Inconsistent Spectral Evolution in Operational Wave Models due to Inaccurate Specification of Nonlinear Interactions

DORUKHAN ARDAG

College of Computing, Engineering and Construction, University of North Florida, Jacksonville, and Civil and Coastal Engineering Department, University of Florida, Gainesville, Florida

DONALD T. RESIO

College of Computing, Engineering and Construction, University of North Florida, Jacksonville, Florida

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ABSTRACT

The introduction of third-generation (3G) models was based on the premise that wave spectra could evolve without prior shape restrictions only if the representation for nonlinear interactions contained as many degrees of freedom as the discretized spectrum being modeled. It is shown here that a different criterion is needed to accurately represent nonlinear spectral evolution within models, a more rigorous criterion such that the number of degrees of freedom in the nonlinear source term must be equal to the intrinsic number of degrees of freedom in the theoretical form of this source term, which is larger than the degrees of freedom in the spectrum. Evolution of spectral shapes produced by the current approximation for nonlinear interactions in 3G models, the discrete interaction approximation (DIA), is compared to the full integral solution for three different time scales: 1) relaxation of the equilibrium range following a perturbation, 2) spectral evolution of the equilibrium range during an interval of constant winds, and 3) the evolution of spectral shape during transition to swell during propagation over long distances. It is shown that the operational nonlinear source term produces significant deviations in the evolution of the wave spectra at all of these scales because of its parametric reduction of the number of degrees of freedom and incorrect energy flux scaling. It is concluded that the DIA does not meet the critical criterion for allowing a spectrum to evolve to spectral shapes consistent with those observed in nature.

1. Introduction

Third-generation (3G) wind-wave models have been used operationally since the late 1980s, following persuasive arguments that these models contained an improved representation of the physics embodied in source terms representing wind-wave generation (Hasselmann et al. 1985, 1988). In those papers, the fundamental reason given for the need to transition from second-generation (2G) models to this new class of model (3G models) was that the parameterized versions of resonant wave-wave interactions S_{nl} in second-generation models did not contain the same number of degrees of freedom as the spectrum itself, and, therefore, constrained the spectral

shape. Hasselmann et al. (1988, p. 1776) argued strongly that the number of degrees of freedom in S_{nl} had to be equal to the “number of degrees of freedom in the spectrum” to allow spectra to evolve with no shape restriction. This criterion is necessary to allow wave spectra to evolve into natural shapes and to avoid parametric constraints on spectral shape evolution. However, while the statement that each source term in a wave model must have at least as many degrees of freedom as the spectrum being modeled is a necessary condition for accurate detailed-balance wave modeling, it is not the sufficient condition.

Sufficiency can only be achieved when each source term has at least as many degrees of freedom as required by the intrinsic physics imbedded within each source term. This context for sufficiency will be reexamined here for the discrete interaction approximation (DIA). This approximation was developed to have as many degrees of freedom as the spectrum being modeled

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Corresponding author: Donald T. Resio, don.resio@unf.edu

times 4, since it assumed that interactions with four points along a particular locus would suffice to provide an accurate estimate of this source term at each point in the spectrum. This particular locus (Phillips 1960) was defined by the shape parameter λ as described in the appendix and represents a very small subset in all of the possible combinations.

Although considerable evaluation of 3G models continues today in terms of integrated wave parameters (significant wave height, mean period, and direction, etc.), little attention has been focused on details of spectral shapes. The question that seems to have been neglected is whether or not interactions between a fixed point and four points near that point is sufficient to obtain an accurate estimate of the complete nonlinear source term at this point in terms of its contribution to spectral evolution. The single figure introduced in support of the DIA (Fig. 1) did not support this assumption, showing clearly that this approximation to the full integral is inconsistent with first principles. The concept that the number of degrees of freedom needed to allow the spectrum to evolve in a realistic fashion is quite reasonable on its own. However, since this approximation used in the DIA creates significant deviations from the first-principles source term for nonlinear interactions, it seems extremely unlikely that additional first-principle source terms (wind input and dissipation) can compensate for these discrepancies and allow spectra to evolve in accordance with the inherent physics governing this process.

It is not surprising that the DIA has failed in essentially all attempts to be calibrated to provide a general agreement with the full integral solution in terms of spectral shape (Resio and Perrie 2008; Perrie and Resio 2009; Ardag 2014). Resio et al. (2016) noted that the number of degrees of freedom required to represent the nonlinear source-term S_{nl} at a single point within a spectrum encompasses a three-dimensional resonant interaction volume imbedded within the six-dimensional overall interaction space. Thus, the complete resonant interaction integral affecting any single spectral component in a discretized wind-wave spectrum with N frequencies and M angles requires a number of degrees of freedom (i.e., the number of contributions from surrounding points within a spectrum) given by $(M \times N) \times (M \times N \times Q)$, where Q represents the number of discrete points along the resonant locus. Since the actual number of terms needed to calculate the full integral accurately depends on the relative magnitude of all terms involved in the full integral, some reduction in the total degrees of freedom is usually possible (Resio and Perrie 1991); consequently, the number of interactions can be reduced somewhat to $(M \times N) \times (\lambda_\theta M \times \lambda_f N \times \lambda_s Q)$, where λ_θ and λ_f represent the portion of the frequency-direction region and λ_s represents the portion of the

FIG. 1. Comparison of the exact one-dimensional distribution S_{nl} with the DIA for a JONSWAP spectrum (Hasselmann et al. 1985).

distance around the resonant locus within the overall three-dimensional space that contribute significantly to the detailed exchanges of action, energy, and momentum within the spectrum at that specific spectral component. Typical values for $\lambda_\theta M$ and $\lambda_f N$ are in the range of 7–12, depending on the spectral resolution and needed accuracy (Ardag 2014), while $\lambda_s Q$ has been shown to need at least 30 points along each locus (Resio and Perrie 1991).

