we take mass centroids of eddy as the eddy center. The mass centroids are defined here as the first statistical moment, weighted by density. The vertical dependence of the propagation of vortices is shown in Fig. 6, which displays the trajectories of the center of vortices at different levels. Previous studies showed a westward component of the motion of eddies due to the beta effect, with a meridional component of motion induced by the nonlinear advection effect. It should be mentioned that the $\beta$ effect is the only driving force of the movement of the vortex as shown in section 3a in our study. It is clear that the path of vortex motion significantly differs from the surface to the bottom in all experiments from Fig. 6. The initial baroclinic vortex moves poleward as a whole, but exhibits different trajectories for each depth, as shown

![Figure 5](image-url)

**Fig. 5.** Evolution of the vortex at 250 m under the different dynamic conditions. Only the final statuses (at day 160) are present. (a) Linear case (Exp1), (b) no beta case (Exp2), (c) linear and no beta case, and (d) large Reynolds number case (Exp4). The pressure anomaly (contoured with positive/negative dashed/solid lines) and normalized potential anomaly (in color) are shown in each panel.
in Fig. 6. After 160 days of integration in the reference experiment (Exp0), the eddy at the surface moved 211.1 km to the west and 114.4 km to the south, with an average translation speed of 1.76 cm s\(^{-1}\) and a direction of 152° from the east. On average over all ocean basins, the eddy translation speed at 30° latitude has been estimated to be 3.5 ± 1.5 cm s\(^{-1}\) from satellite altimetry data. The eddy at the bottom moved 27 km to the east.

FIG. 6. Trajectories of kinetic energy center of vortices at all model levels in the (a) reference experiment Exp0, (b) linear experiment Exp1, and (c) large Reynolds number experiment Exp4.
and 56 km to the south, with an average translation speed of 0.6 cm s$^{-1}$ and a direction of 64° from the east. In the linear case (Exp1), the results show that the surface eddy translated 175 km westward and 132 km southward, with an average translation speed of 1.6 cm s$^{-1}$ and a direction of 127° from the east. Comparing Exp0 and Exp1 reveals that when nonlinearity is included (Exp0), the westward rates increase a little above the linear value. A distinct difference is that the lower eddies in Exp0 move eastward.

The trajectories of vortex centers above a depth of 200 m (nearly the depth of the thermocline layer) are almost consistent, which implies that these eddies in the upper 200-m depth range maintain synchronized movement. Below this depth, the trajectories of vortex centers separate gradually, that is, the deeper the vortex is, the more southward, or even southeastward, the trajectory, while a more interesting phenomenon is that the anticyclonic eddies are nearly consistent with the movement above the nearest cyclonic vortices (see blue dotted lines in Fig. 6 for lower anticyclonic eddies). These findings are not consistent with previous studies in a single-layer model, which predicted that cyclonic and anticyclonic eddies are directed poleward and equatorward, respectively (Flierl 1977; McWilliams and Flierl 1979). In addition, the translation speeds of vortices gradually slow down with increasing depth irrespective of eddy polarity, which can be inferred from shorter translation distance in the deeper water level. The dispersive translation paths in all levels imply that these initially aligned eddies finally develop into misaligned structures in the vertical direction, thus the axis consist of each eddy center is tilt. Comparing linear (Exp1) and reference simulation (Exp0) cases reveals that nonlinearity causes the motion of vortices to become more westward, and make the paths more complicate. In the linear case, the vortices propagate over shorter distances. From Fig. 6c, we observe that higher turbulent environment weakens the southward movement of eddy. Finally, the trajectory of the vortex exhibits cyclonic or anticyclonic curvature during some time segment of its integration, which arises from the interactions between circulations at different levels after the vortex tilts, as discussed in section 3a.

Furthermore, the time series of translation speeds averaged in the vertical direction for upper cyclonic eddies and lower anticyclonic eddies are shown in Fig. 7. Several key points can be found in the figure. First, the westward translation speed in the upper layer is greater than that in the lower layer, with a few exceptions in Exp4. The averaged zonal speed of a cyclonic eddy in the upper layer is about $-1.3$ cm s$^{-1}$ with slight fluctuation, which is similar to $-1.59$ cm s$^{-1}$ obtained using the quasigeostrophic model by McWilliams and Flierl (1979). The anticyclonic eddy in the lower layer translates eastward with an averaged zonal speed of 0.3 cm s$^{-1}$. The theory predicts that the zonal drift of the eddy in a one-layer, reduced gravity model is $c = -(\beta g\delta H/f^2) - (\beta g'\delta H/f^2)$, where $g'$ is the reduced gravity in the lower layer.
gravity, \( H \) is depth, and \( \delta H \) is the vertical displacement scale (Cushman-Roisin et al. 1990). In our model, \( g' \approx 0.01 \) and \( H \) and \( \delta H \) are estimated as 1000 and \(-100\) m, respectively, according to the displacement of the isopycnal for the upper cyclonic eddy. Thus, from the theory the zonal speed is \(-4.3 \text{ cm s}^{-1}\). The calculated zonal speed based on quasigeostrophic theory is larger than that resulting from our 3D model \((-1.3 \text{ cm s}^{-1}\)). However, the use of continuous stratification, a full 3D model, and a different initial eddy structure in this study slightly affects these comparisons. In the linear case (Exp1), the results show that the upper cyclonic eddy and the lower anticyclonic eddy both translate westward at an average speed of \(-1.2 \) and \(-0.67 \text{ cm s}^{-1}\), respectively. The zonal speed of the upper cycloonic and lower anticyclonic eddies increases with a large speed oscillation in Exp4, where the Reynolds number is larger.

Second, the averaged meridional speed for the upper cyclonic eddy is about \(-0.79 \text{ cm s}^{-1}\) in Exp0; the averaged meridional speed for the lower cyclonic eddy is about \(-0.28 \text{ cm s}^{-1}\) with large fluctuation. In Exp1, however, the meridional speed of the upper and lower eddies is nearly equivalent, with an averaged speed of \(0.94 \text{ cm s}^{-1}\).

Third, further examination finds that the westward translation speed of an eddy decreases with depth, and its meridional speed has no observed notable difference in the vertical direction (figures omitted).

Fourth, the motions of vortices are not uniform, and their translation speeds show notable oscillating characteristics, especially for the case of a large Reynolds number, indicating that they exhibit a complicated self-induced processional motion.

Our results obtained for vortex motion seem to exhibit obvious different behaviors compared with what was found in previous studies using the simple model. In our present eddy configuration and 3D simulation, if one attempts to track an eddy in the horizontal plane, multiple possibilities can be found, that is, they drift southward irrespective of the vortex polarity, and they can also probably propagate eastward. The translation of a vortex is the result of three effects: planetary vorticity advection, the secondary circulation due to horizontal deformation of the vortex, and, when stratification is taken into account, the vertical tilting of the vortex core. Morel and McWilliams’s (1997) study further highlighted the trajectories that are highly dependent on the initial vertical structure of the vortex. In a single-moving layer, only a planetary gradient was considered. In our extension to a three-dimensional system with continuous stratification, the first baroclinic structure and vortex stretching were involved in our model; at the same time, radial and vertical potential gradients were also included. These plentiful dynamical factors make the dynamic process of vortex evolution very complicated; there are many kinds of possibilities, as seen in Fig. 6.

c. Overall structure of the eddy

The overall axially symmetric structure of the eddy after 80 days of model integration in the reference experiment (Exp0) is given; for more clarity, the upper cyclonic eddy and lower anticyclonic eddy are shown in Figs. 8 and 9, respectively. The low (high) pressure anomaly dominates the upper (lower) layer (Figs. 8a, 9a), which consists of the first baroclinic eddy structure as initialized; that is, they describe the cyclonic (anticyclonic) eddies in the upper (lower) layer. Correspondingly, there are positive relative vorticity values and negative PV anomalies in the upper levels. However, there are negative relative vorticity values and negative PV anomalies in the lower levels, and positive PV anomalies are present in the middle levels; such PV structures are similar to the configuration used in the quasigeostrophic numerical model of Morel and McWilliams (1997), which they named the R vortex. The distribution of the relative vorticity and pressure anomaly data also clearly reveals the annulus, that is, the main eddy core is surrounded by a ring with the opposite-sign relative vorticity and pressure anomalies, but these annuluses are broken due to shear instability with the evolution of the eddy. A high-density anomaly core exists in the middle levels; the lateral density gradient implies vertical current shear according to thermal wind balance. The azimuthal mean maximum tangential current is located approximately 100 km from the eddy center in the upper layer and tilts inward with depth (Fig. 8e); of course, the location slightly moves outward due to the diffusion of the vortex (figure omitted). The radial velocity shows complicated and interesting features: 1) the combined action of localized convection and baroclinic instability generates a complex secondary circulation. Stronger radial currents appear at mid- to lower depths, and the inflow and outflow are staggered along the radial predating the vertical circulation cells (Figs. 8f, 9f), which can be further verified from the distribution of vertical velocity. 2) Because of the suction of the cyclonic eddy, broad outflow appears in the surface layer. 3) In the deep layer, the anticyclonic eddy drives downwelling (Fig. 9g) in the center of the eddy associated with the outflow (Fig. 9f). The motions of inflow and outflow associated with vertical circulation cells indicate the occurrence of mixing within the eddy. Such secondary circulation, although weak, results in the gradual steepening of the velocity/vorticity profiles in the vortex and enhanced current shear. This is a mechanism that causes a vortex to gradually change.
from a stable to an unstable state, as observed in the multipolar structure as discussed in section 3a.