Although the DIA retains the essential cubic relationship to action density in its representation, it substitutes only four points within the total resonant volume as a surrogate for the complete three-dimensional integral (Fig. 2). This means that four values are used to estimate the contributions from 1470 to 4320 elemental increments, which are found by multiplying the values of λ_θ , λ_f , and λ_s ($7 \times 7 \times 30$ or $12 \times 12 \times 30$), that contribute to accurate numerical estimation. The selection of these four points is based on the Phillips locus for three-wave interactions as a guide for the sampling. Unfortunately, the points do not contain any volume and would produce no transfer unless expanded around the point in the three dimensions shown in Fig. 2. This creates a dimensional problem in the DIA that was closed by a simple dimensional argument, based

FIG. 2. The enclosed volume in the shape shown here illustrates the basic dimensions allowed for four-wave interactions in the Webb (1978) form of the integral. The cylinder shape represents the TSA volume (Resio and Perrie 2008; Perrie and Resio 2009). The four square-shaped points used in the DIA on the Phillips figure-eight diagram added here represent individual points with no defined phase volume. Additional square points added for the multiple DIA method on the figure-eight diagram.

on an assumed Phillips scaling, which has an f^{-5} form, and not based on the full integral form, which produces an f^{-4} equilibrium range due to constant energy fluxes.

In short, the DIA contains far too few degrees of freedom and some implicit dimensional constraints that preclude its ability to represent S_{nl} in a realistic fashion. Thus, it is not surprising that the DIA has been found to provide very inaccurate representations for S_{nl} even in simple parametric situations such as shown in Fig. 1 in the initial development of the DIA. Many other comparisons have shown that the DIA does not reproduce comparable results to the full integral solution; thus, discrepancies in the DIA have to be compensated by tuning other source terms in operational 3G wave models (van Vledder et al. 2001).

Many recent papers have emphasized that operational wave models have not lived up to the expectation expressed in Hasselmann et al. (1988, p. 1776), that “once the source functions have been determined by tests for the fetch-limited, uniform wind case, the model is completely specified and should be applicable to arbitrary wind fields without further tuning.” Instead, comparisons have begun to focus on “holistic” tuning methods, which

optimize multiple model coefficients to agree with sets of observations of integrated wave parameters, such as wave height, period, and direction (Ardhuin et al. 2010; Zieger et al. 2015). While this may be adequate for some applications, for many others it is not, as shown by Rogers and van Vledder (2013), who use statistical arguments to prove that an exact [Webb–Resio–Tracy (WRT)] method indeed leads to more narrow spectra better in agreement with measurement data.

Detailed-balance equations (i.e., equations that are solved separately for both propagation and source terms for each element within a spectrum) are particularly critical for producing realistic spectral shape evolution. The details of spectral shape, in turn, are critical for representing the momentum balance in different regions of wave spectra in the coupled modeling applications, such as the interaction between waves and the atmosphere and between waves and various forms of surface currents. For example, in recent studies of atmospheric coupling, energy levels at frequencies higher than the spectral peak frequency have been shown to play an important role in transfers of energy and momentum between the ocean and atmosphere. A recent example from Fan and Rogers (2016) shows plots of

two-dimensional spectra produced using the model [WAVEWATCH III (WWIII) with hurricane wind field analysis (HWIND)] compared to observations with scanning radar altimeter (SRA) in Hurricane Ivan in 2004 (Fig. 3). Significant deviations between observed and modeled spectra were observed throughout all the frequency ranges of the spectrum for different phases of the storm. In particular, the model did not capture the pronounced angular bimodality in the equilibrium range of the modeling sequence shown here, as noted by the authors in their conclusion section where they specifically state that “the observed spectra appear to be bimodal while the model spectra are single peaked.”

The objective of this paper is to investigate different evolutionary time scales via comparisons of DIA results to results from the complete interaction integral. This should provide an improved perspective regarding the sufficiency of the DIA to represent S_{nl} in operational wave models, particularly in applications in the coupled modeling system. If the DIA is not able to provide accurate shape evolution, the physics of other terms cannot be considered validated at this point, and source terms created to form a detailed-balance equation for wave generation must be changed to achieve an overall detailed balance, given a more accurate representation of S_{nl} . Zakharov et al. (2017) provided a good example of an initial development of such a set of source terms.

2. Historical background

As noted in the introduction, the formulation of the DIA is based on the Phillips spectral form (Hasselmann et al. 1973), an extension of the Pierson–Moskowitz spectrum (Pierson and Moskowitz 1964), which, in turn, had been based on Phillips’ f^{-5} form (Phillips 1958) for the equilibrium range. This family of spectra all retains the inherent dimensional characteristics for wave spectra proposed by Phillips, assuming the dominance of wave breaking in frequencies above the spectral peak,

$$E(f) \approx g^2 f^{-5} \psi\left(\frac{f}{f_p}\right), \quad (1)$$

where g is gravity, $E(f)$ is the directionally integrated spectral energy density at frequency f , and $\psi(f/f_p)$ is a dimensionless shape function.

By the early 1980s, most observationally based studies of wind waves no longer supported the existence of an f^{-5} spectrum but provided strong evidence for an f^{-4} equilibrium form at frequencies greater than about twice the peak frequency (Toba 1973; Forristall 1981; Donelan et al. 1985) where f is wave frequency.

Unlike the implicit assumption in Phillips’ argument that wave breaking would dominate energy in the equilibrium range, which implies a universal equilibrium coefficient, laboratory experiments by Mitsuyasu (1968) and field observations by Toba (1972) found that the equilibrium range coefficient was clearly not constant. The subsequent JONSWAP experiment (Hasselmann et al. 1973) provided additional evidence that the equilibrium range coefficient varied significantly as a function of dynamic parameters such as dimensionless fetch and dimensionless peak frequency.

Toba (1972, 1973) proposed a different form for the equilibrium range, based on dimensional scaling,

$$E(f) = u g f^{-4}, \quad (2)$$

in which u has the units of velocity; however, no theoretical basis was given for this form at that time. Kitaigorodskii (1983) postulated that energy fluxes due to resonant nonlinear interactions were primarily responsible for producing the f^{-4} behavior in the equilibrium range, consistent with earlier theoretical results of Zakharov and Filonenko (1967), who had shown that the condition of zero flux divergence for resonant nonlinear fluxes would naturally produce this spectral behavior. In this context, the balance of energy sources in the equilibrium range could be met only if the sum of the wind input S_{in} and dissipation due to wave breaking S_{ds} were small relative to the ability of wave–wave interactions to restore the spectrum to an f^{-4} form in the equilibrium range,

$$S_{in} + S_{ds} \approx 0, \quad (3)$$

where S_{in} and S_{ds} represent the wind source function and dissipation source function, respectively.