d. Characteristics of the angular mode

A rotational symmetric monopolar vortex cannot have self-induced translational motion on the $f$ plane as shown in Fig. 5. It is well known that beta drift is a basic component of vortex motion. Previous research has also shown that a so-called $\beta$ gyre is formed within the vortex (Reznik et al. 2000) when the planetary vorticity gradient is included. Because eddy motion arises from the interaction between the vortex circulation and the planetary

![Fig. 8. The overall axially symmetric structure of the upper cyclonic eddy after 80 days of model integration in the reference experiment (Exp0). Shown are depth-radius cross sections of the azimuthal mean of (a) pressure anomaly, (b) density anomaly, (c) relative vorticity, (d) potential vorticity anomaly, (e) tangential velocity, (f) radial velocity, (g) vertical velocity, and (h) angular momentum.](image-url)
vorticity gradient, its study is of importance to vortex
dynamics in general. Furthermore, the rotational sym-
metry of a monopolar vortex is destroyed when the $\beta$
effect exists, as discussed in the section 3a. To clarify
the structure and evolution of the vortex, the vortex is
decomposed into an azimuthal mean and angular mode
in a local polar coordinate system attached to a vortex
center, that is,

$$P(r, \theta) = P_0(r) + \sum_{i=1}^{2} a_i(r) \cos(i\theta) + b_i(r) \sin(i\theta)$$

+ Residual. (13)

FIG. 9. The overall axially symmetric structure of the lower anticyclonic eddy after 80 days of model integration in the reference experiment (Exp0). Shown are depth–radius cross sections of the azimuthal mean of (a) pressure anomaly, (b) density anomaly, (c) relative vorticity, (d) potential vorticity anomaly, (e) tangential velocity, (f) radial velocity, (g) vertical velocity, and (h) angular momentum.
The azimuthal mean component $P_0(r)$ represents the rotational symmetry of the vortex, and the other components represent the deformation (asymmetry) parts arising from an interaction between the azimuthal mean component and Earth’s vorticity field. They contribute to a relative flow across the vortex core that causes it to propagate poleward and westward. In our study, the initial no-flow environment eliminates the possible interaction of the vortex with the background flow. Figure 10 shows the temporal evolution of total asymmetric pressure anomaly (first column), and their modes 1 and 2 and the residual component (color shading) at a 250-m depth in the reference experiment (Exp0), in which the azimuthal mean components (white contour lines) are overlaid in each panel. To compare the azimuthal mean components, total asymmetric components and angular mode components all are normalized by maximum pressure anomaly; each row indicates different times (see figure description).

The general features of the asymmetric pressure anomaly are as follows: 1) a dipole-like pattern is significant in the initial stage of eddy evolution; 2) the multipolar structure, which consists of cyclonic and anticyclonic eddies, appears after approximately 20 days of model integration and shows asymmetry; and 3) the small-scale cyclonic and anticyclonic eddies in the asymmetric component shown stagger arrangement along the azimuthal direction. The second and third columns of Fig. 10 show the mode-1 and mode-2 components, respectively, and the final column illustrates the residual component. The mode-1 component mainly represents the evolution of the dipole pattern; such a dipole rotates around the eddy center in the initial stage, as demonstrated by the positive (negative) pressure anomaly that moves from northwest (southeast) to northeast (southwest). To illustrate if the mode structures propagate along the azimuthal direction, an azimuth–time Hovmöller diagram of the pressure anomaly around a radius of 150 km from the eddy center at 250-m depth is shown in

![Fig. 10. Temporal evolution of the asymmetric pressure anomaly in the reference experiment (Exp0); the azimuthal mean components are overlain with white contour lines in each panel. (first column) The total asymmetric component, (second column) the mode-1 component, (third column) the mode-2 component, and (fourth column) the residual component for (top) 10, (middle) 80, and (bottom) 160 days.](image-url)
Fig. 11. It is observed that the dipole structure (mode 1) propagates clockwise before reaching a stable state, while the mode-2 structure propagates in the opposite direction. Another interesting point is that a “double dipole” structure is observed in the mode-1 component with the development of a vortex, which is located in the core and edge regions, respectively. This represents a new finding in our scope of knowledge. The mode-1 signal attenuates
with the evolution of eddies, and the mode-2 and high-mode components are gradually enhanced; the nonlinear effect plays an important role in this process. These results indicate that the large-scale asymmetric component is dominated by a dipole-like pattern in the initial stage. The formation of the dipole mode is easy to understand: a cyclonic vortex advects water parcels to the north on its eastern side and to the south on its western side. According to the conservation of potential vorticity, the parcels advected to the north acquire anticyclonic relative vorticity to compensate for the increased background vorticity, while the parcels advected to the south acquire cyclonic relative vorticity. Thus, the additional vorticity forms a dipole with its axis directed to the north, which indicates the primary direction of translational motion. However, the dipole is also affected by the monopolar velocity field, and its axis rotates clockwise. As a result of this complex nonlinear interaction, the axis of the dipolar component and thus the direction of the translational motion of the entire vortex are to the southwest. However, the double dipole mode and its rotation characteristic complicates the propagation of the eddy and makes the translation velocity of the eddy show high-frequency oscillatory characteristics (Fig. 7), and the trajectories of the eddies are very different from those observed in a previous study, as shown in Fig. 6.

Energetically, the beta gyres develop by extracting kinetic energy from the symmetric circulation of the vortex and depleting the energy of the eddy. Figure 11 also gives the kinetic energy variation of symmetry and asymmetry components. The kinetic energy of symmetry gradually decreases with the evolution of the eddy, and the kinetic energy of the mode-1 signal gradually increases and reaches its maximum value of 100 m$^2$s$^{-2}$ at about day 80, and then shows slight decreasing. Around day 60, the kinetic energy of the mode-1 signal rapidly increases and reaches a maximum value of 480 m$^2$s$^{-2}$ at about day 82. The kinetic energy of the low mode dominates the initial stage of eddy evolution, and then the kinetic energy of the high-mode energy gradually increases and dominates the late stage of the eddy evolution. In the case of linear dynamics (refer to Exp1 and Fig. 12), the results show
some different characteristics, except during the initial stage. Mode 1 is the dominant signal throughout the evolution of the vortex, and the mode-2 and high-mode signals grow slowly. This demonstrates that the nonlinear interaction accelerates the transfer of energy from low modes to high modes.

To affirm that the β effect is the main driving force for the formation of the dipole mode, we also examine the results of Exp2 and Exp3 (figure omitted), respectively, in which the model is run only on the f plane. The dipole-like pattern (mode-1) obviously cannot be detected from the two experiments. To sum up the discussions above, it can be inferred that the development of asymmetric motions mainly depends on the planetary vorticity gradient and it is the only factor to form the dipole mode. Nonlinear advection accelerates the energy transform from low-mode to high-mode structure.

e. Kinetic energy budget

To examine the kinetic energy sources of the asymmetric signals presented in the previous section, we performed a kinetic energy budget. In cylindrical coordinates, the momentum equations can be written as

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \mathbf{p} + \frac{1}{\rho} \mathbf{F} + \mathbf{F}_{D},
\]

where \((u_r, u_\phi, u_z)\) are velocity components in the radial, azimuthal, and vertical directions; \(p\) is the pressure anomaly; \(D_r\) and \(D_\phi\) are horizontal diffusion in the radial and azimuthal directions; and \(F_r\) and \(F_\phi\) are vertical diffusion in the radial and azimuthal directions. The continuity equation is

\[
\frac{\partial u_r}{\partial r} + \frac{1}{r} \frac{\partial u_\phi}{\partial \phi} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r} = 0.
\]

Multiplying the first equation in (14) by \(u_r\) and the second equation by \(u_\phi\), and using the continuity equation (15), we can obtain the kinetic energy

\[
\frac{\partial K}{\partial t} = -\frac{\partial (ru_r K)}{\partial r} - \frac{\partial (ru_\phi K)}{\partial \phi} + \frac{\partial (ru_z K)}{\partial z} - \frac{ru_r}{r} \frac{\partial (ru_r r^2)}{\partial r} - \frac{ru_\phi}{r} \frac{\partial (ru_\phi)^2}{\partial \phi} - \frac{ru_z}{r} \frac{\partial (ru_z)^2}{\partial z} - \frac{ru_r}{r} \frac{\partial (ru_\phi)}{\partial \phi} - \frac{ru_\phi}{r} \frac{\partial (ru_r)}{\partial r} + \rho g u_r \frac{\partial u_z}{\partial z} + \rho g u_\phi \frac{\partial u_z}{\partial \phi} + \rho g u_z \frac{\partial u_z}{\partial r},
\]

where \(K = 1/2(u_r^2 + u_\phi^2)\) is the azimuthal mean kinetic energy (MKE). Subtracting (17) from (15), then taking an azimuthal mean of the resultant equation, one can get the azimuthal mean asymmetric part kinetic energy equation:

\[
\frac{\partial K'}{\partial t} = \text{FDM} + \text{FDE} + \text{BTC} + \text{BCC} + \text{FDP} + \text{PTC} + \text{DISS},
\]
is the flux divergence of $K'$ due to the azimuthal mean vortex;