Shortly after Kitaigorodskii’s work, Phillips (1985) argued that Eq. (3) oversimplified the energy balance in the equilibrium range and that a more general form was needed that included all three source terms,

$$S_{in} + S_{ds} + S_{nl} \approx 0. \quad (4)$$

This latter form may not appear to be substantially different from Eq. (3); however, Eq. (4) implies that the form of the spectrum will not vary substantially for combinations of wind input and wave breaking that do not sum to zero within the equilibrium range; whereas, Eq. (3) implies that the external source term $S_{in} + S_{ds}$ must be relatively small, since the shape of the spectrum would reflect a balance such that,

$$S_{in} + S_{ds} \approx -S_{nl} \neq 0. \quad (5)$$

Although Phillips’ argument has been used to justify the existence of nonnegligible wind and wave breaking

FIG. 3. (left) WWIII model and (right) SRA directional wave spectra at different phases of the hurricane. Differences among the shape and magnitude of the spectrum can be seen here. On the last phase, the actual spectrum is bimodal where the model result is unimodal. The purple line shows u_p/c equals 1. Only the frequencies range within the SRA resolution (<0.17 Hz) are used for the model spectra plot (Fan and Rogers 2016).

source terms in the equilibrium range in third-generation models (Hasselmann et al. 1988; Tolman 2009), it is clear that the existence of significant net external source terms will tend to force the spectrum away from an f^{-4} form; thus, an important question is how strongly the nonzero

external source terms affect the shape of the equilibrium. Verifications of 3G models continue to focus on integral properties of the spectrum such as 1) total energy (sometimes partitioned into separate, topologically determined “wave systems” within the spectrum), 2) location of the

spectral peaks, 3) mean directional heading rather than spectral shape, and 4) integral measures of angular spreading. [Resio et al. \(2016\)](#) showed that specific problems with the DIA contribute substantially to the imbalance in the equilibrium range in these models. ([Tamura et al. 2010, 2014](#)) investigated disparities in spectral shape caused by external source-term imbalances and found deviations in the exponent of the tail; however, the source terms they used were developed in a holistic manner.

Substantial theoretical, experimental, and field evidence continue to accumulate showing that resonant interactions play a dominant role in spectral shape. [Tanaka \(2007\)](#) showed that Webb's alternative integral form for S_{nl} produced results in good agreement with higher-order spectral methods. Laboratory experiments by [Toffoli et al. \(2010\)](#) showed that spectra with initial unimodal angular distributions naturally develop bimodal angular distributions as they propagate under laboratory conditions, consistent with the predicted spectral evolution based on Webb's form for S_{nl} ([Webb 1978](#)). [Resio et al. \(2011\)](#) showed that the development of this frequency-dependent angular spreading in spectra is consistent with the condition of constant nonlinear momentum fluxes within the integral form for S_{nl} , analogous to the development of an f^{-4} equilibrium range by constant energy fluxes. Recent datasets analyzed around the world have continued to contribute strong support for the universality of an f^{-4} equilibrium range despite some uncertainties in earlier experiments ([Rodríguez and Soares 1999](#); [Long and Resio 2007](#); [Romero and Melville 2010](#); [Thomson et al. 2013](#)), at least up to some point where the spectrum tends to transition to an f^{-5} form in some datasets ([Forristall 1981](#); [Long and Resio 2007](#); [Romero and Melville 2010](#)).

To help provide context for our argument that additional degrees of freedom are needed to represent S_{nl} , a brief review is given here. [Hasselmann \(1962\)](#) showed that a minimum of four waves are required for a permanent (resonant) energy transfer in a continuous wave spectrum. [Webb \(1978\)](#) adapted Hasselmann's equation to a more computationally efficient form and showed that the change of energy density at a point $\mathbf{k}_1$ within a spectrum S_{nl} could be written as

$$\frac{\partial n(\mathbf{k}_1)}{\partial t} = \iint_{k_{3x}, k_{3y}} T(\mathbf{k}_1, \mathbf{k}_3) dk_{3x} dk_{3y}, \quad (6)$$

where

$$T(\mathbf{k}_1, \mathbf{k}_3) = 2 \int D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) C(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \left| \frac{\partial W}{\partial n} \right|^{-1} ds, \quad (7)$$

in which

$$\mathbf{k}_4 = \mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 \quad \text{and} \quad \omega_1 + \omega_2 = \omega_3 + \omega_4.$$

Variable C is an algebraically complicated coupling coefficient, W is a phase-space Jacobian, which has $k^{0.5}g^{-0.5}$ dimensions described in ([Tracy and Resio 1982](#); [van Vledder 2006](#)), and D is a density function consisting of action density triplets:

$$D(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) = n(\mathbf{k}_1)n(\mathbf{k}_3)[n(\mathbf{k}_4) - n(\mathbf{k}_2)] + n(\mathbf{k}_2)n(\mathbf{k}_4)[n(\mathbf{k}_3) - n(\mathbf{k}_1)]. \quad (8)$$

Because of the computational complexity of these equations, [Hasselmann et al. \(1985\)](#) introduced a very simplistic approximation to the total integral based on a sample of only four points selected from the complete three-dimensional interaction volume inherent in Eqs. (5)–(7). [Resio et al. \(2016\)](#) have shown that such a representation does not contain the sufficient degrees of freedom to represent the full nonlinear integral and consequently cannot consistently represent detailed-balance exchanges within wind-wave spectra.

[Figure 2](#) shows the three-dimensional integration space (k_{3x}, k_{3y}, s) required for consistency with the full Boltzmann integral with the two-dimensional locus of points contained by the Phillips' "figure eight" restricted to a line on a plane within this three-dimensional integral. In this figure, the vertical dimension is defined by points for which the radial frequencies in the four interacting waves must be such that $\omega_1 + \omega_2 = \omega_3 + \omega_4$ in order for energy to be conserved. The Phillips' integral represents a line within the overall interaction volume and only represents one possible template for sampling small discretized interaction volumes in a single interaction. It is important to note that points along the Phillips figure-eight locus represent only a single resonant line in the general Webb form for four-wave interactions. [Resio et al. \(2016\)](#) show that the integral along a figure eight (or any other line in the resonant interaction space) must be equal to zero, since the three-dimensional volume included within such an integral is equal to zero. Areas included around the four DIA sample points in its approximation, assumed to provide an approximate integration volume, are primarily non-resonant with respect to the center point in this locus. The [appendix](#) contains a short derivation and discussion of additional problems in the DIA introduced by dimensional closure without considering the effect it would have on fluxes of the spectrum.

Since the primary contributions to the total transfer integral contain products of both geometric phase-space terms and the sum of four triplets of actions densities, it

is not possible to obtain a universal multiplier that relates the estimated action/energy transfers in this small subset of points to the contributions from all interactions. As shown in Fig. 2, the multiple discrete interaction approximation (MDIA; Tolman 2004) adds more points along the figure-eight locus. The generalized discrete interaction approximation (GDIA; Tolman 2013) includes additional points that do not fall on this locus (not shown in this figure) but is only optimized empirically in a holistic context for integrated wave parameters, with no clear physical basis for selecting the locations of these points, nor any examination of the impact of this selection on either spectral shape or spectral evolution. Both the MDIA and GDIA retain the dimensional difficulties inherent in the DIA, unless the issue with the physical scaling in the DIA, noted in the appendix, is changed to produce a constant energy flux. A simple inclusion of additional terms without correcting this will still yield nonconstant energy fluxes for an f^{-4} spectrum.

3. Simulation of nonlinear interactions at different scales

In this section, the results of evolutionary solutions for 1) a perturbation in the equilibrium range (tens of wave cycles), 2) the form of the equilibrium range under different forcing (tens–hundreds of wave cycles), and 3) idealized long-distance swell propagation (thousands of wave cycles) are presented. To maintain stability in evolutionary testing with different representations of S_{nl} , simulations at all scales must be executed at a much finer time step than used in operational applications to ensure that the computed spectral evolution is not contaminated by instabilities. In operational systems, the rate of change of action density over a time step can become very large at high frequencies, potentially leading to unbounded instabilities unless constrained by “limiters.” These are discussed in some detail (Monbaliu et al. 2000); however, it is clear that any computationally related, parametric limitations imposed on S_{nl} , rather than based on the physics of the process, will likely not return the spectrum to its appropriate natural form.

Besides the implementation of limiters and some variations in the coefficients used to convert the DIA estimates to full integral estimates, no substantial changes have been made to the operational DIA in the 30 years since it was formulated, although some studies have examined various methods for optimizing the DIA for different spectra (Polnikov 1997; Samiksha et al. 2015), and others have attempted to extend this approximation to include additional points within the three-dimensional interaction volume (Tolman 2004; Tolman and Grumbine 2013). These adaptations still do

not meet the criteria that all significant contributions should be included within the three-dimensional interaction volume of interaction in simulations of spectral evolution.

a. Simulated spectral evolution following a perturbation

As has been noted by both Zakharov and Filonenko (1967) and Hasselmann et al. (1976), wave–wave interactions act as a shape-stabilizing function. Following any perturbation in the overall source-term balance, nonlinear interactions will act to restore the spectrum to a self-similar form. In this part of testing, perturbed and nonperturbed initial spectra were utilized for relaxation simulations. The purpose here is to evaluate the shape-stabilizing effect of the nonlinear interactions using the DIA method without any limiters on the spectral shape, relative to the behavior predicted by the full integral for these interactions. This approach is somewhat similar to that used in a similar simulation by Young and van Vledder (1993).

In Fig. 4, the initial spectrum was generated with a peak frequency of 0.2 Hz, relative peakedness of 1.41, and a transition to a parametric f^{-5} tail is incorporated after $ff_p = 8$ to ensure stability. The term relative peakedness is defined as the ratio compensated spectral energy at the spectral peak $[E(f_p)f_p^4]$ to the compensated energy level in the $E(f)f^{-4}$ equilibrium range (Long and Resio 2007), which yields a constant level and energy flux in an f^{-4} spectrum. A comparison of the f^{-4} -based relative peakedness used here to the f^{-5} -based JONSWAP peakedness relationship is given in Table 1. Since evidence continues to accumulate supporting the f^{-4} form, only the relative peakedness for an f^{-4} spectrum will be used in the remainder of this paper.

This initial spectrum was allowed to evolve with no external sources, except for a boundary condition at high frequencies. The existence of an upper limit in frequencies included within the computational domain impedes fluxes to higher frequencies. As noted by Resio and Perrie (1991), similar to the physical boundaries in heat flux solutions, boundary conditions must be set at both high and low frequencies in the computation of nonlinear wave–wave interaction, since energy fluxes have to be specified at these locations. At low frequencies, it is typically assumed that there is a sufficiently low frequency at which no energy exists throughout a simulation. In this case, the energy flux through the lower-frequency boundary can be set to zero and spectral components at lower frequencies can be excluded in the solution. It is more complicated at higher frequencies, since there is a cascade of energy into higher frequencies. At this point in a solution, the exclusion of interactions, while retaining the interactions with lower frequencies, produces an

FIG. 4. Relaxation of the initial normalized energy spectrum multiplied with f^{-4} after 240 s without any limitation factors. Both methods have 10-s time steps.

artificial discontinuity in the fluxes. Since the dominant (net) flux along this boundary is directed toward higher frequencies, this creates a convergence in the energy flux near the boundary, which eventually produces an instability near this boundary, making it essential to place this boundary at a sufficiently high frequency to avoid undue feedback into the computational range of interest.