$$FDE = -\frac{\partial \left( \frac{u'^2 + u''_r^2}{2} \right)}{\partial r} - \frac{\partial \left( \frac{u'^2 + u''_\phi^2}{2} \right)}{\partial \phi}$$

is the flux divergence of $K'$ due to angular modes;

$$FDP = -\frac{u'_r \partial p}{\partial r} - \frac{u'_\phi \partial p}{\partial \phi}$$

is the flux divergence of $K'$ due to the pressure effect of angular modes;

$$BTC = -\frac{\partial \left( u'_r u'_\phi \right)}{\partial r} - \frac{\partial \left( u'_\phi u''_r \right)}{\partial \phi} - \frac{\partial \left( u'_r u''_\phi \right)}{\partial r} - \frac{\partial \left( u'_\phi u''_\phi \right)}{\partial \phi}$$

is the energy conversion from the azimuthal mean vortex by barotropic processes;

$$BCC = -\frac{\partial u'_r u'_\phi}{\partial z} - \frac{\partial u'_\phi u''_r}{\partial z}$$

is the energy conversion from the azimuthal mean vortex by baroclinic processes;

$$PTC = -g\rho \partial \bar{u}_z$$

is the energy conversion from angular mode potential energy to angular mode kinetic energy;

$$\text{DISS} = \bar{u}_r F'_r + \bar{u}_\phi F'_\phi + u'_r D'_r + u'_\phi D'_\phi$$

is the dissipation of the kinetic energy of the eddy due to both friction and diffusion; and velocity primes $u'_r$, $u'_\phi$, $u'_z$ are the asymmetry velocity components, that is, total velocities minus azimuthal mean velocities. The azimuthal mean and asymmetric signals are defined; $K' = 1/2(\bar{u}_r^2 + \bar{u}_\phi^2)$ is the azimuthal mean kinetic energy of the asymmetric component (MAKE). The energy evolution equation shows the exchange of energy due to barotropic–baroclinic instability. They also exchange energy with the external ocean environment via the divergence of energy fluxes resulting from energy advection and pressure processes.

Figure 13 shows the radial–vertical cross section of the azimuthal mean kinetic energy of symmetric and asymmetric parts for the reference experiment (Exp0) at 80 days. The overlaid dashed lines represent the radius of the eddy at the surface during the initial stage, which is determined from the strongest current speed; this radius is approximately 90 km. The distribution of the kinetic energy of the azimuthal mean cyclonic eddy is similar to the tangential flow shown in Figs. 8 and 9. Two maximal energy centers are located in the upper and lower layers, respectively, which is consistent with the initialized first baroclinic model structure, but two highly energetic centers in the upper layer are unaligned in the vertical direction, tilting outward from the middle to the surface layer. In the lower layer, the active MKE is concentrated at a radius of 100 km at day 80, shifting inward due to eddy lateral diffusion. Two or three active MKE cores form with the evolution of the eddy (figures omitted), which indicates that the initialized monopolar eddy becomes weak or splits into a multi-eddy structure. The growth of the radius of the vortex and the lateral extension of the eddy energy from the evolution of the MKE are also clearly shown. The evolution of the eddy in the upper layer is nearly stable; the angular mode motion energy is weak and slowly increases. The MAKE dominates the middle and lower layers with two remarkably high-energy cores. The MAKE in the middle to lower layers is even stronger than the MKE, where the vertical shear is strong according to the model configuration. In the upper layer (above 1000 m), the MAKE is much smaller than the MKE and is only 1%–10% of the MKE. These weak asymmetric motions mainly originate from outside the annuluses and centers of vortices where shear instability exists. As shown in section 3a, the eddies in the upper layer can move over longer distances with weak decay, and only a small amount of energy is converted into angular mode motion. The eddies in the middle layer decay rapidly and are short-lived (see Fig. 6), and some of their energy is converted into asymmetric motion, as observed from MAKE. Eddies in the lower layer undergo strong deformation and produce plentiful small-scale motions.