Operational models utilize limiters that constrain the total change in any frequency–direction component over a single time step to a fraction of the energy at that location. For tests conducted here, this problem must be overcome by a dissipative mechanism at very high frequencies, sufficiently removed from the region of interest, so that it does not change the spectral form in the range of interest. Simulations typically use a transition to an f^{-5} form at very high frequencies to achieve this goal in careful simulations, consistent with the dissipation term used in Zakharov et al. (2012).

Snapshots for selected times from these simulations are shown for normalized compensated, directionally spectrum compensated normalized by the equilibrium range coefficient β ,

$$\hat{E}(f) = \frac{E(f)f^4}{\beta}, \quad (9)$$

where the equilibrium range coefficient β is given as,

$$\beta = \alpha_4 \mu g (2\pi)^{-3}, \quad (10)$$

in which α_4 is a dimensionless equilibrium range constant (0.006 09) as shown in Resio et al. (2004). A comparison

of the evolution at elapsed simulations times of 120 and 240 s (24–48 spectral peak cycles) show that the DIA and full integral yield quite dissimilar results, with the equilibrium range produced by DIA deviating significantly from an f^{-4} form, while the full integral produces a return to this form. As noted in Resio et al. (2016) and detailed in the appendix, a primary reason for the poor evolutionary response for the DIA simulations is that the Phillips f^{-5} form was used rather than an f^{-4} form in the dimensional closure of the DIA, rather than an argument based on the natural form created by constant energy fluxes.

In Fig. 5, the initial conditions were the same as the previous simulation with a perturbation centered at 0.34 Hz, peak of the perturbation as coefficient times the equilibrium range with a Gaussian frequency distribution, and an isotropic angular distribution positioned closely to the primary peak to ensure cross interactions between the perturbation and the primary peak. In addition to the snapshots at 450 and 900 s (90–180 spectral peak cycles) for both methods, the normalized relaxation rate at the peak of the perturbation is displayed in Fig. 5 to emphasize the dissimilarity between the two results. The relaxation rate is defined as the ratio of the peak perturbation energy at 0.34 Hz to the initial value at the same frequency band throughout the simulation. Hence, it starts from 1 for both

TABLE 1. JONSWAP γ and equivalent relative peakedness γ_r values.

	γ_r	4.32	3.09	2.04	1.23	0.62
	γ	7.0	5.0	3.3	2.0	1.0

FIG. 5. (top) As in Fig. 4, but this time perturbation is introduced to the initial spectra on 0.34 Hz. The FBI time step is 10 s, whereas the DIA simulations are unstable at time steps greater than 1 s. (bottom) The relaxation rates of the compensated energy of the peak of the perturbation for the two runs.

cases, and it should continuously decrease as shown by the line representing the full Boltzmann integral (FBI) method. Since the ability of nonlinear interactions to return a perturbed spectrum to an equilibrium form is a critical property of any S_n approximation, this represents a potentially serious issue for the DIA in applications. As it can be seen from the FBI results, the perturbed energy diffuses within the equilibrium range and increases the equilibrium range constant temporarily, however, since there is no wind input, it would continue to decrease. Additionally, it should be noted that the DIA is inherently more unstable than the full integral. Without limiters, simulations with the DIA required time steps of 1 s to retain stability, while the FBI retained stability with 10-s time steps.

b. Source-term balance within the equilibrium range

As noted earlier in this paper, existing operational models assume that the external source terms do not have to be zero, along the lines suggested by Phillips (1985),

rather than along the lines of Kitaigorodskii (1983). Recently, Thomson et al. (2013) presented additional arguments based on observations at Ocean Station Papa in support of Kitaigorodskii's interpretation of the constraint on the equilibrium range; however, the crux of this issue is how much imbalance between wind input and dissipation would have to exist before such an imbalance would be recognizable in observations. To evaluate this limit, several tests are conducted here with variations in the net momentum gain/loss within the equilibrium range. In this context, there is little advantage in considering the multitude of wind source terms that have been proposed over the years, since each of these would presumably have a “matched” dissipation function that could be tuned to balance the f^{-4} form (i.e., sum to zero over a given range). The simulations presented here pertain to idealized, constant wind speed and direction tests. The tests are intended to investigate the potential deviation of a self-similar spectrum in the equilibrium range induced by nonzero external sources (net wind input plus wave dissipation).

To keep our analysis simple, momentum transferred into the wave field in the equilibrium range is distributed uniformly into a region considered characteristic of this spectral range ($1.5 < f/f_p < f_{\text{cutoff}}$), where f_{cutoff} is chosen to be 4. The total rate of momentum transfer from the wind to the waves into each of these regions is estimated as,

$$M_i = \lambda_i \frac{\rho_a}{\rho_w} C_d U_{10}^2 g^{-1}, \quad (11)$$

where M_i is the total momentum flux into the equilibrium range, λ_i is the percentage of the total momentum transferred into the specified portion of the spectrum, i defines different cases, C_d is the coefficient of drag, and ρ_a and ρ_w are the densities of air and water, respectively. It is found that 5%–6% of the total momentum from the wind to the sea surface is retained by wind waves (Hasselmann et al. 1973; Toba 1978; Mitsuyasu 1985). Thus, λ_i is allowed to vary from +0.05 to −0.05 to investigate the sensitivity of the slope to this imbalance with both the FBI and the DIA. For these tests, the coefficient of drag C_d is taken from Hwang (2011); U_{10} is the wind speed in 10-m-high reference point. So, the combined net external source term ($S_{\text{in}} + S_{\text{ds}}$) for this idealized testing has the form,

$$S_{(\text{in+ds})_i}(f, \theta) = q_i D(\theta) c(f). \quad (12)$$

The variable $D(\theta)$ is the angular distribution function for the net source/sink term that is chosen to be $\cos^2(\theta - \theta_o)$, $c(f)$ is the phase velocity, and $q_i = M_i(\Delta\mathbf{k})^{-1}$, where $(\Delta\mathbf{k})^{-1}$ is the phase area considered within the equilibrium region.

All simulations were initialized with a relative peakedness of 1.4, 10 m s^{-1} wind speed and 0.2 Hz peak frequency and were then allowed to evolve to an equilibrium for half an hour of simulated time (360 spectral peak cycles). Since the main focus was on the f^{-4} range, during these specific tests the switch to the f^{-5} tail (dissipative range) was placed at a value of f/f_p equal to 11 to ensure that there was no feedback into the equilibrium range. The results shown in following figures are normalized the same way in Eq. (9). Figure 6 shows four different results for the FBI and the DIA for positive external sources, and the solid line is an S_{nl} -only run for reference. The rest of the three cases include positive transfer rates corresponding to 5%, 2.5%, and 1% of the total momentum, respectively. As it can be seen from the figure, depending on the amount, positive residual momentum distorts the energy fluxes in the equilibrium range and creates deviation from the self-similar form. The overall effect of the distortions with the DIA test is smaller compared to the FBI.

An additional test was performed with the same initial spectrum combined with a negative source in the equilibrium

range (Fig. 7). Similar deviations are found with decreasing rates as the percentage of the total momentum reduces. This helps demonstrate the point that S_{nl} must be other than zero (i.e., not an f^{-4} equilibrium form) for either positive or negative net external source terms to force the balance to a net equilibrium form in this region of the spectrum and tends to support the argument by Kitaigorodskii (1983) that inequalities between the input and the dissipation term affect the shape throughout the evolution of the spectra.

c. Evolution of a swell spectrum

Recent works by Collard et al. (2009) and Young et al. (2013) present evidence suggesting that nonlinear interactions can be neglected in swell dissipation. Ardhuin et al. (2010) developed a “quasi empirical” method to include swell decay within operational models. These studies focused on wave height and not on spectral shape. During swell propagation, primary energy-containing frequencies typically remain centered on the spectral peak frequency; however, the peak frequency is expected to shift to lower frequencies if significant nonlinear interactions exist along the propagation path.

Quantifying the arrival time of swell using field observations can be difficult since there are numerous physical factors that could lead to deviations (Badulin and Zakharov 2016) both within the generation area and along the propagation path. Here we will focus on an observational study of an extreme storm that caused extensive flood along the Outer Banks of North Carolina because of wave setup and overtopping, the so-called Halloween storm of 1991. The locations of three buoys along the North American coast (44137, 41002, 41010) are shown in Fig. 8. Data from these buoys will be used to examine the behavior of large waves propagating over a distance of about 2000 km (1823 km from 44137 to 41002 and 2065 km from 44137 to 41010). Buoy 44137 represents the source area for these waves and provides the initial wind–sea conditions. The data captured at the other buoys represent the waves propagating over the relatively long distances between the source area (buoy 44137) and buoys 41002 and 41010.

Figure 9 shows the spectra at the time of maximum conditions at each buoy (associated with the first- and second-ranked significant wave heights at each buoy). As can be seen in this figure, there is a slight shift in the peak of the spectrum toward lower frequency at both 41002 and 41010, consistent with some degree of nonlinear transfer of energy to lower frequencies; however, it is possible that such a pattern could also be produced by frequency dispersion. It also appears that the spectra are somewhat broader than expected, so a second analysis using time-shifted energy densities as specific frequencies of 0.06 and 0.1 Hz was performed in which

FIG. 6. The $S_{in} > S_{ds}$ conditions. Different positive rates of the total momentum are induced into the spectra from $1.5 ff_p$ up to $4ff_p$ for the FBI and the DIA. The DIA tail is already deviated; thus the difference is smaller as compared to the FBI results.

the time shift between 44137 and 41002 is estimated from the propagation speed relationship:

$$E(f, t)_{41002} = E(f, t')_{44137}, \quad \text{where} \quad t' = t - \frac{\Delta S}{c_g(f)}. \quad (13)$$

The variable ΔS is the distance between points. Figure 10 shows the results of this procedure. As expected, the arrival time of the shifted peak energy at 41002 for 0.06 Hz precedes the time of the peak energy at 44137 at this frequency. However, the same analysis at

0.10 Hz shows that a significant portion of the energy at 0.10 Hz at 41002 arrives from a period when there was essentially no energy at 44137 at this frequency band. Since buoy 44137 is located very close to the most intense portion of the storm and the wind direction remains relatively constant during the buildup of seas at 44137 (Fig. 11), this discrepancy is hard to attribute to any obvious factor related to propagation effects, which suggests that energy at 0.10 Hz should have arrived well after the time of the peak spectrum shown in Fig. 9.

To examine the potential role of nonlinear interactions, compensated spectra were plotted for the three buoys for

FIG. 7. The $S_{ds} > S_{in}$ conditions. Different negative rates of the total momentum are induced into the spectra from $1.5 ff_p$ up to $4ff_p$ for the FBI and the DIA. The DIA tail is already deviated; thus the difference is smaller as compared to the FBI results.

FIG. 8. Location of three buoys used in this investigation: buoy 44137 (solid square), buoy 41002 (solid diamond), and buoy 41010 (inverted solid triangle).

the period of interest (Fig. 12). Somewhat surprisingly, they all appear to have slopes in the region of the equilibrium range that appear more consistent with a nonlinear flux basis than a frequency dispersion basis, since the latter should be downward sloping in the equilibrium range. Spectra at 44137 show that the local sea portion of the spectrum is located in the vicinity of 0.06 Hz, with some variability in the peakedness in the spectral peak region and a relatively consistent f^{-4} form throughout most of the equilibrium range. Spectra at 41002 and 41010 both exhibit slopes that are less than the wind-sea compensated form. If we envision the balance of energy flux through the equilibrium range, this implies a net input within this range.

The time of the peak conditions and the net effect of frequency dispersion can be understood as a potential sink term in the overall spectral balance in time and space. In this situation, the lower-frequency limit originates at an earlier time than the higher frequency. Since the distance is a constant and the propagation speed increases with decreasing frequency, the time differential becomes smaller. Conversely, for a fixed time increment, the spatial distance becomes larger as frequency decreases will increase,

$$\delta E(f) = c \frac{\Delta E(f)}{\Delta x_{\delta t}} \Delta x_{\delta t}.$$

Thus, more energy will be extracted by frequency dispersion over a fixed time step, and this would create a net lowering of the rate of energy loss with respect to frequency, which is expected to produce a net upward

FIG. 9. Spectra at the buoys used in this study at the time of their maximum significant wave height.

slope in the compensated spectral form as seen in Fig. 12. If wave-wave interactions are negligible, it seems that the opposite should occur, and the energy should be governed by frequency dispersion alone, and the higher frequencies should be truncated for much of the time series.