We calculate and compare the six terms in the angular mode kinetic energy budget equation (14) at day 80; those in the upper and lower layers are shown in Figs. 14 and 15, respectively. In the upper layer, the flux divergence of the asymmetric kinetic energy due to the azimuthal mean vortex (FDM; Fig. 14a) and the asymmetries themselves (FDE; Fig. 14d) is nearly equal to zero above a depth of 800 m; the barotropic and baroclinic conversions (BTC and BCC) transport energy from the vortices to asymmetric motion (Figs. 14b,e). However, the flux divergence of the asymmetric kinetic energy due to the pressure work of the asymmetric motions (FDP in Fig. 14c) and the conversion from EPE to EKE (Ce in Fig. 14f) provide nearly opposite contributions. The result is that the energy of eddies in the upper layer mainly decays through slow dissipation and diffusion processes, and these eddies are long-lived.
In the middle layer, FDP is the main contributor to the eddy kinetic energy budget; it transports the eddy kinetic energy inward from outside the vortices and, at the same time, the pressure work of asymmetric motions (FDP) also transports the asymmetric kinetic energy outward in the inner regions of eddies (Fig. 14c). There is also significant kinetic energy transport from EPE to EKE (Fig. 14f), which is a secondary source of the asymmetric motion kinetic energy budget. The FDM (Fig. 14a) and FDE (Fig. 14b) are of tertiary importance, but the FDE has a larger contribution. The FDE mainly transports the asymmetric motion kinetic energy inward from outside at the middle layers. The BCC transports the mean vortex kinetic energy to asymmetric motion kinetic energy and damps the eddy; however, the angular modes feed their kinetic energy to the mean vortex through barotropic conversion (BTC; Fig. 14b). Such a conversion suppresses the growth of angular modes and represents an upscale-energy cascade mechanism. The barotropic conversion is associated with the radial inflow convergence of the mean vortex, rather than the mean azimuthal flow. The latter is a major sink for the eddy kinetic energy in the lower layer (Fig. 15b), indicating that the convergent inflow, not the instability of nondivergent azimuthal flow, helps the asymmetric motions remain in the lower layer. In contrast to the barotropic conversion discussed above, the baroclinic conversion is related to the vertical shear of the radial flow. In the lower layer (Fig. 15), FDP and Ce are the main sources of energy transported from vortices to asymmetric motions. FDP transports asymmetric motion kinetic energy outside (Fig. 15c), but asymmetric motions obtain kinetic energy by the conversion from EPE to EKE (Fig. 15f). In the bottom layer, BTC and
BCC contribute largely to the development of asymmetric motions (Figs. 15b,e) by extracting the mean vortex kinetic energy. The asymmetric motions in the bottom layer also draw kinetic energy from the mean vortices by FDM and FDE (Figs. 15a,d). Frictional dissipation and horizontal diffusion (DISS) are the main sinks for eddy kinetic energy (figure omitted).

The above energy budget demonstrates that the asymmetric motions mainly receive their kinetic energy from the mean vortex through both barotropic and baroclinic conversions in the upper layer, but at the same time, they lose their kinetic energy due to the pressure work of angular mode motions and the transport of EKE to EPE. In the middle layer, an interesting finding is that barotropic conversion plays an upscale-energy cascade role and gives angular mode kinetic energy to the mean vortex. The flux divergence of the eddy kinetic energy flux due to the azimuthal mean flow and the transfer of eddy potential energy is the main source of the energy transport from vortices to asymmetric motions. The frictional dissipation and horizontal shear of the azimuthal flow of the mean vortex both act as energy sinks in the lower layer. The damping of asymmetric motion kinetic energy that occurs in the bottom layer is a result of the pressure work of asymmetric motion. If we further examine the time series of the integral kinetic energy in the inner vortex (Fig. 16), the six terms present oscillation characteristics. The vortex kinetic energy decays with time, and the angular mode kinetic energy grows in the initial stage, reaching its maximum at approximately 90 days and gradually attenuating after that (Fig. 16a). The pressure work of angular mode motions (FDP) and the conversion from EPE to EKE (Ce) are the major energy sources (sinks) for the evolution of asymmetric motions. The asymmetric motions gain kinetic energy via pressure from the external environment in the initial stage and lose their kinetic energy in a later stage. In the middle stage, the asymmetric motions mainly convert potential energy to kinetic energy. The flux divergence of the asymmetric motion kinetic energy due to the azimuthal mean vortex (FDM) transports asymmetric motion kinetic energy outside of the vortex. The barotropic (baroclinic) conservation and the flux divergence of the asymmetric motion kinetic energy due to asymmetric motion themselves (FDE) all display oscillations during the evolution of the vortex due to complicated nonlinear interaction processes.

f. Parameter dependence

To examine the impact of the background environment on the vortex, we extend previous numerical
simulations to the broader parameter space (Rossby and Froude numbers). In a previous reference experiment, both the Rossby and Froude numbers are taken as 0.03, and the corresponding Burger number is equal to 1; such a problem declares stratification and rotation have an identical effect. Here, we first keep the Rossby number unchanged and still equal to 0.03 (called the small Rossby number case), and let the Froude number change by variable stratification conditions. Two experiments are considered, the small Burger number experiment (Exp5, in which $F_r = 0.09$, $B_u = 0.1$) and the large Burger number experiment (Exp6, in which $F_r = 0.003$, $B_u = 10$). In addition, we increase the Rossby number to 0.3 (called the large Rossby number case), and three experiments (Exp7–9) are done with the Burger number being 0.1, 1, and 10, respectively. All dynamical factors are included in these parameter dependence experiments (Table 2).