The purpose of the exercise presented here is to show that it is important to consider both nonlinear interactions, with an accurate representation for S_{nl} , and frequency-angle dispersion in consideration of long-distance wave propagation, at least for the case of large storms in the Northern Hemisphere where the storms often become semistationary. To provide some perspective for the differences in behavior between the full integral representation and the DIA, Fig. 13 is presented here. The much-idealized case may be viewed by many as unrealistic for real-world conditions; however, idealized comparisons often provide a useful guide for understanding potential problems inherent to model assumptions.

4. Discussion and conclusions

In the first set of tests conducted here, a perturbation was added to an f^{-4} spectrum, and the ability of the FBI and the DIA to return the spectrum to the expected equilibrium range shape was evaluated. While the FBI performed as expected, returning the spectral equilibrium to an f^{-4} form, the DIA performed quite inconsistently and forced the spectrum into a form that deviated strongly from expected results, both with respect to the decay rate of the perturbation and a deviation from the f^{-4} form. Returning to an equilibrium

FIG. 10. Time variation of $E(f)$ at (left) 0.06 and (right) 0.1 Hz.

form following a perturbation is only possible with accurate detailed-balance exchanges that include a sufficient number of interacting components and the proper scaling. This inability of the DIA to accomplish this fundamental tenet laid out for 3G models has been shown (Resio et al. 2016) to be related to two primary factors: 1) insufficient degrees of freedom (number of

terms used) in the DIA required to replicate accurately the full integral form for S_{nl} and 2) application of an inappropriate form for dimensional closure of the DIA that does not produce constant energy fluxes for an f^{-4} spectral form (shown in the appendix). Because of this, the DIA cannot reproduce directionally integrated spectral shapes for four-wave interactions consistent

FIG. 11. Wind speeds and directions at buoy 41317 during Halloween storm from Cardone et al. (1996).

FIG. 12. Compensated spectra for (left) buoy 44137 and (right) buoy 41002. Dark solid lines denote same times as shown in Fig. 9. Dashed lines show subsequent times at 1-h intervals.

with theory (Zakharov and Filonenko 1967) and observations (Toba 1973; Forristall 1981; Donelan et al. 1985; Resio et al. 2004), nor can it reproduce directional spreading consistent with observations and theory (Ewans 1998; Wang and Hwang 2001; Resio et al. 2011).

A second set of tests here investigates the fundamental difference between the argument made by Kitaigorodskii (1983), that external source terms (wind input and dissipation) in the equilibrium range needed to sum to zero, and the argument made by (Phillips 1985), that the external source terms did not need to sum to zero as long as they both had f^{-4} forms such that the sum of these terms and S_{nl} equaled zero. Tests with nonzero external source terms in the equilibrium range, using the full integral solution (FBI), showed that any significant net external source term in this region forced the spectrum to deviate noticeably from the self-similar

f^{-4} form. This supports Kitaigorodskii's criterion for the zero balance in the equilibrium range, rather than Phillips' criterion.

A third test, investigating swell propagation during the Halloween storm (1991), produced extremely interesting results. At two buoys on the path of the swell waves generated by this massive storm (at distances of 1823 and 2065 km from the buoy located in the maximum portion of the storm), wave spectra at the time of peak conditions at the distant buoy exhibited a slight shift toward lower frequencies at the spectral peak. Using a frequency-time-shift method to examine the spectral densities at these peak conditions showed that significant energy at frequencies higher than the peak frequency arrived at the remote buoys well before they would be expected to arrive based on linear propagation. A plot of the compensated spectrum $[E(f)f^4]$ shows

FIG. 13. Performance of different representations of the nonlinear source term with different propagation scenarios (left) FBI neglecting propagation and (right) DIA neglecting propagation.

that the compensated spectra in the area of the storm source had a shape consistent with that of a local wind sea [$E(f)f^4 = \text{constant}$]. At the buoys over 1800 km away from the source, the spectra had an interesting form [$E(f)f^{4-\varepsilon} = \text{constant}$], which suggests that the equilibrium form may be produced by the effects of combined nonlinear interactions and the decreasing negative frequency dispersion effect from low to high frequency. It does not seem possible for the energy densities in the range of frequencies showing up at the remote buoys at the time of the maximum to be explained by frequency dispersion alone. Thus, both swell propagation and nonlinear interactions should be modeled accurately to capture swell arriving from major storm events. Since the evolution of swell is shown to be quite different in a simple idealized test of the DIA and full integral in this paper, this suggests that the DIA-based models likely cannot provide an accurate estimate of swell spectra in such storms.

Hasselmann et al. (1976) hypothesized that the shape of natural wave spectra was controlled by S_{nl} and was essentially self-stabilizing, with two different time scales for relaxation. The “slow” time scale involves the evolution of the spectral peak and the “fast” time scale involves the return of spectral densities within the equilibrium range. In moving toward a new modeling paradigm, it will be important to recognize the existence of multiple scales in spectral evolution and to model each scale appropriately. To some extent, the two-scale approximation (TSA) proposed by Resio and Perrie (2008) represents a first attempt to move in that direction. This is not expected to be a final solution, but it produces much-improved behavior compared to the DIA in terms of spectral evolution. More effort for improving the nonlinear term and the other source terms is underway; however, the point of this paper is to emphasize the need for changing to a more accurate representation of S_{nl} to meet the stated goals of third-generation modeling.

A summary of the key results shown in this paper are given below:

- 1) The DIA, executed at a time step that avoided the need for limiters, was shown to yield unsatisfactory spectral shape evolution for all cases investigated. For the case of a spectrum with no perturbation and no net external source term, incorrect fluxes within the DIA forces the spectrum to an equilibrium form with a significantly shallower slope than the full integral; for the case of a perturbation and no net external source term, this problem is exacerbated. These problems suggest that the flux imbalance in the DIA would have to be compensated by artificially compensated combinations

of wind input and wave breaking to produce the f^{-4} equilibrium range slopes observed in nature.

- 2) Tests with varying net positive external inputs in the equilibrium range combined with the full integral solution show that even an input as small as $\pm 2\%$ – 3% of the total input into the wave field in the equilibrium range produces noticeable deviations from the f^{-4} equilibrium range slope. The same tests with the DIA produced even larger deviations from the full integral results than those with no net external sources. Such inaccuracies in the DIA could cause problems in high-wind conditions (Amrutha et al. 2016) or yield underestimation in integrated wave parameters for the regions in which the source terms are not well calibrated (Kazeminezhad and Siadatmousavi 2017). Also, since the high-frequency range in spectra plays a very important role in both remote sensing applications and in many coupled modeling systems, these problems could create significant impediments to the use of existing 3G models for such applications.
- 3) The inability of 3G models to predict increased energy levels and period associated with the spectral peak could severely impact accurate predictions of swell arrival time and the shape and magnitude of these spectra. Long-distant swell modulates the computed energy spectra, and their propagation in geographic space and time is vital for wave activity in coastal areas (Nayak et al. 2013); especially for the local wind fields, opposing swells can change the structure of the pressure field above the surface or even transfer upward momentum fluxes (Grachev and Fairall 2001; Sullivan et al. 2008). Additionally, the lack of spectral peakedness produced by models using the DIA would be expected to affect estimates of higher-order wave moments such as kurtosis, found to be important for freak waves, as well as magnitude of three-wave interactions in coastal areas (Salmon et al. 2016).
- 4) An idealized test conducted here shows that the DIA does not capture important features on the S_{nl} source term needed to simulate the decay of swell over long distances. Although these tests neglect the effects of frequency dispersion, the results are still indicative of a deficiency in the DIA that would require an additional artificial source term to correct.
- 5) Given that the DIA does not produce accurate spectral evolution, it is not clear that the initial premise for using these models, the need for wave models to return a wave spectrum to its appropriate natural form from any perturbation, has been met.

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APPENDIX

Dimensional Examination of the DIA Method

Spectral evolution simulations using the DIA show an inherent accumulation of energy at the higher frequencies and a deviation from an f^{-4} spectral tail. This consistent behavior suggests a possible issue with the dimensional closure and/or energy fluxes. In order to examine the dimensional closure, the following form is obtained by simplifying the DIA equation to display dimensional terms only in the frequency space (Hasselmann et al. 1985):

$$\delta S_{nl} \approx C g^{-4} f^{11} F^3. \quad (A1)$$

The variable C is a dimensionless, numerical constant, and F is one-dimensional energy density that has two different dimensional forms, given by,

$$F(f) \approx \begin{cases} g^2 f^{-5} \psi\left(\frac{f}{f_p}\right) \Rightarrow f^{-5} \text{ (breaking)} \\ u g f^{-4} \psi\left(\frac{f}{f_p}\right) \Rightarrow f^{-4} \text{ (nonlinear fluxes)} \end{cases}. \quad (A2)$$

Either of these possible forms of (A2) into (A1) yields,

$$\delta S_{nl} \approx L^2, \quad (A3)$$

where L is the length scale and consistent with the spectral energy that is transferred. This result shows that the DIA meets the dimensional closure for the transfer integral. Therefore, it is likely that the issue lies in the scaling of the energy fluxes. Resio (1987) shows that the inherent transfer of energy (action) from one point to another has to scale with the wavenumber to produce a constant cascade of energy to high frequencies as shown in Fig. A1. The geometric wavenumber space that forms the basis for these fluxes is shown in some detail in Resio (1987). Only an f^{-4} directionally integrated spectrum in conjunction with the geometric dimensions of the full integral can produce constant flux through the equilibrium range. The introduction of the f^{-5} shape in the DIA derivation leads to an incorrect power law for the geometric factors imbedded within the nonlinear integral. Even though the DIA form is dimensionally consistent with the full integral, its use of the $g^2 f^{-5}$ produces a different dependence on frequency in energy

FIG. A1. One-to-one correspondence of integration areas in polar grid (Resio 1987). Units of distance here are normalized by the width of the source line extending from $y = -0.5$ to 0.5 (black line on the left-hand side). As can be seen here, even at a distance of 2 times the width of the line source, energy arriving at distance $2x$ is originating over a finite distance. For example, a 10° angle band receives energy that comes from a region covering 17.4% of the total source line, not from a single point; therefore, to obtain a finite width of angles at the receiving point, such as needed to have finite energy within the angle band, it is necessary to consider a range of locations along the source line and cannot be mapped back into a single point.

fluxes, that is, the energy flux $\sim [E(f)]^3 G(f)$, where $G(f)$ is a function that represents the inherent dependence of the energy transfers on frequency. A geometric derivation of this flux equation (Resio 1987) shows that in deep water, $G(f) \sim f^{12}$, which cannot be matched for spectra with a basic f^{-5} shape. Thus, even though the $g^2 f^{-5}$ form for energy fluxes is dimensionally consistent with the $u g f^{-4}$ form, use of the f^{-5} scaling produces inconsistent energy fluxes within the interaction integral, since the dimensions of $G(f)$ are incorrect in the DIA formulation yields fluxes that have the form

$$[E(f)]^3 G(f) \sim f^{-15} f^{+12} \sim f^{-3}, \quad (A4)$$

Compared to the fluxes in an f^{-4} spectrum

$$[E(f)]^3 G(f) \sim f^{-12} f^{+12} \sim \text{constant}. \quad (A5)$$

This shows that the flux diminishes as a function of frequency in the DIA scaling relative to what is required to produce an f^{-4} equilibrium form. As the frequency

increases in the DIA scaling, this leads to a convergence of energy, consistent with the observed energy accumulation at high frequencies observed in the numerical tests shown in this paper.

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