Figure 17 shows the evolution of the vortex at 250-m depth for the small Rossby number cases with different Burger numbers. For comparison, the reference experiment (it is also belongs to the small Rossby number case, in which $R_o = 0.003$, $B_u = 1$) is also present (middle row). Obviously, a different stratification condition exerts significant impacts on the evolution of the vortex, and the same initial vortices' structures show different patterns at the final stage. In the smaller Burger number condition (Exp5), the eddy hardly moved, that is, the initialized isolated eddy nearly remains where it was (top row in Fig. 17). The $\beta$ eddy did not form, while it is a noticeable occurrence in the reference simulation (Exp0) with the moderate Burger number ($B_u = 1$). Another notable feature is that eddies were torn into small-scale vortex filaments, which were wrapped around the core vortex. Examining the vertical slice structures of the vortex at the final stage (figure omitted), it is found that the column vortex holds the steady first baroclinic structure as an initial declaration, and the potential energy stored by the hump of the isopycnal is rarely released. For the larger Burger number case (bottom row in Fig. 17), the eddy moves fast westward, and the wave pattern arising from dispersion of the eddy is observed, which other papers refer to as wake Rossby waves (McWilliams and Flierl 1979; Early et al. 2011). The dispersive nature of the different Rossby wave components is evident in the north–south asymmetry that develops with time. The simulation realization shows the flat characteristic of isopycnals at the final stage of eddy evolution, and the released potential
energy via eddies is propagated out by radiative Rossby waves.

We also analyze the large Rossby number case ($R_o = 0.3$) and make comparisons with the small Rossby cases; the corresponding pictures are shown in Fig. 18. For the larger Rossby cases, an enhanced nonlinear effect stabilizes the evolution of eddies, the lateral diffusion becomes weaker by comparing Exp0 and Exp8 (Fig. 5a and middle row in Fig. 18), and the $\beta$ eddy in the larger Rossby cases develops stronger than that in the smaller Rossby cases. The first baroclinic Rossby waves are clearly visible from both the plane and slice diagram for the larger Burger number experiments, but the larger Rossby number augments the north–south asymmetry of Rossby waves. Moreover, the nonlinear effect with the stronger flow makes the evolution of eddies more complicated with strong 3D small-scale motions, especially in the moderate Burger condition, where a higher mode baroclinic structure grows out of the first baroclinic vortex.

![Figure 16](image-url)

**Fig. 16.** Time series of the integral kinetic energy in the inner vortex. (a) Azimuthal mean kinetic energy and azimuthal mean asymmetric motion kinetic energy; (b) six terms of the asymmetric motion kinetic energy budget. Note that the Ce and FDP are scaled by a factor of $10^{-3}$. 

- **OCTOBER 2019 LIAO ET AL. 2595**
- **Unauthenticated | Downloaded 12/30/23 07:33 PM UTC**
4. Discussion and conclusions

Researchers have long studied the evolution of vortices, including their geographic distribution (Chelton et al. 2007), amplitude decay (Smith and Reid 1982; Early et al. 2011), and propagation (Nof 1981; Cushman-Roisin et al. 1990). A number of numerical, laboratory, and analytical investigations have recognized that the evolution of a quasigeostrophic vortex on a beta plane consists of three different stages (Reznik et al. 2000; Early et al. 2011). The first stage is the formation of the beta gyre within the vortex, in which an initially axisymmetric eddy evolves a secondary dipole circulation. Over a certain time scale (Fiorino and Elsberry 1989; Reznik et al. 2000), the eddy feels the beta effect. For a cyclone, the beta effect generates anticyclonic (cyclonic) vorticity to the east (west) of the initial vortex according to the conservation of potential vorticity. The dipole with its axis directed to the north is advected by the vortex, resulting in the reflection of the dipole axis. As a result of this complex nonlinear interaction, the flow associated with the dipole structure causes the largely meridional deflection of the eddy. The second stage is the influence of high-order azimuthal harmonics, and the evolution of the eddy is mainly dictated by wave radiation and nonlinearity. The energy loss due to the excitation of waves reduces the vortex amplitude and thus decelerates the motion of the vortex (Sutyrin et al. 1994). The third stage is the disappearance of the eddy. During the final stage, the distortion of the vortex becomes strong, and its amplitude decreases to the background level, that is, the coherent structure of the eddy dies out.

To better understand the characteristics and behaviors of eddies in a stratified ocean, the long-term evolution of a 3D vortex is studied by numerical simulation using a primitive equation model. The model setup is arguably idealistic and is the simplest one for studying evolution of 3D vortex. But the simulations show surprisingly rich and complex dynamics, depending on the parameters, like a double dipolar structure within the eddy, the formation of a beta eddy, the radiation of wave Rossby waves, and so on. In our 3D model, the evolution of the vortex in the initial stage is dominated by the development of the dipole mode ($\beta$ gyre); a new finding is that a double dipole structure is observed with the development of vortex, which is located in the core and edge regions, respectively. Such a double dipole mode and its rotation characteristic complicate the propagation of the eddy and make its translation velocity show high-frequency oscillatory characteristics. In addition, the evolution of the eddy strictly depends on the Burger number. The environment with the small Burger number stabilizes the eddy, but it is easily torn into small-scale vortex filaments. An opposite polarity eddy grows out and forms a counterrotation vortex pair in the case of the moderate Burger number condition. For a large Burger number, the wavelike pattern arising from dispersion of the eddy is observed, which is referred to as wave Rossby waves. In the second stage, the nonlinearity plays an important role, which accelerates the
transfer of energy from a low angular mode to high angular mode. The evolution of the vortex is dominated by the development of the $\beta$ gyre, shear instability, and nonlinear effect. The eddy undergoes a transformation and shows anisotropy due to the planetary vorticity gradient. Over vertical distance, eddies exhibit different behaviors: eddies in the upper level exhibit strong stability, and the main eddy cores can maintain their shapes for longer times and be identified. At the levels around the node of the first baroclinic mode, eddies decay rapidly where they form mixed barotropic and baroclinic instabilities. Eddies in the lower layer evolve into multipolar structures.

The $\beta$ effect provides the propulsion for eddy motion by a secondary circulation. The propagation of a vortex is dominated by the development of the $\beta$ gyre, and such secondary circulations interact with eddies. Nonlinearity speeds up the motion of the eddy. The different translation speeds at each depth imply that these initially aligned eddies finally develop into misaligned structures in the vertical direction, which also leads to different trajectories from the surface to the bottom. The translation speeds of vortices gradually slow down with increasing depth. Our results for vortex motion seem to exhibit some different behaviors compared with those simulated in previous studies using simple models. The zonal speed from our 3D model is less than the theory speed obtained from the quasigeostrophic model.

In a single-layer model, quasigeostrophic monopolar vortices propagate westward and excite Rossby waves because of the $\beta$ effect, which causes cyclones to drift northward and anticyclones to drift southward. In our 3D model, all trajectories show that eddies have a southward shift regardless of their polarity. In the zonal direction, the eddy translates westward, but it also is observed to move eastward. In an ocean exhibiting 3D stratification, the planetary vorticity advection, horizontal deformation of the vortex, and vertical tilting of the vortex core can drastically modify the propagation of an eddy and show complicated propagation patterns.

Examining the overall structure during the evolution of a 3D eddy, a high-density anomaly core exists in the middle levels; the lateral density gradient implies vertical current shear according to thermal wind balance. The azimuthal mean maximum tangential current tilts inward with depth. Stronger radial currents appear at mid- to lower depths with inflow or outflow and are associated with vertical velocity consisting of vertical circulation cells, which results in the gradual steepening of velocity/vorticity profiles in the vortex and enhanced current shear. This mechanism causes the vortex to gradually change from a stable state to an unstable state and leads to the rapid decay of the eddy or its evolution into a multipolar structure. Mixing is intensified due to the existence of vertical circulation cells within the eddy. In the vertical direction, the main eddy core is not aligned; the axis consisting of vortex centers at different depths appears to undergo tilting.

According to the kinetic energy budget, active asymmetric motions mainly occur in the middle to lower levels. The vortex kinetic energy decays with time, and
the asymmetric motion kinetic energy grows in the initial stage, reaching its maximum at approximately 90 days. The pressure work of angular mode motions is the main contributor to the eddy kinetic energy budget; it transports the eddy kinetic energy inward from outside the vortices, and the kinetic transport from EPE to EKE is a secondary source for the angular mode kinetic energy budget. The baroclinic conversion (BCC) transports mean vortex kinetic energy to angular mode kinetic energy and damps the eddy; however, angular modes feed their kinetic energy to the mean vortex through barotropic conversion (BTC). Such a conversion suppresses the growth of angular modes and represents an upscale-energy cascade mechanism. In the bottom layer, BTC and BCC contribute largely to the development of angular mode motions by extracting the mean vortex kinetic energy.

Acknowledgments. We are grateful to the reviewers for their constructive comments. This study was supported by the National Key Research and Development Program of China (Grant 2017YFA0604100) and National Nature Science Foundation of China (Grant 41376033), the Fundamental Research Funds for the Central Universities (Grant 2017B04314), and the China Ocean Mineral Resources Research and Development Association program (DY135-E2-3-05). A part of the work were done during Liao’s visits to Department of Atmospheric and Oceanic Science at University of California, Los Angeles, Los Angeles, California, and sponsored by the China Scholarship Council.

REFERENCES


